Mesoscopic fluctuations in stochastic spacetime

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Mesoscopic effects associated with wave propagation in spacetime with metric stochasticity are studied. We show that the scalar and spinor waves in a stochastic spacetime behave similarly to the electrons in a disordered system. Viewing this as the quantum transport problem, mesoscopic fluctuations in such a spacetime are discussed. The conductance and its fluctuations are expressed in terms of a nonlinear sigma model in the closed time path formalism. We show that the conductance fluctuations are universal, independent of the volume of the stochastic region and the amount of stochasticity.

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I. INTRODUCTION

A. Metric fluctuations in semiclassical gravity

Since the Einstein-Hilbert action is insensitive to fluctuations near the Planck scale, we expect large fluctuations in this regime to occur which may require reconsideration of the concept of the microstructure of spacetime itself. The concept of metric fluctuations thus introduced originally by Wheeler [1,2] has been studied and modified in various different contexts. Spacetime foams or other string-theory-motivated microstructures of spacetime can be treated as possible stochastic sources [3]. Since these fluctuations become dominant only near the Planck scale and cannot be directly observable, it is important to identify their possible influence on the matter fields propagating in such a background with an energy much lower than the Planck value which is possibly detectable by high energy astrophysical observation.

Since quantum gravity is still an unsolved problem, in conventional semiclassical gravity, a spacetime is left unquantized [4]. A recent study in semiclassical gravity, however, reveals many pathological features in this approach. One of the most serious problems is violation of the positive energy theorem due to large quantum fluctuations, which leads to violation of causality by allowing the creation of traversable wormholes [5]. It is also noticed that positivity can be recovered by imposing additional smearing in the case of Minkowski spacetime [6]. This smearing, originating in microscopic quantum fluctuations, may also cure other problems such as initial or black hole singularity problems. Other studies also suggest that the small stochastic fluctuations of the spacetime metric lying on the deterministic background spacetime are not only the useful phenomenological modification of the semiclassical Einstein equation but also the inevitable consequence of more fundamental processes or of the backreaction induced by matter fluctuations [7,8]. In an astrophysical context, squeezed states, evolved from primordial gravitational waves by parametric amplification during the cosmological expansion [9], are considered to be one of the origins of the stochastic gravitational waves believed to exist in the present universe. Whether or not the trace of such primodial fluctuations as the quatum noise lies within the detectable range of the Laser Interferometric Gravitational Wave Observatory detector (LIGO) [10] is a curious question under debate [11,12].

B. Semiclassical gravity and mesoscopic physics

Semiclassical gravity, though far different in energy scale, shares many common features with mesoscopic physics [13,14]. The quantum transport properties of metallic systems are known to be divided into several different regimes depending on the qualitatively different contribution of scatterers. For a length scale less than the mean free path l_M, the wave propagates ballistically, similar to the free coherent wave; for a length scale larger than the coherence length L_{coh}, the scale at which the mutual coherence of waves is lost due to inelastic scattering, classical Boltzmann transport theory is valid. In the mesoscopic scale $l_M<L<L_{coh}$, multiple scattering has to be taken into account and the coherence between different paths becomes important. The effect of the environment also has to be considered, and the dissipation and decoherence due to inelastic scattering play an important role. Furthermore, in the mesoscopic regime close to L_{coh}, the quantum-classical correspondence of the propagating wave becomes relevant; in the same regime close to l_M, the possible influence of the microscopic constituent of a medium will manifest itself in the transport properties of the wave. Owing to recent progress in nanoscale technology [15], many phenomena in this regime are amenable to experiments. In light of this analogy, various effects associated with electromagnetic wave propagation in the Friedmann-Robertson-Walker universe and the Schwarzschild spacetime with metric stochasticity were studied in [16] based on the formal equivalence of Maxwell's equations in a stochastic spacetime with those in random media. Localization of photon and anomalous particle creation can occur in such stochastic spacetimes.

In this paper, we show that the scalar and spinor fields propagating in a stochastic Minkowski spacetime can be considered as electrons propagating in a disordered potential. The randomness couples to the frequency of the wave additively for the conformal metric fluctuations. Thus the effect of the stochasticity resembles that of a random potential. We use the closed time path formalism, which allows us to study

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equilibrium and nonequilibrium quantum field theory in a unified framework [17,18]. The closed time path partition function is defined for the stochastic quantum system from which we obtain the effective interaction between fundamental fields in the form of a four-point vertex after averaging over stochasticity. Diagonalization of the matrix Green function is employed. This is an essential step for making use of the nonlinear sigma model developed for the disordered transport problem [19–23]. We use the Hubbard-Storatonovich transformation and write the collective excitations in terms of auxiliary fields. The conductance and its fluctuations are expressed by these auxiliary fields.

The paper is organized as follows: In Sec. II, we start by showing that the effects of conformal metric fluctuations on scalar and Dirac fields can be simply represented by the effects of a fluctuating mass. In Sec. III, we develop the nonlinear sigma model to express higher order fluctuations of fundamental fields in terms of the correlation function of collective fields. The conductance associated with propagation of particles in such a spacetime can be defined analogous to the one in electric circuits by the Kubo formula as the correlation function of currents. Similar to the mesoscopic quantum transport problem, we show that the conductance fluctuations are universal, independent of the size of the stochastic region and the amount of stochasticity initially assumed. The amplitude of the conductance fluctuations is constant up to leading order in the weak disorder expansion. In Sec. IV, a summary of results is given followed by a brief discussion.

II. WAVE PROPAGATION AND SCALAR AND DIRAC FIELDS IN STOCHASTIC SPACETIMES

The Lagrangian density for the complex free scalar field in curved spacetime has the form

$$\mathcal{L}_S=\sqrt{-g}[g_{\mu\nu}\partial^\mu\phi^\dagger\partial^\nu\phi-(m_S^2+\xi R)\phi^2], \tag{1}$$

where ξ is a dimensionless nonminimal coupling parameter. $\xi=1/6$ is called conformal coupling and $\xi=0$ is minimal coupling. In the presence of a slight amount of inhomogeneity in a flat spacetime background characterized by $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$, the Lagrangian given above can be split into a flat space term and a perturbation term as $\mathcal{L}_S=\mathcal{L}_{S0}+\mathcal{L}_{SI}$:

$$\mathcal{L}_{S0}=\eta^{\mu\nu}\partial_\mu\phi^\dagger\partial_\nu\phi-m_S^2\phi^2 \tag{2}$$

and $\mathcal{L}_{SI}=-T_S^{\mu\nu}h_{\mu\nu}$, where $T_S^{\mu\nu}$ is the stress-energy tensor. In a conformal coupling case ($\xi=1/6$), the action, Eq. (1), is conformally invariant except for the mass term. Thus, a conformal type of metric fluctuations can be attributed to the effect of a fluctuating mass. Accordingly, if we write the stress-energy tensor in the form

$$T_S^{\mu\nu}=\partial^\mu\phi^\dagger\partial^\nu\phi-\frac{1}{2}\eta^{\mu\nu}[\eta^{\lambda\rho}\partial_\lambda\phi^\dagger\partial_\rho\phi+m_S^2\phi^2]$$
$$-2\xi\Big[\partial^\mu\phi^\dagger\partial^\nu\phi-\eta^{\mu\nu}\eta^{\lambda\rho}\partial_\lambda\phi^\dagger\partial_\rho\phi+\phi^\dagger\partial^\mu\partial^\nu\phi$$
$$-\frac{1}{4}\eta^{\mu\nu}\phi^\dagger\Box\phi-\frac{3}{4}\eta^{\mu\nu}m_S^2\phi^2\Big] \tag{3}$$

for an isotropic and conformal type of stochasticity $g_{\mu\nu}=\eta_{\mu\nu}e^{v(x)}\sim\eta_{\mu\nu}+\eta_{\mu\nu}v(x)$, where $v(x)$ is a stochastic field, the interaction term can be simply written as

$$\mathcal{L}_{SI}=-m_S^2v(x)\phi^2(x). \tag{4}$$

In this case the total Hamiltonian is given by

$$H=\int d^3x[\pi^2+(\nabla\phi)^2+m_S^2\phi^2+m_S^2v(x)\phi^2] \tag{5}$$

or, in a momentum representation,

$$H=\sum_p\Big[\phi(-p)(p^2+m_S^2)\phi(p)$$
$$+m_S^2\sum_q v(q)\phi(p)\phi(p+q)\Big]. \tag{6}$$

Furthermore, if we restrict our case to elastic scattering, we have

$$H=\sum_{\vec{p}}\Big[\phi(-\vec{p})(\vec{p}^{\,2}+m_S^2-\omega^2)\phi(\vec{p})$$
$$+m_S^2\sum_{\vec{q}} v(\vec{q})\phi(\vec{p})\phi(\vec{p}+\vec{q})\Big], \tag{7}$$

where $\omega\equiv p^0$. This has the form equivalent to electron propagation in a disordered potential.

The equation of motion in this model will be

$$\nabla^2\phi+(\omega^2-m_S^2)\phi+v_S(x)\phi=0, \tag{8}$$

where $v_S(x)\equiv m_S^2v(x)$. Comparing Eq. (8) with the equation of motion for the electromagnetic wave [16],

$$\nabla^2\phi+\omega^2\phi-\omega^2v_S(x)\phi=0, \tag{9}$$

we see that the randomness couples additively to the frequency in the scalar wave, whereas it couples multiplicatively in the electromagnetic wave. Since the electromagnetic wave obeys Maxwell's equations which are conformally invariant, the equation of motion is rather similar to that of wave propagation in random media, while for the scalar wave subjected to conformal metric fluctuations the equation of motion has the form of massive particles in a random potential. We expect that the property of fields under the influence of more general types of metric fluctuations will possess both aspects. The difference of the form between Eq. (8) and Eq. (9) leads to the different density of

states and transport properties. We also point out that although the restriction of the metric fluctuations to the conformal type simplifies the arguments significantly, many arguments in the rest of the paper are applicable to a more general class of fluctuations.

The Lagrangian density for the Dirac field in curved spacetime has the form

$$\mathcal{L}_D = b\left[\frac{i}{2}\bar{\psi}\not{\nabla}\psi - m_D\bar{\psi}\psi\right], \tag{10}$$

where $\not{\nabla} \equiv \gamma_a \nabla^a$, $\gamma^a \equiv b^a{}_\mu \gamma^\mu$, and $b = \det b^a{}_\mu$ for vierbein fields $b^a{}_\mu$. The Lagrangian given above splits into a flat space contribution and a perturbation term as $\mathcal{L} = \mathcal{L}_{D0} + \mathcal{L}_{DI}$ similar to the scalar field case:

$$\mathcal{L}_{D0} = \frac{i}{2}\bar{\psi}\not{\nabla}\psi - m_D\bar{\psi}\psi \tag{11}$$

and $\mathcal{L}_{DI} = -T_D^{\mu\nu}h_{\mu\nu}$, where $T_D^{\mu\nu}$ is the stress-energy tensor, which can be written as

$$T_D^{\mu\nu} = \frac{i}{2}[\bar{\psi}\gamma^\mu\nabla^\nu\psi - \nabla^\nu\bar{\psi}\gamma^\mu\psi]. \tag{12}$$

For a massless Dirac field, the action, Eq. (10), is conformally invariant. Thus, a conformal type of metric fluctuations can also be considered as the effect of a fluctuating mass as in the scalar field. For an isotropic, conformal type of stochasticity $v(x)$ defined above, the interaction term can be manifestly given as

$$\mathcal{L}_{DI} = \frac{i}{2}v(x)\bar{\psi}(x)\not{\nabla}\psi(x) = m_D v(x)\bar{\psi}(x)\psi(x), \tag{13}$$

where we used the equation of motion to obtain the second expression. The equation of motion in this model will be

$$[i\not{\partial} - m_D - v_D(x)]\psi(x) = 0, \tag{14}$$

where $v_D(x) \equiv m_D v(x)$. The corresponding equations for the Green functions are

$$-[\Box + m_S^2 + v_S(x)]G(x,x') = \delta^4(x-x') \quad \text{for a scalar field,} \tag{15}$$

$$[i\not{\partial} - m_D - v_D(x)]S(x,x') = \delta^4(x-x') \quad \text{for a Dirac field.} \tag{16}$$

We expect that the autocorrelation functions of the stochastic fields have the following forms:

$$\Delta_S(x-y) = \langle v_S(x)v_S(y)\rangle = u_S\mathcal{F}_L(|\vec{x}-\vec{x}'|/L)\mathcal{F}_T(|t-t'|/T),$$

$$\Delta_D(x-y) = \langle v_D(x)v_D(y)\rangle = u_D\mathcal{F}_L(|\vec{x}-\vec{x}'|/L)\mathcal{F}_T(|t-t'|/T), \tag{17}$$

where $\mathcal{F}_L$ and $\mathcal{F}_T$ are rapidly decaying functions and L and T are characteristic correlation ranges of the stochastic field in space and time, respectively. L and T can be regarded as the minimum length and time appearing as a result of the coarse graining of microscopic dynamics characterizing the finite resolution of spacetime. The functions $\mathcal{F}_L$ and $\mathcal{F}_T$ are normalized such that $\int \mathcal{F}_L d^3x = \int \mathcal{F}_T dt = 1$. For a possible choice of the form of fluctuations,

$$\langle v(x)v(y)\rangle = \frac{l_{pl}^\alpha}{|\vec{x}-\vec{x}'|^\alpha}e^{-|\vec{x}-\vec{x}'|/l_{pl}} \tag{18}$$

for a constant α, where $L = l_{pl} = 1.6\times 10^{-33}$ cm is the Planck length, we have

$$u_S = 4\pi m_S^4 l_{pl}^3, \tag{19}$$

independent of α. The disorder-averaged retarded (advanced) Green function $\langle G_{R(A)}(p,p')\rangle = G_{R(A)}(p)$ can be written in the form

$$G_{R(A)}(p) = \frac{1}{p^2 - m_R^2 \mp i\,\mathrm{sgn}(p_0)\Sigma_I(p)}, \tag{20}$$

where m_R is the renormalized mass and $\Sigma_I(p)$ is an imaginary part of the self-energy. The upper (lower) sign corresponds to the retarded (advanced) Green function. For the stochastic field that obeys Eq. (18), $v(x)$ can be approximated as a white noise potential if $|\vec{p}| \ll M_{pl}$, where $M_{pl} = 10^{19}$ GeV is the Planck mass. Consequently, the self-energy $\Sigma_I(p)$ is independent of the momentum and depends only on the frequency in such a limit. That is, the low energy elastic scattering is insensitive to the length scale that characterizes the fine structure of the medium. In this limit, the effect of the medium is to renormalize the frequency of the wave propagating through it. The real part of the self-energy can be absorbed in the frequency term in this limit and will not be considered in this paper. Such a medium is called the effective medium. Under this condition, the mean free path of this system is given by $l_M = 4\pi/u_S$ in the Born approximation. Combining with Eq. (19), we obtain $l_M = m_S^{-4}l_{pl}^{-3}$. For a mass m_S much smaller than the Planck mass ($m_S \ll M_{pl}$), this naive calculation illustrates that the mean free path is much larger than the Planck length ($l_M \gg l_{pl}$). Similar results hold for the spinor field as well. The coherence length L_{coh}, on the other hand, is determined by many factors including the other possible interactions, the time dependence in the randomness, and external fields and will not be specified here.

III. MESOSCOPIC EFFECTS IN STOCHASTIC MINKOWSKI SPACETIME

In this section, we consider quantization of the noninteracting scalar field that obeys the stochastic equation of motion discussed in the last section. The parallel argument for the Dirac field is given in Appendix A. The stochastic action for the scalar field has the form

$$S_{v_S}[\phi,\phi^\dagger] = \int d^4x[\partial^\mu\phi^\dagger\partial_\mu\phi - m_S^2\phi^2 - v_S\phi^2]. \tag{21}$$

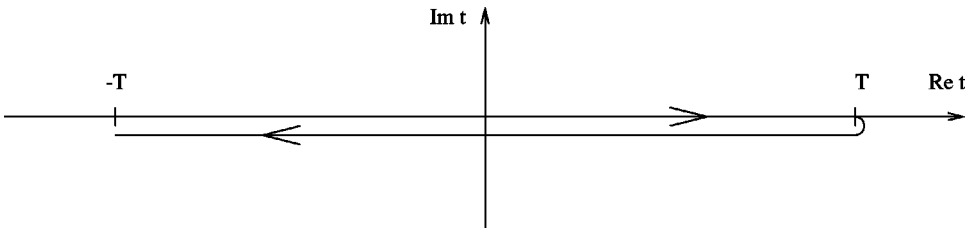

FIG. 1. The contour in the closed time path integral formalism.

This action is invariant under the global U(1) gauge transformation

$$\phi(x)\rightarrow e^{i\theta}\phi(x),$$
$$\phi^{\dagger}(x)\rightarrow\phi^{\dagger}(x)e^{-i\theta}. \tag{22}$$

The corresponding Noether current is

$$J_{\mu}(x)=i\phi^{\dagger}\partial_{\mu}\phi-i\partial_{\mu}\phi^{\dagger}\phi. \tag{23}$$

A. Closed time path formalism for the stochastic system

The quantization of the stochastic system described in Eq. (9) can be treated in the closed time path formalism. Here we assume that the whole system consists of a complex scalar quantum field ϕ and a classical stochastic field v_S. Then the density matrix for the whole system is given by

$$\rho_S[\phi_1,\phi_1^{\dagger},\phi_2,\phi_2^{\dagger},v_S,t]=\langle\phi_1,\phi_1^{\dagger},v_S|\hat{\rho}_S(t)|\phi_2,\phi_2^{\dagger},v_S\rangle, \tag{24}$$

where $|\phi,\phi^{\dagger},v_S\rangle$ is an eigenstate of the field operator $\hat{\phi}$ for a particular realization of v_S such that

$$\hat{\phi}(x)|\phi,\phi^{\dagger},v_S\rangle=\phi(x)|\phi,\phi^{\dagger},v_S\rangle. \tag{25}$$

To obtain the reduced density matrix $\rho_S[\phi_1,\phi_1^{\dagger},\phi_2,\phi_2^{\dagger},t]$ for the system, we average over v_S as

$$\rho_S[\phi_1,\phi_1^{\dagger},\phi_2,\phi_2^{\dagger},t]=\langle\rho_S[\phi_1,\phi_1^{\dagger},\phi_2,\phi_2^{\dagger},v_S,t]\rangle_v. \tag{26}$$

The closed time path partition function for this system is given by

$$\begin{aligned}Z[J,J^{\dagger},v_S]&=\int d\phi_f d\phi_f^{\dagger}\langle 0_-|\tilde{T}\exp[-iJ_2\cdot\hat{\phi}]|\phi_f,\phi_f^{\dagger},v_S\rangle\\&\quad\times\langle\phi_f,\phi_f^{\dagger},v_S|T\exp[iJ_1\cdot\hat{\phi}]|0_-\rangle\\&=\int d\phi_f d\phi_f^{\dagger}D\phi_1 D\phi_1^{\dagger}D\phi_2 D\phi_2^{\dagger}\\&\quad\times\exp\{i(S[\phi_1,\phi_1^{\dagger}]-S[\phi_2,\phi_2^{\dagger}]\\&\quad+J_1\cdot\phi_1-J_2\cdot\phi_2)\}\exp\{i(S_I[\phi_1,\phi_1^{\dagger},v_S]\\&\quad-S_I[\phi_2,\phi_2^{\dagger},v_S])\}\end{aligned} \tag{27}$$

for $\hat{\rho}_S(t_i)=|0_-\rangle\langle 0_-|$ and $S_I[\phi,\phi^{\dagger},v_S]\equiv-\int d^4x\, v_S\phi^2$, $J\cdot\phi\equiv\int d^4x[J^{\dagger}(x)\phi(x)+\phi^{\dagger}(x)J(x)]$ and $\tilde{T}$ denotes an anti-time-ordered product. The path integral above is defined along two paths, one forward in time and the other backward in time (Fig. 1). For an arbitrary initial state given by $\rho_S[\phi_{1i},\phi_{1i}^{\dagger},\phi_{2i},\phi_{2i}^{\dagger}]$, the partition function takes the form

$$\begin{aligned}Z[J,J^{\dagger},v_S]&=\int d\phi_f d\phi_f^{\dagger}D\phi_1 D\phi_1^{\dagger}D\phi_2 D\phi_2^{\dagger}\\&\quad\times\exp\{i(S[\phi_1,\phi_1^{\dagger}]-S[\phi_2,\phi_2^{\dagger}]\\&\quad+J_1\cdot\phi_1-J_2\cdot\phi_2)\}\rho_S[\phi_{1i},\phi_{1i}^{\dagger},\phi_{2i},\phi_{2i}^{\dagger}]\\&\quad\times F[\phi_1,\phi_1^{\dagger},\phi_2,\phi_2^{\dagger},v_S],\end{aligned} \tag{28}$$

where

$$\begin{aligned}&F[\phi_1,\phi_1^{\dagger},\phi_2,\phi_2^{\dagger},v_S]\\&\quad=\exp\{i(S_I[\phi_1,\phi_1^{\dagger},v_S]-S_I[\phi_2,\phi_2^{\dagger},v_S])\}\end{aligned} \tag{29}$$

is a stochastic influence functional.

In the absence of the stochastic field,

$$\begin{aligned}Z[J,J^{\dagger}]&=\int d\phi_f d\phi_f^{\dagger}D\phi_1 D\phi_1^{\dagger}D\phi_2 D\phi_2^{\dagger}\\&\quad\times\exp\{i(S[\phi_1,\phi_1^{\dagger}]-S[\phi_2,\phi_2^{\dagger}]\\&\quad+J_1\cdot\phi_1-J_2\cdot\phi_2)\}\rho_S[\phi_{1i},\phi_{1i}^{\dagger},\phi_{2i},\phi_{2i}^{\dagger}]\\&=\exp[-iJ^{\dagger}GJ],\end{aligned} \tag{30}$$

where now the Green function G acquired the 2×2 matrix structure as

$$G=\begin{pmatrix}G_{11}&G_{12}\\G_{21}&G_{22}\end{pmatrix} \tag{31}$$

and $J^{\dagger}=(J_1^{\dagger},-J_2^{\dagger})$. For an initial thermally equilibrium state with the temperature $T=1/\beta$, each component of the matrix Green function in the momentum representation is given by

$$\begin{aligned}G_{11}(p)&=-G_{22}^{*}(p)=\theta(p_0)G_F(p)+\theta(-p_0)G_F^{*}(p)\\&\quad-2\pi i\,\mathrm{sgn}(p_0)n_B(p)\,\delta(p^2-m_S^2),\\G_{12}(p)&=-2\pi i\,\mathrm{sgn}(p_0)n_B(p)\,\delta(p^2-m_S^2),\\G_{21}(p)&=-2\pi i\,\mathrm{sgn}(p_0)e^{\beta(p_0-\mu)}n_B(p)\,\delta(p^2-m_S^2),\end{aligned} \tag{32}$$

where $G_F(p)=(p^2-m_S^2+i\epsilon)^{-1}$ is the vacuum Feynman propagator, $n_B(p)\equiv(e^{\beta(p_0-\mu)}-1)^{-1}$ is the Bose distribu-

tion function, and μ is the chemical potential. G can be diagonalized by multiplying matrices u_B from each side as $G=u_B G_d u_B^{-1}\eta$, with

$$G_d=\begin{pmatrix} G_R & 0 \\ 0 & G_A \end{pmatrix}, \tag{33}$$

where G_R and G_A are retarded and advanced Green functions, respectively. Here $u_B(p)$ is the thermal Bogoliubov matrix and η is the 2×2 Lorentz matrix which have the following forms [24,25]:

$$u_B(p)=\sqrt{n_B(p)}e^{\beta(p_0-\mu)/2}\begin{pmatrix} 1 & e^{-\beta(p_0-\mu)} \\ 1 & 1 \end{pmatrix},$$
$$\eta=\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{34}$$

Defining the thermal doublet $\phi=(\phi_1,\phi_2)$ and its conjugate $\bar{\phi}\equiv(\phi_1^\dagger,\phi_2^\dagger)\eta$, we can write

$$\phi^\dagger G^{-1}\phi=\phi^\dagger \eta u_B G_d^{-1}u_B^{-1}\phi$$
$$=\bar{\phi}u_B G_d^{-1}u_B^{-1}\phi. \tag{35}$$

By redefining the field variables by global Bogoliubov transformations $u_B^{-1}\phi\to\phi$ and $\bar{\phi}u_B\to\bar{\phi}$, one can write the partition function in Eq. (30) as

$$Z[J,\bar{J}]=\int D\bar{\phi}D\phi\,\exp\{iS_0[\phi,\bar{\phi}]+i\bar{J}\phi+i\bar{\phi}J\}\rho_S[\phi_i,\bar{\phi}_i], \tag{36}$$

where

$$S_0[\phi,\bar{\phi}]=\bar{\phi}G_d^{-1}\phi$$
$$=\int d^3x\,d\omega$$
$$\times\{\bar{\phi}(x,\omega)[\omega^2+\vec{\partial}^2-m_S^2+i\epsilon(\omega)\eta]\phi(x,\omega)\}. \tag{37}$$

Here $\epsilon(\omega)\equiv\epsilon\,\mathrm{sgn}(\omega)$ for some infinitesimal constant ϵ. When the stochastic field v_S has no time dependence as in Eq. (7), the close time path action is given by

$$S_{v_S}[\phi,\bar{\phi}]=\int d^3x\,d\omega\{\bar{\phi}(\vec{x},\omega)[\omega^2+\vec{\partial}^2-m_S^2$$
$$-v_S(\vec{x})+i\epsilon(\omega)\eta]\phi(\vec{x},\omega)\}.$$

Next we express the Green function and the partition function for the scalar field in terms of the nonlinear sigma model. After averaging out the partition function in Eq. (28) without the source term with respect to the stochastic potential v_S which obeys the Gaussian probability distribution

$$P[v_S]=\mathcal{N}\exp\left[-\frac{1}{2}\int d^4x\,d^4y\,v_S(x)\Delta_S^{-1}(x-y)v_S(y)\right], \tag{38}$$

with the normalization constant $\mathcal{N}$, we obtain the reduced action

$$Z=\langle Z[v_S]\rangle_{v_S}=\int Dv_S P[v_S]D\bar{\phi}D\phi\,\exp\{iS_{v_S}[\bar{\phi},\phi]\}$$
$$=\int D\bar{\phi}D\phi\,\exp\{iS_0[\bar{\phi},\phi]+iS_I[\bar{\phi},\phi]\}, \tag{39}$$

where

$$S_I[\bar{\phi},\phi]=\frac{i}{2}\int d^4x\,d^4y\,\bar{\phi}(x)\phi(x)\Delta_S(x-y)\bar{\phi}(y)\phi(y). \tag{40}$$

Note that for a local correlation $\Delta_S(x-y)\sim\delta^4(x-y)$, we obtain the effective ϕ^4 theory similar to the one obtained from spacetime foam [3].

Now we extract the slow modes by the Hubbard-Stratonovich transformation. Introducing the auxiliary bilocal matrix field $\sigma(x,y)$ as [19]

$$e^{iS_I[\bar{\phi},\phi]}=\int D\sigma\,\exp\left[-\frac{1}{2}\int d^4x\,d^4y\right.$$
$$\left.\times\mathrm{Tr}[\sigma(x,y)\Delta_S^{-1}(x-y)\sigma(y,x)]\right]$$
$$\times\exp\{iS_{HS}[\sigma,\bar{\phi},\phi]\}, \tag{41}$$

where

$$S_{HS}[\sigma,\bar{\phi},\phi]=-\int d^4x\,d^4y\,\bar{\phi}(x)\sigma(x,y)\phi(y) \tag{42}$$

and the trace is taken over thermal indices. The partition function can be written as

$$Z=\int D\sigma D\bar{\phi}D\phi\,\exp\left[-\frac{1}{2}\int\mathrm{Tr}[\sigma\Delta_S^{-1}\sigma]\right]$$
$$\times\exp\{iS_0[\bar{\phi},\phi]+iS_{HS}[\sigma,\bar{\phi},\phi]\}, \tag{43}$$

where σ is Hermitian by construction, i.e., $\sigma^\dagger(x-y)=\sigma(y-x)$. In energy representation, Eq. (42) becomes

$$S_{HS}[\sigma,\bar{\phi},\phi]=-\int d^3x\,d^3x'\frac{d\omega}{2\pi}\frac{d\omega'}{2\pi}\bar{\phi}(\vec{x},\omega)$$
$$\times\sigma_{\omega\omega'}(\vec{x},\vec{x}')\phi(\vec{x}',\omega') \tag{44}$$

and

$$S_0[\bar{\phi},\phi]+S_{HS}[\sigma,\bar{\phi},\phi]=\int d^3x\,d^3x'\frac{d\omega}{2\pi}\frac{d\omega'}{2\pi}\bar{\phi}(\vec{x},\omega)\times\{[\omega^2+\vec{\partial}^2-m_S^2+i\epsilon(\omega)\eta]\times\delta(\vec{x}-\vec{x}')\delta(\omega-\omega')-\sigma_{\omega\omega'}(\vec{x},\vec{x}')\}\phi(\vec{x}',\omega'). \quad (45)$$

After integrating out $\bar{\phi}$ and ϕ, we obtain

$$Z=\int D\sigma\exp\left[-\frac{1}{2}\int \mathrm{Tr}[\sigma\Delta_S^{-1}\sigma]\right]\times\exp\left[-\int d^3x\,d^3x'\frac{d\omega}{2\pi}\frac{d\omega'}{2\pi}\mathrm{Tr}\log\{[\omega^2+\vec{\partial}^2-m_S^2+i\epsilon(\omega)\eta]\delta(\vec{x}-\vec{x}')\delta(\omega-\omega')-\sigma_{\omega\omega'}(\vec{x},\vec{x}')\}\right]. \quad (46)$$

The nonlinear sigma model is commonly used in many different areas in physics to study collective excitations and the dynamical symmetry breaking property of the system. Nevertheless, it has not been studied previously in the closed time path method in detail. When applied to the disordered systems, it is required that all fermion loop diagrams cancel in order to include the effects of elastic scattering due to impurities which carry no energy. Such techniques as replica formalism [19] or supersymmetric extension [26] are commonly used for this purpose. A general class of real time path ordered methods [27–31] is also known to have such a property owing to the energy integral. The closed time path formalism employed here has the advantage compared to other methods that it is naturally extendable to the nonequilibrium setting. The trace part in Eq. (46) can be expanded in terms of the σ field as

$$\begin{aligned}&\mathrm{Tr}\log\{[\omega^2+\vec{\partial}^2-m_S^2+i\epsilon(\omega)\eta]\times\delta(\omega-\omega')\delta(\vec{x}-\vec{x}')-\sigma_{\omega\omega'}(\vec{x},\vec{x}')\}\\&=\mathrm{Tr}\log[G^{-1}-\sigma]\\&=\mathrm{Tr}\log[G^{-1}]+\sum_{n=1}\frac{(-1)^n}{n}\mathrm{Tr}[G\sigma]^n, \quad (47)\end{aligned}$$

where $G(x,x')\equiv(\Box+m_S^2)^{-1}$ is the free boson propagator.

Now we assume that the spatial correlation of disorder decays sufficiently fast so that the σ field can be treated as a local field variable in space. This implies that the stochastic potential v_S has the time-independent form

$$P[v_S]=\mathcal{N}\exp\left[-\frac{1}{2u_S}\int d^3x\,v_S^2(\vec{x})\right]. \quad (48)$$

In this case the partition function is reduced to

$$Z=\int D\sigma\exp\left[-\frac{1}{2u_S}\int d^3x\,\mathrm{Tr}\,\sigma^2(\vec{x})\right]\times\exp\left[-\int d^3x\frac{d\omega}{2\pi}\frac{d\omega'}{2\pi}\mathrm{Tr}\log\{[\omega^2+\vec{\partial}^2-m_S^2+i\epsilon(\omega)\eta]\times\delta(\omega-\omega')-\sigma_{\omega\omega'}(\vec{x})\}\right]. \quad (49)$$

The equation of motion obtained from Eq. (49) corresponds to the one obtained from the coherent potential approximation:

$$\sigma_{\omega\omega'}(\vec{P})=\frac{u_S}{\omega^2-\vec{P}^2-m_S^2+i\epsilon(\omega)\eta-\sigma_{\omega\omega'}(\vec{P})}. \quad (50)$$

The real part of the σ field gives the mass and frequency renormalization and will not be considered hereafter. The imaginary part of the σ field $\sigma_I(\omega)$ gives the scattering rate. Writing the homogeneous solution of Eq. (50) as $\sigma_{\omega\omega'}(\vec{P})=-i\sigma_I(\omega)\mathrm{sgn}(\omega)\delta(\vec{P})\delta(\omega-\omega')$, one obtains the relation

$$\sigma_I(\omega)=\frac{u_S\pi N(\omega)}{2|\omega|}, \quad (51)$$

where $N(\omega)$ is the density of states of the scalar field.

The quadratic term in Eq. (47) can be written as

$$\frac{1}{2}\int d^3x_1d^3y_1d^3x_2d^3y_2\frac{d\omega}{2\pi}\frac{d\omega'}{2\pi}\mathrm{Tr}[G^{aa}(\vec{x}_1,\vec{y}_1,\omega)\times\sigma^{ab}_{\omega\omega'}(\vec{y}_1,\vec{x}_2)G^{bb}(\vec{x}_2,\vec{y}_2,\omega')\sigma^{ba}_{\omega'\omega}(\vec{y}_2,\vec{x}_1)]. \quad (52)$$

In the momentum representation, the above expression becomes

$$\frac{1}{2}\int\frac{d^4P}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[G^{bb}\left(k+\frac{P}{2}\right)\sigma_k^{ba}(-P)\times G^{aa}\left(k-\frac{P}{2}\right)\sigma_k^{ab}(P)\right]. \quad (53)$$

The Fourier component of the σ field is defined as

$$\sigma(x,y)=\int\frac{d^4P}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\sigma_k(P)e^{-ikr}e^{-iPX}, \quad (54)$$

where $r\equiv x-y$ and $X\equiv(x+y)/2$. Thus we obtain the kinetic term of the σ field in Eq. (46)

$$\frac{1}{2}\int\frac{d^4P}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left\{\sigma_k^{ba}(-P)\left[\frac{1}{u_S}-G^{aa}\left(k-\frac{P}{2}\right)\times G^{bb}\left(k+\frac{P}{2}\right)\right]\sigma_k^{ab}(P)\right\}. \quad (55)$$

If the spatial dependence in the σ field is local, the σ field is independent of the momentum $\vec{k}$, i.e., $\sigma_k(P)=\sigma_{k_0}(P)$, and we can integrate out $\vec{k}$ and obtain

$$\frac{1}{2}\int \frac{d^4P}{(2\pi)^4}\frac{dk_0}{2\pi}\mathrm{Tr}\left\{\sigma^{ba}_{k_0}(-P)\left[\frac{1}{u_S}-\int_{\vec{k}}G^{aa}\left(k-\frac{P}{2}\right)\right.\right.$$
$$\left.\left.\times G^{bb}\left(k+\frac{P}{2}\right)\right]\sigma^{ab}_{k_0}(P)\right\}. \tag{56}$$

By expanding inside the brackets in Eq. (56) with respect to P_0 and $\vec{P}$, one can show that, in the limit $\sigma_I(k_0)\gg k_0P_0$, the off-diagonal term of the free propagator of the σ field in the thermal indices gives the massless excitation which has the form of the diffusion propagator [32]:

$$\langle\sigma^{ab}_{k_0}(P)\sigma^{ba}_{k_0}(-P)\rangle=\frac{2\sigma_I^2(k_0)}{\pi N(k_0)}\frac{1}{D(k_0)\vec{P}^2-iP_0\eta^{aa}}, \tag{57}$$

where $D(k_0)$ is the diffusion constant. Diagramatically this form is obtained by including all the ladder diagrams in the particle-hole propagator. The diffusion constant is related to the dc conductivity C_0 through the Einstein relation $C_0=N(k_0)D(k_0)$. The diagonal term in Eq. (56) gives the massive excitation that can be integrated out. However, it does not contribute directly to the infrared divergence responsible for the universal behavior of the conductance fluctuations and will not be considered in this work.

In this U(2) nonlinear sigma model, the matrix field σ takes its value in the coset space $\mathrm{U}(2)/\mathrm{U}(1)\times\mathrm{U}(1)$. After making a transformation $\sigma\rightarrow V^{-1}\sigma V$ with $V=\theta(\omega)1+\theta(-\omega)\sigma_2$, where matrices 1 and σ_2 act on the thermal indices, the saddle point solution changes its form as $\mathrm{sgn}(\omega)\,\eta\rightarrow\eta$. In this representation, the field σ around the saddle point can be parametrized as [21]

$$\sigma=\begin{pmatrix}\sqrt{1-qq^\dagger} & q\\ q^\dagger & -\sqrt{1-q^\dagger q}\end{pmatrix}$$
$$=\begin{pmatrix}1&0\\0&-1\end{pmatrix}+\begin{pmatrix}0&q\\q^\dagger&0\end{pmatrix}-\frac{1}{2}\begin{pmatrix}qq^\dagger&0\\0&-q^\dagger q\end{pmatrix}\cdots. \tag{58}$$

The matrix fields $q=q_{\omega\omega'}$ and $q^\dagger=q^\dagger_{\omega'\omega}$ carry two frequency indices. Inserting this expression into Eq. (56), we obtain the form of the free propagator of q field as

$$\langle q_{k_0}(P)q^\dagger_{k_0}(-P)\rangle=\frac{2}{\pi N(k_0)}\frac{1}{D(k_0)\vec{P}^2-iP_0}. \tag{59}$$

$\sigma_I(\omega)$ in the numerator was absorbed in the redefinition of q. Thus the partition function for the linear part of the sigma model with respect to q field is

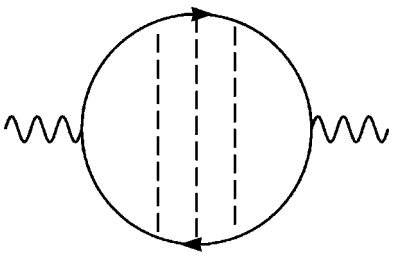

FIG. 2. The Feynman diagram for the conductance. The metric fluctuations are included as a ladder contribution which is represented by the dotted line.

$$Z=\int Dq^\dagger Dq\,\exp\left\{-\pi\int\frac{d^4P}{(2\pi)^4}\frac{dk_0}{2\pi}N(k_0)q^\dagger_{k_0}(-P)\right.$$
$$\left.\times[D(k_0)\vec{P}^2-iP_0]q_{k_0}(P)\right\}. \tag{60}$$

Higher order vertex terms will follow corresponding to the expansion in Eq. (58). The fields q and $q^\dagger$ are free from the constraint and take all possible values. Time dependence in the stochasticity can be ascribed to the intrinsic property of the effective medium and treated as the frequency-dependent random potential. The modification associated with this change can be absorbed in the diffusion constant as the term proportional to $\partial v_S/\partial\omega$. If this correction is too large, a different approach is required. The usual prescription to account for the effect of inelastic scattering is to include the inelastic scattering rate Δ in the denominator of the diffusion propagator so that we replace $D(k_0)\vec{P}^2-iP_0$ in Eq. (60) simply by $D(k_0)\vec{P}^2-iP_0+\Delta$. Here Δ will set the time scale beyond which the phase memory of a scattered wave is lost and the transport behavior becomes classical.

B. Mesoscopic fluctuations of scalar fields

The conductivity associated with the Noether current in the presence of the external field with a frequency κ can be written by the current-current correlation function by the Kubo formula [33] as

$$C_\kappa(\vec{x},\vec{y})\equiv\frac{\pi}{\kappa}\int_{-\infty}^{\infty}d\omega\,\Omega_\kappa(\omega)\sum_{mn}j_{mn}(\vec{x})j_{nm}(\vec{y})$$
$$\times\delta(\omega+\kappa-\omega_n)\,\delta(\omega-\omega_m), \tag{61}$$

where $j_{mn}\equiv i\phi^\dagger_m\overleftrightarrow{\partial}\phi_n$ is the Noether current expressed by two energy eigenstates and $\Omega_\kappa(\omega)$ is a smearing function that depends on the characteristics of the system and the environment. Near equilibrium, it can be written as $\Omega_\kappa(\omega)=\rho_S[\omega]-\rho_S[\omega+\kappa]$, where $\rho_S[\omega]$ is the initial density matrix. If the metric fluctuations are independent of temperature as we assume, the effect of temperature on the conductivity only arises from this term. Here we assume that $\Omega_\kappa(\omega)$ is normalized such that $\int d\omega\,\Omega_\kappa(\omega)=\kappa$. In Fig. 2, the conductance is represented by the Feynman diagram. In the leading order weak disorder expansion, averaging over disorder is

taken into account by including all the ladder diagrams. The conductivity can be written in the more familiar form in terms of Green functions as [34]

$$C_\kappa(\vec{x},\vec{y}) \equiv -\frac{1}{4\pi\kappa}\int_{-\infty}^{\infty} d\omega\, \Omega_\kappa(\omega)[G_R(\vec{x},\vec{y},\omega)-G_A(\vec{x},\vec{y},\omega)] \times \overleftrightarrow{\partial}_x \overleftrightarrow{\partial}_y [G_R(\vec{y},\vec{x},\omega+\kappa)-G_A(\vec{y},\vec{x},\omega+\kappa)]. \quad (62)$$

With the expression of Green functions in terms of thermal fields,

$$G_R(x,y) \equiv -i\,\theta(x_0-y_0)\langle[\hat{\phi}(x),\hat{\phi}^\dagger(y)]\rangle = i\langle \phi_1(x)\phi_1^\dagger(y)\rangle,$$
$$G_A(x,y) \equiv i\,\theta(y_0-x_0)\langle[\hat{\phi}(x),\hat{\phi}^\dagger(y)]\rangle = -i\langle \phi_2(x)\phi_2^\dagger(y)\rangle, \quad (63)$$

we write Eq. (62) as

$$C_\kappa(\vec{x},\vec{y}) \equiv \frac{-1}{4\pi\kappa}\int_{-\infty}^{\infty} d\omega\, \Omega_\kappa(\omega) \sum_{abcd} \langle \phi^a(\vec{x},\omega)\overleftrightarrow{\partial}_x \phi^{\dagger b} \times (\vec{x},\omega+\kappa)\phi^c(\vec{y},\omega+\kappa)\overleftrightarrow{\partial}_y \phi^{\dagger d}(\vec{y},\omega)\rangle. \quad (64)$$

Note that even though the thermal indices in the sum run over all possible values, only pairwise equal terms contribute to the conductivity.

This expression can be obtained directly from the partition function in terms of the functional derivative by introducing the external source term $\vec{A}^\kappa$ in the form

$$S[A,\overline{\phi},\phi] = i\int d^3x \frac{d\omega}{2\pi}\frac{d\omega'}{2\pi}\overline{\phi}(\vec{x},\omega)\vec{A}^\kappa_\omega(\vec{x})\cdot\overleftrightarrow{\partial}\delta_\kappa\phi(\vec{x},\omega'), \quad (65)$$

with

$$\delta_\kappa \equiv \delta(\omega'-\omega+\kappa)\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}. \quad (66)$$

The matrix in δ_κ acts on the thermal indices. Note that δ_κ is nilpotent, i.e., $\delta_\kappa^2 = 0$. The nonlocal conductivity is given by

$$C_\kappa(\vec{x},\vec{y}) = \frac{-\pi}{\kappa V}\frac{\delta^2 W[A]}{\delta\vec{A}^\kappa(\vec{x})\delta\vec{A}^{-\kappa}(\vec{y})}, \quad (67)$$

where V is the spatial volume of the system and the integral over energy indices are understood. In the presence of the source term, the partition function for the matrix field σ in Eq. (46) becomes

$$Z[A] = e^{iW[A]} = \int D\sigma \exp\left[-\frac{1}{2}\int \mathrm{Tr}[\sigma\Delta_S^{-1}\sigma]\right] \times \exp\left[\int d^3x\, d^3x' \frac{d\omega}{2\pi}\frac{d\omega'}{2\pi} \times \mathrm{Tr}\log(\{[\omega^2+\vec{\partial}^2-m_S^2+i\epsilon(\omega)\eta]\delta(\omega-\omega') + i\vec{A}^\kappa_\omega\cdot\overleftrightarrow{\partial}\delta_\kappa\}\delta(\vec{x}-\vec{x}') - \sigma_{\omega\omega'}(\vec{x},\vec{x}'))\right]. \quad (68)$$

Making use of the gauge symmetry in Eq. (68), for a constant vector field $\vec{A}^\kappa$, the source term above can be generated by the following gauge transformation:

$$\phi(\vec{x}) \to e^{i\vec{x}\cdot\vec{A}^\kappa\delta_\kappa}\phi(\vec{x}),$$
$$\overline{\phi}(\vec{x}) \to \overline{\phi}(\vec{x})e^{-i\vec{x}\cdot\vec{A}^\kappa\delta_\kappa}. \quad (69)$$

Correspondingly, the Green function $G(\vec{x},\vec{y},\omega)$ and the σ field transform as

$$G(\vec{x},\vec{y}) \to U^{-1}(\vec{x})G(\vec{x},\vec{y})U(\vec{y}) \quad (70)$$

and

$$\sigma(\vec{x},\vec{y}) \to U^{-1}(\vec{x})\sigma(\vec{x},\vec{y})U(\vec{y}), \quad (71)$$

where $U(\vec{x}) \equiv e^{-i\vec{x}\cdot\vec{A}^\kappa\delta_\kappa}$. This gauge symmetry will induce the gauge coupling in the effective Lagrangian through the covariant derivative $\vec{\nabla} \equiv \vec{\partial} + i\vec{A}^\kappa\delta_\kappa$:

$$\mathrm{Tr}(\vec{\nabla}\sigma\vec{\nabla}\sigma) = \mathrm{Tr}(\vec{\partial}\sigma\vec{\partial}\sigma) + 2i\,\mathrm{Tr}(\vec{A}^\kappa\delta_\kappa\sigma\vec{\partial}\sigma) - \mathrm{Tr}(\vec{A}^{\kappa_1}\delta_{\kappa_1}\sigma\vec{A}^{\kappa_2}\delta_{\kappa_2}\sigma). \quad (72)$$

Now we obtain the expression of the conductivity in terms of the σ field,

$$C_\kappa = \frac{-\pi}{\kappa V}\frac{\delta^2 W[A]}{\delta\vec{A}^\kappa\delta\vec{A}^{-\kappa}} = \frac{-\pi}{\kappa V}\left[\int d^3x_1\langle \mathrm{Tr}[\delta_\kappa\sigma(x_1)\delta_{-\kappa}\sigma(x_1)]\rangle - \int d^3x_1 d^3x_2\langle \mathrm{Tr}[\delta_\kappa\sigma(x_1)\vec{\partial}\sigma(x_1)] \times \mathrm{Tr}[\delta_{-\kappa}\sigma(x_2)\vec{\partial}\sigma(x_2)]\rangle\right], \quad (73)$$

and its fluctuation,

$$C^2_{\kappa_1\kappa_2} = \frac{\pi^2}{V^2\kappa_1\kappa_2}\frac{\delta^4 W[A]}{\delta\vec{A}^{\kappa_1}\delta\vec{A}^{-\kappa_1}\delta\vec{A}^{\kappa_2}\delta\vec{A}^{-\kappa_2}} = \frac{1}{V}[C^{(1)}-C^{(2)}+C^{(3)}], \quad (74)$$

where

$$C^{(1)} = \frac{2\pi^2}{\kappa_1\kappa_2 V}\int d^3x_1 d^3x_2\langle \mathrm{Tr}[\delta_{\kappa_1}\sigma(\vec{x}_1)\delta_{-\kappa_1}\sigma(\vec{x}_1)] \times \mathrm{Tr}[\delta_{\kappa_2}\sigma(\vec{x}_2)\delta_{-\kappa_2}\sigma(\vec{x}_2)]\rangle,$$

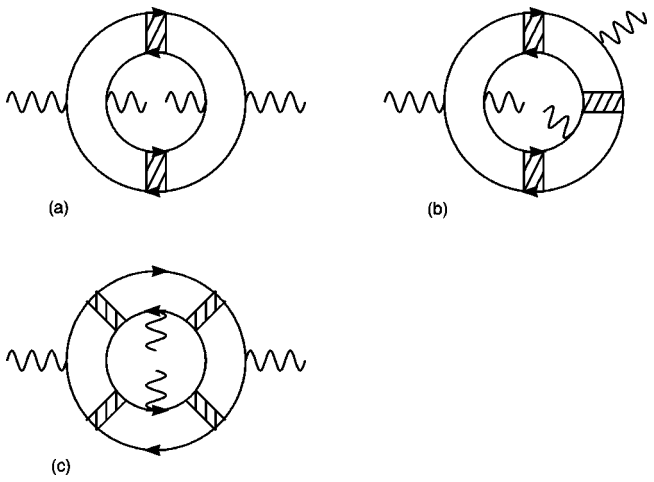

FIG. 3. The conductance fluctuations are represented by Feynman diagrams. The shaded regions are diffusion propagators. (a), (b), and (c) contain two, three, and four diffusions and correspond to $C^{(1)}$, $C^{(2)}$, and $C^{(3)}$ in Eq. (74), respectively.

$$C^{(2)}=\frac{4\pi^2}{\kappa_1\kappa_2 V}\int d^3x_1 d^3x_2 d^3x_3 \langle \mathrm{Tr}[\,\delta_{\kappa_1}\sigma(\vec{x}_1)\vec{\partial}\sigma(\vec{x}_1)]$$
$$\times \mathrm{Tr}[\,\delta_{-\kappa_1}\sigma(\vec{x}_2)\vec{\partial}\sigma(\vec{x}_2)]\mathrm{Tr}[\,\delta_{\kappa_2}\sigma(\vec{x}_3)\delta_{-\kappa_2}\sigma(\vec{x}_3)]\rangle,$$

$$C^{(3)}=\frac{\pi^2}{\kappa_1\kappa_2 V}\int d^3x_1 d^3x_2 d^3x_3 d^3x_4 \langle \mathrm{Tr}[\,\delta_{\kappa_1}\sigma(\vec{x}_1)\vec{\partial}\sigma(\vec{x}_1)]$$
$$\times \mathrm{Tr}[\,\delta_{-\kappa_1}\sigma(\vec{x}_2)\vec{\partial}\sigma(\vec{x}_2)]\mathrm{Tr}[\,\delta_{\kappa_2}\sigma(\vec{x}_3)\vec{\partial}\sigma(\vec{x}_3)]$$
$$\times \mathrm{Tr}[\,\delta_{-\kappa_2}\sigma(\vec{x}_4)\vec{\partial}\sigma(\vec{x}_4)]\rangle.$$

Here we further included the energy indices in the definition of the trace as

$$\mathrm{Tr}(\mathcal{O})\equiv \pi\int dk_0/(2\pi)^2 N(k_0)D(k_0)\int dP_0 \Sigma_a \mathcal{O}^{aa}(k_0,P_0).$$

We are interested in the dc conductivity evaluated in the limit $\kappa_1,\kappa_2\to 0$. Note that $\Omega_\kappa(\omega)$ is generally a peak function which describes a wave packet peaked around the specific mode $\omega=\omega_0$. Furthermore, if we assume for simplicity that it is given by the step function $\Omega_\kappa(\omega)=1$ (for $\omega_0<\omega<\omega_0+\kappa$) and 0 (otherwise), then taking the dc limit $\kappa\to 0$ extracts the particular mode ω_0. The effect of finite temperature T can be viewed as an additional smearing due to the width of $\Omega_\kappa(\omega)$. From Appendix B, in the dc limit, the formula above gives the simple result

$$C_0^2=\lim_{\kappa_1,\kappa_2\to 0} C^2_{\kappa_1\kappa_2}=\frac{c}{VM_{IR}}, \tag{75}$$

where $c=7.295\ldots$ is a constant and M_{IR} is the infrared cutoff of the momentum integral. In Fig. 3, the corresponding Feynman diagrams for the conductance fluctuations are given. Other diagrams that contain crossed diagrams contribute as higher order terms in the weak disorder expansion. Here we assume that the region with the fluctuating metric is restricted in the finite cube with the edge length L and that the rest of the spacetime is flat. This enables us to handle the problem as a scattering process. Taking L as the parameter that lies in the *mesoscopic scale*, the analogy with the electric circuit becomes elucidated. If we use the relation $C_0=gL^{2-d}$ for the conductivity in d dimension for the dimensionless conductance g, we obtain the conductance fluctuations in terms of the conductivity fluctuations as $g^2=C^2L^{2d-4}$. Then from Eq. (75), in three dimensions, using $M_{IR}\sim\pi/L$, we obtain

$$g^2\sim 2.322\ldots. \tag{76}$$

This value is universal in the sense that it is independent of the amount of stochasticity initially assumed and the size of the stochastic region [35,36]. The conductance is also directly related to the transmission matrix. Indeed one can show that the conductance measures the intensity of wave transmission [34] and the fluctuations of conductance correspond to the fluctuations of wave intensity.

IV. DISCUSSION

In this paper, we showed the analogy between the field propagation in Minkowski spacetime with a small stochasticity in the metric and the wave in disordered systems. While the electromagnetic field propagation in a stochastic spacetime is similar to that in random media, the scalar and spinor field propagation was shown to be similar to the electron in a disordered potential. Both cases can be treated similarly; however, the following difference should be noted. In the former, the field remains massless and the randomness affects the refraction property of light differently depending on the frequency of the wave. In particular, low energy scattering is suppressed. In the latter, a random mass causes scattering with any energy. Mesoscopic fluctuations associated with wave propagation were characterized by the nonlinear sigma model in the closed time path method. We introduced the collective fields by the Hubbard-Storatonovich transformation and integrated out the fundamental field variables and obtained the nonlinear sigma model written in terms of the collective fields only. The conductivity and its fluctuations were expressed by these fields. For the time-independent or slowly dependent stochasticity, the fluctuations of the dc conductivity were shown to be universal and of order unity. The origin of this universality is traced back to the infrared divergence due to the Nambu-Goldstone boson which appears as a result of symmetry breaking.

Although the induced effects on the propagation of waves in the presence of metric fluctuations are themselves of theoretical and observational importance as long as the backreaction of the matter fluctuations is small, a self-consistent treatment is necessary for Planck scale processes [37]. This line of consideration is important particularly of the metric fluctuations produced in the cosmological processes.

Relativistic quantum field theoretic calculations of the transport coefficients have been developed during the past decade [38–40]. The electrical conductivity in the early universe controls the generation of primordial magnetic field which is believed to be the origin of the strong magnetic

fields presently observed in spiral galaxies [41,42]. The method developed in Sec. III also gives the field theoretic basis for the study of, for example, mesoscopic fluctuations due to random magnetic fields.

The coarse graining of microscopic degree of freedom necessarily induces the nonlocal correlation in the stochastic fields. Moreover, unitarity in the whole system guarantees the relation between the dissipation kernel and the noise kernel in the form of the fluctuation-dissipation theorem upon coarse graining. In the present work, the nonlocal, noncommutative origin of the stochastic fields and the effect of dissipation are ignored and only the classical aspects are considered. The possible manifestation of the quantum nature of underlying microscopic gravitational dynamics in the mesoscopic effects remains to be clarified [43]. The fluctuating metric is also relevant for inferring possible decoherence effects in the quantum interference of propagating particles [44–46]. The closed time path method gives a suitable framework to discuss such effects. Our results of conductance fluctuations assume that the time scale of metric fluctuations is relatively long. In such a case, the time scale of fluctuations appears in the coherence time scale to restrict the validity of the arguments and diagrammatic calculations based on the coherence between different modes beyond this time scale. This prescription is quite successful in explaining many mesoscopic experimental results such as electron scattering in a helium gas. Thus, the heavy defects randomly created in the phase transition in the early universe can be the origin of such fluctuations of the metric.

Since we have not specified the origin of stochasticity in the metric in this work, explicit derivation of such stochasticity from the fundamental model of gravity is desired. The spacetime uncertainty proposed in the context of string theory [47] may have similar effects as discussed in this work on low energy physics. Branes or other solitonic objects that appear in string theory acquire heavy mass in the weak string coupling limit and become another possible source of stochasticity. Along with the possible decoherence associated with a fluctuating metric, the mesoscopic effects treated here may have an observable consequence on future experiments [48,49]. These microscopic origins of metric fluctuations are intrinsically beyond the validity of semiclassical gravity. Therefore the metric fluctuations introduced as a modification of the semiclassical Einstein equation in this paper possibly capture the essential effects of near Planck scale physics on the sub-Planckian scale physics effectively while the self-consistency based only on the conventional semiclassical Einstein equation may not have a predictive power on such a phenomenon. Clarifying the difference between these approaches needs more careful study. The effects of metric fluctuations in cosmological and black hole spacetimes have been considered by many authors, for example, in [50–55]. To identify how the mesoscopic effects discussed in this paper manifest themselves in such curved spacetimes is of particular interest. Studies in these directions are currently in progress.

ACKNOWLEDGMENTS

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APPENDIX A: MESOSCOPIC FLUCTUATIONS OF DIRAC FIELDS

In this appendix we consider the quantization of the Dirac field which obeys the equation of motion in Eq. (14). The action for this system has the following form:

$$S_{v_D}[\psi,\bar{\psi}]=\int d^4x\,\bar{\psi}(x)[i\not{\partial}-m_D-v_D(x)]\psi(x). \quad \text{(A1)}$$

This action is invariant under the global U(1) gauge transformation

$$\begin{aligned}\psi(x)&\to e^{i\eta}\psi(x),\\ \bar{\psi}(x)&\to\bar{\psi}(x)e^{-i\eta},\end{aligned} \quad \text{(A2)}$$

and the corresponding Noether current is

$$J_\mu(x)=i\bar{\psi}\gamma_\mu\psi. \quad \text{(A3)}$$

In the absence of a stochastic field, the closed time path partition function has a form corresponding to Eq. (30):

$$\begin{aligned}Z[J,J^\dagger]=&\int d\psi_f d\bar{\psi}_f D\psi_1 D\bar{\psi}_1 D\psi_2 D\bar{\psi}_2\\ &\times\exp\{i(S[\psi_1,\bar{\psi}_1]-S[\psi_2,\bar{\psi}_2]\\ &+J_1\cdot\psi_1-J_2\cdot\psi_2)\}\rho_D[\psi_{1i},\bar{\psi}_{1i},\psi_{2i},\bar{\psi}_{2i}]\\ =&\exp[-iJ^\dagger SJ],\end{aligned} \quad \text{(A4)}$$

where $J\cdot\psi\equiv\int d^4x[J^\dagger(x)\psi(x)+\psi^\dagger(x)J(x)]$. The matrix Green function S has the form

$$S=\begin{pmatrix}S_{11}&S_{12}\\ S_{21}&S_{22}\end{pmatrix}, \quad \text{(A5)}$$

whose components are

$$\begin{aligned}S_{11}(p)&=-S_{22}^*(p)\\ &=\theta(p_0)S_F(p)+\theta(-p_0)S_F^*(p)\\ &\quad-2\pi i\,\mathrm{sgn}(p_0)n_F(p)\,\delta(p^2-m_D^2),\\ S_{12}(p)&=2\pi i\,\mathrm{sgn}(p_0)n_F(p)(\not{p}+m_D)\,\delta(p^2-m_D^2),\\ S_{21}(p)&=-2\pi i\,\mathrm{sgn}(p_0)e^{\beta(p_0-\mu)}n_F(p)(\not{p}+m_D)\\ &\quad\times\delta(p^2-m_D^2),\end{aligned} \quad \text{(A6)}$$

where $S_F(p)=(\not{p}-m_D+i\epsilon)^{-1}$ is the fermion vacuum Feynman propagator, $n_F(p)\equiv(e^{\beta(p_0-\mu)}+1)^{-1}$ is the Fermi distribution function, and $J^\dagger=(J_1^\dagger,-J_2^\dagger)$. Here S can be diagonalized by multiplying matrices u_F from both sides as $S=u_F S_d u_F^{-1}\eta$, with

$$S_d=\begin{pmatrix} S_R & 0 \\ 0 & S_A \end{pmatrix}, \tag{A7}$$

where S_R and S_A are retarded and advanced Dirac propagators. Here $u_F(p)$ is the thermal Bogoliubov matrix which has the following form:

$$u_F(p)=\sqrt{n_F(p)}e^{\beta(p_0-\mu)/2}\begin{pmatrix} 1 & -e^{-\beta(p_0-\mu)} \\ 1 & 1 \end{pmatrix}. \tag{A8}$$

Using the above property, one can write

$$\psi^\dagger S^{-1}\psi=\psi^\dagger \eta u_F S_d^{-1}u_F^{-1}\psi=\bar{\psi}u_F S_d^{-1}u_F^{-1}\psi, \tag{A9}$$

where $\psi=(\psi_1,\psi_2)$ is the thermal fermion doublet and we define its conjugate $\bar{\psi}\equiv(\psi_1^\dagger,\psi_2^\dagger)\eta$. By changing the field variables by the global Bogoliubov transformations $u_F^{-1}\psi\rightarrow\psi$ and $\bar{\psi}u_F\rightarrow\bar{\psi}$, one can write the partition function in Eq. (A4) without the source term as

$$Z=\int d\psi_f d\bar{\psi}_f D\psi D\bar{\psi}\exp\{iS_0[\psi,\bar{\psi}]\}\rho_D[\psi,\bar{\psi}], \tag{A10}$$

where

$$\begin{aligned} S_0[\psi,\bar{\psi}]&=\bar{\psi}S_d^{-1}\psi \\ &=\int d^3x\frac{d\omega}{2\pi} \\ &\quad\times[\bar{\psi}(x,\omega)(\omega\gamma^0-i\vec{\partial}\cdot\vec{\gamma}-m_D+i\epsilon\eta\gamma_0)\psi(x,\omega)]. \end{aligned} \tag{A11}$$

In the presence of the random variable v_D which is assumed to obey the probability distribution given in Eq. (38), we average the partition function over v_D and obtain the reduced action

$$\begin{aligned} \langle Z[v_D]\rangle&=\int Dv_D P[v_D]D\bar{\psi}D\psi\exp\{iS_{v_D}[\psi,\bar{\psi}]\} \\ &=\int D\bar{\psi}D\psi\exp\{iS_0[\psi,\bar{\psi}]+iS_I[\psi,\bar{\psi}]\}, \end{aligned} \tag{A12}$$

where

$$S_I[\psi,\bar{\psi}]=\frac{i}{2}\int d^4x\,d^4y\;\bar{\psi}(x)\psi(x)\Delta_D(x-y)\bar{\psi}(y)\psi(y). \tag{A13}$$

By introducing the auxiliary bilocal matrix field $\sigma_D(x,y)$ as

$$e^{iS_I[\psi,\bar{\psi}]}=\int D\sigma_D\exp\left[\frac{1}{2}\int d^4x\,d^4y\;\mathrm{Tr}[\sigma_D(x,y)\Delta_D^{-1}(x-y)\sigma_D(y,x)]\right]\exp\{iS_{HS}[\sigma_D,\psi,\bar{\psi}]\}, \tag{A14}$$

where

$$S_{HS}[\sigma_D,\psi,\bar{\psi}]=-\int d^4x\,d^4y\;\bar{\psi}(x)\sigma_D(x,y)\psi(y), \tag{A15}$$

the partition function can be written as

$$Z=\int D\sigma_D D\bar{\psi}D\psi\exp\left[\frac{1}{2}\int\mathrm{Tr}[\sigma_D\Delta_D^{-1}\sigma_D]\right]\exp\{iS_0[\psi,\bar{\psi}]+iS_{HS}[\sigma_D,\psi,\bar{\psi}]\}. \tag{A16}$$

In energy representation, Eqs. (A15) and (A16) have the form

$$S_{HS}[\sigma_D,\psi,\bar{\psi}]=-\int d^3x\,d^3x'\frac{d\omega}{2\pi}\frac{d\omega'}{2\pi}\bar{\psi}(\vec{x},\omega)\sigma_{D\omega\omega'}(\vec{x},\vec{x}')\psi(\vec{x}',\omega') \tag{A17}$$

and

$$\begin{aligned} S_0[\psi,\bar{\psi}]+S_{HS}[\sigma_D,\psi,\bar{\psi}]&=\int d^3x\,d^3x'\frac{d\omega}{2\pi}\frac{d\omega'}{2\pi}\bar{\psi}(\vec{x},\omega)[(\omega\gamma^0-i\vec{\partial}\cdot\vec{\gamma}-m_D+i\epsilon\eta\gamma_0) \\ &\quad\times\delta(\vec{x}-\vec{x}')\delta(\omega-\omega')-\sigma_{D\omega\omega'}(\vec{x},\vec{x}')]\psi(\vec{x}',\omega'). \end{aligned} \tag{A18}$$

After integrating out $\bar{\psi}$ and ψ, we obtain

$$Z=\int D\sigma_D \exp\left[-\frac{1}{2}\int \mathrm{Tr}[\sigma_D \Delta_D^{-1}\sigma_D]\right]$$
$$\times\exp\left[\int d^3x\, d^3x' \frac{d\omega}{2\pi}\frac{d\omega'}{2\pi}\mathrm{Tr}\log\{[\omega\gamma^0 - i\vec{\partial}\cdot\vec{\gamma} - m_D + i\epsilon\eta\gamma_0]\delta(\vec{x}-\vec{x}')\delta(\omega-\omega') - \sigma_{D\omega\omega'}(\vec{x},\vec{x}')\}\right] \tag{A19}$$

and, for the time-independent stochastic field as in Eq. (48),

$$Z=\int D\sigma_D \exp\left[-\frac{1}{2u_D}\int d^3x\, \mathrm{Tr}\,\sigma_D^2(x)\right]\exp\left[\int d^3x\, \mathrm{Tr}\log\{(\omega\gamma^0 - i\vec{\partial}\cdot\vec{\gamma} - m_D + i\epsilon\eta\gamma_0)\delta(\omega-\omega') - \sigma_{D\omega\omega'}(x)\}\right]. \tag{A20}$$

The equation of motion can be obtained from Eq. (A20) by functional derivative with respect to σ_D. Following the steps from Eq. (52) to Eq. (55), the kinetic term in the σ_D field in Eq. (A19) can be given similarly to Eq. (55) which enables us to constuct the effective field theory in terms of the collective field.

By the Kubo formula the conductivity can be written in terms of the Green functions as in Eq. (62):

$$C_\kappa(\vec{x},\vec{y}) \equiv -\frac{1}{4\pi\kappa}\int_0^\infty d\omega\, \Omega_\kappa(\omega)\mathrm{Tr}\{[S_R(\vec{x},\vec{y},\omega) - S_A(\vec{x},\vec{y},\omega)]\vec{\gamma}[S_R(\vec{y},\vec{x},\omega+\kappa) - S_A(\vec{y},\vec{x},\omega+\kappa)]\vec{\gamma}\}. \tag{A21}$$

We obtain

$$C_\kappa(\vec{x},\vec{y}) \equiv \frac{-1}{4\pi\kappa}\int_0^\infty d\omega\ \Omega_\kappa(\omega)\sum_{abcd}\langle \psi^{\dagger a}(\vec{x},\omega)\vec{\gamma}\psi^b(\vec{x},\omega+\kappa)\psi^{\dagger c}(\vec{y},\omega+\kappa)\vec{\gamma}\psi^d(\vec{y},\omega)\rangle. \tag{A22}$$

This expression can be also obtained directly from the partition function by functional derivative after introducing a source term in the form

$$i\int d^3x \frac{d\omega}{2\pi}\frac{d\omega'}{2\pi}\overline{\psi}(\vec{x},\omega)\vec{A}^\kappa\cdot\vec{\gamma}\delta_\kappa\psi(\vec{x},\omega'), \tag{A23}$$

where $\vec{A}^\kappa$ is the external source field and δ_κ was defined in Eq. (66). Then the conductivity is given similarly by Eq. (67). The gauge transformation

$$\psi(\vec{x}) \to e^{-i\vec{x}\cdot\vec{A}^\kappa\delta_\kappa}\psi(\vec{x}), \quad \overline{\psi}(\vec{x}) \to \overline{\psi}(\vec{x})e^{i\vec{x}\cdot\vec{A}^\kappa\delta_\kappa}, \tag{A24}$$

as we saw in Eq. (69), generates the gauge coupling in the effective theory represented by

$$Z=\int D\sigma_D \exp\left[-\frac{1}{2}\int \mathrm{Tr}[\sigma_D\Delta^{-1}\sigma_D]\right]\exp\left[\int d^3x\, d^3y \frac{d\omega}{2\pi}\frac{d\omega'}{2\pi}\right.$$
$$\left.\times\mathrm{Tr}\log\{[(\omega\gamma^0 - i\vec{\partial}\cdot\vec{\gamma} - m_D + i\epsilon\eta\gamma_0)\delta(\omega-\omega') + i\vec{A}^\kappa\cdot\vec{\gamma}\delta_\kappa]\delta(\vec{x}-\vec{y}) - \sigma_{D\omega\omega'}(\vec{x},\vec{y})\}\right]. \tag{A25}$$

This allows us to write the mesoscopic fluctuations in terms of σ_D fields. We can discuss universal fluctuations parallel to the scalar fields in this formalism. For an initially thermal equilibrium state, $\Omega_\kappa(\omega) = \rho_D[\omega] - \rho_D[\omega+\kappa]$, where $\rho_D[\omega] = n_F(\omega)$ is the Fermi distribution, the dc limit $\kappa \to 0$ extracts the Fermi energy ω_F, which reminds us of the electron transport problem.

APPENDIX B: CONDUCTANCE FLUCTUATIONS

$$C^{(1)}=\frac{2\pi^4}{\kappa_1\kappa_2}\frac{1}{(2\pi)^5}\int d\omega\, d\omega'\,\Omega_\kappa(\omega-\kappa)\Omega_\kappa(\omega')N^2(k_0)D^2(k_0)\int d^3P[\langle q_{\omega-\kappa_1\omega'}(\vec{P})q^\dagger_{\omega'\omega-\kappa_1}(-\vec{P})\rangle+\text{H.c.}]$$
$$\times[\langle q_{\omega'+\kappa_2\omega}(\vec{P})q^\dagger_{\omega\omega'+\kappa_2}(-\vec{P})\rangle+\text{H.c.}]. \tag{B1}$$

For $\kappa_1\kappa_2\to 0$,

$$C^{(1)}\to\frac{2^5}{(2\pi)^2}\int_{M_{IR}}^\infty\frac{dp}{p^2}=\frac{8}{\pi^2M_{IR}}, \tag{B2}$$

$$C^{(2)}=\frac{4\pi^5}{\kappa_1\kappa_2}\frac{1}{(2\pi)^5}\int d\omega\, d\omega'\,\Omega_\kappa(\omega-\kappa)\Omega_\kappa(\omega')N^3(k_0)D^3(k_0)\int d^3P\vec{P}^2[\langle q_{\omega-\kappa_1\omega'}(\vec{P})q^\dagger_{\omega'\omega-\kappa_1}(-\vec{P})\rangle$$
$$\times\langle q_{\omega'+\kappa_2\omega}(-\vec{P})q^\dagger_{\omega\omega'+\kappa_2}(\vec{P})\rangle\langle q_{\omega-\kappa_2\omega'+\kappa_1}(\vec{P})q^\dagger_{\omega'+\kappa_1\omega-\kappa_2}(-\vec{P})\rangle+\text{H. c.}]. \tag{B3}$$

For $\kappa_1\kappa_2\to 0$,

$$C^{(2)}\to\frac{2^7}{(2\pi)^2}\int_{M_{IR}}^\infty\frac{dp}{p^2}=\frac{32}{\pi^2M_{IR}}, \tag{B4}$$

$$C^{(3)}=\frac{\pi^6}{\kappa_1\kappa_2}\frac{1}{(2\pi)^5}\int d\omega\, d\omega'\,\Omega_\kappa(\omega-\kappa)\Omega_\kappa(\omega')N^4(k_0)D^4(k_0)$$
$$\times\int d^3P\vec{P}^4[\langle q_{\omega-\kappa_1\omega'}(\vec{P})q^\dagger_{\omega'\omega-\kappa_1}(-\vec{P})\rangle\langle q_{\omega'+\kappa_1\omega}(-\vec{P})q^\dagger_{\omega\omega'+\kappa_1}(\vec{P})\rangle\langle q_{\omega'\omega}(-\vec{P})q^\dagger_{\omega\omega'}(\vec{P})\rangle$$
$$\times\langle q_{\omega-\kappa_1\omega'+\kappa_2}(\vec{P})q^\dagger_{\omega'+\kappa_2\omega-\kappa_1}(-\vec{P})\rangle+\text{H. c.}]+\text{three other terms}. \tag{B5}$$

For $\kappa_1\kappa_2\to 0$,

$$C^{(3)}\to\frac{2^5\times 9}{(2\pi)^2}\int_{M_{IR}}^\infty\frac{dp}{p^2}=\frac{72}{\pi^2M_{IR}}. \tag{B6}$$

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