

Chiral symmetry restoration and axial vector renormalization for Wilson fermions

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Lattice gauge theories with Wilson fermions break chiral symmetry. In the $U(1)$ axial vector current this manifests itself in an anomaly. On the other hand it is generally expected that the axial vector flavor mixing current is nonanomalous. We give a short, but strict proof of this to all orders of perturbation theory, and show that chiral symmetry restoration implies a unique multiplicative renormalization constant for the current. This constant is determined entirely from an irrelevant operator in the Ward identity. The basic ingredients going into the proof are the lattice Ward identity, charge conjugation symmetry and the power counting theorem. We compute the renormalization constant to one loop order. It is largely independent of the particular lattice realization of the current.

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I. INTRODUCTION

Any realization of fermions on the lattice has to respect the constraints imposed by the Nielsen-Ninomiya theorem [1]. Whereas Wilson fermions break chiral symmetry explicitly, Ginsparg-Wilson fermions [2,3] have an exact chiral symmetry on the lattice that is generated by composite local lattice operators [4]. In both cases, the continuum chiral flavor mixing symmetry and the anomaly are to be properly reproduced as the cutoff is removed.

In a recent paper [5] it was shown that under very general conditions on the lattice Dirac operator, which, in particular, are satisfied both for Wilson and for Ginsparg-Wilson fermions, the axial anomaly is correctly generated in the continuum limit. The main conditions are gauge invariance, absence of doublers, and locality on the lattice in a more general sense. The origin of the anomaly is traced back to an irrelevant, local lattice operator in the axial vector Ward identity.

For Ginsparg-Wilson fermions, the composite operator in the chiral transformation, which ensures an exact flavor mixing symmetry on the lattice, stays irrelevant under renormalization [6]. As a consequence, the axial vector current does not require renormalization. On the other hand, for Wilson fermions it is not obvious, although widely believed, that the chiral symmetry becomes restored in the continuum limit. To one-loop order this has been verified in the literature [7–10]. In this paper we give a short but strict proof of this assertion to all orders of perturbation theory, based on lattice power counting for massless theories [11]. Although we explicitly refer to Wilson fermions, the result is as general as that for the anomaly generation given in [5]. As we shall show, the only role played by the irrelevant, symmetry breaking operator in the flavor mixing axial vector Ward identity is to give rise to a unique multiplicative renormalization Z_j of the axial vector current, ensuring that chiral symmetry is restored in the continuum limit.

We compute the one-loop contribution to Z_j from the ir-

relevant operator as a function of the Wilson parameter. The result is largely independent of the particular lattice regularization of the current. The values of Z_j agree with those obtained for a particular choice of the current in an earlier calculation [10].

II. GENERAL FRAMEWORK

Although our general proof will be given for QED, it generalizes in an obvious way to non-Abelian gauge theories with massless fermions.

A. Renormalized lattice QED

The action for renormalized QED is given by

$$S(A, \psi, \bar{\psi}) = S_W(U) + S_f(U, \psi, \bar{\psi}) + S_{gf}(A). \quad (1)$$

$S_W(U)$ is e.g. the Wilson plaquette action:

$$S_W(U) = Z_A \frac{1}{2g^2} \sum_{x \in aZ^4} \sum_{\mu \neq \nu=0}^3 [1 - U(x; \mu) \times U(x + a\hat{\mu}; \nu) U(x + a\hat{\nu}; \mu)^{-1} U(x; \nu)^{-1}], \quad (2)$$

with g the renormalized gauge coupling constant and $U(x, \mu) = \exp[iagA_\mu(x)] \in U(1)$. The fermion action is given by

$$S_f = a^4 \sum_{x \in aZ^4} Z_\psi \bar{\psi}(x) (D[U] + m_0) \psi(x), \quad (3)$$

with ψ a 2-flavor Dirac spinor field and with $D[U]$ the Wilson Dirac operator:

$$D[U]\psi(x) = \frac{1}{2a} \sum_{\mu=0}^3 [(\gamma_{\mu}-r)U(x;\mu)\psi(x+a\hat{\mu}) - (\gamma_{\mu}+r)U(x-a\hat{\mu};\mu)^{-1}\psi(x-a\hat{\mu}) + 2r\psi(x)]. \quad (4)$$

m_0 is the bare fermion mass, which for massless fields must be tuned to its critical value of $O(g^2)$. S_{gf} denotes the gauge fixing action. For concreteness we choose the Lorentz gauge

$$S_{gf}(A) = a^4 \sum_{x \in aZ^4} \frac{\lambda}{2} \left(\sum_{\mu=0}^3 \frac{1}{a} \hat{\partial}_{\mu}^* A_{\mu}(x) \right)^2, \quad (5)$$

with $\lambda > 0$ the gauge fixing parameter. Here and in the following, $\hat{\partial}_{\mu}$ and $\hat{\partial}_{\mu}^*$ denote the forward and backward lattice difference operators, respectively:

$$\hat{\partial}_{\mu} f(x) = f(x+a\hat{\mu}) - f(x), \quad \hat{\partial}_{\mu}^* f(x) = f(x) - f(x-a\hat{\mu}), \quad (6)$$

where $\hat{\mu}$ is the unit vector in μ direction.

The generating functional W of the connected correlation functions is given by

$$\exp W(J, \eta, \bar{\eta}) = \int \prod_x \left(d\psi(x) d\bar{\psi}(x) \prod_{\mu} dA_{\mu}(x) \right) \times \exp\{-S(A, \psi, \bar{\psi}) + S_c(A, \psi, \bar{\psi}; J, \eta, \bar{\eta})\} \quad (7)$$

with source term S_c :

$$S_c(A, \psi, \bar{\psi}; J, \eta, \bar{\eta}) = a^4 \sum_x \left\{ \sum_{\mu} J_{\mu}(x) A_{\mu}(x) + \bar{\eta}(x) \psi(x) + \bar{\psi}(x) \eta(x) \right\}. \quad (8)$$

The vertex functional Γ is obtained by a Legendre transformation

$$W(J, \eta, \bar{\eta}) = \Gamma(A, \psi, \bar{\psi}) + a^4 \sum_x \left(\sum_{\mu} J_{\mu}(x) A_{\mu}(x) + \bar{\eta}(x) \psi(x) + \bar{\psi}(x) \eta(x) \right), \quad (9)$$

where

$$a^4 A_{\mu}(x) = \frac{\partial W}{\partial J_{\mu}(x)}, \quad a^4 \psi(x) = \frac{\partial W}{\partial \bar{\eta}(x)}, \quad (10)$$

$$a^4 \bar{\psi}(x) = -\frac{\partial W}{\partial \eta(x)}.$$

By $\tilde{\Gamma}^{(n,m)}(k,l)$ we denote the momentum space vertex function of n fermion pairs and m gauge fields, with their col-

lected momenta denoted by k and l , respectively. For Q any composite local lattice operator, we write $\tilde{\Gamma}_Q^{(n,m)}(q;k,l)$ for the vertex function with one insertion of Q , with q its momentum. Momentum conservation is implied. Massless fermions require that

$$\text{tr} \tilde{\Gamma}^{(1,0)}(k=0) = 0 \quad (11)$$

to be achieved by tuning m_0 , where the trace is taken in spinor space. Z_A and Z_{ψ} are uniquely determined by appropriate normalization conditions at non-exceptional momenta, e.g. by

$$\left. \frac{i}{4} \frac{\partial}{\partial \bar{k}_0} \text{tr} \gamma_0 \tilde{\Gamma}^{(1,0)}(k) \right|_{\bar{k}} = 1, \quad \left. -\frac{1}{2} \frac{\partial}{\partial \hat{k}_0} \tilde{\Gamma}_{11}^{(0,2)}(k) \right|_{\bar{k}} = \hat{\mu}, \quad (12)$$

where $\bar{k} = (\bar{\mu} \neq 0, 0, 0, 0)$, $\hat{k} = (2/a) \sin(ka/2)$, $\bar{k} = (1/a) \sin(ka)$.

B. Symmetries

Below we make explicit reference to the charge conjugation symmetry

$$\Gamma(\mathcal{A}^C, \psi^C, \bar{\psi}^C) = \Gamma(\mathcal{A}, \psi, \bar{\psi}), \quad (13)$$

where

$$\mathcal{A}_{\mu}^C(x) = -\mathcal{A}_{\mu}(x), \quad \psi^C(x) = C \bar{\psi}(x)^T, \quad (14)$$

$$\bar{\psi}^C(x) = -\psi(x)^T C^{-1}.$$

The superscript T denotes transposition and C the charge conjugation matrix satisfying

$$C^{-1} \gamma_{\mu} C = -\gamma_{\mu}^T, \quad \mu = 0, \dots, 3. \quad (15)$$

Furthermore, applying a gauge transformation leads to the local Ward identity,

$$i \sum_{\mu=0}^3 \frac{1}{a} \hat{\partial}_{\mu}^* \frac{\partial \Gamma}{\partial A_{\mu}(x)} + \left[g \bar{\psi}(x) \frac{\partial \Gamma}{\partial \bar{\psi}(x)} - g \psi(x) \frac{\partial \Gamma}{\partial \psi(x)} \right] - i \lambda a \sum_{\mu, \nu=0}^3 \hat{\partial}_{\nu}^* \hat{\partial}_{\nu} \hat{\partial}_{\mu}^* A_{\mu}(x) = 0. \quad (16)$$

It implies that the renormalized action is of the form as stated above.

III. CHIRAL SYMMETRY BREAKING AND SYMMETRY RESTORATION

Chiral symmetry is broken by the Wilson Dirac operator. Under a local, flavor mixing chiral transformation

$$\delta \psi(x) = i \epsilon(x) \sigma_{\alpha} \gamma_5 \psi(x), \quad \delta \bar{\psi}(x) = i \epsilon(x) \bar{\psi}(x) \gamma_5 \sigma_{\alpha}, \quad (17)$$

where σ_{α} , $\alpha = 1, 2, 3$, denote the Pauli matrices acting in flavor space, the action transforms according to

$$\delta S = a^4 \sum_x i \epsilon(x) \left(- \sum_{\mu} \frac{1}{a} \hat{\partial}_{\mu}^* j_{\mu\alpha}(x) + \Delta_{\alpha}(x) + 2m_0 \mathcal{P}_{\alpha}(x) \right), \quad (18)$$

with

$$\mathcal{P}_{\alpha}(x) = Z_{\psi} \bar{\psi}(x) \gamma_5 \sigma_{\alpha} \psi(x). \quad (19)$$

The gauge invariant local operators j_{μ} and Δ are not uniquely determined by Eq. (18). In general, Δ is a local lattice operator which is classically irrelevant, that is,

$$\lim_{a \rightarrow 0} \Delta_{\alpha}(x) = 0. \quad (20)$$

It has UV degree 4 and IR degree 5. A convenient representation of $j_{\mu\alpha}$ and of Δ_{α} is given by

$$j_{\mu\alpha}(x) = Z_{\psi} \frac{1}{2} [\bar{\psi}(x) (\gamma_{\mu} + s) \gamma_5 \sigma_{\alpha} U(x; \mu) \psi(x + a \hat{\mu}) + \bar{\psi}(x + a \hat{\mu}) (\gamma_{\mu} - s) \gamma_5 \sigma_{\alpha} U(x; \mu)^{-1} \psi(x)], \quad (21)$$

$$\begin{aligned} \Delta_{\alpha}(x) = & -Z_{\psi} \frac{1}{2a} \sum_{\mu=0}^3 \{ (r-s) \bar{\psi}(x) \gamma_5 \sigma_{\alpha} [U(x; \mu) \psi(x + a \hat{\mu}) \\ & + U(x - a \hat{\mu}; \mu)^{-1} \psi(x - a \hat{\mu}) - 2\psi(x)] + (r+s) \\ & \times [\bar{\psi}(x + a \hat{\mu}) U(x; \mu)^{-1} + \bar{\psi}(x - a \hat{\mu}) U(x - a \hat{\mu}; \mu) \\ & - 2\bar{\psi}(x)] \gamma_5 \sigma_{\alpha} \psi(x) \}, \end{aligned}$$

where r is the Wilson parameter, and s some arbitrary real but otherwise fixed constant. The following discussion of renormalization does not depend on a particular choice of s .

We add to the source part of the action S_c a term

$$a^4 \sum_x \sum_{\alpha=1}^3 \left(\sum_{\mu=0}^3 G_{\mu\alpha}(x) j_{\mu\alpha}(x) + F_{\alpha}(x) [\Delta_{\alpha}(x) + 2m_0 \mathcal{P}_{\alpha}(x)] \right), \quad (22)$$

and denote the corresponding vertex functional by $\Gamma'(A, \psi, \bar{\psi}; G, F)$. Obviously, $\Gamma'(A, \psi, \bar{\psi}; G=0, F=0) = \Gamma(A, \psi, \bar{\psi})$. Then Eq. (18) implies that Γ' satisfies the axial vector current Ward identity

$$\begin{aligned} \sum_{\mu} \frac{1}{a} \hat{\partial}_{\mu}^* \frac{\partial \Gamma'}{\partial a^4 G_{\mu\alpha}(x)} + \left\{ \frac{\partial \Gamma'}{\partial a^4 \psi(x)} \sigma_{\alpha} \gamma_5 \psi(x) \right. \\ \left. - \bar{\psi}(x) \sigma_{\alpha} \gamma_5 \frac{\partial \Gamma'}{\partial a^4 \bar{\psi}(x)} \right\} = \frac{\partial \Gamma'}{\partial a^4 F_{\alpha}(x)} + O(F, G). \end{aligned} \quad (23)$$

The functional identity (23) is equivalent to the infinite set of momentum space Ward identities:

$$\begin{aligned} i \sum_{\mu=0}^3 \hat{q}_{\mu} \tilde{\Gamma}_{j_{\mu\alpha}}^{(n,m)}(q; k, l) - \tilde{\Gamma}_{QED}^{(n,m)}(k, l) \\ = \tilde{\Gamma}_{\Delta_{\alpha}}^{(n,m)}(q; k, l) + 2m_0 \tilde{\Gamma}_{\mathcal{P}_{\alpha}}^{(n,m)}(q; k, l). \end{aligned} \quad (24)$$

Here we have written $\tilde{\Gamma}_{QED}^{(n,m)}(k, l)$ for the pure QED part, which is a linear combination of $\tilde{\Gamma}^{(n,m)}(k, l)$ with $\gamma_5 \sigma_{\alpha}$ attached to the various external fermion lines, but with no composite operator inserted. According to the renormalization prescription of QED, it is UV finite and universal in the continuum limit.

QED is already renormalized, but because of $\Delta \neq 0$, the axial vector current j_{μ} requires additional renormalization. This renormalization is multiplicative. That is, there exists a renormalization constant Z_j such that

$$\tilde{\Gamma}_{j_{\mu\alpha R}}^{(n,m)}(q; k, l) = Z_j \tilde{\Gamma}_{j_{\mu\alpha}}^{(n,m)}(q; k, l) \quad (25)$$

is finite in the continuum limit, for all n and m . The renormalized current satisfies the Ward identities

$$i \sum_{\mu=0}^3 \hat{q}_{\mu} \tilde{\Gamma}_{j_{\mu\alpha R}}^{(n,m)}(q; k, l) - \tilde{\Gamma}_{QED}^{(n,m)}(k, l) = \tilde{\Gamma}_{\Delta_{\alpha R}}^{(n,m)}(q; k, l), \quad (26)$$

where

$$\begin{aligned} \tilde{\Gamma}_{\Delta_{\alpha R}}^{(n,m)}(q; k, l) = \tilde{\Gamma}_{\Delta_{\alpha}}^{(n,m)}(q; k, l) + \left[2m_0 \tilde{\Gamma}_{\mathcal{P}_{\alpha}}^{(n,m)}(q; k, l) \right. \\ \left. + i(Z_j - 1) \sum_{\mu=0}^3 \hat{q}_{\mu} \tilde{\Gamma}_{j_{\mu\alpha}}^{(n,m)}(q; k, l) \right] \end{aligned} \quad (27)$$

are the renormalized vertex functions with one Δ_{α} insertion. Because of

$$m_0 = O(g^2) \quad \text{and} \quad Z_j - 1 = O(g^2), \quad (28)$$

the part in brackets on the right hand side of Eq. (27) is equivalently obtained by adding local counterterms to the lattice source action S_c , that is, in Eq. (22), the term in square brackets is replaced by

$$\Delta_{\alpha}(x) + 2m_0 \mathcal{P}_{\alpha}(x) + (Z_j - 1) \frac{1}{a} \sum_{\mu=0}^3 \hat{\partial}_{\mu}^* j_{\mu\alpha}(x). \quad (29)$$

We now show that for the particular choice of Z_j , these counterterms provide precisely overall Taylor subtractions at zero momentum, for all correlation functions with one Δ insertion, according to their overall ultraviolet lattice divergence degrees. Together with Eq. (12), because Δ is an irrelevant local lattice operator, this then implies that

$$\lim_{a \rightarrow 0} \tilde{\Gamma}_{\Delta_{\alpha R}}^{(n,m)}(q; k, l) = 0, \quad (30)$$

to all orders of perturbation theory, and for all n and m [11]. The renormalized axial vector current becomes conserved in the continuum limit.

For the proof of this assertion, we recall that Δ is a local operator of IR degree 5. This implies that $\tilde{\Gamma}_{\Delta R}^{(1,1)}$ is continuous at zero momentum and $\tilde{\Gamma}_{\Delta R}^{(1,0)}$ is once continuously differentiable at zero momentum. [These are the only vertex functions with one Δ insertion that require overall UV subtractions, with overall (lattice) divergence degrees 0 and 1, respectively.]

First, charge conjugation symmetry implies that

$$\tilde{\Gamma}_{\Delta R}^{(1,1)}(0) = 0. \quad (31)$$

Furthermore, for the massless theory, satisfying Eq. (11), we obtain from Eq. (26), with $n=1$, $m=0$,

$$\tilde{\Gamma}_{\Delta R}^{(1,0)}(0) = 0, \quad (32)$$

because $\tilde{\Gamma}_{j\mu\alpha R}^{(1,0)}(q;k,l)$ is at most logarithmically infrared divergent if all momenta are sent to zero. Again, using charge conjugation symmetry, we know that, for small momenta,

$$\tilde{\Gamma}_{\Delta R}^{(1,0)}(q;k) = i \sum_{\mu=0}^3 \gamma_{\mu} q_{\mu} \gamma_5 \sigma_{\alpha} z_{\Delta} + o(q,k) \quad \text{as } q,k \rightarrow 0, \quad (33)$$

with (infrared) finite constant z_{Δ} . Hence, order by order, Z_j is uniquely determined by the requirement that

$$\tilde{\Gamma}_{\Delta R}^{(1,0)}(q;k) = o(q,k) \quad \text{as } q,k \rightarrow 0. \quad (34)$$

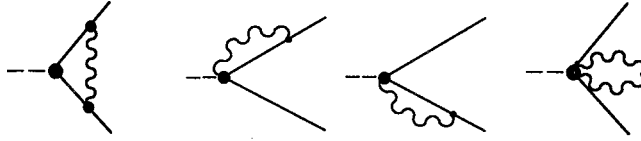
This completes the proof.

IV. AXIAL VECTOR RENORMALIZATION CONSTANT IN ONE-LOOP ORDER

To one-loop order the renormalization constant Z_j is determined from the condition (34), where $\tilde{\Gamma}_{\Delta R}^{(1,0)}$ is given according to Eq. (27) by

$$\begin{aligned} [\tilde{\Gamma}_{\Delta R}^{(1,0)}(q;k)]_{1-loop} &= [\tilde{\Gamma}_{\Delta R}^{(1,0)}(q;k)]_{1-loop} + 2m_0 \gamma_5 \sigma_{\alpha} \\ &+ i(Z_j - 1) \sum_{\mu=0}^3 \hat{q}_{\mu} \gamma_{\mu} \gamma_5 \sigma_{\alpha}. \end{aligned} \quad (35)$$

The lattice Feynman diagrams that contribute to the first term on the right hand side are listed below. The vertices that correspond to the Δ insertion are given in Appendix B:



(36)

To implement condition (34) we expand the right hand side of Eq. (35) around vanishing momentum up to first order. After a tedious but straight forward calculation one finds that

$$T_1 \tilde{\Gamma}_{\Delta R}^{(1,0)}(q;k) = i [-\zeta_{\Delta} + (Z_j - 1)] \sum_{\mu=0}^3 q_{\mu} \gamma_{\mu} \gamma_5 \sigma_{\alpha}, \quad (37)$$

where T_1 denotes the Taylor expansion to order 1 at zero momentum. ζ_{Δ} is given by the following (r -dependent) expression:

$$\zeta_{\Delta} = -C g^2 r^2 \int_{-\pi}^{\pi} \frac{d^4 l}{(2\pi)^4} \frac{h_{\sigma}(l)}{(\tilde{l}^2 + \hat{M}_r^2)^2}, \quad (38)$$

with $C=1$ for $U(1)$ and $C=(N^2-1)/(2N)$ for $SU(N)$, and

$$\begin{aligned} h_{\sigma}(l) &= \cos l_{\sigma} \left(\tilde{l}^2 - \tilde{l}_{\sigma}^2 + \frac{\hat{l}^2}{2} \eta_{\sigma}(l) \right) - \tilde{l}_{\sigma}^2 \eta_{\sigma}(l) + \frac{\tilde{l}^2}{2} \\ &+ \frac{1}{4} r^2 \hat{l}^2 \left(\hat{l}^2 \cos^2 \frac{l_{\sigma}}{2} - \tilde{l}_{\sigma}^2 \right), \end{aligned} \quad (39)$$

where

$$\hat{l}_{\mu} = 2 \sin \frac{l_{\mu}}{2}, \quad \hat{l}^2 = \sum_{\mu=0}^3 \hat{l}_{\mu}^2, \quad \tilde{l}_{\mu} = \sin l_{\mu}, \quad \tilde{l}^2 = \sum_{\mu=0}^3 \tilde{l}_{\mu}^2, \quad (40)$$

$$\hat{M}_r = \frac{r}{2} \hat{l}^2, \quad \eta_{\sigma}(l) = 2 \cos^2 \frac{l_{\sigma}}{2} - \sum_{\mu=0}^3 \cos^2 \frac{l_{\mu}}{2}.$$

σ is any one of the indices $0, \dots, 3$. Condition (34) now determines Z_j to be

$$Z_j = 1 + \zeta_{\Delta}. \quad (41)$$

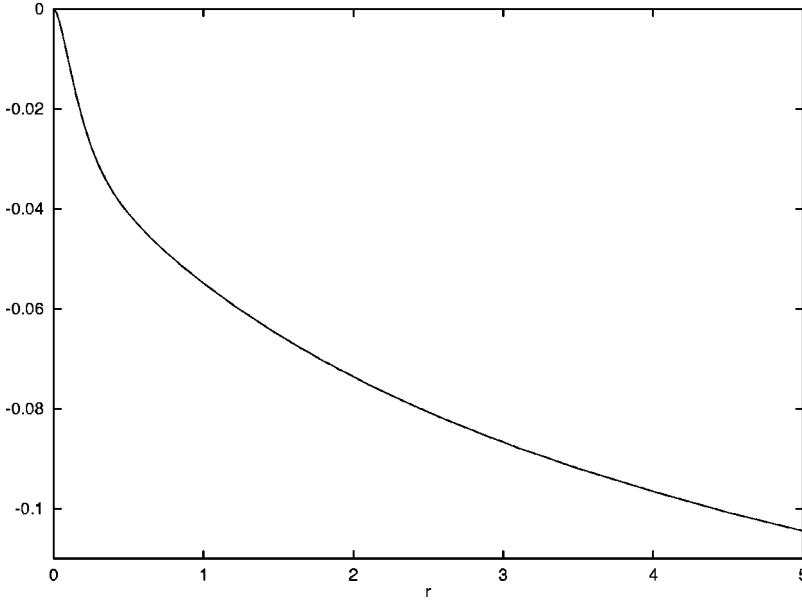


FIG. 1. $\zeta_\Delta(r)/(Cg^2)$ as function of the Wilson parameters r . We have $\zeta_\Delta(-r)=\zeta_\Delta(r)$ and $\zeta_\Delta(r)\neq 0$ whenever $r\neq 0$.

ζ_Δ does not depend on a particular realization of the axial vector current $j_{\mu\alpha}$, Eq. (21); that is, it is independent of the parameter s .

Note that the (finite) renormalization constant of the axial vector current is solely determined from diagrams involving the insertion of a classically irrelevant operator. The integral (38) is evaluated numerically. The dependence of ζ_Δ on the Wilson parameter is shown in Fig. 1. From this figure we see that ζ_Δ is a monotonically decreasing function of $r>0$. There exists no non-zero value of r for which the axial current is not renormalized. In particular, for the commonly used value $r=1$ we obtain $\zeta_\Delta=-0.0549Cg^2$.

V. CONCLUSION

In this paper we have investigated the renormalization of the flavor mixing axial vector current for massless gauge theories with Wilson fermions. The corresponding axial vector Ward identity involves a symmetry breaking lattice operator Δ which is local and classically irrelevant. Using the lattice power counting theorem for massless field theories, we have shown that Δ uniquely determines the renormalization constant of the axial vector current in such a way that the chiral symmetry becomes restored in the continuum limit, to all orders of perturbation theory.

We have computed the renormalization constant to one-loop order. It is largely independent of a particular lattice realization of the current and non-vanishing whenever the Wilson parameter $r\neq 0$. The values agree with those obtained for a particular choice of the axial vector current in [10].

Although we have considered Wilson fermions, the result of symmetry restoration in the continuum limit is quite general. It holds for any lattice Dirac operator that satisfies a general set of conditions. These conditions are gauge invariance and charge conjugation symmetry, absence of doublers, and locality in the more general sense as stated in [5,12].

APPENDIX A: SMALL MOMENTUM BEHAVIOR

The infrared properties of Δ ensure that the vertex functions $\tilde{\Gamma}_{\Delta R}^{(1,1)}(q;k,l)$ and $\tilde{\Gamma}_{\Delta R}^{(1,0)}(q;k)$ are continuous and once continuously differentiable at zero momentum, respectively. Their regular parts are obtained from the small momentum behavior of that part of the vertex functional Γ' that is linear in the source F . In momentum space it reads

$$\begin{aligned} & \int \frac{d^4q}{(2\pi)^4} \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \sum_{\mu\alpha} \tilde{F}_{\mu\alpha}(q) \\ & \times \{ (2\pi)^4 \delta(q+k_1+k_2) \tilde{\psi}(k_1) \\ & \times [i(\rho_r k_2 + \rho_l k_1)_\mu \gamma_\mu + \eta] \gamma_5 \sigma_\alpha \tilde{\psi}(k_2) \\ & + \xi \tilde{\psi}(k_1) \gamma_\mu A_{\mu}(-q-k_1-k_2) \gamma_5 \sigma_\alpha \tilde{\psi}(k_2) \} \end{aligned} \quad (\text{A1})$$

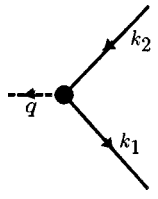
with c numbers ρ_r , ρ_l , η , and ξ . Applying the transformation (14) yields the same expression with

$$\rho_r \leftrightarrow \rho_l, \quad \eta \rightarrow \eta, \quad \xi \rightarrow -\xi. \quad (\text{A2})$$

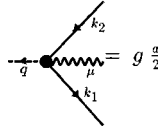
The symmetry (13) thus implies that $\rho_r=\rho_l$ and $\xi=0$. The vanishing of η is implied by the chiral Ward identity. This implies the statements (31)–(33).

APPENDIX B: FEYNMAN RULES

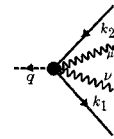
We state the Feynman rules for the insertion of one Δ_α operator, Eq. (21), that are required for the computation of the axial vector current renormalization constant Z_j to one-loop order. For simplicity the rules are given for gauge group $U(1)$:



$$= \frac{a}{2} \left[(r-s) \widehat{k_2^2} + (r+s) \widehat{k_1^2} \right] \gamma_5 \sigma_\alpha \quad (\text{B1})$$



$$= g \frac{a}{2} \left[(r-s) (q + \widehat{k_1 + k_2})_\mu - (r+s) (q - \widehat{k_1 - k_2})_\mu \right] \gamma_5 \sigma_\alpha \quad (\text{B2})$$



$$= g^2 a \left[(r-s) c_{q+k_1+k_2,\mu} + (r+s) c_{q-k_1-k_2,\mu} \right] \delta_{\mu\alpha} \gamma_5 \sigma_\alpha \quad (\text{B3})$$

where

$$c_{k,\mu} = \cos \frac{k_\mu a}{2}, \quad \widehat{k}_\mu = \frac{2}{a} \sin \frac{k_\mu a}{2}. \quad (\text{B4})$$

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