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Skewness as a probe of non-Gaussian initial conditions

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We compute the skewness of the matter distribution arising from nonlinear evolution and from non-Gaussian initial perturbations. We apply our result to a very generic class of models with non-Gaussian initial conditions and we estimate analytically the ratio between the skewness due to nonlinear clustering and the part due to the intrinsic non-Gaussianity of the models. We finally extend our estimates to higher moments.

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The source of the initial density fluctuations which have led to the formation of structure, observed in the Universe today, is unknown. Determining its nature will certainly be of utmost importance for the fruitful relation between high energy physics and cosmology.

In models which presently attract the most attention, initial density fluctuations are generated during an inflationary phase. In the simplest inflationary models, the initial fluctuations obey Gaussian statistics. If this picture is correct, the deviations from Gaussianity we observe today were induced by nonlinear gravitational instability [1–4]. However, it is also conceivable that the present deviations from Gaussianity have two components: gravitationally induced and intrinsic, coming from the initial conditions rather than nonlinear dynamics [5–8]. Here, we investigate to what extent an intrinsic component can be ''washed out'' by nonlinear dynamics and on which scales it could be either detected or constrained from above in future galaxy surveys.

We start by deriving a general expression for the so-called skewness parameter S_3, including the effect of an initial non-Gaussianity, nonlinear evolution and smoothing. We then estimate the normalized N-point cumulant S_N for a wide class of models and compare it with the result obtained in Gaussian models due to mild nonlinearities.

If the galaxies trace the spatial mass distribution, galaxy surveys [9] can be used to estimate the cumulants of the mass density contrast field, given by

$$M_N(R)\equiv\langle(\delta_R)^N(\mathbf{x},\eta_0)\rangle_c \qquad (1)$$

of the smoothed density field $\delta_R(\mathbf{x},\eta)\equiv\int d^3\mathbf{x}'\,W_R(|\mathbf{x}-\mathbf{x}'|)\,\delta(\mathbf{x}',\eta)$, where $\delta(\mathbf{x},\eta)$ is the density contrast, η and η_0 the conformal time and its value today, and W_R is a window function (e.g., Gaussian or top hat) of width R. The brackets in Eq. (1) denote an ensemble average and the subscript c indicates that we deal with the connected part of the N-point function. For a Gaussian field, all cumulants of order $N>2$ vanish: $M_N=0$. M_2 is the variance while M_3 is a measure of the asymmetry of the distribution, known as skewness. We will also use the more common normalized cumulant,

$$S_N(R)=M_N(R)/(M_2(R))^{(N-1)}.$$

In the weakly nonlinear regime, this ratio is time-independent to lowest nonvanishing order in perturbation theory for all models with Gaussian initial conditions [2–4]. To calculate the general expression for $M_3(R)$ in the weakly nonlinear regime, we follow the method developed in [4]. Expanding $\delta(\mathbf{x},\eta)$ in a perturbative series, $\delta_1+\delta_2+\mathcal{O}(3)$ and solving the system of coupled Euler, Poisson, and continuity equations at second order leads, in Fourier space, to $\Delta_1(\eta,\mathbf{k})=D(\eta,\mathbf{k})$ and

$$\Delta_2(\eta,\mathbf{k})=(2\pi)^{-3/2}\int d^2\mathbf{q}J(\mathbf{q},\mathbf{k}-\mathbf{q})D(\eta,\mathbf{q})D(\eta,\mathbf{k}-\mathbf{q})$$

where we consider only the fastest growing modes and we use the convention

$$\Delta_N(\eta,\mathbf{k})=(2\pi)^{-3/2}\int \delta_N(\eta,\mathbf{x})e^{-i\mathbf{k}\cdot\mathbf{x}}d^3\mathbf{x}.$$

At late times where a possible source term or seed has decayed, the time and space dependence of the function D can be factorized, $D(\eta,\mathbf{k})=D_+(\eta)\epsilon(\mathbf{k})$, where D_+ is the standard linear growing mode [1]. Perturbation theory gives [4]

$$J(\mathbf{k},\mathbf{q})=\tfrac{2}{3}(1+\kappa)+(q/k)P_1(\mu)+\tfrac{2}{3}(\tfrac{1}{2}-\kappa)P_2(\mu), \qquad (2)$$

where the P_l is the Legendre polynomial of order l, $\mu\equiv\mathbf{k}\cdot\mathbf{q}/kq$. The quantity κ is a weak function of Ω; for $\Omega>0.01$, $\kappa\approx(3/14)\Omega^{-0.03}$ [3]. The smoothing applies order by order. In Fourier space, we have $\Delta_R(\eta,\mathbf{k})=D(\eta,\mathbf{k})W_k$, W_k being the Fourier transform of the window function. To fifth order, the skewness is

$$M_3=\langle\delta_{R,1}^3\rangle+3\langle\delta_{R,1}^2\delta_{R,2}\rangle+\mathcal{O}(5). \qquad (3)$$

We introduce the two-, three-, and four-point power spectra as

$$\langle 12\rangle\equiv\mathcal{P}_2(k_1)\,\delta(\mathbf{k}_1+\mathbf{k}_2),$$

$$\langle 123\rangle\equiv\mathcal{P}_3(\mathbf{k}_1,\mathbf{k}_2)\,\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3),$$

$$\langle 1234\rangle_c\equiv\mathcal{P}_4(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\,\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4). \qquad (4)$$

(The Dirac δ is a simple consequence of statistical homogeneity which we assume throughout.) Here $\langle 12\cdots N\rangle \equiv\langle D(\eta,\mathbf{k}_1)D(\eta,\mathbf{k}_2)\cdots D(\eta,\mathbf{k}_N)\rangle$. The functions $\mathcal{P}_2$ and $\mathcal{P}_3$ are also known as the power spectrum and the bispectrum, respectively. Inserting the Fourier transforms of δ_1 and δ_2 after smoothing in Eq. (3), expressing the correlators of D in terms of the power spectra (4), and performing one integration using the Dirac function in Eq. (4), we obtain

$$M_3(R)=\int\frac{d^3\mathbf{k}d^3\mathbf{q}}{(2\pi)^6}\mathcal{P}_3(\mathbf{k},\mathbf{q})W_kW_qW_{|\mathbf{k}+\mathbf{q}|}+\int\frac{d^3\mathbf{k}d^3\mathbf{q}}{(2\pi)^6}\mathcal{P}_2(k)\mathcal{P}_2(q)W_kW_qW_{|\mathbf{k}+\mathbf{q}|}J(\mathbf{k},\mathbf{q})+\int\frac{d^3\mathbf{k}d^3\mathbf{q}d^3\mathbf{p}}{(2\pi)^6}\mathcal{P}_4(\mathbf{k},\mathbf{q}-\mathbf{k},\mathbf{p})W_qW_pW_{|\mathbf{q}+\mathbf{p}|}\times J(\mathbf{k},\mathbf{q}-\mathbf{k}). \tag{5}$$

For a Gaussian field, $\mathcal{P}_4=\mathcal{P}_3=0$ and the only nonvanishing contribution comes from the second term in the above expression. For a top hat window, this term gives $M_3=(34/7-\gamma)M_2^2$, with $\gamma=-d\log M_2(R)/d\log R$ [4]. Note also that $\gamma(R)$ is the logarithmic slope of the two-point correlation function of the density fluctuations, the Fourier transform of $\mathcal{P}_2(k)$. It is usually assumed that $\gamma>0$ (condition of hierarchical clustering, see, e.g., [1]).

The class of models we want to analyze are those where fluctuations in the dark matter are induced by the energy and momentum of an inhomogeneously distributed component which contributes only a small fraction to the total energy momentum tensor and which interacts only gravitationally with the cosmic fluid. Such a component is denoted as ''seed'' [10]. As stressed above, we need to compute the N-point power spectra of the density field at the end of the linear regime. The comoving linear density fluctuation D of the cosmic matter-radiation fluid evolves according to [11,10]

$$\ddot{D}+H(1-6w+3c_s^2)\dot{D}+k^2c_s^2D-\tfrac{3}{2}(1+8w-3w^2-6c_s^2)H^2D=\mathcal{S}(\mathbf{k},\eta), \tag{6}$$

with $\mathcal{S}\equiv(1+w)4\pi G(f_\rho+3f_P)$, f_ρ and f_P being the inhomogeneous energy density and pressure of the seeds. When the seed is a scalar field ϕ with vanishing potential, $f_\rho+3f_P=\dot{\phi}^2$. G is Newton's constant, a denotes the cosmic scale factor, a dot refers to the derivative with respect to conformal time, $H\equiv\dot{a}/a$. The variables $w\equiv P/\rho$ and $c_s^2\equiv\dot{P}/\dot{\rho}$ are respectively the enthalpy and the adiabatic sound speed of the cosmic fluid.

Equation (6) can be solved by a Green's function $\mathcal{G}$,

$$D(\mathbf{k},\eta)=\int_{\eta_i}^{\eta}\mathcal{G}(\mathbf{k},\eta,\eta')\mathcal{S}(\mathbf{k},\eta')d\eta', \tag{7}$$

where η_i is some early initial time deep in the radiation era. For the linear part of the reduced N-point function we then obtain

$$\langle D(\mathbf{k}_1,\eta)\cdots D(\mathbf{k}_N,\eta)\rangle_c=\int_{\eta_i}^{\eta}d\eta_1\cdots d\eta_N\,\mathcal{G}(\mathbf{k}_1,\eta,\eta_1)\cdots\mathcal{G}(\mathbf{k}_N,\eta,\eta_N)\langle\mathcal{S}(1)\cdots\mathcal{S}(N)\rangle_c, \tag{8}$$

where $(i)\equiv(\mathbf{k}_i,\eta_i)$. We define the connected N-point function of the source by

$$\langle\mathcal{S}(1)\dots\mathcal{S}(N)\rangle_c\equiv F_N(\mathbf{k}_1,\dots\mathbf{k}_N;\eta_1\dots\eta_N)\,\delta\left(\sum\mathbf{k}_i\right).$$

Again, the δ function of the sum of all momenta is a consequence of the statistical homogeneity.

We now assume that the reduced N-point function of the source can be replaced by its ''perfectly coherent approximation'' given by

$$F_N(\mathbf{k}_1,\dots,\mathbf{k}_{N-1};\eta_1,\dots,\eta_N)\simeq\mathrm{sgn}(F_N)\sqrt[N]{|F_N(\mathbf{k}_1,\dots,\mathbf{k}_{N-1};\eta_1,\dots,\eta_1)\dots F_N(\mathbf{k}_1,\dots,\mathbf{k}_{N-1};\eta_N,\dots,\eta_N)|} \tag{9}$$

[here and below, $\mathbf{k}_N$ is always given by $\mathbf{k}_N=-(\mathbf{k}_1+\cdots+\mathbf{k}_{N-1})$]. This approximation is exact if the evolution equation for $\mathcal{S}$ is linear and the randomness is entirely due to initial conditions. Then the source term is of the form $\mathcal{S}(\mathbf{k},\eta)=R(\mathbf{k})s(k,\eta)$, where only R is a random variable and s is a deterministic solution to the linear evolution equation of $\mathcal{S}$ which can be taken out of the average $\langle\,\rangle$. This is the key property which renders the N-point function decoherent. Then F_N can be written as

$$F_N(\mathbf{k}_1,\dots,\mathbf{k}_{N-1};\eta_1,\dots,\eta_N)\simeq s(1)\cdots s(N)\langle R(\mathbf{k}_1)\cdots R(\mathbf{k}_N)\rangle_c \tag{10}$$

which is clearly of the form (9).

An important example are models with no sources but with non-Gaussian initial conditions for D. Such models, like, e.g., the recent χ^2 Peebles model [12], are always perfectly coherent and therefore included in our analysis: in this case $D(\mathbf{k},\eta)=R(\mathbf{k})d(k,\eta)$, where R is a non-Gaussian random variable given by the initial condition and d is a deterministic homogeneous solution of Eq. (6). Clearly, if we choose $\mathcal{S}(\mathbf{k},\eta)=R(\mathbf{k})\delta(\eta-\eta_{\rm in})$ and $\mathcal{G}(k,\eta,\eta')=d(k,\eta)$, then D is of the form (7). Therefore, models where the non-Gaussianity is purely due to initial conditions are always perfectly coherent. As the equation of motion for D is second order, the homogeneous solution has in principle two modes, $D=R_1(\mathbf{k})d_1(k,\eta)+R_2(\mathbf{k})d_2(k,\eta)$, but since we shall evaluate the N-point functions deeply in the matter era, the decaying mode will have disappeared and may thus be neglected in our analysis.

Models where the source term is due to a scalar field which evolves linearly in time are not perfectly coherent, since $\mathcal{S}$ is given by the components of the energy momentum tensor which are quadratic in the fields. Numerical calculations, however, have shown that this nonlinearity is not severe and perfect coherence is a relatively good approximation [13,14]. One example of this kind are axionic seeds in pre–big-bang cosmology [15–17] for which decoherence has been tested and is found to be on the level of less than 5% for the cosmic microwave background power spectrum. In Fig. 1 the functions $D_2(k,\eta)$ and $D_3(k,k,\eta)$ as obtained by a full numerical calculation are compared to their coherent approximation (9) for the large-N limit of global $O(N)$ symmetric scalar fields. This is another example where the scalar field evolution is linear and the only nonlinearity in the source term is due to the energy momentum tensor being quadratic in the field [18,14,13].

For topological defects, especially for cosmic strings, the perfectly coherent approximation misses several important features (like the ''smearing out'' of secondary acoustic peaks). However, we believe that our generic scaling result holds also in this case, as is indicated by numerical simulations of global texture: even though global texture show considerable decoherence [13], the same scaling law for higher moments which we derive below has been discovered numerically [7].

Under the perfectly coherent approximation, Eq. (8) can be factorized as the product of the N solutions, $D_{N_j}(\mathbf{k}_1,\ldots,\mathbf{k}_{N-1},\eta)$ of Eq. (6) with source term $[F_N(\mathbf{k}_1,\ldots,\mathbf{k}_{N-1},\eta,\ldots,\eta)]^{1/N}$, where $\mathbf{k}_j$ is the wave number $\mathbf{k}$ appearing in the term $c_s^2k^2$ on the left-hand side of Eq. (6) and the other wave numbers have to be considered like parameters of the source term,

$$\begin{aligned}\langle D(\mathbf{k}_1,\eta)\cdots D(\mathbf{k}_N,\eta)\rangle_c &\simeq\left[\prod_{j=1}^{N} D_{N_j}(\mathbf{k}_1,\ldots,\mathbf{k}_{N-1},\eta)\right]\delta\left(\sum \mathbf{k}_i\right)\\ &\equiv\mathcal{P}_N(\mathbf{k}_1,\ldots,\mathbf{k}_{N-1},\eta)\,\delta\left(\sum \mathbf{k}_i\right).\end{aligned}\tag{11}$$

To continue, we assume that F_N is a simple power law in the k_i on super-Hubble scales and that it decays after Hubble crossing. This behavior is certainly correct for all examples discussed in the literature so far. We can then make the following ansatz:

$$F_N\simeq\begin{cases}\left(\prod_{i=1}^{N}\dfrac{k_i^\alpha}{k_0^\alpha}\right)(f(\eta)\eta)^N\eta^{-3} & \text{if } k_i\eta\leqslant 1,\forall i,\\ 0, & \text{otherwise.}\end{cases}\tag{12}$$

Here f is a dimensionless function and k_0 is an arbitrary scale. For scale invariant seeds (e.g., topological defects) f is just a constant and $\alpha=0$. For axion seeds generated during a pre–big-bang phase, α depends on the spectral index of the axion field, which in turn is determined by the evolution law of the extra dimension [16]. For the Peebles model α is given by the power spectrum of the scalar field ϕ and f is a delta function. Since F_N is symmetrical in the variables $\mathbf{k}_j$ we can order them such that $k_1\geqslant k_2\geqslant\cdots\geqslant k_N$.

Let us discuss the temporal behavior of the variables D_{N_j}. As long as $k_1\eta<1$, the term $c_s^2k_j^2D$ can be neglected in Eq. (6) and the Green's function is a power law. At $k_1\eta\sim1$ the source term decays and as long as a perturbation remains super horizon, it just grows like η^2, so that for $k_j\eta<1$

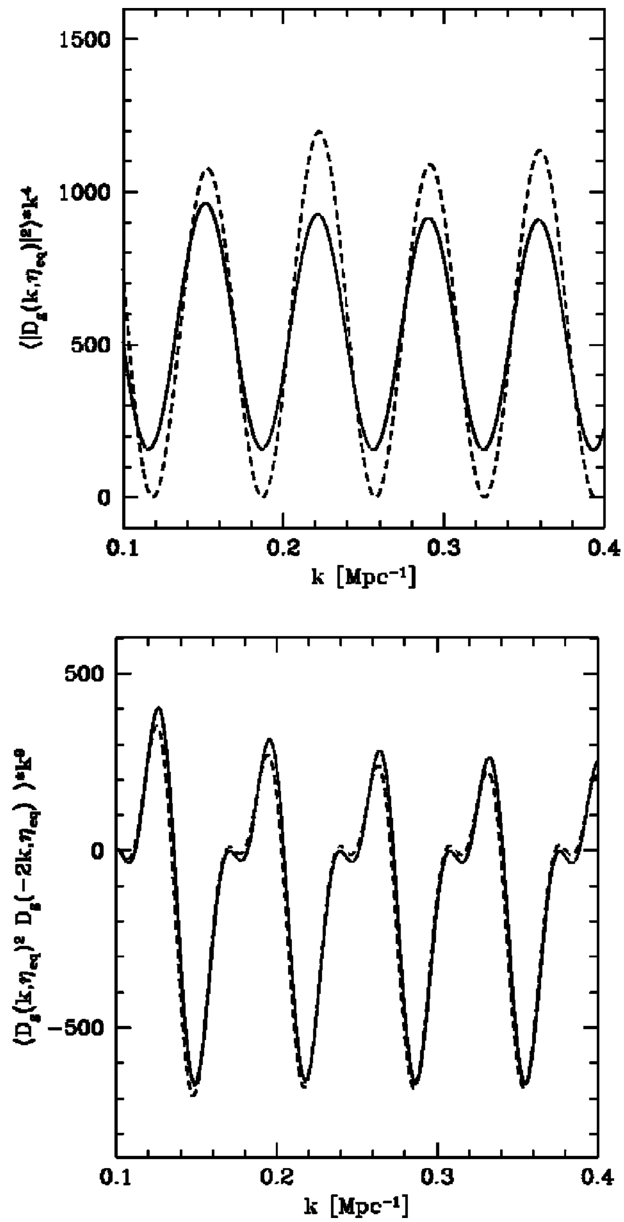

FIG. 1. The coherent approximation (dashed line) and the full decoherent result (solid line) for the two- (top) and three-point (bottom) functions of the large-N limit of global $O(N)$ symmetric scalar fields is shown at the end of the radiation era. The sign in the coherent approximation for the three-point function is chosen to agree with the sign for the decoherent three-point function.

$<k_1\eta$,

$$D_{N_j}\approx g(1/k_1)k_1^{-2+3/N}(\eta k_1)^2\prod_{n=1}^{N}(k_n/k_0)^{\alpha}$$

where $g(\eta)=\frac{4\pi G}{\eta^{2-3/N}}\int_{\eta_{\rm in}}^{\eta}G(\eta,\eta')f(\eta')\eta'^{(2-3/N)}\frac{d\eta'}{\eta'}$,

and we have to take the part of the integral above which remains finite when $\eta_{\rm in}\to 0$.

Once the perturbation enters the horizon it either starts oscillating with roughly constant amplitude or continues to grow $\propto\eta^2$, depending on whether k_j enters during the radiation or matter dominated era. At late time, $\eta\gg\eta_{\rm eq}$ and $k\eta\gg 1$, we therefore obtain

$$D_{N_j}\approx g(1/k_1)k_1^{-2+3/N}(k_1/k_j)^2\times\prod_{n=1}^{N}(k_n/k_0)^{\alpha/N}\begin{cases}\left(\frac{\eta}{\eta_{\rm eq}}\right)^2 & \text{if } k_j\eta_{\rm eq}>1,\\ (\eta k_j)^2 & \text{if } k_j\eta_{\rm eq}<1,\end{cases}$$

where $\eta_{\rm eq}$ is the time of equality between the matter and radiation densities. Defining $0\leqslant j_{\rm eq}\leqslant N$ so that $k_j\eta_{\rm eq}>1$ for all $j\leqslant j_{\rm eq}$ we obtain for the connected N-point function

$$\mathcal{P}_N(\mathbf{k}_1,\ldots,\mathbf{k}_{N-1},\eta)\simeq g(1/k_1)^N k_1^3\eta^{2N}\times\prod_{n=1}^{N}\left(\frac{k_n}{k_0}\right)^{\alpha}\prod_{j=1}^{j_{\rm eq}}\left(\frac{1}{k_j\eta_{\rm eq}}\right)^2. \quad (13)$$

Using this result for the ordinary power spectrum $\mathcal{P}_2$, we can express $\mathcal{P}_N$ is terms of products of $\mathcal{P}_2$ as

$$\mathcal{P}_N(\mathbf{k}_1,\ldots,\mathbf{k}_{N-1},\eta)\simeq k_1^{3(1-N/2)}\prod_{j=1}^{N}\left(\sqrt{\mathcal{P}_2(k_j,\eta)}\frac{g(1/k_j)k_1^{3/2}}{g(1/k_1)k_j^{3/2}}\right). \quad (14)$$

For the class of models considered and under the assumption of perfect coherence, we have determined the connected N-point power spectra in the linear regime which are the input of the skewness (5).

M_3 has two contributions: A linear one due to the initial non-Gaussianity (contained in $\mathcal{P}_3$) and one due to nonlinear clustering which induces skewness even in an originally Gaussian distribution of perturbations; it contains a Gaussian part ($\mathcal{P}_2^2$) and a non-Gaussian term ($\mathcal{P}_4$). We decompose the skewness as $M_3=M_3^{(L)}+M_3^{(NL)}$. We want to estimate the ratio of these two contributions. Under our approximation (14), the first term of Eq. (5) reduces to

$$M_3^{(L)}=\int\frac{d^3\mathbf{q}d^3\mathbf{q}'}{(2\pi)^6}W_kW_qW_{|\mathbf{k}+\mathbf{k}'|}\sqrt{\mathcal{P}_2(k)\mathcal{P}_2(q)\mathcal{P}_2(|\mathbf{k}+\mathbf{q}|)}\times k_{\rm max}^{-3/2}\left[\frac{g(1/k_{\rm max})^3(kq|\mathbf{k}+\mathbf{q}|)^{3/2}}{g(1/k)g(1/q)g(1/|\mathbf{k}+\mathbf{q}|)(kq|\mathbf{k}+\mathbf{q}|)^{3/2}}\right], \quad (15)$$

where $k_{\rm max}\equiv\max\{k,q,|\mathbf{k}+\mathbf{q}|\}$. $M_3^{(NL)}$ is given by the second and third terms in Eq. (5).

To estimate analytically the ratio $M_3^{(L)}/M_3^{(NL)}=S_3^{(L)}/S_3^{(NL)}$, we make the following approximations: (i) We assume that $\mathcal{P}_2$ is a simple power law within the range of scales of interest, namely all the modes which enter the horizon during the radiation era, this is $0.1h^{-1}\,{\rm Mpc}\lesssim 2\pi/k\lesssim 20h^{-2}\,{\rm Mpc}$, namely $\mathcal{P}_2(k)=k^{-3}(k/k_*)^{\gamma}$. (ii) We also assume that $g(\eta)\propto\eta^r$. (iii) We replace the window function by a simple cutoff at $k=1/R$. (iv) For symmetry reasons we may integrate over the triangle $q\leqslant k\leqslant R$ and then multiply the result by 2. (v) Since in our integration region, $q\leqslant k$, we replace $|\mathbf{k}+\mathbf{q}|$ by k.

With these approximations the angular dependence of the integrand disappears and the integrals over $\mathbf{k}$ and $\mathbf{q}$ in Eq. (5) can be trivially performed leading to

$$M_3^{(L)}(R)\simeq\frac{4(k_*R)^{-3\gamma/2}}{(2\pi)^4 3\gamma(3+\gamma/2+r)}$$

$$\text{for}\quad \gamma>0\quad\text{and}\quad 3+\gamma/2+r>0,$$

$$M_3^{(NL)}(R)\simeq\frac{(k_*R)^{-2\gamma}}{(2\pi)^4\gamma^2}\quad\text{for}\quad\gamma>0, \quad (16)$$

where we have just considered the Gaussian contribution $\mathcal{P}_2^2$ to $M_3^{(NL)}$.

Since k_* is just the scale beyond which the density contrast $\langle D(\mathbf{x})^2\rangle_{R=1/k}\sim\mathcal{P}_2(k)k^3$ is larger than unity and nonlinearities become important, we define the nonlinearity scale $R_{\rm lin}=1/k_*$. The ratio between the skewness due to the non-Gaussianity in the linear perturbation and the one due to dynamical nonlinearities is then

$$\frac{S_3^{(L)}}{S_3^{(NL)}}\sim\frac{4\gamma}{3(3+\gamma/2-r)}\left(\frac{R}{R_{\rm lin}}\right)^{\gamma/2}. \quad (17)$$

This is our main result. It is readily checked that the non-Gaussian contribution $\mathcal{P}_4$ to $M_3^{(NL)}$ behaves just like the contribution $M_3^{(NL)}$ and thus only modifies the prefactor in Eq. (17), which should not be taken too seriously in view of the relatively crude approximations which we have employed to obtain our result.

This computation of the skewness is easily generalized to higher moments. As our computation shows, linear non-Gaussianities scale like

$$M_N^{(L)}(R)\propto(R/R_{\rm lin})^{-N\gamma/2}. \quad (18)$$

The dominant nonlinear contribution to the *connected* N-point function which is also present in Gaussian theories contains $N-2$ second order terms D_2 [2] and therefore scales like

$$M_N^{(\rm NL,Gauss)}(R)\propto(R/R_{\rm lin})^{-(N-1)\gamma}. \quad (19)$$

The lowest order nonlinearity for a generic non-Gaussian model, however, just comes from the non-Gaussian term with $N+1$ factors of D. The non-Gaussian nonlinear corrections therefore generically scale like

$$M_N^{(NL,\text{no Gauss})}(R) \propto (R/R_{\text{lin}})^{-(N+1)\gamma/2}. \tag{20}$$

Only for $N=3$ the two terms (19) and (20) scale in the same way. For all higher N's the non-Gaussian contribution dominates in the mildly nonlinear regime, $R \geqslant R_{\text{lin}}$. From Eq. (20) we infer that on large scales the ratios for all reduced N-point functions very generically scale like

$$S_N^{(L)}(R)/S_N^{(NL)}(R) \propto (R/R_{\text{lin}})^{\gamma/2}. \tag{21}$$

This expression agrees with other analytic predictions [5] as well as numerical simulations in a global texture model [7]. The agreement with the texture simulations which are decoherent suggests that the validity of our result extends beyond the conditions under which Eq. (21) was derived. More important than decoherence is that the source term decays at late times and therefore the density perturbations just evolve according to the homogeneous solution. This implies that at late times the N-point functions behave like the homogeneous growing mode to the Nth power, while the reduced N-point function induced by nonlinear clustering from Gaussian perturbations scales like the growing mode to the $2(N-1)$th power. Since topological defect sources decay on subhorizon scales, we conclude that the derived scaling behavior is also valid for them (this argument will be expanded in our follow up publication [11]).

Our result implies that on small scales ($R \lesssim R_{\text{lin}}$), the dominant contribution to the cumulants comes from nonlinear Newtonian gravitational clustering, and the Gaussian term actually dominates. Intrinsic deviations from Gaussianity are difficult to detect on small scales. Hence, we should look for signs of intrinsic non-Gaussianity at large scales ($R > R_{\text{lin}}$). This suggestion was expressed earlier based on qualitative physical arguments [5]; however, our present result is derived from first principles for a specific class of initial conditions—coherent seeds.

If galaxies trace mass, the measurements of the two-point correlation function suggest $R_{\text{lin}} \sim 10h^{-1}$ Mpc and $\gamma(R) \approx 1.8$ for $10\,\text{kpc} \lesssim hR \lesssim 15\,\text{Mpc}$ (here h is the usual parameterization for the Hubble constant in units of 100 $\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$); the slope γ becomes steeper at larger separations R [1,9]. A frequently considered theoretical possibility for long-wave tail of the initial $\mathcal{P}_2(k)$, called the Zel'dovich-Harrison spectrum, would give $\gamma=4$ at large separations. Hence, we can expect all S_N to ''blow up'' with increasing scale for the class of non-Gaussian models considered here, in contrast with models with Gaussian initial conditions. The available measurements of $S_3(R)$ and $S_4(R)$ do not show such a rise with scale and have already been used to constrain texture models [7]. Likewise, there are indications that the existent data from the Automatic Plate Measuring (APM) Galaxy Survey may already extend to sufficiently large scales to constrain the χ^2 Peebles model [19,20]. With surveys presently underway like the Sloan Digital Sky Survey [21], the prospects for using the approach outlined here to probe the statistics of the cosmological initial conditions will become even better.

In this work we derived a scaling law for the ''intrinsic to induced'' skewness ratio (17) for coherent seeds. We also showed how to generalize this law to higher cumulants. We plan to follow these calculations with more detailed predictions for coherent seed models and to confront our analytic results with numerical simulations as well as observational data from galaxy surveys [11]. Let us also repeat that the derived scaling laws seem to be more general than their derivation as they have been obtained numerically for global texture which are decoherent seeds. We actually believe that the origin of the scaling laws is not coherence but mainly the decay of the sources at late time and we therefore conjecture that they hold also for topological defects.

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