Modulation effect for supersymmetric dark matter detection with asymmetric velocity dispersion

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The detection of the theoretically expected dark matter is central to particle physics and cosmology. Current fashionable supersymmetric models provide a natural dark matter candidate which is the lightest supersymmetric particle (LSP). Such models combined with fairly well understood physics, such as the quark substructure of the nucleon and the nuclear structure (form factor and/or spin response function), permit the evaluation of the event rate for LSP-nucleus elastic scattering. The thus obtained event rates are, however, very low or even undetectable. So it is imperative to exploit the modulation effect, i.e. the dependence of the event rate on the Earth's annual motion. In this paper we study such a modulation effect both in nondirectional and directional experiments. We calculate both the differential and the total rates using symmetric as well as asymmetric velocity distributions. We find that in the symmetric case the modulation amplitude is small, less than 0.07. The inclusion of asymmetry, with a realistic enhanced velocity dispersion in the galactocentric direction, yields an enhanced modulation effect, with an amplitude which for certain parameters can become as large as 0.46.

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I. INTRODUCTION

In recent years the consideration of exotic dark matter has become necessary in order to close the universe [1,2]. Furthermore in in order to understand the large scale structure of the universe it has become necessary to consider matter made up of particles which were non-relativistic at the time of freeze out. This is the cold dark matter (CDM) component. The Cosmic Background Explorer (COBE) data [3] suggest that CDM is at least 60% [4]. On the other hand, during the last few years evidence has appeared which suggests the presence of a cosmological constant, which may dominate the universe. As a matter of fact recent data from the Supernova Cosmology Project suggest [5,6] that the situation can be adequately described by a baryionic component $\Omega_B=0.1$ along with the exotic components $\Omega_{CDM}=0.3$ and $\Omega_\Lambda=0.6$. In another analysis Turner [7] gives $\Omega_m=\Omega_{CDM}+\Omega_B=0.4$. Since the nonexotic component cannot exceed 40% of the CDM [2,8], there is room for exotic WIMP's (weakly interacting massive particles). In fact the DAMA experiment [9] has claimed the observation of one signal in direct detection of a WIMP, which with better statistics has subsequently been interpreted as a modulation signal [10].

The above developments are in line with particle physics considerations. Thus, in the currently favored supersymmetric (SUSY) extensions of the standard model, the most natural WIMP candidate is the lightest supersymmetric particle (LSP). In the most favored scenarios the LSP can be simply described as a Majorana fermion, a linear combination of the neutral components of the gauginos and Higgsinos [2,11–20].

Since this particle is expected to be very massive, $m_\chi \geqslant 30\,\mathrm{GeV}$, and extremely nonrelativistic with average kinetic energy $T\leqslant 100\,\mathrm{keV}$, it can be directly detected [11,12] mainly via the recoiling of a nucleus (A,Z) in the elastic scattering process:

$$\chi+(A,Z)\to\chi+(A,Z)^* \tag{1}$$

(χ denotes the LSP). In order to compute the event rate one needs the following ingredients:

(1) An effective Lagrangian at the elementary particle (quark) level obtained in the framework of supersymmetry as described in Refs. [2], Bottino *et al.* [17] and [20].

(2) A procedure in going from the quark to the nucleon level, i.e. a quark model for the nucleon. The results depend crucially on the content of the nucleon in quarks other than u and d. This is particularly true for the scalar couplings as well as the isoscalar axial coupling [13,21,22].

(3) Compute the relevant nuclear matrix elements [23–26] using as reliable as possible many body nuclear wave functions. By putting as accurate nuclear physics input as possible, one will be able to constrain the SUSY parameters as much as possible. The situation is a bit simpler in the case of the scalar coupling, in which case one only needs the nuclear form factor.

Since the obtained rates are very low, one would like to be able to exploit the modulation of the event rates due to the Earth's revolution around the Sun. To this end one adopts a folding procedure assuming some distribution [2,28] of velocities for the LSP.

The purpose of our present review is to focus on this last point along the lines suggested by our recent Letter [15]. We will expand our previous results and give some of the missing calculational details. For the reader's convenience, however, we will give a very brief description of the basic ingredients on how to calculate LSP-nucleus scattering cross section. We will not, however, elaborate on how one gets the needed parameters from supersymmetry. The calculation of these parameters has become pretty standard. One starts with representative input in the restricted SUSY parameter space

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as described in the literature, e.g. Bottino *et al.* [17] and Kane *et al.*, Castano *et al.* and Arnowitt and co-workers [18].

After this we will specialize our study to the case of the nucleus ^{127}I, which is one of the most popular targets [9,29,30]. To this end we will consider both a symmetric Maxwell-Boltzmann distribution [2] as well as asymmetric distributions like the one suggested by Drukier *et al.* [28]. We will, of course, include an appropriate nuclear form factor. We will examine the modulation effect in the directional as well as the non directional experiments, both in the differential as well as the total event rates. We will present our results as a function of the LSP mass, m_χ, for various detector energy thresholds, in a way which can be easily understood by the experimentalists.

II. BASIC INGREDIENTS FOR LSP NUCLEUS SCATTERING

Because of lack of space, we are not going to elaborate here further on the construction of the effective Lagrangian derived from supersymmetry, but refer the reader to the literature [11,12,14,17,32]. The effective Lagrangian can be obtained in first order via Higgs boson exchange, *s*-quark exchange and *Z* exchange. In a formalism familiar from the theory of weak interactions we write

$$L_{eff}=-\frac{G_F}{\sqrt{2}}\{(\bar{\chi}_1\gamma^\lambda\gamma_5\chi_1)J_\lambda+(\bar{\chi}_1\chi_1)J\} \tag{2}$$

where

$$J_\lambda=\bar{N}\gamma_\lambda(f_V^0+f_V^1\tau_3+f_A^0\gamma_5+f_A^1\gamma_5\tau_3)N \tag{3}$$

and

$$J=\bar{N}(f_s^0+f_s^1\tau_3)N. \tag{4}$$

We have neglected the uninteresting pseudoscalar and tensor currents. Note that, as a result of the Majorana nature of the LSP, $\bar{\chi}_1\gamma^\lambda\chi_1=0$ (identically). The parameters $f_V^0, f_V^1, f_A^0, f_A^1, f_S^0, f_S^1$ depend on the SUSY model employed. In SUSY models derived from minimal supergravity (SUGRA) the allowed parameter space is characterized at the grand unified theory (GUT) scale by five parameters, two universal mass parameters, one for the scalars, m_0, and one for the fermions, $m_{1/2}$, as well as the parameters $\tan\beta$, one of A_0 (or m_t^{pole}) and the sign of μ [18]. Deviations from universality at the GUT scale have also been considered and found to be useful [19]. We will not elaborate further on this point since the above parameters involving universal masses have already been computed in some models [11,32] and effects resulting from deviations from universality will be found elsewhere (see Nath and Arnowitt in Ref. [19] and Bottino *et al.* in Ref. [17]). For some choices in the allowed parameter space the obtained couplings can be found in a previous paper [32].

The differential cross section can be cast in the form [16]

$$d\sigma(u,\upsilon)=\frac{du}{2(\mu_r b\upsilon)^2}\left[\left(\bar{\Sigma}_S+\bar{\Sigma}_V\frac{\upsilon^2}{c^2}F^2(u)\right)+\bar{\Sigma}_{spin}F_{11}\right] \tag{5}$$

with

$$\bar{\Sigma}_S=\sigma_0\left(\frac{\mu_r}{m_N}\right)^2\left\{A^2\left[\left(f_S^0-f_S^1\frac{A-2Z}{A}\right)^2\right]\right\} \tag{6}$$

$$\bar{\Sigma}_{spin}=\sigma_0\left(\frac{\mu_r}{m_N}\right)^2\left[[f_A^0\Omega_0(0)]^2\frac{F_{00}(u)}{F_{11}(u)}+2f_A^0f_A^1\Omega_0(0)\Omega_1(0)\frac{F_{01}(u)}{F_{11}(u)}+[f_A^1\Omega_1(0)]^2\right] \tag{7}$$

$$\bar{\Sigma}_V=\sigma_0\left(\frac{\mu_r}{m_N}\right)^2A^2\left(f_V^0-f_V^1\frac{A-2Z}{A}\right)^2\times\left[1-\frac{1}{(2\mu_r b)^2}\frac{2\eta+1}{(1+\eta)^2}\frac{\langle 2u\rangle}{\langle\upsilon^2\rangle}\right] \tag{8}$$

where m_N is the proton mass, $\eta=m_x/m_NA$, μ_r is the reduced mass and

$$\sigma_0=\frac{1}{2\pi}(G_Fm_N)^2\simeq 0.77\times 10^{-38}\,\mathrm{cm}^2. \tag{9}$$

We should remark that even though the quantity $\bar{\Sigma}_{spin}$ can be a function of *u*, in actual practice it is independent of *u*. The same is true of the less important term $\bar{\Sigma}_V$. In the above expressions $F(u)$ is the nuclear form factor and

$$F_{\rho\rho'}(u)=\sum_{\lambda,\kappa}\frac{\Omega_\rho^{(\lambda,\kappa)}(u)}{\Omega_\rho(0)}\frac{\Omega_{\rho'}^{(\lambda,\kappa)}(u)}{\Omega_{\rho'}(0)},\quad \rho,\rho'=0,1, \tag{10}$$

are the spin form factors [16] (ρ,ρ' are isospin indices) and

$$u=q^2b^2/2, \tag{11}$$

b being the harmonic oscillator size parameter and *q* the momentum transfer to the nucleus. The quantity *u* is also related to the experimentally measurable energy transfer *Q* via the relations

$$Q=Q_0u,\quad Q_0=\frac{1}{Am_Nb^2}. \tag{12}$$

The detection rate for a particle with velocity $\boldsymbol{\upsilon}$ and a target with mass *m* detecting in the direction $\mathbf{e}$ will be denoted by $R(\rightarrow\mathbf{e})$. Then one defines the undirectional rate R_{undir} via the equations

$$R_{undir}=\frac{dN}{dt}=\frac{\rho(0)}{m_\chi}\frac{m}{Am_N}\sigma(u,\upsilon)[|\upsilon\cdot\hat{e}_x|+|\upsilon\cdot\hat{e}_y|+|\upsilon\cdot\hat{e}_z|]. \tag{13}$$

$\rho(0)=0.3\,\mathrm{GeV/cm}^3$ is the LSP density in our vicinity. This density has to be consistent with the LSP velocity distribution (see next section).

The differential undirectional rate can be written as

$$dR_{undir}=\frac{\rho(0)}{m_\chi}\frac{m}{Am_N}d\sigma(u,\upsilon)[|\upsilon\cdot\hat{e}_x|+|\upsilon\cdot\hat{e}_y|+|\upsilon\cdot\hat{e}_z|] \tag{14}$$

where $d\sigma(u,\upsilon)$ is given by Eq. (5)

The directional rate in the direction $\hat{e}$ is defined by

$$R_{dir}=R(\rightarrow e)-R(\rightarrow -e)=\frac{\rho(0)}{m_\chi}\frac{m}{Am_N}\upsilon\cdot e\sigma(u,\upsilon) \tag{15}$$

and the corresponding differential rate is given by

$$dR_{dir}=\frac{\rho(0)}{m_\chi}\frac{m}{Am_N}\upsilon\cdot e d\sigma(u,\upsilon). \tag{16}$$

III. CONVOLUTION OF THE EVENT RATE

We have seen that the event rate for LSP-nucleus scattering depends on the relative LSP-target velocity. In this section we will examine the consequences of the Earth's revolution around the Sun (the effect of its rotation around its axis is expected to be negligible), i.e. the modulation effect. In practice this has been accomplished by assuming a consistent LSP velocity dispersion, such as a Maxwell distribution [2]. More recently other non-isothermal approaches, in the context velocity peaks and caustic rings, have been proposed; see e.g Sikivie *et al.* [27]. Investigation of the modulation effect in such models is underway, but it is not the subject of this work. In the present paper following the work of Drukier *et al.* (see Ref. [28]), we will assume that the velocity distribution is only axially symmetric, i.e. of the form

$$f(\upsilon',\lambda)=N(y_{esc},\lambda)(\sqrt{\pi}\upsilon_0)^{-3}[f_1(\upsilon',\lambda)-f_2(\upsilon',\upsilon_{esc},\lambda)] \tag{17}$$

with

$$f_1(\upsilon',\lambda)=\exp\left[-\frac{(\upsilon_x')^2+(1+\lambda)[(\upsilon_y')^2+(\upsilon_z')^2]}{\upsilon_0^2}\right] \tag{18}$$

$$f_2(\upsilon',\upsilon_{esc},\lambda)=\exp\left[-\frac{\upsilon_{esc}^2+\lambda[(\upsilon_y')^2+(\upsilon_z')^2]}{\upsilon_0^2}\right] \tag{19}$$

where

$$\upsilon_0=\sqrt{(2/3)\langle\upsilon^2\rangle}=220\,\mathrm{km/s}; \tag{20}$$

i.e., υ_0 is the velocity of the Sun around the center of the Galaxy. υ_{esc} is the escape velocity in the gravitational field of the Galaxy, $\upsilon_{esc}=625\,\mathrm{km/s}$ [28]. In the above expressions λ is a parameter, which describes the asymmetry and takes values between 0 and 1 and N is a proper normalization constant [16]. For $y_{esc}\rightarrow\infty$ we get the simple expression $N^{-1}=\lambda+1$.

The z axis is chosen in the direction of the disk's rotation, i.e. in the direction of the motion of the Sun, the y axis is perpendicular to the plane of the galaxy and the x axis is in the radial direction. Since the axis of the ecliptic [12] lies very close to the y,z plane, the velocity of the Earth around the Sun is given by

$$\upsilon_E=\upsilon_0+\upsilon_1=\upsilon_0+\upsilon_1(\sin\alpha\hat{\mathbf{x}}-\cos\alpha\cos\gamma\hat{\mathbf{y}}+\cos\alpha\sin\gamma\hat{\mathbf{z}}) \tag{21}$$

where α is the phase of the earth's orbital motion, $\alpha=2\pi(t-t_1)/T_E$, where t_1 is around the second of June and $T_E=1$ yr.

One can now express the above distribution in the laboratory frame [16] by writing $\upsilon'=\upsilon+\upsilon_E$.

IV. EXPRESSIONS FOR THE DIFFERENTIAL EVENT RATE IN THE PRESENCE OF VELOCITY DISPERSION

We will begin with the undirectional rate.

A. Expressions for the undirectional differential event rate

The mean value of the undirectional event rate of Eq. (15) is given by

$$\left\langle\frac{dR_{undir}}{du}\right\rangle=\frac{\rho(0)}{m_\chi}\frac{m}{Am_N}\int f(\upsilon,\upsilon_E)[|\upsilon\cdot\hat{e}_x|+|\upsilon\cdot\hat{e}_y|+|\upsilon\cdot\hat{e}_z|]\frac{d\sigma(u,\upsilon)}{du}d^3\upsilon. \tag{22}$$

From now on we will omit the subscript $undir$ in the case of the undirectional rate. The above expression can be more conveniently written as

$$\left\langle\frac{dR}{du}\right\rangle=\frac{\rho(0)}{m_\chi}\frac{m}{Am_N}\sqrt{\langle\upsilon^2\rangle}\left\langle\frac{d\Sigma}{du}\right\rangle \tag{23}$$

where

$$\left\langle\frac{d\Sigma}{du}\right\rangle=\int\frac{[\|\upsilon\cdot\hat{e}_x\|+\|\upsilon\cdot\hat{e}_y\|+\|\upsilon\cdot\hat{e}_z\|]}{\sqrt{\langle\upsilon^2\rangle}}\times f(\upsilon,\upsilon_E)\frac{d\sigma(u,\upsilon)}{du}d^3\upsilon. \tag{24}$$

Introducing the parameter

$$\delta=\frac{2\upsilon_1}{\upsilon_0}=0.27, \tag{25}$$

expanding in powers of δ and keeping terms up to linear in it we can manage to perform the angular integrations [16] and get

$$\left\langle \frac{d\Sigma}{du} \right\rangle = \bar{\Sigma}_S \bar{F}_0(u) + \frac{\langle v^2 \rangle}{c^2} \bar{\Sigma}_V \bar{F}_1(u) + \bar{\Sigma}_{spin} \bar{F}_{spin}(u) \quad (26)$$

where the $\bar{\Sigma}_i$, $i=S,V,spin$, are given by Eqs. (6)–(8).

The quantities $\bar{F}_0,\bar{F}_1,\bar{F}_{spin}$ are obtained from the corresponding form factors via the equations

$$\bar{F}_k(u) = F^2(u) \bar{\Psi}_k(u) \frac{(1+k)a^2}{2k+1}, \quad (27)$$

$$k=0,1, \quad (28)$$

$$\bar{F}_{spin}(u) = F_{11}(u) \Psi_0(u) \quad (29)$$

$$\tilde{\Psi}_k(u) = [\tilde{\psi}_{(0),k}(a\sqrt{u}) + 0.135 \cos\alpha \tilde{\psi}_{(1),k}(a\sqrt{u})] \quad (30)$$

with

$$a = \frac{1}{\sqrt{2}\mu_r b v_0} \quad (31)$$

and

$$\tilde{\psi}_{(l),k}(x) = N(y_{esc},\lambda) e^{-\lambda} \{ e^{-1} \tilde{\Phi}_{(l),k}(x) - \exp[-y_{esc}^2] \tilde{\Phi}'_{(l),k}(x) \} \quad (32)$$

$$\tilde{\Phi}_{(l),k}(x) = \frac{2}{\sqrt{6\pi}} \int_x^{y_{esc}} dy y^{2k-1} \exp[-(1+\lambda)y^2] \times [\tilde{F}_l(\lambda,(\lambda+1)2y) + \tilde{G}_l(\lambda,y)] \quad (33)$$

$$\tilde{\Phi}'_{(l),k}(x) = \frac{2}{\sqrt{6\pi}} \int_x^{y_{esc}} dy y^{2k-1} \times \exp(-\lambda y^2) \tilde{G}'_l(\lambda,y). \quad (34)$$

In the above expressions,

$$\tilde{G}_0(0,y)=0, \quad \tilde{G}_1(0,y)=0 \quad (35)$$

$$\tilde{F}_0(\lambda,x) = (\lambda+1)^{-2}[x\sinh(x) - \cosh(x) + 1 + xI_1(x)] \quad (36)$$

$$\tilde{F}_1(\lambda,x) = (1+\lambda)^{-2}[(2+\lambda)(\{x^2/[2(2+\lambda)]+1\}\cosh(x) - x\sinh(x) - 1) + x^2 I_2[x] - (\lambda+1)xI_1(x)]. \quad (37)$$

Note that here $x=(\lambda+1)2y$. $I_m(x)$ is the modified Bessel function of order m. The functions $\tilde{G}$ cannot be obtained analytically, but they can easily be expressed as a rapidly convergent series in $y=v/v_0$, which will not be given here.

The functions $\tilde{G}'_i(\lambda,y)$, associated with the small second term of the velocity distribution, are obtained similarly [16].

The undirectional differential rate takes the form

$$\left\langle \frac{dR}{du} \right\rangle = \bar{R} t T(u)[1+\cos\alpha H(u)]. \quad (38)$$

In the above expressions $\bar{R}$ is the rate obtained in the conventional approach [11] by neglecting the folding with the LSP velocity and the momentum transfer dependence of the differential cross section, i.e. by

$$\bar{R} = \frac{\rho(0)}{m_\chi} \frac{m}{Am_N} \sqrt{\langle v^2 \rangle} \left[\bar{\Sigma}_S + \bar{\Sigma}_{spin} + \frac{\langle v^2 \rangle}{c^2} \bar{\Sigma}_V \right] \quad (39)$$

where $\bar{\Sigma}_i$, $i=S,V,spin$, have been defined above; see Eqs. (6)–(8).

The factor $T(u)$ takes care of the u dependence of the unmodulated differential rate. It is defined so that

$$\int_{u_{min}}^{u_{max}} du T(u) = 1; \quad (40)$$

i.e., it is the relative differential rate. u_{min} is determined by the energy cutoff due to the performance of the detector. u_{max} is determined by the escape velocity v_{esc} via the relation

$$u_{max} = \frac{y_{esc}^2}{a^2}. \quad (41)$$

On the other hand, $H(u)$ gives the energy transfer dependent modulation amplitude. The quantity t takes care of the modification of the total rate due to the nuclear form factor and the folding with the LSP velocity distribution. Since the functions $\bar{F}_0(u),\bar{F}_1$ and $\bar{F}_{spin}$ have a different dependence on u, the functions $T(u),H(u)$ and t depend on the SUSY parameters. If, however, we ignore the small vector contribution and assume that (i) the scalar and axial (spin) dependence on u is the same, as seems to be the case for light systems [33,34], or (ii) only one mechanism (S, V, *spin*) dominates, the parameter $\bar{R}$ contains the dependence on all SUSY parameters. The other factors depend only on the LSP mass and the nuclear parameters. More specifically considering only the scalar interaction we get $\bar{R} \to \bar{R}_S$ and

$$tT(u) = a^2 F^2(u) \tilde{\psi}_{(0),0}(a\sqrt{u}). \quad (42)$$

For the spin interaction we get a similar expression except that $\bar{R} \to \bar{R}_{spin}$ and $F^2 \to F_{11}$. Finally for completeness we will consider the less important vector contribution. We get $\bar{R} \to \bar{R}_V$ and

$$tT(u)=F^2(u)\left[\tilde{\psi}_{(0),1}(a\sqrt{u}) - \frac{1}{(2\mu_r b)^2}\frac{2\eta+1}{(1+\eta)^2}u\tilde{\psi}_{(0),0}(a\sqrt{u})\right]\frac{2a^2}{3}. \quad (43)$$

The quantity $T(u)$ depends on the nucleus through the nuclear form factor or the spin response function and the parameter a. The modulation amplitude takes the form

$$H(u)=0.135\frac{\tilde{\psi}_{(1),k}(a\sqrt{u})}{\tilde{\psi}_{(0),k}(a\sqrt{u})}, \quad l=1,3. \quad (44)$$

Thus in this case $H(u)$ depends only on $a\sqrt{u}$, which coincides with the parameter x of Ref. [31], i.e. only on the momentum transfer, the reduced mass and the size of the nucleus.

Returning to the differential rate it is sometimes convenient to use the quantity $T(u)H(u)$ rather than H, since $H(u)$ may appear artificially increasing function of u due to the faster decrease of $T(u)$ [$H(u)$ was obtained after division by $T(u)$].

B. Expressions for the directional differential event rate

The mean value of the directional differential event rate of Eq. (16) is defined by

$$\left\langle\frac{dR}{du}\right\rangle_{dir}=\frac{\rho(0)}{m_\chi}\frac{m}{Am_N}\int f(\upsilon,\upsilon_E)\upsilon\cdot e\frac{d\sigma(u,\upsilon)}{du}d^3\upsilon \quad (45)$$

where $\hat{\mathbf{e}}$ is the unit vector in the direction of observation. It can be more conveniently expressed as

$$\left\langle\frac{dR}{du}\right\rangle_{dir}=\frac{\rho(0)}{m_\chi}\frac{m}{Am_N}\sqrt{\langle\upsilon^2\rangle}\left\langle\frac{d\Sigma}{du}\right\rangle_{dir} \quad (46)$$

where

$$\left\langle\frac{d\Sigma}{du}\right\rangle_{dir}=\int\frac{\upsilon\cdot e}{\sqrt{\langle\upsilon^2\rangle}}f(\upsilon,\upsilon_E)\frac{d\sigma(u,\upsilon)}{du}d^3\upsilon. \quad (47)$$

Working as in the previous subsection we get [16]

$$\left\langle\frac{d\Sigma}{du}\right\rangle_{dir}=\left[\bar{\Sigma}_S F_0(u)+\frac{\langle\upsilon^2\rangle}{c^2}\bar{\Sigma}_V F_1(u)+\bar{\Sigma}_{spin}F_{spin}(u)\right] \quad (48)$$

where the $\bar{\Sigma}_i, i=S,V,spin$ are given by Eqs. (6)–(8). The quantities F_0,F_1,F_{spin} are obtained from the equations

$$F_k(u)=F^2(u)\Psi_k(u)\frac{(1+k)a^2}{2k+1}, \quad k=0,1, \quad (49)$$

$$F_{spin}(u)=F_{11}(u)\Psi_0(u) \quad (50)$$

$$\Psi_k(u)=\frac{1}{2}\{[\psi_{(0),k}(a\sqrt{u})+0.135\times\cos\alpha\psi_{(1),k}(a\sqrt{u})]\mathbf{e}_z\cdot\mathbf{e}-0.117\times\cos\alpha\psi_{(2),k}(a\sqrt{u})\mathbf{e}_y\cdot\mathbf{e}+0.135\times\sin\alpha\psi_{(3),k}(a\sqrt{u})\mathbf{e}_x\cdot\mathbf{e}\} \quad (51)$$

with

$$\psi_{(l),k}(x)=N(y_{esc},\lambda)e^{-\lambda}\{e^{-1}\Phi_{(l),k}(x)-\exp[-y_{esc}^2]\Phi'_{(l),k}(x)\} \quad (52)$$

$$\Phi_{(l),k}(x)=\frac{2}{\sqrt{6\pi}}\int_x^{y_{esc}}dy y^{2k-1}\exp[-(1+\lambda)y^2]\times[F_l(\lambda,2(\lambda+1)y)+G_l(\lambda,y)] \quad (53)$$

$$\Phi'_{(l),k}(x)=\frac{2}{\sqrt{6\pi}}\int_x^{y_{esc}}dy y^{2k-1}\times\exp(-\lambda y^2)G'_l(\lambda,y). \quad (54)$$

In the above expressions,

$$F_i(\lambda,\chi)=\chi\cosh\chi-\sinh\chi, \quad i=0,2,3, \quad (55)$$

$$F_1(\lambda,\chi)=2(1-\lambda)\left[\left(\frac{(\lambda+1)\chi^2}{4(1-\lambda)}+1\right)\sinh\chi-\chi\cosh\chi\right]. \quad (56)$$

TABLE I. The dependence of the modulation amplitude h on the velocity of the Earth in the symmetric case ($\lambda=0$) and $Q_{min}=0$.

	LSP mass (GeV)						
Velocity	10	30	50	80	100	125	250
z comonent	0.0453	0.0320	0.0179	0.0075	0.0041	0.0015	-0.0033
All	0.0723	0.0558	0.0383	0.0252	0.0208	0.0173	0.0112

The purely asymmetric quantities G_i satisfy

$$G_i(0,y)=0, \quad i=0,1,2,3. \tag{57}$$

The qualities $G_i'(0,y)=0$, $i=0,1,2,3$, refer to the second term of the velocity distribution and were obtained in an analogous fashion.

If we consider each mode (scalar, spin vector) separately the directional rate takes the form

$$\left\langle \frac{dR}{du}\right\rangle_{dir}=\frac{\bar{R}}{2}t^0R^0[(1+\cos\alpha H_1(u))\mathbf{e}_z\cdot\mathbf{e}-\cos\alpha H_2(u)\mathbf{e}_y\cdot\mathbf{e} + \sin\alpha H_3(u)\mathbf{e}_x\cdot\mathbf{e}]. \tag{58}$$

In other words the directional differential modulated amplitude is described in terms of the three parameters, $H_l(u)$, $l=1$, 2 and 3. The unmodulated amplitude $R^0(u)$ is again normalized to unity. The parameter t^0 entering Eq. (58) takes care of whatever modifications are needed due to the convolution with the LSP velocity distribution in the presence of the nuclear form factors.

From Eqs. (48)–(58) we see that if we consider each mode separately, the differential modulation amplitudes $H_l(u)$ take the form

$$H_l(u)=0.135\frac{\psi_k^{(l)}(a\sqrt{u})}{\psi_k^{(0)}(a\sqrt{u})}, \quad l=1,3,$$

$$H_2(u)=0.117\frac{\psi_k^{(2)}(a\sqrt{u})}{\psi_k^{(0)}(a\sqrt{u})}. \tag{59}$$

Thus in this case the H_l depend only on $a\sqrt{u}$, which coincides with the parameter x of Ref. [29]. This means that H_l essentially depend only on the momentum transfer, the reduced mass and the size of the nucleus. We note that in the case $\lambda=0$ we have $H_2=0.117$ and $H_3=0.135$.

TABLE II. The quantities t and h for $\lambda=0$ in the case of the target ${}_{53}I^{127}$ for various LSP masses and Q_{min} in keV (for definitions see text). Only the scalar contribution is considered.

		LSP mass (GeV)						
Quantity	Q_{min}	10	30	50	80	100	125	250
t	0.0	1.599	1.134	0.765	0.491	0.399	0.328	0.198
h	0.0	0.072	0.056	0.038	0.025	0.021	0.017	0.011
t	10.	0.000	0.276	0.307	0.236	0.200	0.170	0.108
h	10.	0.000	0.055	0.028	0.014	0.010	0.007	0.001
t	20.	0.000	0.058	0.117	0.110	0.098	0.086	0.058
h	20.	0.000	0.084	0.044	0.024	0.017	0.013	0.005

TABLE III. The same quantities with Table I for $\lambda=0.5$.

		LSP mass (GeV)						
Quantity	Q_{min}	10	30	50	80	100	125	250
t	0.0	1.690	1.241	0.861	0.558	0.453	0.372	0.224
h	0.0	0.198	0.151	0.107	0.083	0.076	0.071	0.063
t	10.	0.000	0.267	0.337	0.268	0.229	0.194	0.122
h	10.	0.000	0.344	0.175	0.113	0.097	0.087	0.072
h	20.	0.000	0.000	0.121	0.123	0.111	0.098	0.066
h	20.	0.000	0.000	0.267	0.150	0.124	0.106	0.081

It is sometimes convenient to use the quantity R_l rather than H_l defined by

$$R_l=R^0H_l, \quad l=1,2,3. \tag{60}$$

The reason is that H_l, being the ratio of two quantities, may appear superficially large due to the denominator becoming small.

Once again, if one mechanism dominates, the parameters R_0 and R_l are independent of the particular SUSY model considered, except the LSP mass. In fact we find for the scalar interaction we get $\bar{R}\rightarrow\bar{R}_S$ and

$$t^0R^0(u)=a^2F^2(u)\psi_0^{(0)}(a\sqrt{u}). \tag{61}$$

For the spin interaction we get a similar expression except that $\bar{R}\rightarrow\bar{R}_{spin}$ and $F^2\rightarrow F_{\rho,\rho'}$. Finally for completeness we will consider the less important vector contribution. We get $\bar{R}\rightarrow\bar{R}_V$ and

$$t^0R^0(u)=F^2(u)\left[\psi_1^{(0)}(a\sqrt{u}) - \frac{1}{(2\mu_rb)^2}\frac{2\eta+1}{(1+\eta)^2}u\psi_0^{(0)}(a\sqrt{u})\right]\frac{2a^2}{3}. \tag{62}$$

TABLE IV. The same quantities with Table I for $\lambda=1.0$.

		LSP mass (GeV)						
Quantity	Q_{min}	10	30	50	80	100	125	250
t	0.0	1.729	1.299	0.919	0.600	0.487	0.399	0.240
h	0.0	0.314	0.247	0.181	0.141	0.131	0.123	0.112
t	10.	0.000	0.252	0.353	0.289	0.247	0.209	0.132
h	10.	0.000	0.579	0.291	0.187	0.163	0.147	0.124
t	20.	0.000	0.000	0.120	0.131	0.120	0.106	0.071
h	20.	0.000	0.000	0.455	0.249	0.205	0.177	0.137

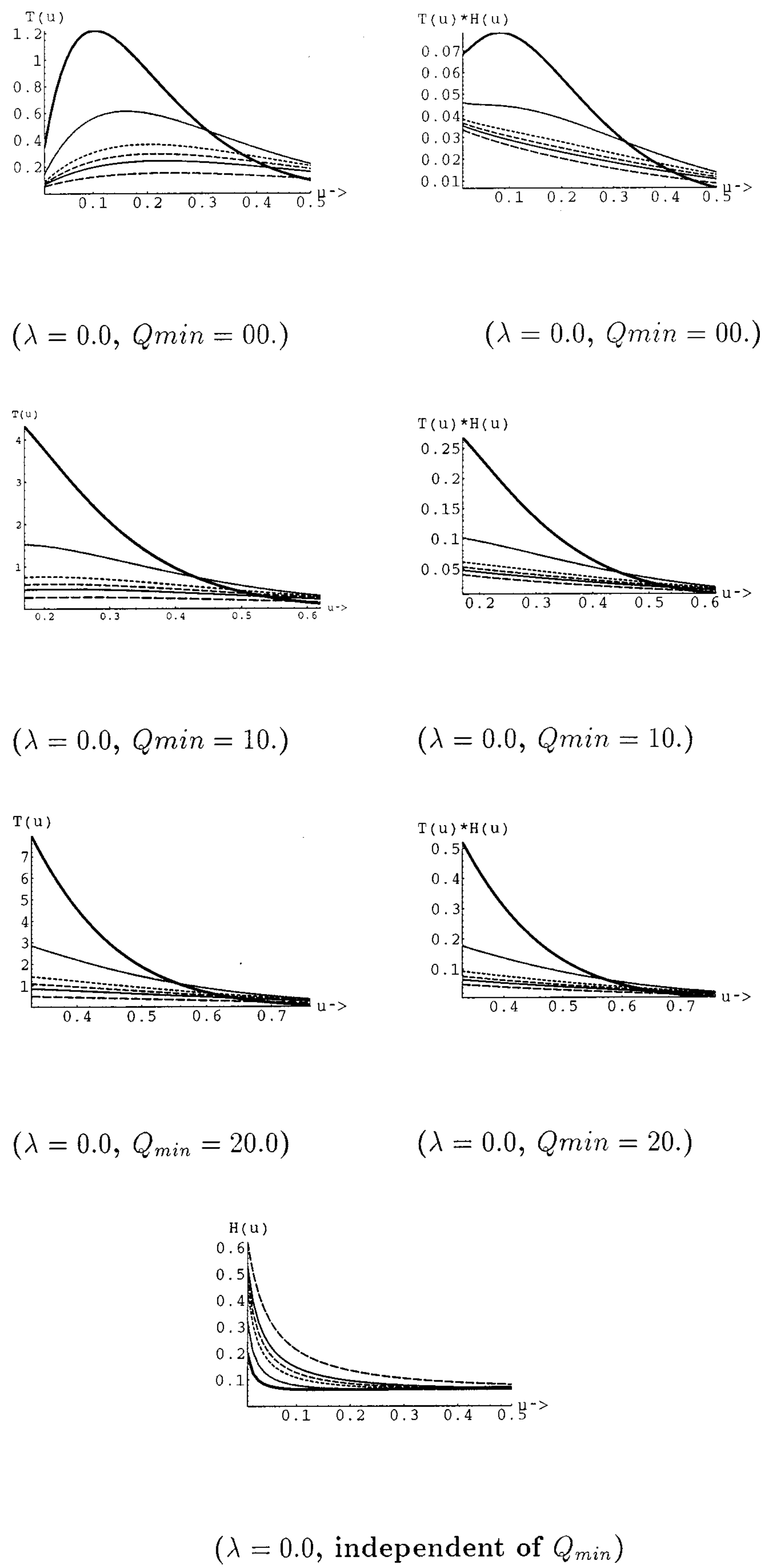
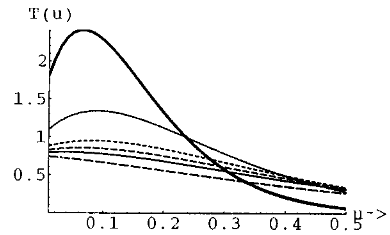
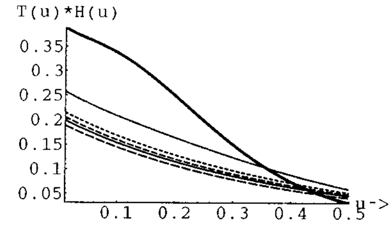
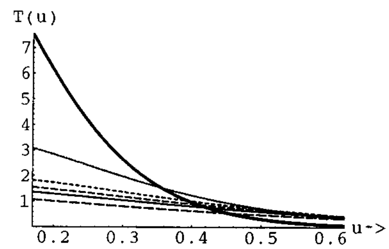


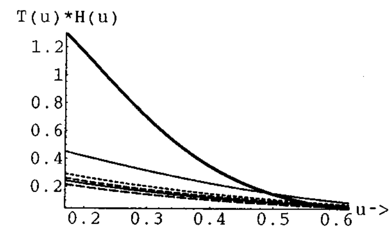
FIG. 1. The quantities T,H and TH entering the undirectional differential rate for $\lambda = 0.0$ and various values of energy cut off in keV. For definitions see text. The energy transfer Q is given by $Q = uQ_0$, $Q_0 = 60$ keV.



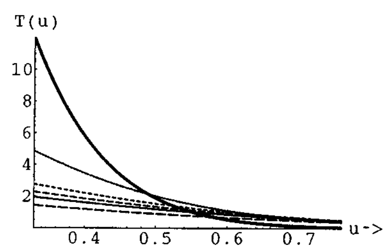


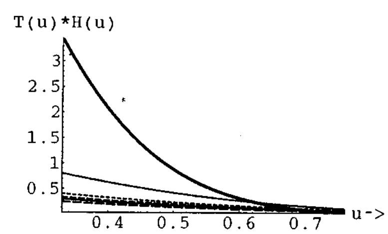
$(\lambda = 0.5,\ Qmin = 00.)$ $(\lambda = 0.5,\ Qmin = 00.)$



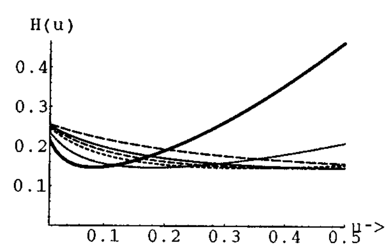


$(\lambda = 0.5,\ Qmin = 10.)$ $(\lambda = 0.5,\ Qmin = 10.)$



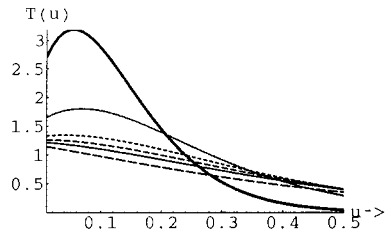


$(\lambda = 0.5,\ Q_{min} = 20.0)$ $(\lambda = 0.5,\ Qmin = 20.)$

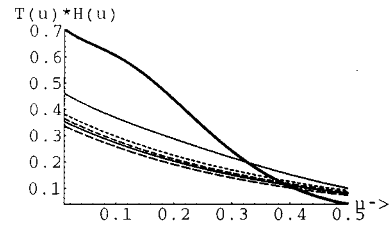


$(\lambda = 0.5,\ \textbf{independent of}\ Q_{min})$

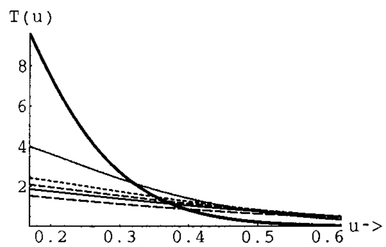
FIG. 2. The same as in Fig. 1 for $\lambda = 0.5$.



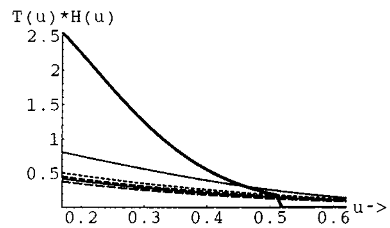
($\lambda = 1.0,\ Qmin = 00.$)



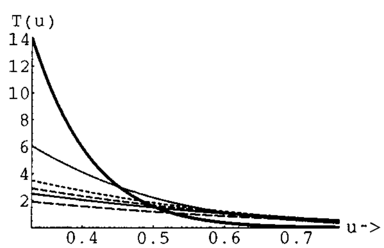
($\lambda = 1.0,\ Qmin = 00.$)



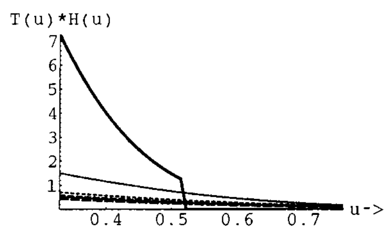
($\lambda = 1.0,\ Qmin = 10.$)



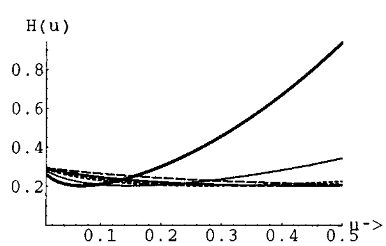
($\lambda = 1.0,\ Qmin = 10.$)



($\lambda = 1.0,\ Q_{min} = 20.0$)

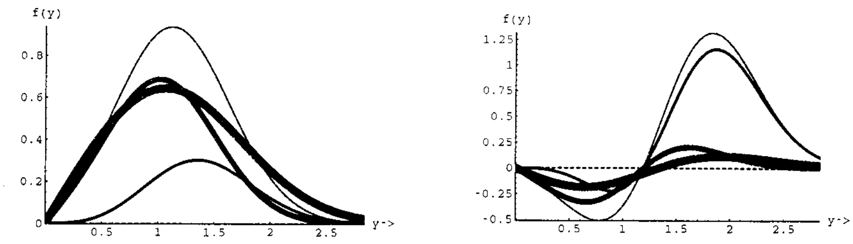


($\lambda = 1.0,\ Qmin = 20.$)



($\lambda = 1.0$, **independent of** Q_{min})

FIG. 3. The same as in Fig. 1 for $\lambda = 1.0$.



$(\ f(y) \leftrightarrow \tilde{F}_0, \tilde{G}_0,\ \tilde{F}_0 + \tilde{G}_0\ )$ $(\ f(y) \leftrightarrow \tilde{F}_1, \tilde{G}_1,\ \tilde{F}_1 + \tilde{G}_1\ )$

FIG. 4. The quantities $f(y)$ described in the text associated with $\tilde{F}_i(\lambda,2(\lambda+1)y), \tilde{G}_i(\lambda,y)$ and $\tilde{F}_i(\lambda,2(\lambda+1)y)+\tilde{G}_i(\lambda,y)$, $i=0,1$. The intermediate thickness solid line corresponds to $\tilde{F}_i$, $i=0,1$, the fine line to $\tilde{G}_i$, $i=0,1$, and the dashed line to the sum of the two, all drawn for $\lambda=1.0$. The thickest line corresponds to $\lambda=0$, in which case $\tilde{G}_i=0$, $i=0,1$.

The quantity R_0 depends on the nucleus through the form factor or the spin response function.

From Eq. (58) one finds for all directions that

$$\left\langle \frac{dR}{du} \right\rangle_{dir,all} = \frac{\bar{R}}{2} t^0 R^0(u) \{[1+H_m(u)]\cos[\alpha-\alpha_H(u)]\}$$

$$H_m = [(H_1+H_2)^2+H_3^2]^{1/2} - \mathrm{Min}(H_1-H_2,H_3),$$

$$\alpha_H = \tan^{-1}\left[\frac{H_3(u)}{H_1(u)+H_2(u)}\right]. \tag{63}$$

V. TOTAL MODULATED EVENT RATES

We will distinguish two possibilities, namely the directional and the nondirectional case. Integrating Eq. (58) we obtain for the total undirectional rate

$$R = \bar{R}t[(1+h(a,Q_{min})\cos\alpha)] \tag{64}$$

where Q_{min} is the energy transfer cutoff imposed by the detector. The modulation can be described in terms of the parameter h.

The effect of folding with LSP velocity on the total rate is taken into account via the quantity t. The SUSY parameters have been absorbed in $\bar{R}$. From our discussion in the case of differential rate it is clear that strictly speaking the quantities t and h also depend on the SUSY parameters. They do not depend on them, however, if one considers the scalar, spin etc. modes separately.

Let us now examine the directional rate. Integrating Eq. (49) we obtain

$$R_{dir} = \bar{R}(t^0/2)\{[1+h_1(a,Q_{min})\cos\alpha]\mathbf{e}_z\cdot\mathbf{e} - h_2(a,Q_{min})\cos\alpha\,\mathbf{e}_y\cdot\mathbf{e} + h_3(a,Q_{min})\sin\alpha\,\mathbf{e}_x\cdot\mathbf{e}\}. \tag{65}$$

TABLE V. The quantities t^0, h_1 and h_m for $\lambda=0$ in the case of the target $_{53}I^{127}$ for various LSP masses and Q_{min} in keV (for definitions see text). Only the scalar contribution is considered. Note that in this case h_2 and h_3 are constants equal to 0.117 and 0.135 respectively.

		LSP mass (GeV)						
Quantity	Q_{min}	10	30	50	80	100	125	250
t^0	0.0	1.960	1.355	0.886	0.552	0.442	0.360	0.212
h_1	0.0	0.059	0.048	0.037	0.029	0.027	0.025	0.023
h_m	0.0	0.164	0.144	0.124	0.111	0.107	0.104	0.100
t^0	10.	0.000	0.365	0.383	0.280	0.233	0.194	0.119
h_1	10.	0.000	0.086	0.054	0.038	0.033	0.030	0.025
h_m	10.	0.000	0.214	0.155	0.127	0.119	0.113	0.104
t^0	20.	0.000	0.080	0.153	0.136	0.11	0.102	0.065
h_1	20.	0.000	0.123	0.073	0.048	0.041	0.036	0.028
h_m	20.	0.000	0.282	0.190	0.145	0.132	0.123	0.109

TABLE VI. The same as in Table V, but for the value of the asymmetry parameter $\lambda=0.5$.

		LSP mass (GeV)						
Quantity	Q_{min}	10	30	50	80	100	125	250
t^0	0.0	2.309	1.682	1.153	0.737	0.595	0.485	0.288
h_1	0.0	0.138	0.128	0.117	0.108	0.105	0.103	0.100
h_2	0.0	0.139	0.137	0.135	0.133	0.133	0.133	0.132
h_3	0.0	0.175	0.171	0.167	0.165	0.163	0.162	0.162
h_m	0.0	0.327	0.307	0.284	0.266	0.261	0.257	0.250
t^0	10.	0.000	0.376	0.468	0.365	0.308	0.259	0.160
h_1	10.	0.000	0.174	0.139	0.120	0.114	0.110	0.103
h_2	10.	0.000	0.145	0.138	0.135	0.134	0.134	0.133
h_3	10.	0.000	0.188	0.174	0.167	0.165	0.164	0.162
h_m	10.	0.000	0.400	0.328	0.290	0.278	0.270	0.256
t^0	20.	0.000	0.063	0.170	0.171	0.153	0.134	0.087
h_1	20.	0.000	0.216	0.162	0.133	0.124	0.118	0.107
h_2	20.	0.000	0.155	0.143	0.137	0.136	0.135	0.133
h_3	20.	0.000	0.209	0.182	0.171	0.168	0.166	0.164
h_m	20.	0.000	0.487	0.374	0.316	0.299	0.286	0.265

TABLE VII. The same as in Table V, but for the value of the asymmetry parameter $\lambda=1.0$.

		LSP mass (GeV)						
Quantity	Q_{min}	10	30	50	80	100	125	250
t^0	0.0	2.429	1.825	1.290	0.837	0.678	0.554	0.330
h_1	0.0	0.192	0.182	0.170	0.159	0.156	0.154	0.150
h_2	0.0	0.146	0.144	0.141	0.139	0.139	0.138	0.138
h_3	0.0	0.232	0.222	0.211	0.204	0.202	0.200	0.198
h_m	0.0	0.456	0.432	0.404	0.382	0.375	0.379	0.361
t^0	10.	0.000	0.354	0.502	0.410	0.349	0.295	0.184
h_1	10.	0.000	0.241	0.197	0.174	0.167	0.162	0.154
h_2	10.	0.000	0.157	0.146	0.142	0.140	0.140	0.138
h_3	10.	0.000	0.273	0.231	0.213	0.208	0.205	0.200
h_m	10.	0.000	0.565	0.464	0.413	0.398	0.387	0.370
t^0	20.	0.000	0.047	0.169	0.186	0.170	0.150	0.100
h_1	20.	0.000	0.297	0.226	0.190	0.179	0.172	0.159
h_2	20.	0.000	0.177	0.153	0.144	0.142	0.141	0.139
h_3	20.	0.000	0.349	0.256	0.224	0.216	0.211	0.203
h_m	20.	0.000	0.709	0.550	0.448	0.424	0.408	0.380

The above equation is a bit complicated. Optimistically one may try to measure R_{dir} in some sort of averaging over all directions, i.e.

$$
\begin{aligned}
R_{dir,all} = & \bar{R}(t^0/2)[1+h_1(a,Q_{min})\cos\alpha + h_2(a,Q_{min})|\cos\alpha| \\
& + h_3(a,Q_{min})|\sin\alpha|] \\
= & \bar{R}(t^0/2)[1+h_m(a,Q_{min})\cos(\alpha-\alpha_h)]. \qquad (66)
\end{aligned}
$$

The effect of folding with LSP velocity on the total rate is taken into account via the quantity t^0. All other SUSY parameters have been absorbed in $\bar{R}$, under the assumptions discussed above in the case of undirectional rates.

We see that the modulation of the directional total event rate can be described in terms of three parameters h_l, $l=1,2,3$. In the special case of $\lambda=0$ we essentially have one parameter, namely h_1, since then we have $h_2=0.117$ and $h_3=0.135$.

Given the functions $h_l(a,Q_{min})$ one can plot the the expression in Eq. (66) as a function of the phase of the earth α. Alternatively one can employ h_m and a_h.

VI. DISCUSSION OF THE RESULTS

We have calculated the differential as well as the total event rates (directional and nondirectional) for elastic LSP-nucleus scattering including realistic nuclear form factors. We specialized our results for the target ^{127}I. Only the coherent mode due to the scalar interaction was considered. The spin contribution will appear elsewhere. Special attention was paid to the modulation effect due to the annual motion of the Earth. To this end we included not only the component of the Earth's velocity in the direction of the Sun's motion, as has been done so far, but all of its components. In addition both spherically symmetric [2] as well as only axially symmetric [28] LSP velocity distributions were examined. Furthermore, we considered the effects of the detector energy cutoffs by studying two typical cases $Q_{min}=10$ and 20 keV. We focused our attention on those aspects which do not depend on the parameters of supersymmetry other then the LSP mass.

The parameter $\bar{R}$, normally calculated in SUSY theories, was not considered in this work. The interested reader is referred to the literature [14,19] and, in our notation, to our previous work [11,12,32].

A. Undirectional rates

Let us begin with the total rates, which are described in terms of the quantities t and h. In Table I we show the dependence of h on the Earth's velocity, in the symmetric case

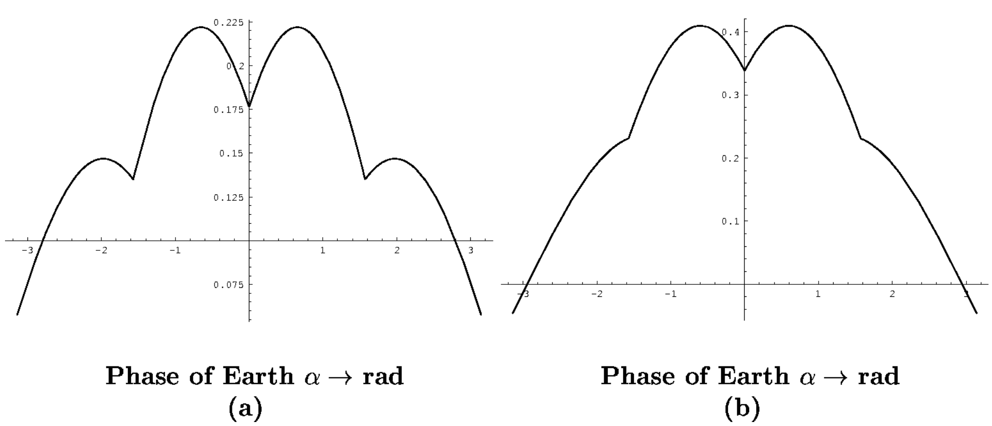


FIG. 5. The total directional modulation amplitude as a function of the phase of the Earth, α, in two cases: (a) for $h_1=0.059$, $h_2=0.117$ and $h_3=0.135$ and (b) $h_1=0.192$, $h_2=0.146$ and $h_3=0.231$. Note that in case (b) the minimum is negative. The results shown are for the target $_{53}I^{127}$ (for the definitions see text).

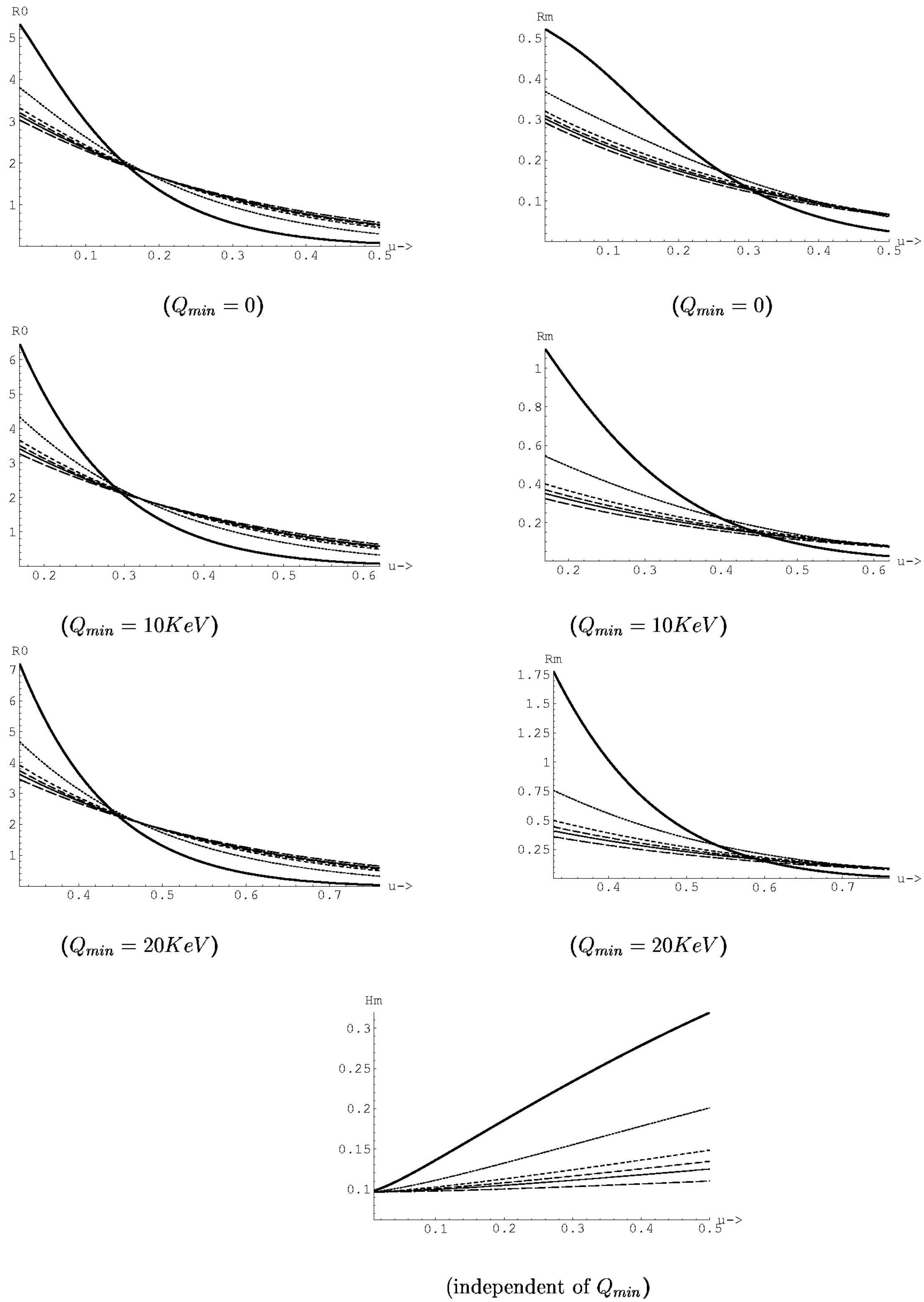


FIG. 6. The relative differential event rate R_0 and the amplitudes for modulation R_m and H_m vs u for the target ${}_{53}\mathrm{I}^{127}$ in the case of symmetric velocity distribution, $\lambda = 0$ (for the definitions see text).

$(\lambda = 0)$. We see that the modulation amplitude increases for about 50% when all components of the earth's velocity are included.

In Table II we show how the quantities t and h depend on the detector energy cutoff and the LSP mass for the symmetric case. In Tables III and IV we show the same quantities for $\lambda = 0.5$ and $\lambda = 1.0$ respectively. From these tables we see a dramatic increase of the modulation when the realistic axially symmetric velocity distribution is turned on. This means that the modulation amplitude can be exploited by the ex-

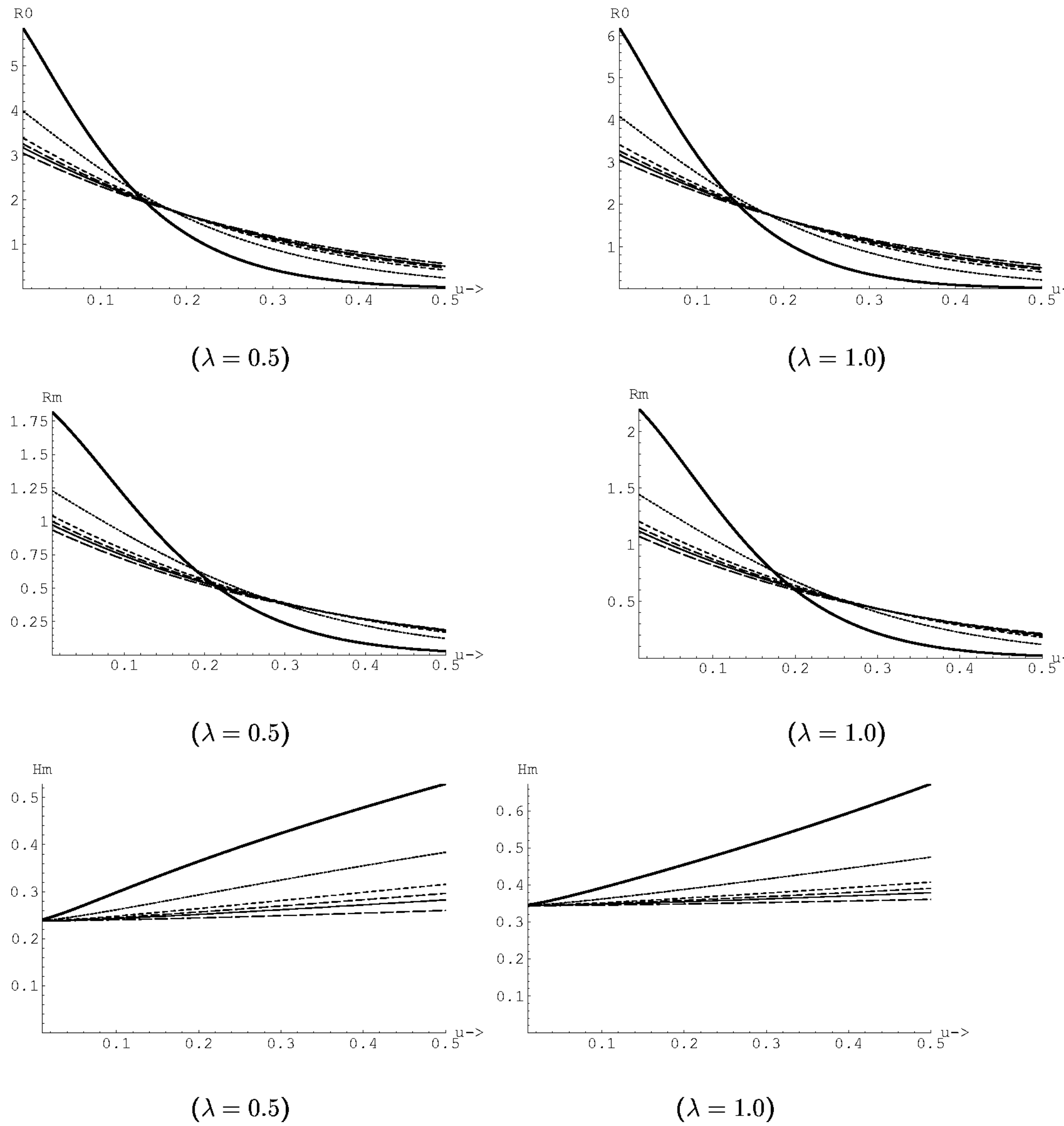


FIG. 7. The same as in Fig. 2 in the asymmetric case ($\lambda = 0.5$ and $\lambda = 1.0$). Only the case $Q_{min} = 0$ is exhibited.

perimentalists. We further notice that the modulation amplitude increases somewhat with cutoff energy. This is due to the fact that the modulation amplitude decreases less rapidly with the cutoff energy Q_{min} than the unmodulated amplitude. This effect may be of use to the experimentalists, even though it occurs at the expense of the total rate.

By looking at Tables II–IV we see that t decreases with an increase in the reduced mass. This means that the kinematic advantage given to the cross section by the large μ_r [see Eqs. (6)–(8)] is lost when the nuclear form factor and the convolution with the velocity distribution are taken into account. Let us now examine the differential rates, which are described by the functions $T(u)$, $H(u)$ and $T(u) \times H(u)$. These are shown for various LSP masses and Q_{min} in Fig. 1 ($\lambda = 0.0$), Fig. 2 ($\lambda = 0.5$) and Fig. 3 ($\lambda = 1.0$) We remind the reader that the dimensionless quantity u is related to the energy transfer Q via Eq. (13) with $Q_0 = 60\,\mathrm{keV}$ for ^{127}I. The curves shown correspond to LSP masses as follows: (i) Solid line $\Leftrightarrow m_\chi = 30\,\mathrm{GeV}$. (ii) Thin solid line $\Leftrightarrow m_\chi = 30\,\mathrm{GeV}$. (iii) Dotted line $\Leftrightarrow m_\chi = 50\,\mathrm{GeV}$. (iii) Dashed line $\Leftrightarrow m_\chi = 80\,\mathrm{GeV}$. (iv) Intermediate dashed line $\Leftrightarrow m_\chi = 100\,\mathrm{GeV}$. (vi) Fine solid line $\Leftrightarrow m_\chi = 125\,\mathrm{GeV}$. (vi) Long dashed line $\Leftrightarrow m_\chi = 250\,\mathrm{GeV}$. If some curves of the above list seem to have been omitted, it is understood that they fall on top of (vi). Note that, as a result of our normalization of T, the area under the corresponding curve is unity. This normalization was adopted to bring the various graphs on scale, since the absolute values may change much faster as a function of the LSP mass.

In order to understand the dependence of the total and differential rates on λ, we will examine the functions $f(y)$, which are equal to the quantity $N(\lambda,y)\,(2/\sqrt{6\pi})$ multiplied by the integrand of Eq. (33). The latter crucially depends on the functions $\tilde{F}_i(\lambda,2(\lambda+1)y)$ and $\tilde{G}_i(\lambda,y)$, $i=1,2$. The functions $f(y)$ are shown in Fig. 4 for $\lambda = 0,1$. We see that in the case of $\lambda = 1.0$ the positive section of the function is enhanced.

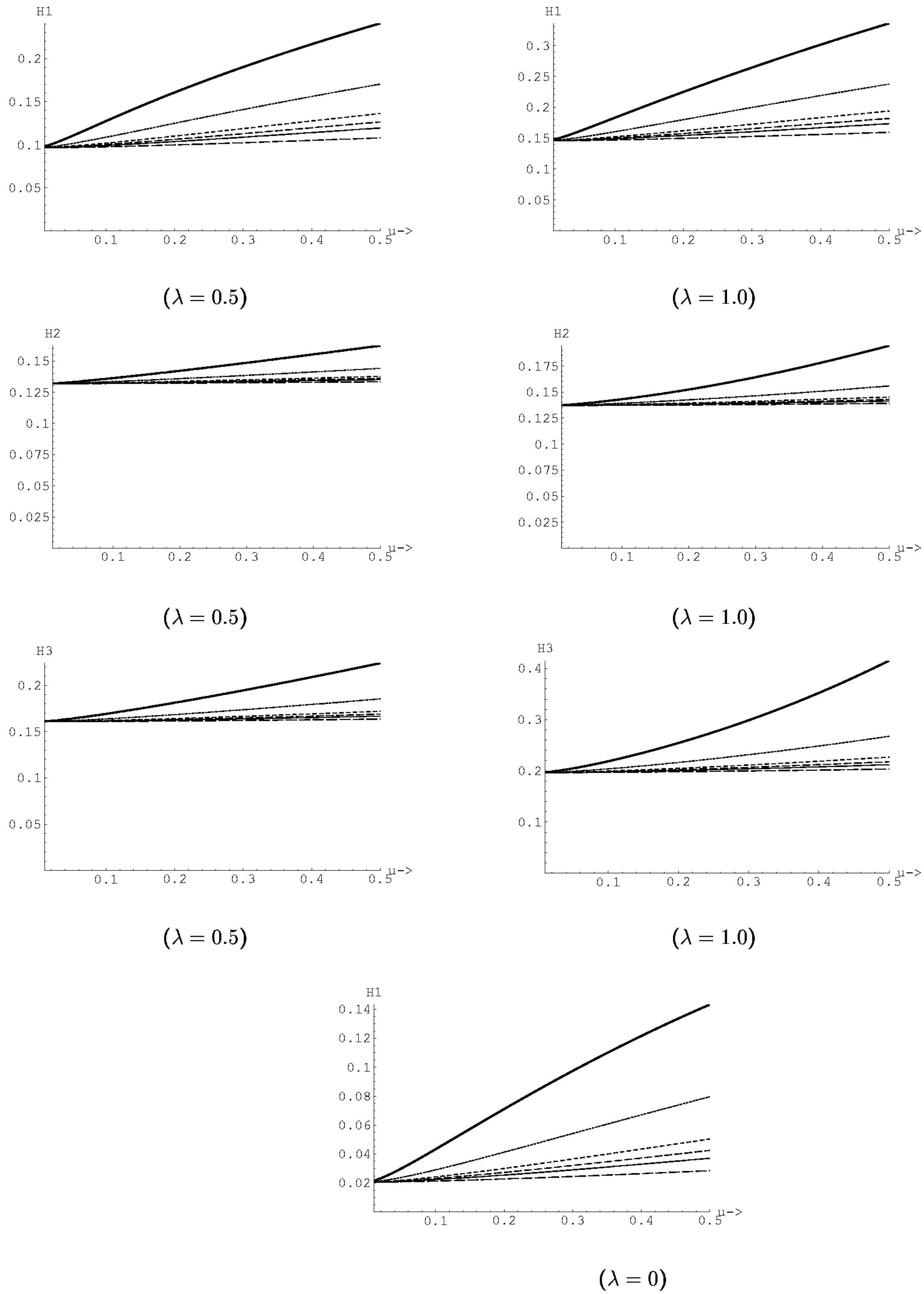


FIG. 8. The same as in Fig. 3 for the quantities H_1, H_2 and H_3. These quantities do not depend on Q_{min}, except for the fact that one should look at $u > u_{min}$.

B. Directional rates

Once again we distinguish two cases, the total and the differential rates.

1. Directional total event rates

The detection directional total event rates are perhaps beyond the goals of the present experiments. We will, however, include it in the present discussion. The unmodulated rates can be can be parametrized in terms of the parameter t^0. This describes the modification of the total directional nonmodulated event rate due to the convolution with the velocity distribution. The modulation is now described by the three parameters h_1, h_2, h_3 [see Eq. (65)], which are shown in Tables V–VII. We mention again that h_2 and h_3 are constant, 0.117 and 0.135 respectively, in the symmetric case. On the other hand, h_1 and h_3 substantially increase in the presence of asymmetry. The precise value of the directional rate depends on the direction of observation. One can find optimal orientations, but we are not going to elaborate further.

A crude picture for the modulation can be given by the quantity given by Eq. (66). This quantity is no longer a simple sinusoidal function. For some interesting cases the situation is shown in Fig. 5.

An idea about what is happening can be given by h_m and α_h. The first gives the difference between the maximum and the minimum values of modulated amplitude. The second involves the phase shift from the second of June, which is no longer the the date of the maximum.

The second of June gives the location of the maximum, when only the component of the earth‘s velocity along the Suns direction of motion is considered or when h_3 is neglected. In almost all cases considered in this work, however, h_3 is important and in fact the obtained shift is on the average about $\pm$ 35 days from the second of June.

2. Directional differential event rates

The directional differential rate is also very hard to detect, but perhaps a bit more practical than the total rates described in the previous subsection. It is given by Eq. (58) in terms of four functions of u, namely $R_0(u)$ and $H_i(u)$, $i=1,2,3$.

A gross description of the modulation can be given via the functions $a_H(u)$ and $H_m(u)$. The phase shift α_H has been found to be a constant and about 0.7, which corresponds to a shift about ± 35 days from the second of June. Since , however, H_m is defined as the ratio of two quantities, following the strategy of the previous subsection, we also present the quantity $R_m = R_0 H_m$. These functions are shown in Fig. 6 for LSP masses in the range 30–250 GeV, $\lambda=0$ and $Q_{min}=0$, 10 and 20 keV. Note that the quantity H_m is itself independent of the cutoff except that one should look at the u relevant to the allowed energy transfer interval.

The curves shown correspond to LSP masses as in the undirectional case. Again due to the normalization of R_0, the area under the corresponding curve is unity.

The above quantities for $\lambda=0.5$ and 1.0 are shown in Fig. 7, but only for $Q_{min}=0$. Their dependence on the energy transfer cutoff shows behavior similar to that of the undirectional case. In any case for $Q_{min}=10$, 20 keV, the functions R_0 and R_m show a behavior similar to that for $Q_{min}=0$, except that they start from higher energy transfer. We should remind the reader that in all cases R_0 represents the relative differential rate; i.e., it is normalized so that the area under the corresponding curve is unity for all LSP masses. One, therefore, should take into account the factor t^0 of Tables V–VII.

Coming back to Eq. (58) for the directional differential rate one clearly needs, in addition to R_0, the functions $H_l(u)$, $l=1,2,3$, which are plotted in Fig. 8. In the case of $\lambda=0$ only H_1 is plotted, since the other two are in this case constant ($H_2=0.117$, $H_3=0.135$). We see that in the presence of asymmetry, e.g. $\lambda=0.5$ and 1.0, all functions, but especially H_1 and H_3, are substantially increased.

VII. CONCLUSIONS

In the present paper we have expanded the the results obtained in of our recent Letter [15]. We have calculated all the parameters which can describe the modulation of the direct detection rate for supersymmetric dark matter. The differential as well as the total event rates were obtained both for the nondirectional as well as directional experiments. All components of the Earth’s velocity were taken into account, not just its component along the Sun’s direction of motion. Realistic axially symmetric velocity distributions, with enhanced dispersion in the galactocentric direction, were considered. The obtained results were compared to the up to now employed Maxwell-Boltzmann distribution.

We presented our results in a suitable fashion so that they do not depend on parameters of supersymmetry other than the the LSP mass. Strictly speaking the obtained results describe the coherent process in the case of ^{127}I, but we do not expect large changes, if the axial vector current is considered. Recall that the dependence on supersymmetry is contained in the parameter $\bar{R}$ not discussed in the present paper. The nuclear form factor was taken into account and the effects of the detector energy cutoff were also considered.

Our results, in particular the parameters t and t_0 (see Tables II–IV and V–VII), indicate that for large reduced mass, the kinematical advantage of μ_r [see Eqs. (6)–(8)] is partly lost when the nuclear form factor and the convolution with the velocity distribution are taken into account.

In the case of the undirectional total event rates we find that in the symmetric case the modulation amplitude for zero energy cutoff is less than 0.07. It gets substantially increased in the case of asymmetric velocity distribution reaching values up to 0.31 for $\lambda=1$. In the presence of the detector energy cutoff it can increase even further, up to 0.46, but this occurs at the expense of the total number of counts. The modulation amplitude in the case of the differential rate is shifted by the asymmetry at higher energy transfers and, for maximum asymmetry $\lambda=1$, gets about doubled compared to the symmetric case ($\lambda=0$). This amplitude does not depend on the the energy cutoff, but the lower energy transfers must, of course, be excluded if such a cutoff exists.

Analogous conclusions can be drawn about the directional differential event rate. The presence of asymmetry more than triples the differential modulation amplitude (from about 10% to about 35%). There exist now regions of the energy transfer such that the modulation amplitude can become as large as 50%.

Finally it is important that one should consider all components of the Earth’s motion, not just its velocity along the Sun’s motion, especially if the directional signals are to be measured.

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