$K\bar{K}$ families in spectroscopy of exotic mesons?

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Applying a classical massive string model, we find that the exotic mesons $f_0(980)$, $f_1(1420)$, $f_J(1710)$, and $f_J(2220)$ lie on a Regge trajectory, where the spins of $f_J(1710)$ and $f_J(2220)$ are 2 and 4, respectively. According to this Regge trajectory, another member of the family, $f_3(2000)$ with spin=3 and mass $\cong$2 GeV, might exist.

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Spectroscopy of mesons has attracted growing interest in recent years. In addition to ordinary $q\bar{q}$ mesons, multiquark $qq\bar{q}\bar{q}$ states, meson-meson pair states, and lowest-lying glueballs are expected as well [1].

The existence of gluon self-coupling in QCD suggests that gluonia (or glueballs) and hybrids ($q\bar{q}g$) might exist. Another possible kind of non-$q\bar{q}$ mesons is multiquark states, which can be either baglike or clusters of mesons. Many of the best non-$q\bar{q}$ candidates lie close to important thresholds, which suggests that they might be bound states of a meson pair, such as $f_0(980)$, $a_0(980)$, $f_1(1420)$, $f_2(1520)$, $f_J(1710)$, and so on.

Several proposals have been made to describe the $f_0(980)$ and the $a_0(980)$ mesons as $q\bar{q}$ mesons [2], multiquark $qq\bar{q}\bar{q}$ states [3], or $K\bar{K}$ molecules [4]. According to the results from the Crystal Barrel [5], an isoscalar $f_0(1370)$ and isovector $a_0(1450)$ with masses and widths are more appropriate for the ground state $q\bar{q}$ nonet. If the $f_0(980)$ and the $a_0(980)$ are included in the $^3P_0$ $q\bar{q}$ scalar nonet, the low values for their masses and widths have called these assignments into question. It seems that a solution of the bound state of the multiquark $qq\bar{q}\bar{q}$ was not found in the region of effective mass 1 GeV by analysis of the quark potential model [4]. A recent theoretical calculation [6] based on the $K\bar{K}$ molecule structure can successfully explain the disappearance of a dip and the appearance of a peak in the region of the $f_0(980)$ resonance for the Brookhaven experiment [7] on the reaction $\pi^- p \rightarrow \pi^0\pi^0 n$ at $P_{lab}^{\pi^-}=18.3$ GeV/c as transfer momentum increases. The $\gamma\gamma$ decay widths of the $f_0(980)$ and the $a_0(980)$ resonances have also been used to discriminate the properties among the models, which have typically been presented as a support for the $K\bar{K}$ molecule structures of these states [8–10].

It is interesting to note that the $f_0(980)$ and the $a_0(980)$ are close to the $K(494)\bar{K}(494)$ threshold, the $f_1(1420)$, the $K(494)\bar{K}^*(892)$ threshold, and the $f_J(1710)$, the $K^*(892)\bar{K}^*(892)$ threshold. What is the relation between these mesons and $K\bar{K}$ molecules? Why does the $K\bar{K}$ mode have two states, the $f_0(980)$ and the $a_0(980)$? The spin of the $f_J(1710)$, especially, is still a question to be solved [1]. If it is a ground-state glueball, its spin should be 0 [11]. However, the WA76 experiment on 300 GeV/c pp interactions observed a structure that favors spin 2 at the same mass [12]. Another interesting point is that the $f_1(1420)$ is above the $K(494)\bar{K}^*(892)$ threshold and the $f_J(1710)$ below the $K(494)\bar{K}^*(1410)$ (with spin=1), the $K^*(892)\bar{K}^*(892)$ and the $\omega\phi$ (with spin=0 or 2) thresholds. The $f_J(2220)$, especially, is below the $K^*(892)\bar{K}^*(1410)$ (with spin=0 or 2) and above the $K(494)\bar{K}^*(1680)$ (with spin=1) thresholds. If the $f_1(1420)$ was simply regarded as a threshold enhancement of $K(494)\bar{K}^*(892)$ pair, or a $K\bar{K}\pi$ molecule [13], and the $f_J(1710)$ a composition of the $K(494)\bar{K}^*(1410)$, the $K^*(892)\bar{K}^*(892)$ and the $\omega\phi$ [14], it would be difficult for one to determine the spins of the $f_J(1710)$ (0, 1, or 2) and $f_J(2220)$ (0, 1, 2, or 3) and know the $f_J(2220)$ nature [a threshold enhancement of the $K(494)\bar{K}^*(1680)$, a bound state of the $K^*(892)\bar{K}^*(1410)$, or admixture of these two states] only based on the thresholds. This confusing picture points to a pressing need for a framework to examine these mesons in a unified way.

The invariant mass M of the mesonic states is related to the occupation number ($a_r^{\mu+}$ and a_r^{μ}) of Fock space by the spectral equation [15,16]

$$\alpha' M^2 = \sum_{r=0}^{\infty} r a_r^{\mu+} a_{r\mu}, \tag{1}$$

which may be interpreted by using the models of both a continuous string [17] and a relativistic oscillator [18,19]. The latter is often used in nuclear and particle physics. Thus, previous experience showed the value of the string model as a tool for including the systematics of meson states.

It is well known that the string model is usually used to describe quark confinement in QCD, where the current quark-antiquark with small masses are at the two end points of the string respectively. When one pulls the current quarks away, the energy of the string shall increase rapidly enough so as to produce a new current quark and a new current antiquark at the broken point. The new current quark (antiquark) and the original current antiquark (quark) become a new pair connected by the string. Thus, it is no way to observe the free quark. It is different situation between meson-

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TABLE I. Exotic mesons grouped in the $f_0(980)$ rotational family (Regge trajectory), where $J=-\mu^2/(4\alpha')+\alpha' M^2$.

J	0	1	2	3	4
M(GeV), $\alpha'=1.0$ GeV^2	0.98	1.40	1.72	1.99	2.227
M(GeV), $\alpha'=0.98$ GeV^2	0.98	1.421	1.744	2.015	2.254
Mesons	$f_0(980)$	$f_1(1420)$	$f_J(1710)$	$f_3(2000)$?	$f_J(2220)$
Experiment (MeV)	980 ± 10	1426.2 ± 1.2	1712 ± 5	?	2225 ± 6

meson pair. The energy is not enough to produce a new meson and a new antimeson at broken point when the string is broken. Thus, one can observe free mesons. Therefore, it is possible to use the string model for the meson-meson pair in some energy region. Our calculation below indicates that it might be $E<2.5$ GeV.

Let $x^\mu(\sigma,\tau)$ denote space-time coordinates of world-sheet of a string. $x^\mu(\sigma,\tau)$ form a mapping of the 2-dimensional string parameter space (σ,τ) into space-time. The Lagrangian of the massive string [20] can be expressed by

$$\mathcal{L}(x^\mu,\dot{x}^\mu,x'^\mu)=\frac{1}{2\pi\alpha'}\sqrt{(\dot{x}^\mu x'_\mu)^2-(\dot{x}^\mu)^2((x'_\nu)^2-\mu^2)},\tag{2}$$

where μ is a constant to be determined below, $1/2\pi\alpha'$ the energy per unit length in the rest system of a point along the string. And

$$\dot{x}^\mu=\frac{\partial x^\mu}{\partial\tau},\quad x'_\mu=\frac{\partial x_\mu}{\partial\sigma}.\tag{3}$$

The motion equation and the boundary condition of the string determined by Hamilton's principle may be expressed as

$$\frac{\partial}{\partial\tau}\frac{\partial\mathcal{L}}{\partial\dot{x}^\mu}+\frac{\partial}{\partial\sigma}\frac{\partial\mathcal{L}}{\partial x'^\mu}=0,\quad \left.\frac{\partial\mathcal{L}}{\partial x'^\mu}\right|_{\sigma=0,\pi}=0.\tag{4}$$

The corresponding conjugate momentum density may be written as

$$P^\mu=\frac{\partial\mathcal{L}}{\partial\dot{x}_\mu}=\frac{-1}{2\pi\alpha'}\frac{\dot{x}^\mu((x'_\nu)^2-\mu^2)-x'^\mu(\dot{x}^\nu x'_\nu)}{\{(\dot{x}^\mu x'_\mu)^2-(\dot{x}^\mu)^2((x'_\nu)^2-\mu^2)\}^{1/2}},\tag{5}$$

$$\Pi^\mu=\frac{\partial\mathcal{L}}{\partial x'_\mu}=\frac{-1}{2\pi\alpha'}\frac{x'^\mu(\dot{x}_\nu)^2-\dot{x}^\mu(\dot{x}^\nu x'_\nu)}{\{(\dot{x}^\mu x'_\mu)^2-(\dot{x}^\mu)^2((x'_\nu)^2-\mu^2)\}^{1/2}}.\tag{6}$$

Inserting Eqs. (5) and (6) into Eq. (4), we see that Eqs. (4) are nothing other than the statement about the conservation of the energy and momentum of the string: First formula in Eq. (4) means that the flow of energy momentum is locally conserved inside the string, second one in Eq.(4) means that no energy and momentum flows in or out of end of the string. Thus we may look for a set of rigid rotator solutions with $x^\mu(\sigma,\tau)=\{\tau,\rho(\sigma)\cos\omega\tau,\rho(\sigma)\sin\omega\tau,0\}$ to Eq. (4) [20]. After considerable algebraic manipulation, one finds that

$$\rho''+\omega^2\mu^2\rho-\omega^2\rho(\rho\rho''-\rho'^2)=0,\quad \rho'(\sigma)|_{\sigma=0,\pi}=0.\tag{7}$$

Thus the solutions to Eq. (7) are

$$\rho_n(\sigma)=\left(\frac{n^2-\mu^2\omega^2}{\omega^2 n^2}\right)^{1/2}\cos n\sigma,\quad n=1,2,\ldots.\tag{8}$$

Using the solutions, one may easily compute the energy and angular momentum of the string,

$$M=\int_0^\pi P^0 d\sigma=\frac{n}{2\alpha'\omega},$$
$$J=\int_0^\pi (x^1P^2-x^2P^1)d\sigma=\frac{n^2-\omega^2\mu^2}{4\alpha' n\omega^2}.\tag{9}$$

It is thus clear that the angular momentum J and the mass M squared of the string are directly proportional to each other,

$$J=\alpha' M^2/n-\frac{\mu^2}{4\alpha' n},\quad n=1,2,\ldots.\tag{10}$$

Therefore, the prediction of the model is that all known mesons do fall on straight lines known as Regge trajectories in a plot of spin vs squared mass. Every trajectory differs in constant μ of intercept for the different meson, where we take it as $\mu^2/4=0.96$ GeV^{-2}.

It is noted in Eq. (10) that there are several different modes for $n=1,2,\ldots$. Every mode should represent a different meson family [21]. We take the leading trajectory ($n=1$) to describe the $f_0(980)$ family. The result is shown in Table I, where the spin of the $f_J(1710)$ taken to be as 2, which is in agreement with experimental data of Armstrong *et al*. Thus the natural assignment for the $f_J(2220)$ is $J=4$. This state has been seen at SPEAR in the $K\bar{K}$ systems produced in the radiative decay of $J/\psi(1S)$ [22,23], where the spin 2 or 4 was suggested by authors. The $f_1(1420)$ particle, as pointed out by Longacre [13], is not a hybrid $q\bar{q}g$ meson or a four-quark state described in Refs. [24] and [25], but is a $K\bar{K}$ excited state in our results. Another $f_3(2000)$ with the quantum numbers $I^G(J^{PC})=0^+(3^{++})$ and $M\cong2$ GeV is not found in the Particle Data Group booklet [1].

From Eq. (10), we see that the Regge trajectory is strongly dependent on the intercept from the $f_0(980)$ mass,

$m_{f_0} = \sqrt{\mu^2/4\alpha'} = \sqrt{0.96}$ GeV $=0.98$ GeV. In other words, the rotational family is determined by the $f_0(980)$ constituents. If the $f_0(980)$ is a $K\bar{K}$ molecule, the $f_1(1420)$, the $f_J(1710)$ and the $f_J(2220)$ may be the $K\bar{K}$ excited states. This may be the reason why they all possess the hidden strangeness. The fact that resonances and boundstates with low internal energy (Q value) are easily produced means that the interpretation of these states as excited states of two mesons seems quite reasonable. This conclusion is further strengthened by observing that the opening of a new channel produces an effective attractive interaction in the open channel. Therefore, the missing $K\bar{K}$ excited state, the $f_3(2000)$, might exist.

If $n=2$ in Eq. (17) is assigned to describe the $a_0(980)$ family (see Table II). For $J=1$, the mass should be about 1.72 GeV. Therefore $\rho(1700)$ might be a candidate of the $a_0(980)$ family. For $J=2$, member of this family should have the mass $\cong 2.23$ GeV and the quantum numbers $I^G(J^{PC}) = 1^-(2^{++})$. This family may need to be studied further.

TABLE II. The $a_0(980)$ rotational family, where $2J = -\mu^2/(4\alpha') + \alpha' M^2$.

J	0	1	2
M(GeV), $\alpha' = 1.0$ GeV^2	0.98	1.72	2.227
M(GeV), $\alpha' = 0.98$ GeV^2	0.98	1.744	2.254
Mesons	$a_0(980)$	$\rho(1700)$	$a_2(2230)$?
Experiment (MeV)	983.4 ± 0.9	1700 ± 20	?

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