Scalar fields in an anisotropic closed universe

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We study in this article a class of homogeneous, but anisotropic cosmological models in which shear viscosity is included. Within the matter content we consider a component (the quintessence component) determined by the baryotropic equations of state, $p=\alpha\rho$, with $\alpha<0$. We establish conditions under which a closed axisymmetrical cosmological model may look flat at low redshift.

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I. INTRODUCTION

Current observations of luminosity-redshift relations of type Ia supernovas [1] and measurements of the anisotropy cosmic background radiation and mass power spectrum [2] provide evidence that the total matter density of the universe coincides with its critical value. This agrees with the theoretical arguments derived from inflation [3], where it is suggested that our universe should become flat soon after a short period of inflation.

Since astronomical observations give rise to the bound $\Omega_M \lesssim 0.3$, in which baryons and cold dark matter are included, we are in front of a problematic situation. There exist a sort of ''missing energy'' that should represent something around 70% of the critical value.

It has been argued that the simplest explication, a cosmological constant (vacuum energy density) is consistent with these results [4]. Other alternatives have been considered. For instance, bulk pressure that is significantly negative, i.e., $\alpha \lesssim -1/3$, where $p=\alpha\rho$ is the effective equation of state, in which p is the pressure and ρ is the energy density. Here, this sort of matter could correspond to a network of topological defects [5] (such that strings or walls) or an evolving scalar field (referred as quintessence) [6] $Q(t)$, in which case the pressure and the energy density become defined by $p_Q = \frac{1}{2}\dot{Q} - V(Q)$ and $\rho_Q = \frac{1}{2}\dot{Q}^2 + V(Q)$, respectively. Here, $V(Q)$ represents the scalar potential associated to the scalar field Q and the overdots specify derivatives with respect to time.

The main difference between these two sort of models, i.e., the cosmological constant and the scalar field with a negative pressure, is that the latter is spatially inhomogeneous and thus can cluster gravitationally, where the former is totally spatial uniform. In this respect, the fluctuation of the scalar field could have an important effect on large scale structure of the universe [7].

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Since the total energy density equals the critical density, then the spatial part of the metric is supposed to correspond to a flat Friedmann-Robertson-Walker (FRW) metric. However, It has been mentioned that the observations referred to above, i.e., those related to type Ia supernovas, do not rule out a different type of geometry [8]. There, it was advanced that these measurements allow an open universe in which the cosmological constant is vanished.

From the theoretical point of view, it seems that quantum field theory is more consistent on compact spatial surfaces that in hyperbolic spaces [9]. On the other hand, in quantum cosmology the ''birth'' of universes have been described under the assumption that the three-geometry is characterized by a close spatial surface. In this way, motivated by quantum cosmology and by the short period of inflation that the universe underwent at early time in its evolution, we describe in this paper the conditions under which a closed universe model may look flat at low redshift. This kind of situation has been considered in the literature [10]. There, a closed universe with $\Omega_0<1$ was studied. Here, Ω_0 represents the density parameter associated to the total mass of the universe. Openness is obtained by adding to the matter density texture or tangled strings with equation of state $p=-\rho/3$ [11]. Here, the additional energy density is redshifted as a^{-2}, similar to the curvature term in a closed universe, where a is the scale factor. Kolb [12] studied this sort of matter, arising to the important conclusion that a closed universe may expand forever at constant speed.

It is natural to assume the geometry at very early epoch more general than just the isotropic and homogeneous FRW. Although the universe, on large scale, seems homogeneous and isotropic at present, there is no observational data that guarantees the isotropy in an era prior to the recombination. In fact, it is possible to begin with an anisotropic universe which isotropizes during its evolution.

In relation with the matter that we could take into account in an anisotropic background, may have many possible sources. For instance, populations of collisionless particles, gravitons, electric, or magnetic fields, or by topological defects [13].

The anisotropic dynamics can in general encode either relative velocity effects or dissipative effects or both [14]. In this respect, it is possible to start with an anisotropic universe that eventually isotropizes at later time in the evolution of the universe due to dissipative processes involving the matter that it contains. Also, this kind of model seems to be more appropriate when adiabatic theory of galaxy formation is considered [15]. Thus, it seems quite natural to include in this study a matter component with this kind of property, in a background which in essence is anisotropic [16].

The aim of the present paper is to study a closed anisotropic cosmological model, with a metric corresponding to Kanstowski-Sachs [17], where the matter content is composed by an imperfect fluid together with a scalar field whose equation of state parameter α remains negative during the evolution of the universe.

II. THE FIELD EQUATIONS

We start by considering the effective Einstein Lagrangian given by

$$\mathcal{L}=\frac{1}{\kappa}R+\frac{1}{2}(\partial_\mu Q)^2-V(Q)+\mathcal{L}_M, \tag{1}$$

where, $\kappa=16\pi G$, with G the Newton's gravitational constant, R the scalar curvature, Q the quintessence scalar field with associated potential $V(Q)$, and $\mathcal{L}_M$ represents the matter Lagrangian density. We assume that the matter Lagrangian density $\mathcal{L}_M$ is associated to a fluid (characterized by the pressure and energy density p_M and ρ_M, respectively) which presents a shear viscosity. By taking a preferred timelike vector field (four velocity) u^α, which satisfies $u^\alpha u_\alpha=1$ and it is a Ricci eigenvector, we can write the following matter energy-momentum tensor:

$$T_{\alpha\beta}=(\rho_M+p_M)u_\alpha u_\beta-p_M g_{\alpha\beta}+2\,\eta_M\sigma_{\alpha\beta}, \tag{2}$$

where η_M and $\sigma_{\alpha\beta}$ are the shear viscosity (or coefficient of dynamic viscosity, $\eta_M\geqslant 0$) and the traceless shear tensor, respectively. The shear tensor has the form

$$\sigma_{\alpha\beta}=h^\gamma_\alpha u_{(\gamma;\delta)}h^\delta_\beta-\frac{1}{3}\theta h_{\alpha\beta}, \tag{3}$$

where $\theta=u^\alpha{}_{;\alpha}$ is the scalar expansion and $h_{\alpha\gamma}$ is the projection tensor defined from the expression $h_{\alpha\beta}=g_{\alpha\beta}-u_\alpha u_\beta$, with signature for the metric $(+,-,-,-)$.

In this paper we consider a spatially homogeneous background spacetime of Kantowski-Sachs type, which, as far as it is known, is the only spatially homogeneous model that is not included in the Bianchi classification, thus we have

$$ds^2=dt^2-a^2(t)[d\theta^2+\sin^2(\theta)d\phi^2]-b^2(t)dr^2, \tag{4}$$

where a and b are the scale factors which describe the anisotropy of the model. This sort of metric combines spherical symmetry with a translational symmetry in the ''radial'' direction. The metric (4) has been studied by many authors that have considered different sort of matter components. As an example it has been considered a homogeneous shear-free cosmological model with an imperfect fluid matter content [18]. On the other hand, a energy-effective-action related to string theory has been studied [19]. Here, when the pseudoscalar axion field is time dependent only, it reduces to that of a stiff perfect-fluid cosmology. Also, a scalar field for a convex positive scalar potential [20], was taken into account among others.

Since the metric (4) is spatially homogeneous the scalar field Q can only depend on time, and thus the time-time component of Einstein's field equations is

$$\left(\frac{\dot a}{a}\right)^2+\left(\frac{\dot a}{a}\right)\left(\frac{\dot b}{b}\right)+\frac{1}{a^2}=\frac{2\kappa}{3}(\rho_M+\rho_Q), \tag{5}$$

where, as was mentioned above, the dots stand for derivatives with respect to the cosmological time t. From the metric (4), and considering the comoving frame, i.e., $u^\alpha=\delta^\alpha_0$, we find that the components of the shear tensor are given by

$$\sigma_{11}=\frac{2}{3}b^2\left(\frac{\dot a}{a}-\frac{\dot b}{b}\right),$$

$$\sigma_{22}=\frac{1}{3}a^2\left(\frac{\dot b}{b}-\frac{\dot a}{a}\right), \tag{6}$$

$$\sigma_{33}=\frac{1}{3}\sin^2(\theta)\,a^2\left(\frac{\dot b}{b}-\frac{\dot a}{a}\right).$$

Here, $\sigma_{00}=0$ and $\sigma^\alpha{}_\alpha=0$. The other components of Einstein's field equations are

$$2\frac{\ddot a}{a}+\left(\frac{\dot a}{a}\right)^2+\left(\frac{1}{a}\right)^2=-\kappa(p_M+p_Q)-\frac{4}{3}\kappa\,\eta_M\left(\frac{\dot a}{a}-\frac{\dot b}{b}\right), \tag{7}$$

and

$$\frac{\ddot b}{b}+\frac{\ddot a}{a}+\frac{\dot a\,\dot b}{a\,b}=-\kappa(p_M+p_Q)+\frac{2}{3}\kappa\,\eta_M\left(\frac{\dot a}{a}-\frac{\dot b}{b}\right). \tag{8}$$

In order to solve this set of equations, we need to supply this set with equation of state for the matter content and the scalar field. We assume that the matter content satisfies the relation $p_M=\gamma\rho_M$, where γ may be a time dependent quantity and its (present) value depends on the characteristics of matter content. In the following we assume that this constant lies in the range $0\leqslant\gamma\leqslant 1$, where the extremes correspond to dust and stiff fluid, respectively. In the same way, we shall assume that the scalar field Q satisfies a similar effective equation of state, i.e., $p_Q=\alpha\rho_Q$, where now the parameter α is assumed to be negative.

In order to have a universe which is closed, but still have a matter density content corresponding to a flat universe, we impose the following relations:

$$\kappa\,\rho_Q=a^{-2} \tag{9}$$

and

$$\eta' = \eta_M + \frac{1}{2}\frac{\rho_Q}{\overline{\sigma}}, \tag{10}$$

where

$$\overline{\sigma} = \left(\frac{\dot{a}}{a} - \frac{\dot{b}}{b}\right)$$

and α has been chosen to be equal to $-1/3$ [21]. Under these conditions the Einstein's field equations become

$$\left(\frac{\dot{a}}{a}\right)^2 + \left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{b}}{b}\right) = \frac{2\kappa}{3}\rho_M, \tag{11}$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = -\kappa\gamma\rho_M - \frac{4}{3}\kappa\eta'\overline{\sigma}, \tag{12}$$

and

$$\frac{\ddot{b}}{b} + \frac{\ddot{a}}{a} + \frac{\dot{a}}{a}\frac{\dot{b}}{b} = -\kappa\gamma\rho_M + \frac{2}{3}\kappa\eta'\overline{\sigma}. \tag{13}$$

This set of equations is similar to that of a matter fluid with shear viscosity immersed in a background corresponding to a flat axisymmetric cosmological model.

III. SOLUTION TO THE FIELD EQUATIONS AND SOME CONSEQUENCES

In the following we will describe solutions to the set of equations (11)–(13) in the cases in which $\eta' = 0$, i.e., where there is not generation of entropy and $\eta' \neq 0$, where exist generation of entropy. For the former solutions we calculate the angular distance-redshifts relations (specifically, for the stiff model with $\eta' = 0$) which are compared with its analogous results corresponding to the flat spacetime, and for the latter, we calculate the generation of entropy (for the cases in which $\eta' \neq 0$).

A. Cases $\eta' = 0$

In this case we describe two possible solutions. One of these is the vacuum Kasner solution in which $p_M = \rho_M = 0$, and a stiff fluid with equation of state $p_M = \rho_M \neq 0$, i.e., $\gamma = 1$. In the former case it is found that $a(t) = a_0$ and $b(t) = t$ are possible solutions to the field equations. Here, the scalar field Q remains constant and thus $\rho_Q = \text{const} = V_0$ and $p_Q = -(1/3)V_0$. On the other hand, η_M becomes $\eta_M = \frac{1}{2}(V_0/t)$. It seems that we could have another Kasner solution, such that $a(t) = t^{2/3}$ and $b(t) = t^{-1/3}$. However, this sort of solution gives rise to a shear viscosity which is essentially negative, since $\eta = -(1/2\kappa)(1/t^{1/3})$. Therefore, we disregard this type of solution.

In the latter case, in which $p_M = \rho_M$, i.e., $\gamma = 1$, we find a possible solution in which

$$a(t) = t^n, \quad b(t) = t^{1-2n}, \tag{14}$$

where n is a positive number. Here, we get

$$p_M = \rho_M = \frac{n(2-3n)}{\kappa t^2} \tag{15}$$

and

$$\eta_M = \frac{1}{2\kappa}\frac{1}{(1-3n)}\frac{1}{t^{2n-1}}. \tag{16}$$

In order to obtain $\eta > 0$ we demand $n < 1/3$. Here, it is found that the scalar field growth as

$$Q(t) = \sqrt{\left(\frac{2}{3\kappa}\right)}\frac{1}{1-n}t^{1-n}, \tag{17}$$

and its corresponding potential is given by

$$V(Q) = \left(\frac{2^{2n-1}}{3\kappa}\right)^{1/(1-n)} Q^{-2n/(1-n)}. \tag{18}$$

Notice that this potential decreases when Q increases, since $n < 1/3$.

At this moment, we would like to calculate the luminosity distance $d_L(z)$ as a function of the redshift z. This concept plays a crucial role in describing the geometry and matter content of the universe. From the metric (4) we observe that, light emitted by the object of luminosity $\mathcal{L}$ and located at the coordinate distance θ, at a time t is received by an observer (assumed located at $\theta = 0$) at the time $t = t_0$. The time coordinates are related by the cosmological redshift z in the θ direction by the expression, $1 + z = a(t_0)/a(t) \equiv a_0/a(t)$. The luminosity flux reaching the observer is $\mathcal{F} = \mathcal{L}/4\pi d_L^2$, where d_L is the luminosity distance to the object, given by $d_L(z) = a_0 \sin[\theta(z)](1+z)$.

In order to obtain an explicit expression for the angular size, let us now consider an object aligned to the ϕ direction and proper length l, so that its ''up'' and ''down'' coordinates are $(\theta, \phi + \delta\phi, 0)$ and $(\theta, \phi, 0)$. The proper length of the object is obtained by setting $t = \text{const}$ in the line-element metric (4), $ds^2 = -l^2 = -a^2(t)\sin^2(\theta)\delta\phi^2$. Thus, the angular size becomes

$$\delta\phi = \frac{l}{d_L(z)}(1+z)^2, \tag{19}$$

with d_L defined above.

From the solutions represented by Eq. (14) we obtain for the angular size

$$\delta\phi_n = \frac{l}{a_0} \frac{(1+z)}{\sin\left[\frac{n}{1-n}\frac{1}{a_0 H_0}\left[1-(1+z)^{-\frac{1-n}{n}}\right]\right]}, \tag{20}$$

where H_0 is a parameter defined by $H_0 = n/a_0^{1/n}$.

Figure 1 shows the angular size as a function of the redshift in the range $0.05 \leqslant z \leqslant 2.80$ for three different values of the parameters n. Here, we have used the value $a_0 H_0$

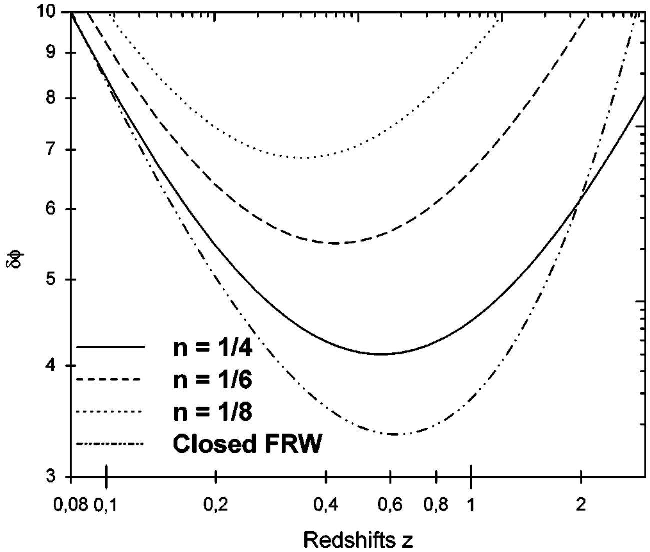

FIG. 1. This plot shows the angular size (in unit of l/a_0) as a function of the redshift z, for three different values of the parameters n, $n=1/4,1/6,1/8$. The dash-dot-dot line corresponds to the isotropic closed FRW model, with a matter component defined by the equation of state $\alpha=-1/3$.

$=\sqrt{2/5}$. In this plot we have added the graph of the angular size corresponding to the isotropic closed FRW model, with a matter dominated by a quintessence component defined by $\alpha=-1/3$. Notice that, for different values of the parameter n, those curves near to the value $n=1/3$ become closer to that corresponding to the isotropic FRW model.

In Fig. 2 we show the angular sizes as a function of the redshift z for flat and closed anisotropic universe models. Notice that at low redshift both curves become similar. We could distinguish them at $z\gtrsim 0.5$.

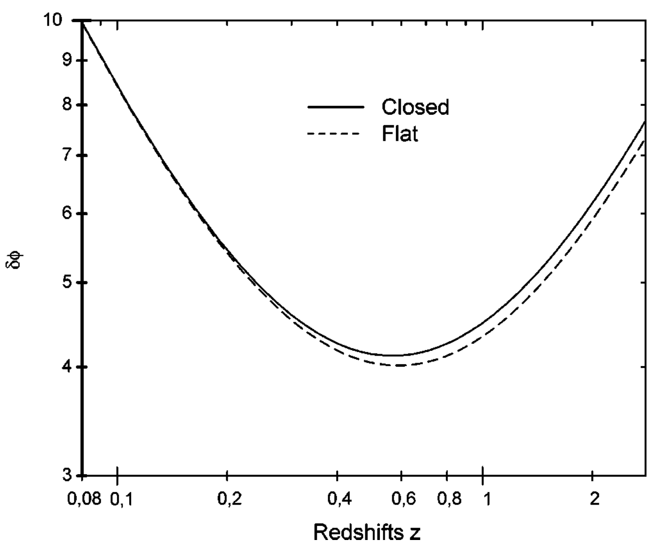

FIG. 2. This plot shows the angular size (in unit of l/a_0) as a function of the redshifts z, for flat and closed anisotropic models. The parameter n was chosen to be $n=1/4$. Notice that at low redshift the models become indistinguishable.

B. Cases $\eta' \neq 0$

As before, in this case we consider two different solutions. We start by describing a quasianisotropic and an exponential growing solutions.

In the former case, we found as a possible solution $a(t)=t^{2/3}$ and $b(t)=t^{2/3}[1+(t/t_0)^{-n}]$, where n and t_0 are two arbitrary constants. Notice that at large time $b(t)$ approach to $a(t)$ and thus the universe isotropizes. Thus, asymptotically the universe approaches to an homogeneous isotropic flat universe which is filled by dust, i.e., $b(t)\sim a(t)=t^{2/3}$.

For this solution it is found that

$$\rho_M=\frac{4}{3\kappa t^2}\left[1-\frac{n}{1+\left(\frac{t}{t_0}\right)^n}\right], \tag{21}$$

$$p_M=\frac{2n(1-n)}{3\kappa t^2\left[1+\frac{n}{1+\left(\frac{t}{t_0}\right)^n}\right]}, \tag{22}$$

and

$$\eta_M=\frac{1}{2n\kappa t}\left[n\kappa(n-1)+t^{2/3}\left[1+\frac{n}{1+\left(\frac{t}{t_0}\right)^n}\right]\right]. \tag{23}$$

Notice that γ becomes a time dependent quantity in this case

$$\gamma(t)=\frac{n}{2(1-n)}\left[1-n+\left(\frac{t}{t_0}\right)^n\right]^{-1}. \tag{24}$$

Notice also that $\gamma(t)\to 0$ for $t\to\infty$, in agreement with the remark described above.

The effective shear viscosity becomes $\eta'=(n-1)/2\kappa t$, and in order to be positive the parameter n should be bounded from below, i.e., $n\geqslant 1$. The corresponding solution for the scalar field is

$$Q(t)=\sqrt{\frac{6}{\kappa}}\left(\frac{t}{t_0}\right)^{1/3}\equiv Q_0\left(\frac{t}{t_0}\right)^{1/3}, \tag{25}$$

and the potential becomes

$$V(Q)=V_0\left(\frac{Q_0}{Q}\right)^4, \tag{26}$$

where the constant V_0 is given by $V_0=\frac{1}{9}Q_0^{-3}$. This sort of solution was described in Ref. [21] where scalar fields in FRW metric were studied.

A second possible solution is that in which the scale factor a grows exponentially, i.e.,

$$a(t)=e^{Ht} \tag{27}$$

and

$$b(t)=e^{-Ht/2}\sin\left(\frac{3Ht}{2}\right), \tag{28}$$

where H is a constant to be determined later on.

This solution corresponds to a universe filled with a viscous dust, since $p_M=0$ and $\eta'\neq 0$ at any time. The energy density and the effective shear viscosity become

$$\rho_M=3H^2\cot\left(\frac{3Ht}{2}\right). \tag{29}$$

and

$$\eta'=\frac{3H}{2[\cot((3Ht/2))-1]}, \tag{30}$$

respectively.

In order to have $\eta'\geqslant 0$ we must impose that $0\leqslant 3Ht/2\leqslant\pi/4$. This result in an age for the universe given by $t_0=(\pi/6)\,H^{-1}$, which could be used for fixing the value of H.

Notice that the solutions (27) and (28) give rise to the following Hubble expansion rates:

$$H_1=\frac{\dot a}{a}=H \tag{31}$$

and

$$H_2=\frac{\dot b}{b}=\frac{H}{2}\left[3\cot\left(\frac{3Ht}{2}\right)-1\right], \tag{32}$$

and thus the Hubble horizon related to the θ-ϕ plane remains constant.

The corresponding scalar field is found to evolve as

$$Q(t)=\frac{1}{H}\sqrt{\frac{2}{3\kappa}}[e^{-Ht_0}-e^{Ht}]+Q_0, \tag{33}$$

where Q_0 is the value of $Q(t)$ at $t=t_0$. The corresponding scalar potential $V(Q)$ becomes

$$V(Q)=V_0\left[1-\sqrt{\frac{3\kappa}{2}}(Q-Q_0)\right], \tag{34}$$

where V_0 is a constant defined by $V_0=(2/3\kappa)\,e^{-2Ht_0}$. From this expression we see that this potential decreases when Q increases, similar to the other case.

It is well known that the production of entropy could be related to the anisotropy of the universe [22,23]. In the following we proceed to calculate this production in the cases described above. In order to do this, we introduce the entropy current four-vector S^μ as

$$S^\mu=n_b k_B \lambda u^\mu, \tag{35}$$

where as before u^μ represents the four-velocity, n_b the baryon number density, k_B is the Boltzmann's constant, and λ the nondimensional entropy per baryon. It could be shown that [24]

$$S^\mu{}_{;\mu}=\frac{2\eta}{T}\sigma_{\mu\nu}\,\sigma^{\mu\nu}, \tag{36}$$

where in the first case we get

$$S^\mu{}_{;\mu}=\frac{2n^2(n-1)}{3\kappa T t^3[1+(t/t_0)^n]^2}. \tag{37}$$

The left-hand side of this expression gives, in the comoving frame of reference

$$S^\mu{}_{;\mu}=k_B n_b\dot\lambda, \tag{38}$$

where we have used the conservation equation for baryon number $(n_b u^\mu)_{;\mu}=0$. Thus, we get

$$\dot\lambda=\frac{2n^2(n-1)}{3n_b k_B T t^3[1+(t/t_0)^n]^2}. \tag{39}$$

Note that this expression decrease when t increase and becomes zero for $t\to\infty$, similar to η_M, p_M, and ρ_M.

From expression (42) evaluated at $t=t_{1000}$ (equivalent to 1000 s) and $t=t_{\rm rec}$ (time at recombination) we get

$$\frac{\dot\lambda_{1000}}{\dot\lambda_{\rm rec}}=\frac{n_b^{\rm rec}}{n_b^{1000}}\frac{T_{\rm rec}}{T_{1000}}\left(\frac{t_{\rm rec}}{t_{1000}}\right)^3\left[\frac{1+t_{\rm rec}^n/t_0^n}{1+t_{1000}^n/t_0^n}.\right]^2 \tag{40}$$

With the data given in Ref. [25] and taking $n=2$, we obtain for t_0 the value $t_0\approx 4\times 10^{10}$ s. This value, together with the age of the universe, $t_c\approx 5\times 10^{17}$ s, allows us to obtain the ratio between the shear, σ, and the scalar expansion, θ, given by

$$\left(\frac{\sigma}{\theta}\right)_{t_c}=\frac{n}{\sqrt{3}\,(2(t_c/t_0)^n+2-n)}\approx 5\times 10^{-15}, \tag{41}$$

which is inside of the bound expressed by Cosmic Background Explorer (COBE) measurements, that gives $(\sigma/\theta)_{t_c}\leqslant 6.9\times 10^{-10}$ [26,27].

In the second case, and following a similar process we find that

$$\dot\lambda=\frac{9H^3}{4n_b k_B\kappa T}\left[\cot\left(\frac{3Ht}{2}\right)-1\right]. \tag{42}$$

By using the observational data specified above we get that

$$\frac{\dot\lambda_{1000}}{\dot\lambda_{\rm rec}}\simeq 6\times 10^{-25}. \tag{43}$$

Thus, the generation of entropy has decreased more than 10^{25} times the value at recombination during the period from $t_{\rm rec}$ to $t\simeq 1000$ s.

IV. CONCLUSIONS

We have studied an anisotropic universe cosmological model described by the metric (4). We included in our model negative anisotropic pressures motivated by quintessence cosmological scenarios. This component was represented by a scalar field Q, whose equation of state was considered to be given by $p_Q = \alpha \rho_Q$, where the parameter α was considered to be equal to $-1/3$.

In order that our closed universe scenario could resemble a flat model, we imposed the conditions specified by Eqs. (9) and (10). Under these conditions, we have determined, in different cases, explicit expressions for the scalar potential $V(Q)$. In all these cases we have found that this potential decreases as a function of the scalar field Q. In this respect, it would be interesting to study the cosmological consequences that this sort of potential may have during the evolution of the universe. Especially, the influence that it carried during the rapid expansion (inflation) that the universe is believed to present at early time of its evolution.

In the cases in which the shear viscosity was vanished we have determined the angular sizes for different values of the parameters. Here, we found, similar to the isotropic case, that our closed model looks similar to a flat model at low redshifts.

On the other hand, solutions in which the shear viscosity was not vanished, we have determined the generation of entropy. Here, we have found that our results agree with the bound imposed by the observational data.

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