Exclusive semileptonic B decays to radially excited D mesons

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Exclusive semileptonic B decays to radially excited charmed mesons are investigated at the first order of heavy quark expansion. The arising leading and subleading Isgur-Wise functions are calculated in the framework of the relativistic quark model. It is found that the $1/m_Q$ corrections play an important role and substantially modify the results. An interesting interplay between different corrections is found. As a result the branching ratio for the $B\to D' e\nu$ decay is essentially increased by $1/m_Q$ corrections, while the one for $B\to D^{*\prime} e\nu$ is only slightly influenced by them.

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I. INTRODUCTION

The investigation of semileptonic decays of B mesons to excited charmed mesons represents a task interesting both from the experimental and theoretical points of view. The current experimental data on the semileptonic B decays to the ground state D mesons indicate that a substantial part ($\approx 40\%$) of the inclusive semileptonic B decays should go to excited D meson states. The first experimental data on some exclusive B decay channels to excited charmed mesons are becoming available now [1–3] and more data are expected in near future. Thus the comprehensive theoretical study of these decays is necessary. The presence of the heavy quark in the initial and final meson states in these decays considerably simplifies their theoretical description. A good starting point for this analysis is the infinitely heavy quark limit, $m_Q\to\infty$ [4]. In this limit the heavy quark symmetry arises, which strongly reduces the number of independent weak form factors [5]. The heavy quark mass and spin then decouple and all meson properties are determined by light-quark degrees of freedom alone. This leads to a considerable reduction of the number of independent form factors which are necessary for the description of heavy-to-heavy semileptonic decays. For example, in this limit only one form factor is necessary for the semileptonic B decay to S-wave D mesons (both for the ground state and its radial excitations), while the decays to P states require two form factors [5]. It is important to note that the heavy quark symmetry requires that in the infinitely heavy quark limit matrix elements between a B meson and an excited D meson should vanish at the point of zero recoil of the final excited charmed meson in the rest frame of the B meson. In the case of B decays to radially excited charmed mesons this is the result of the orthogonality of radial parts of wave functions, while for the decays to orbital excitations this is the consequence of orthogonality of their angular parts. However, some of the $1/m_Q$ corrections to these decay matrix elements can give nonzero contributions at zero recoil. As a result the role of these corrections could be considerably enhanced, since the kinematical range for B decays to excited states is a rather small region around zero recoil. Recent calculations of semileptonic B decays to orbitally excited (P-wave) charmed mesons with the account of the $1/m_Q$ corrections support this observation [6,7]. Our calculations [7] in the framework of the relativistic quark model show that some rates of B decays to orbitally excited charmed mesons receive contributions from first order $1/m_Q$ corrections approximately of the same value as a leading order contribution. In this paper we extend our analysis to B decays to radially excited D mesons.

Our relativistic quark model is based on the quasipotential approach in quantum field theory with a specific choice of the quark-antiquark interaction potential. It provides a consistent scheme for the calculation of all relativistic corrections at a given v^2/c^2 order and allows for the heavy quark $1/m_Q$ expansion. In preceding papers we applied this model to the calculation of the mass spectra of orbitally and radially excited states of heavy-light mesons [8], as well as to a description of weak decays of B mesons to ground state heavy and light mesons [9,10]. The heavy quark expansion for the ground state heavy-to-heavy semileptonic transitions [11] has been found to be in agreement with model-independent predictions of the heavy quark effective theory (HQET).

The paper is organized as follows. In Sec. II we carry out the heavy quark expansion for the weak decay matrix elements between a B meson and radially excited charmed meson states up to the first order in $1/m_Q$ using HQET. In our analysis we follow HQET derivations for the matrix elements between ground states [12,13] and for the matrix element between a B meson and orbitally excited charmed meson [6], as well as a general analysis of these matrix elements in Ref. [14]. In Sec. III we describe our relativistic quark

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model. The heavy quark expansion for decay matrix elements is then carried out up to the first order $1/m_Q$ corrections and compared to the model-independent HQET results in Secs. IV and V. We determine the leading and subleading Isgur-Wise functions and give our predictions for decay branching ratios in the heavy quark limit and with the account of $1/m_Q$ corrections. The electron spectra for the considered decays are also presented. Section VI contains our conclusions.

II. DECAY MATRIX ELEMENTS AND THE HEAVY QUARK EXPANSION

The matrix elements of the vector current ($J^V_\mu=\bar{c}\gamma_\mu b$) and axial vector current ($J^A_\mu=\bar{c}\gamma_\mu\gamma_5 b$) between B and radially excited D' or $D^{*\prime}$ mesons can be parametrized by six hadronic form factors:

$$\frac{\langle D'(v')|\bar{c}\gamma^\mu b|B(v)\rangle}{\sqrt{m_{D'}m_B}}=h_+(v+v')^\mu+h_-(v-v')^\mu,$$

$$\langle D'(v')|\bar{c}\gamma^\mu b\gamma_5|B(v)\rangle=0,$$

$$\frac{\langle D^{*\prime}(v',\epsilon)|\bar{c}\gamma^\mu b|B(v)\rangle}{\sqrt{m_{D^{*\prime}}m_B}}=ih_V\varepsilon^{\mu\alpha\beta\gamma}\epsilon^*_\alpha v'_\beta v_\gamma,$$

$$\frac{\langle D^{*\prime}(v',\epsilon)|\bar{c}\gamma^\mu\gamma_5 b|B(v)\rangle}{\sqrt{m_{D^{*\prime}}m_B}}=h_{A_1}(w+1)\epsilon^{*\mu}-(h_{A_2}v^\mu+h_{A_3}v'^\mu)(\epsilon^*\cdot v), \tag{1}$$

where $v(v')$ is the four-velocity of the $B(D^{(*)\prime})$ meson, ϵ^μ is a polarization vector of the final vector charmed meson, and the form factors h_i are dimensionless functions of the product of velocities $w=v\cdot v'$. The double differential decay rates expressed in terms of the form factors are

$$\frac{d^2\Gamma_{D'}}{dw\, d\cos\theta}=3\Gamma_0 r^3(w^2-1)^{3/2}\sin^2\theta[(1+r)h_+-(1-r)h_-]^2,$$

$$\frac{d^2\Gamma_{D^{*\prime}}}{dw\, d\cos\theta}=3\Gamma_0 r^{*3}\sqrt{w^2-1}\{\sin^2\theta[(w-r^*)h_{A_1}+(w^2-1)(h_{A_3}+r^*h_{A_2})]^2+(1-2r^*w+r^{*2})\times[(1+\cos^2\theta)[h^2_{A_1}+(w^2-1)h^2_V]-4\cos\theta\sqrt{w^2-1}h_{A_1}h_V]\}, \tag{2}$$

where $\Gamma_0=G_F^2|V_{cb}|^2m_B^5/(192\pi^3)$, $r=m_{D'}/m_B$, $r^*=m_{D^{*\prime}}/m_B$, and θ is the angle between the charged lepton and the charmed meson in the rest frame of the virtual W boson.

Now we expand the form factors h_i in powers of $1/m_Q$ up to first order and relate the coefficients in this expansion to universal Isgur-Wise functions. This is achieved by evaluating the matrix elements of the effective current operators arising from the HQET expansion of the weak currents. For simplicity we limit our analysis to the leading order in α_s and use the trace formalism [15]. Following Ref. [6], we introduce the matrix

$$H_v=\frac{1+\not{v}}{2}[P^{*\mu}_v\gamma_\mu-P_v\gamma_5], \tag{3}$$

composed from the fields P_v and $P^{*\mu}_v$ that destroy mesons in the $j^P=\frac{1}{2}^-$ doublet[1] with four-velocity v. At leading order of the heavy quark expansion ($m_Q\to\infty$) the matrix elements of the weak current between the ground and radially excited states destroyed by the fields in H_v and H'_v, respectively, are given by

$$\bar{c}\Gamma b\to\bar{h}^{(c)}_{v'}\Gamma h^{(b)}_v=\xi^{(n)}(w)\mathrm{Tr}\{\bar{H}'_{v'}\Gamma H_v\}, \tag{4}$$

where $h^{(Q)}_v$ is the heavy quark field in the effective theory. The leading order Isgur-Wise function $\xi^{(n)}(w)$ vanishes at the zero recoil ($w=1$) of the final meson for any Γ, because of the heavy quark symmetry and the orthogonality of the radially excited state wave function with respect to the ground state one.

At first order of the $1/m_Q$ expansion there are contributions from the corrections to the HQET Lagrangian

$$\delta\mathcal{L}=\frac{1}{2m_Q}\mathcal{L}^{(Q)}_{1,v}\equiv\frac{1}{2m_Q}[O^{(Q)}_{\mathrm{kin},v}+O^{(Q)}_{\mathrm{mag},v}], \tag{5}$$

$$O^{(Q)}_{\mathrm{kin},v}=\bar{h}^{(Q)}_v(iD)^2h^{(Q)}_v,\quad O^{(Q)}_{\mathrm{mag},v}=\bar{h}^{(Q)}_v\frac{g_s}{2}\sigma_{\alpha\beta}G^{\alpha\beta}h^{(Q)}_v$$

and from the tree-level matching of the weak current operator onto effective theory which contains a covariant derivative $D^\lambda=\partial^\lambda-ig_st_aA^\lambda_a$

[1]Here j is the total light quark angular momentum, and the superscript P denotes the meson parity.

$$\bar{c}\Gamma b \to \overline{h}_{v'}^{(c)}\left(\Gamma - \frac{i}{2m_c}\overleftarrow{\not{D}}\Gamma + \frac{i}{2m_b}\Gamma\vec{\not{D}}\right)h_v^{(b)}. \qquad (6)$$

The matrix elements of the latter operators can be parametrized as

$$\overline{h}_{v'}^{(c)} i\overleftarrow{D}_\lambda \Gamma h_v^{(b)} = \mathrm{Tr}\{\xi_\lambda^{(c)}\overline{H}_{v'}\Gamma H_v\},$$

$$\overline{h}_{v'}^{(c)}\Gamma i\vec{D}_\lambda h_v^{(b)} = \mathrm{Tr}\{\xi_\lambda^{(b)}\overline{H}_{v'}\Gamma H_v\}. \qquad (7)$$

The most general form for $\xi_\lambda^{(Q)}$ is [12]

$$\xi_\lambda^{(Q)} = \xi_+^{(Q)}(v+v')_\lambda + \xi_-^{(Q)}(v-v')_\lambda - \xi_3^{(Q)}\gamma_\lambda. \qquad (8)$$

The equation of motion for the heavy quark, $i(v\cdot D)h^{(Q)} = 0$, yields the relations between the form factors $\xi_i^{(Q)}$

$$\xi_+^{(c)}(1+w) + \xi_-^{(c)}(w-1) + \xi_3^{(c)} = 0$$

$$\xi_+^{(b)}(1+w) - \xi_-^{(b)}(w-1) + \xi_3^{(b)} = 0. \qquad (9)$$

The additional relations can be obtained from the momentum conservation and the definition of the heavy quark fields $h_v^{(Q)}$, which lead to the equation $i\partial_\nu(\overline{h}_{v'}^{(c)}\Gamma h_v^{(b)}) = (\bar{\Lambda}v_\nu - \bar{\Lambda}^{(n)}v'_\nu)\overline{h}_{v'}^{(c)}\Gamma h_v^{(b)}$, implying that

$$\xi_\lambda^{(c)} + \xi_\lambda^{(b)} = (\bar{\Lambda}v_\lambda - \bar{\Lambda}^{(n)}v'_\lambda)\xi^{(n)}. \qquad (10)$$

Here $\bar{\Lambda}(\bar{\Lambda}^{(n)}) = M(M^{(n)}) - m_Q$ is the difference between the heavy ground state (radially excited) meson and heavy quark masses in the limit $m_Q \to \infty$. This equation results in the following relations:

$$\xi_+^{(c)} + \xi_+^{(b)} + \xi_-^{(c)} + \xi_-^{(b)} = \bar{\Lambda}\xi^{(n)},$$

$$\xi_+^{(c)} + \xi_+^{(b)} - \xi_-^{(c)} - \xi_-^{(b)} = -\bar{\Lambda}^{(n)}\xi^{(n)},$$

$$\xi_3^{(c)} + \xi_3^{(b)} = 0. \qquad (11)$$

The relations (9) and (11) can be used to express the functions $\xi_{-,+}^{(Q)}$ in terms of $\tilde{\xi}_3(\equiv \xi_3^{(b)} = -\xi_3^{(c)})$ and the leading order function $\xi^{(n)}$:

$$\xi_-^{(c)} = \left(\frac{\bar{\Lambda}^{(n)}}{2} + \frac{1}{2}\,\frac{\bar{\Lambda}^{(n)} - \bar{\Lambda}}{w-1}\right)\xi^{(n)},$$

$$\xi_-^{(b)} = \left(\frac{\bar{\Lambda}}{2} - \frac{1}{2}\,\frac{\bar{\Lambda}^{(n)} - \bar{\Lambda}}{w-1}\right)\xi^{(n)},$$

$$\xi_+^{(c)} = \left(-\frac{\bar{\Lambda}^{(n)}}{2} + \frac{1}{2}\,\frac{\bar{\Lambda}^{(n)} + \bar{\Lambda}}{w+1}\right)\xi^{(n)} + \frac{1}{w+1}\tilde{\xi}_3,$$

$$\xi_+^{(b)} = \left(\frac{\bar{\Lambda}}{2} - \frac{1}{2}\,\frac{\bar{\Lambda}^{(n)} + \bar{\Lambda}}{w+1}\right)\xi^{(n)} - \frac{1}{w+1}\tilde{\xi}_3. \qquad (12)$$

The matrix elements of the $1/m_Q$ corrections resulting from insertions of higher-dimension operators of the HQET Lagrangian (5) have the structure [12]

$$i\int dx T\{\mathcal{L}_{1,v'}^{(c)}(x)[\overline{h}_{v'}^{(c)}\Gamma h_v^{(b)}](0)\} = 2\chi_1^{(c)}\mathrm{Tr}\{\overline{H}_{v'}\Gamma H_v\} + 2\,\mathrm{Tr}\left\{\chi_{\alpha\beta}^{(c)}\overline{H}_{v'}i\sigma^{\alpha\beta}\frac{1+\not{v}'}{2}\Gamma H_v\right\},$$

$$i\int dx T\{\mathcal{L}_{1,v}^{(b)}(x)[\overline{h}_{v'}^{(c)}\Gamma h_v^{(b)}](0)\} = 2\chi_1^{(b)}\mathrm{Tr}\{\overline{H}_{v'}\Gamma H_v\} + 2\,\mathrm{Tr}\left\{\chi_{\alpha\beta}^{(b)}\overline{H}_{v'}\Gamma\frac{1+\not{v}}{2}i\sigma^{\alpha\beta}H_v\right\}. \qquad (13)$$

The corrections coming from the kinetic energy term $O_{\rm kin}$ do not violate spin symmetry and, hence, the corresponding functions $\chi_1^{(Q)}$ effectively correct the leading order function $\xi^{(n)}$. The chromomagnetic operator $O_{\rm mag}$, on the other hand, explicitly violates spin symmetry. The most general decomposition of the tensor form factor $\chi_{\alpha\beta}^{(Q)}$ is [12,13]

$$\chi_{\alpha\beta}^{(c)} = \chi_2^{(c)}v_\alpha\gamma_\beta - \chi_3^{(c)}i\sigma_{\alpha\beta},$$

$$\chi_{\alpha\beta}^{(b)} = \chi_2^{(b)}v'_\alpha\gamma_\beta - \chi_3^{(b)}i\sigma_{\alpha\beta}. \qquad (14)$$

The functions $\chi_i^{(b)}$ contribute to the decay form factors (1) only in the linear combination $\chi_b = 2\chi_1^{(b)} - 4(w-1)\chi_2^{(b)} + 12\chi_3^{(b)}$. Thus five independent functions $\tilde{\xi}_3$, χ_b and $\tilde{\chi}_i(\equiv \chi_i^{(c)})$, as well as two mass parameters $\bar{\Lambda}$ and $\bar{\Lambda}^{(n)}$, are necessary to describe first order $1/m_Q$ corrections to matrix elements of *B* meson decays to radially excited *D* meson states. The resulting structure of the decay form factors is

$$h_+ = \xi^{(n)} + \varepsilon_c[2\tilde{\chi}_1 - 4(w-1)\tilde{\chi}_2 + 12\tilde{\chi}_3] + \varepsilon_b\chi_b,$$

$$h_- = \varepsilon_c\left[2\tilde{\xi}_3 - \left(\bar{\Lambda}^{(n)} + \frac{\bar{\Lambda}^{(n)} - \bar{\Lambda}}{w-1}\right)\xi^{(n)}\right] - \varepsilon_b\left[2\tilde{\xi}_3 - \left(\bar{\Lambda} - \frac{\bar{\Lambda}^{(n)} - \bar{\Lambda}}{w-1}\right)\xi^{(n)}\right],$$

$$h_V=\xi^{(n)}+\varepsilon_c\left[2\tilde{\chi}_1-4\tilde{\chi}_3+\left(\overline{\Lambda^{(n)}}+\frac{\overline{\Lambda^{(n)}}-\bar{\Lambda}}{w-1}\right)\xi^{(n)}\right]+\varepsilon_b\left[\chi_b+\left(\bar{\Lambda}-\frac{\overline{\Lambda^{(n)}}-\bar{\Lambda}}{w-1}\right)\xi^{(n)}-2\tilde{\xi}_3\right],$$

$$h_{A_1}=\xi^{(n)}+\varepsilon_c\left[2\tilde{\chi}_1-4\tilde{\chi}_3+\frac{w-1}{w+1}\left(\overline{\Lambda^{(n)}}+\frac{\overline{\Lambda^{(n)}}-\bar{\Lambda}}{w-1}\right)\xi^{(n)}\right]+\varepsilon_b\left\{\chi_b+\frac{w-1}{w+1}\left[\left(\bar{\Lambda}-\frac{\overline{\Lambda^{(n)}}-\bar{\Lambda}}{w-1}\right)\xi^{(n)}-2\tilde{\xi}_3\right]\right\},$$

$$h_{A_2}=\varepsilon_c\left\{4\tilde{\chi}_2-\frac{2}{w+1}\left[\left(\overline{\Lambda^{(n)}}+\frac{\overline{\Lambda^{(n)}}-\bar{\Lambda}}{w-1}\right)\xi^{(n)}+\tilde{\xi}_3\right]\right\},$$

$$h_{A_3}=\xi^{(n)}+\varepsilon_c\left[2\tilde{\chi}_1-4\tilde{\chi}_2-4\tilde{\chi}_3+\frac{w-1}{w+1}\left(\overline{\Lambda^{(n)}}+\frac{\overline{\Lambda^{(n)}}-\bar{\Lambda}}{w-1}\right)\xi^{(n)}-\frac{2}{w+1}\tilde{\xi}_3\right]+\varepsilon_b\left[\chi_b+\left(\bar{\Lambda}-\frac{\overline{\Lambda^{(n)}}-\bar{\Lambda}}{w-1}\right)\xi^{(n)}-2\tilde{\xi}_3\right], \tag{15}$$

where $\varepsilon_Q=1/(2m_Q)$.

In the following sections we apply the relativistic quark model to the calculation of leading and subleading Isgur-Wise functions.

III. RELATIVISTIC QUARK MODEL

In the quasipotential approach, a meson is described by the wave function of the bound quark-antiquark state, which satisfies the quasipotential equation [16] of the Schrödinger type [17]:

$$\left(\frac{b^2(M)}{2\mu_R}-\frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_M(\mathbf{p})=\int\frac{d^3q}{(2\pi)^3}V(\mathbf{p},\mathbf{q};M)\Psi_M(\mathbf{q}), \tag{16}$$

where the relativistic reduced mass is

$$\mu_R=\frac{M^4-(m_q^2-m_Q^2)^2}{4M^3}. \tag{17}$$

Here $m_{q,Q}$ are the masses of light and heavy quarks, and $\mathbf{p}$ is their relative momentum. In the center-of-mass system the relative momentum squared on mass shell reads

$$b^2(M)=\frac{[M^2-(m_q+m_Q)^2][M^2-(m_q-m_Q)^2]}{4M^2}. \tag{18}$$

The kernel $V(\mathbf{p},\mathbf{q};M)$ in Eq. (16) is the quasipotential operator of the quark-antiquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive energy states. An important role in this construction is played by the Lorentz structure of the confining quark-antiquark interaction in the meson. In constructing the quasipotential of the quark-antiquark interaction we have assumed that the effective interaction is the sum of the usual one-gluon exchange term and the mixture of vector and scalar linear confining potentials. The quasipotential is then defined by [18]

$$\begin{aligned}V(\mathbf{p},\mathbf{q};M)&=\bar{u}_q(p)\bar{u}_Q(-p)\mathcal{V}(\mathbf{p},\mathbf{q};M)u_q(q)u_Q(-q)\\&=\bar{u}_q(p)\bar{u}_Q(-p)\left\{\frac{4}{3}\alpha_sD_{\mu\nu}(\mathbf{k})\gamma_q^\mu\gamma_Q^\nu\right.\\&\left.\quad+V_{\rm conf}^V(\mathbf{k})\Gamma_q^\mu\Gamma_{Q;\mu}+V_{\rm conf}^S(\mathbf{k})\right\}u_q(q)u_Q(-q),\end{aligned} \tag{19}$$

where α_s is the QCD coupling constant, $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge and $\mathbf{k}=\mathbf{p}-\mathbf{q}$; γ_μ and $u(p)$ are the Dirac matrices and spinors

$$u^\lambda(p)=\sqrt{\frac{\epsilon(p)+m}{2\epsilon(p)}}\begin{pmatrix}1\\ \dfrac{\boldsymbol{\sigma}\mathbf{p}}{\epsilon(p)+m}\end{pmatrix}\chi^\lambda, \tag{20}$$

with $\epsilon(p)=\sqrt{\mathbf{p}^2+m^2}$. The effective long-range vector vertex is given by

$$\Gamma_\mu(\mathbf{k})=\gamma_\mu+\frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu, \tag{21}$$

where κ is the Pauli interaction constant characterizing the nonperturbative anomalous chromomagnetic moment of quarks. Vector and scalar confining potentials in the nonrelativistic limit reduce to

$$V_{\rm conf}^V(r)=(1-\varepsilon)(Ar+B),\quad V_{\rm conf}^S(r)=\varepsilon(Ar+B), \tag{22}$$

reproducing

$$V_{\rm conf}(r)=V_{\rm conf}^S(r)+V_{\rm conf}^V(r)=Ar+B, \tag{23}$$

where ε is the mixing coefficient.

The quasipotential for the heavy quarkonia, expanded in v^2/c^2, can be found in Refs. [18,19] and for heavy-light mesons in [8]. All the parameters of our model, such as quark masses, parameters of the linear confining potential, mixing coefficient ε, and anomalous chromomagnetic quark moment κ, were fixed from the analysis of heavy quarkonia

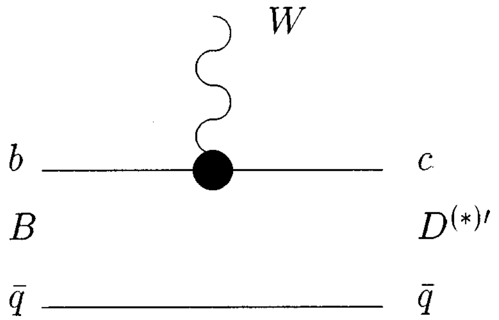

FIG. 1. Lowest order vertex function $\Gamma^{(1)}$ contributing to the current matrix element (25).

masses [18] and radiative decays [20]. The quark masses $m_b=4.88$ GeV, $m_c=1.55$ GeV, $m_s=0.50$ GeV, $m_{u,d}=0.33$ GeV, and the parameters of the linear potential $A=0.18$ GeV^2 and $B=-0.30$ GeV have the usual quark model values. In Ref. [11] we have considered the expansion of the matrix elements of weak heavy quark currents between pseudoscalar and vector meson ground states up to the second order in inverse powers of the heavy quark masses. It has been found that the general structure of the leading, first, and second order $1/m_Q$ corrections in our relativistic model is in accord with the predictions of HQET. The heavy quark symmetry and QCD impose rigid constraints on the parameters of the long-range potential in our model. The analysis of the first order corrections [11] allowed us to fix the value of the Pauli interaction constant $\kappa=-1$. The same value of κ was found previously from the fine splitting of heavy quarkonia ^3P_J- states [18].[2] Note that the long-range chromomagnetic spin-dependent interaction in our model is proportional to $(1+\kappa)$ and thus vanishes for $\kappa=-1$ in agreement with the flux tube model [22]. The value of the mixing parameter of vector and scalar confining potentials $\varepsilon=-1$ has been found from the analysis of the second order corrections [11]. This value is very close to the one determined from radiative decays of heavy quarkonia [20].

In order to calculate the exclusive semileptonic decay rate of the B meson, it is necessary to determine the corresponding matrix element of the weak current between meson states. In the quasipotential approach, the matrix element of the weak current $J^W=\bar{c}\gamma_\mu(1-\gamma^5)b$ between a B meson and a radially excited $D^{(*)\prime}$ meson takes the form [23]

$$\langle D^{(*)\prime}|J^W_\mu(0)|B\rangle=\int\frac{d^3p\,d^3q}{(2\pi)^6}\bar{\Psi}_{D^{(*)\prime}}(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q})\Psi_B(\mathbf{q}), \tag{24}$$

where $\Gamma_\mu(\mathbf{p},\mathbf{q})$ is the two-particle vertex function and $\Psi_{B,D^{(*)\prime}}$ are the meson wave functions projected onto the positive energy states of quarks and boosted to the moving reference frame. The contributions to Γ come from Figs. 1 and 2.[3] In the heavy quark limit $m_{b,c}\to\infty$ only $\Gamma^{(1)}$ contributes, while $\Gamma^{(2)}$ contributes at $1/m_Q$ order. They look like

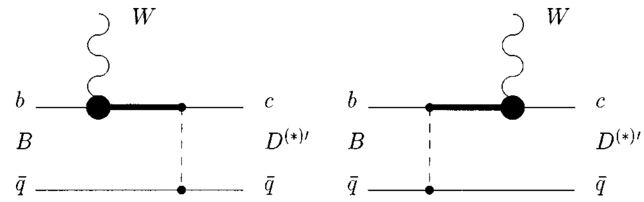

FIG. 2. Vertex function $\Gamma^{(2)}$ taking the quark interaction into account. Dashed lines correspond to the effective potential $\mathcal{V}$ in Eq. (19). Bold lines denote the negative-energy part of the quark propagator.

$$\Gamma^{(1)}_\mu(\mathbf{p},\mathbf{q})=\bar{u}_c(p_c)\gamma_\mu(1-\gamma^5)u_b(q_b)(2\pi)^3\delta(\mathbf{p}_q-\mathbf{q}_q), \tag{25}$$

and

$$\Gamma^{(2)}_\mu(\mathbf{p},\mathbf{q})=\bar{u}_c(p_c)\bar{u}_q(p_q)\times\Bigg\{\gamma_{Q\mu}(1-\gamma_Q^5)\frac{\Lambda_b^{(-)}(k)}{\epsilon_b(k)+\epsilon_b(p_c)}\gamma_Q^0\mathcal{V}(\mathbf{p}_q-\mathbf{q}_q)+\mathcal{V}(\mathbf{p}_q-\mathbf{q}_q)\frac{\Lambda_c^{(-)}(k')}{\epsilon_c(k')+\epsilon_c(q_b)}\gamma_Q^0\gamma_{Q\mu}\times(1-\gamma_Q^5)\Bigg\}u_b(q_b)u_q(q_q), \tag{26}$$

where the superscripts "(1)" and "(2)" correspond to Figs. 1 and 2, $Q=c$ or b, $\mathbf{k}=\mathbf{p}_c-\mathbf{\Delta}$; $\mathbf{k}'=\mathbf{q}_b+\mathbf{\Delta}$; $\mathbf{\Delta}=\mathbf{p}_{D^{(*)\prime}}-\mathbf{p}_B$;

$$\Lambda^{(-)}(p)=\frac{\epsilon(p)-(m\gamma^0+\gamma^0(\boldsymbol{\gamma}\mathbf{p}))}{2\epsilon(p)}.$$

Here [23]

$$p_{c,q}=\epsilon_{c,q}(p)\frac{p_{D^{(*)\prime}}}{M_{D^{(*)\prime}}}\pm\sum_{i=1}^{3}n^{(i)}(p_{D^{(*)\prime}})p^i,$$

$$q_{b,q}=\epsilon_{b,q}(q)\frac{p_B}{M_B}\pm\sum_{i=1}^{3}n^{(i)}(p_B)q^i,$$

and $n^{(i)}$ are three four-vectors given by

$$n^{(i)\mu}(p)=\left\{\frac{p^i}{M},\ \delta_{ij}+\frac{p^ip^j}{M(E+M)}\right\},\quad E=\sqrt{\mathbf{p}^2+M^2}.$$

[2]It has been known for a long time that the correct reproduction of the spin-dependent part of the quark-antiquark interaction requires either assuming the scalar confinement or equivalently introducing the Pauli interaction with $\kappa=-1$ [21,18,19] in the vector confinement.

[3]The contribution $\Gamma^{(2)}$ is the consequence of the projection onto the positive-energy states. Note that the form of the relativistic corrections resulting from the vertex function $\Gamma^{(2)}$ is explicitly dependent on the Lorentz structure of the $q\bar{q}$ interaction.

It is important to note that the wave functions entering the weak current matrix element (24) are not in the rest frame in general. For example, in the B meson rest frame, the $D^{(*)\prime}$ meson is moving with the recoil momentum $\mathbf{\Delta}$. The wave function of the moving $D^{(*)\prime}$ meson $\Psi_{D^{(*)\prime}\mathbf{\Delta}}$ is connected with the $D^{(*)\prime}$ wave function in the rest frame $\Psi_{D^{(*)\prime}\mathbf{0}}$ by the transformation [23]

$$\Psi_{D^{(*)\prime}\mathbf{\Delta}}(\mathbf{p})=D_c^{1/2}(R^W_{L_{\mathbf{\Delta}}})D_q^{1/2}(R^W_{L_{\mathbf{\Delta}}})\Psi_{D^{(*)\prime}\mathbf{0}}(\mathbf{p}), \qquad (27)$$

where R^W is the Wigner rotation, $L_{\mathbf{\Delta}}$ is the Lorentz boost from the meson rest frame to a moving one, and the rotation matrix $D^{1/2}(R)$ in spinor representation is given by

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} D^{1/2}_{c,q}(R^W_{L_{\mathbf{\Delta}}})=S^{-1}(\mathbf{p}_{c,q})S(\mathbf{\Delta})S(\mathbf{p}), \qquad (28)$$

where

$$S(\mathbf{p})=\sqrt{\frac{\epsilon(p)+m}{2m}}\left(1+\frac{\boldsymbol{\alpha p}}{\epsilon(p)+m}\right)$$

is the usual Lorentz transformation matrix of the four-spinor.

IV. LEADING AND SUBLEADING ISGUR-WISE FUNCTIONS

Now we can perform the heavy quark expansion for the matrix elements of B decays to radially excited D mesons in the framework of our model and determine leading and subleading Isgur-Wise functions. We substitute the vertex functions $\Gamma^{(1)}$ and $\Gamma^{(2)}$ given by Eqs. (25) and (26) in the decay matrix element (24) and take into account the wave function transformation (27). The resulting structure of this matrix element is rather complicated, because it is necessary to integrate both over d^3p and d^3q. The δ function in expression (25) permits us to perform one of these integrations and thus this contribution can be easily calculated. The calculation of the vertex function $\Gamma^{(2)}$ contribution is more difficult. Here, instead of a δ function, we have a complicated structure, containing the $Q\bar{q}$ interaction potential in the meson. However, we can expand this contribution in inverse powers of heavy (b,c) quark masses and then use the quasipotential equation in order to perform one of the integrations in the current matrix element. We carry out the heavy quark expansion up to first order in $1/m_Q$. It is easy to see that the vertex function $\Gamma^{(2)}$ contributes already at the subleading order of the $1/m_Q$ expansion. Then we compare the arising decay matrix elements with the form factor decomposition (1) and determine the corresponding form factors. We find that, for the chosen values of our model parameters (the mixing coefficient of vector and scalar confining potential $\varepsilon=-1$ and the Pauli constant $\kappa=-1$), the resulting structure at leading and subleading order in $1/m_Q$ coincides with the model-independent predictions of HQET given by Eq. (15). We get the following expressions for leading and subleading Isgur-Wise functions:

$$\xi^{(1)}(w)=\left(\frac{2}{w+1}\right)^{1/2}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}^{(0)}_{D^{(*)\prime}}\left(\mathbf{p}+\frac{2\epsilon_q}{M_{D^{(*)\prime}}(w+1)}\mathbf{\Delta}\right)\psi^{(0)}_B(\mathbf{p}), \qquad (29)$$

$$\tilde{\xi}_3(w)=\left(\frac{\bar{\Lambda}^{(1)}+\bar{\Lambda}}{2}-m_q+\frac{1}{6}\,\frac{\bar{\Lambda}^{(1)}-\bar{\Lambda}}{w-1}\right)\left(1+\frac{2}{3}\,\frac{w-1}{w+1}\right)\xi^{(1)}(w), \qquad (30)$$

$$\tilde{\chi}_1(w)\cong\frac{1}{20}\,\frac{w-1}{w+1}\,\frac{\bar{\Lambda}^{(1)}-\bar{\Lambda}}{w-1}\xi^{(1)}(w)+\frac{\bar{\Lambda}^{(1)}}{2}\left(\frac{2}{w+1}\right)^{1/2}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}^{(1)si}_{D^{(*)\prime}}\left(\mathbf{p}+\frac{2\epsilon_q}{M_{D^{(*)\prime}}(w+1)}\mathbf{\Delta}\right)\psi^{(0)}_B(\mathbf{p}), \qquad (31)$$

$$\tilde{\chi}_2(w)\cong-\frac{1}{12}\,\frac{1}{w+1}\,\frac{\bar{\Lambda}^{(1)}-\bar{\Lambda}}{w-1}\xi^{(1)}(w), \qquad (32)$$

$$\tilde{\chi}_3(w)\cong-\frac{3}{80}\,\frac{w-1}{w+1}\,\frac{\bar{\Lambda}^{(1)}-\bar{\Lambda}}{w-1}\xi^{(1)}(w)+\frac{\bar{\Lambda}^{(1)}}{4}\left(\frac{2}{w+1}\right)^{1/2}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}^{(1)sd}_{D^{(*)\prime}}\left(\mathbf{p}+\frac{2\epsilon_q}{M_{D^{(*)\prime}}(w+1)}\mathbf{\Delta}\right)\psi^{(0)}_B(\mathbf{p}), \qquad (33)$$

$$\chi_b(w)\cong\bar{\Lambda}\left(\frac{2}{w+1}\right)^{1/2}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}^{(0)}_{D^{(*)\prime}}\left(\mathbf{p}+\frac{2\epsilon_q}{M_{D^{(*)\prime}}(w+1)}\mathbf{\Delta}\right)[\psi^{(1)si}_B(\mathbf{p})-3\psi^{(1)sd}_B(\mathbf{p})], \qquad (34)$$

where $\mathbf{\Delta}^2=M^2_{D^{(*)\prime}}(w^2-1)$. Here we used the expansion for the S-wave meson wave function

$$\psi_M=\psi^{(0)}_M+\bar{\Lambda}_M\varepsilon_Q(\psi^{(1)si}_M+d_M\psi^{(1)sd}_M)+O(1/m_Q^2),$$

where $\psi^{(0)}_M$ is the wave function in the limit $m_Q\to\infty$, $\psi^{(1)si}_M$ and $\psi^{(1)sd}_M$ are the spin-independent and spin-dependent first order $1/m_Q$ corrections, $d_P=-3$ for pseudoscalar, and $d_V=1$ for vector mesons. The symbol $\cong$ in the expressions

TABLE I. Masses of radially excited $D^{(*)\prime}$ mesons and the mass parameters $\bar{\Lambda}$ in our model.

Parameter	State	Value (GeV) [8]	Exp. (GeV) [25]
$m_{D'}$	$D'(2S_0)$	2.579	
$m_{D^{*\prime}}$	$D^{*\prime}(2S_1)$	2.629	2.637(9)
$\bar{\Lambda}$	$B,D(1S)$	0.51	
$\bar{\Lambda}^{(1)}$	$D(2S)$	0.94	
$m_{D_s'}$	$D_s'(2S_0)$	2.670	
$m_{D_s^{*\prime}}$	$D_s^{*\prime}(2S_1)$	2.716	
$\bar{\Lambda}_s$	$B_s,D_s(1S)$	0.61	
$\bar{\Lambda}_s^{(1)}$	$D_s(2S)$	1.05	

(31)–(34) for the subleading functions $\tilde{\chi}_i(w)$ means that the corrections suppressed by an additional power of the ratio $(w-1)/(w+1)$, which is equal to zero at $w=1$ and less than 1/6 at $w_{\max}$, were neglected. Since the main contribution to the decay rate comes from the values of form factors close to $w=1$, these corrections turn out to be unimportant.

It is clear from the expression (29) that the leading order contribution vanishes at the point of zero recoil ($\mathbf{\Delta}=0,w=1$) of the final $D^{(*)\prime}$ meson, since the radial parts of the wave functions $\Psi_{D^{(*)\prime}}$ and Ψ_B are orthogonal in the infinitely heavy quark limit. The $1/m_Q$ corrections to the current (12) also do not contribute at this kinematical point for the same reason. The only nonzero contributions at $w=1$ come from corrections to the Lagrangian[4] $\tilde{\chi}_1(w)$, $\tilde{\chi}_3(w)$, and $\chi_b(w)$. From Eqs. (15) one can find for the form factors contributing to the decay matrix elements at zero recoil

$$h_+(1)=\varepsilon_c[2\tilde{\chi}_1(1)+12\tilde{\chi}_3(1)]+\varepsilon_b\chi_b(1),$$

$$h_{A_1}(1)=\varepsilon_c[2\tilde{\chi}_1(1)-4\tilde{\chi}_3(1)]+\varepsilon_b\chi_b(1). \qquad (35)$$

Such nonvanishing contributions at zero recoil result from the first order $1/m_Q$ corrections to the wave functions [see Eq. (34) and the last terms in Eqs. (31), (33)]. Since the kinematically allowed range for these decays is not broad ($1\leqslant w\leqslant w_{\max}\approx 1.27$) the relative contribution to the decay rate of such small $1/m_Q$ corrections is substantially increased. Note that the terms $\varepsilon_Q(\bar{\Lambda}^{(n)}-\bar{\Lambda})\xi^{(n)}(w)/(w-1)$ have the same behavior near $w=1$ as the leading order contribution, in contrast to decays to the ground state $D^{(*)}$ mesons [24], where $1/m_Q$ corrections are suppressed with respect to the leading order contribution by the factor $(w-1)$ near this point (this result is known as Luke's theorem [12]). Since inclusion of first order heavy quark corrections to B decays to the ground state $D^{(*)}$ mesons results in approximately a 10–20% increase of decay rates [11,13], one could expect that the influence of these corrections on decay rates to radially excited $D^{(*)\prime}$ mesons will be more essential. Our numerical analysis supports these observations.

[4]There are no normalization conditions for these corrections contrary to the decay to the ground state $D^{(*)}$ mesons, where the conservation of vector current requires their vanishing at zero recoil [12].

TABLE II. Leading and subleading Isgur-Wise functions and their slopes $\rho^2_{\xi_i}=-(1/\xi_i)(\partial/\partial w)\xi_i|_{w=1}$ and $\rho^2_{\chi_i}=-(1/\chi_i)(\partial/\partial w)\chi_i|_{w=1}$ at zero recoil. We factored out $(w-1)$ from the leading order form factor and defined $\xi^{(1)}(w)=(w-1)\Xi(w)$. The values of the functions $\tilde{\xi}_3(1)$, $\chi_i(1)$ are given in units $(\bar{\Lambda}^{(1)}+\bar{\Lambda})/2$.

	$\Xi(w)$	$\tilde{\xi}_3(w)$	$\tilde{\chi}_1(w)$	$\tilde{\chi}_2(w)$	$\tilde{\chi}_3(w)$	$\chi_b(w)$
Value at $w=1$	2.2	0.21	0.18	-0.054	-0.023	-0.098
Slope at $w=1$	2.6	-3.3	1.9	3.1	1.0	2.1

V. NUMERICAL RESULTS AND PREDICTIONS

In Table I we present the masses of radially excited D' and $D^{*\prime}$ as well as mass parameters $\bar{\Lambda}$ calculated in the framework of our model [8]. Our prediction for the $D^{*\prime}$ mass is in good agreement with the DELPHI measurement [25]. Other radially excited states have not been observed yet. Thus we use our predictions for numerical calculations. The values of leading and subleading Isgur-Wise functions (29)–(34) and their slopes at the point of zero recoil of the final $D^{(*)\prime}$ meson are given in Table II. In Fig. 3 we plot our results for the leading order Isgur-Wise function $\xi^{(1)}(w)$ and the current correction function $\tilde{\xi}_3(w)$. The functions $\tilde{\chi}_1(w)$, $\tilde{\chi}_2(w)$, $\tilde{\chi}_3(w)$, and $\chi_b(w)$ are plotted in Fig. 4. We see that the functions, parametrizing chromomagnetic corrections to the HQET Lagrangian, are rather small in accord with the HQET based expectations.

We can now calculate the decay branching ratios by integrating double differential decay rates in Eq. (2). Our results for decay rates both in the infinitely heavy quark limit and taking account of the first order $1/m_Q$ corrections as well as their ratio

$$R=\frac{\mathrm{Br}(B\to D^{(*)\prime}e\nu)_{\text{with }1/m_Q}}{\mathrm{Br}(B\to D^{(*)\prime}e\nu)_{m_Q\to\infty}}$$

are presented in Table III. We find that both $1/m_Q$ corrections to decay rates arising from corrections to HQET Lagrangian (31)–(34), which do not vanish at zero recoil, and corrections to the current (30), (12), vanishing at zero recoil, give significant contributions. In the case of $B\to D'e\nu$ decay both types of these corrections tend to increase the decay rate leading to approximately a 75% increase of the $B\to D'e\nu$ decay rate. On the other hand, these corrections give opposite contributions to the $B\to D^{*\prime}e\nu$ decay rate: the corrections to the current give a negative contribution, while corrections to the Lagrangian give a positive one of approximately the same value. This interplay of $1/m_Q$ corrections only slightly ($\approx 10\%$) increases the decay rate with respect to the infinitely heavy quark limit. As a result the branching ratio for $B\to D'e\nu$ decay exceeds the one for B

TABLE III. Decay rates Γ (in units of $|V_{cb}/0.04|^2 \times 10^{-15}$ GeV) and branching ratios BR (in %) for B (B_s) decays to radially excited $D^{(*)\prime}$ ($D_s^{(*)\prime}$) mesons in the infinitely heavy quark mass limit and taking account of first order $1/m_Q$ corrections. $\Sigma(B\to D^{(*)\prime} e\nu)$ and $\Sigma(B_s\to D_s^{(*)\prime} e\nu)$ represent the sum over the channels. R is a ratio of branching ratios taking account of $1/m_Q$ corrections to branching ratios in the infinitely heavy quark mass limit.

	$m_Q\to\infty$		With $1/m_Q$		
Decay	Γ	Br	Γ	Br	R
$B\to D' e\nu$	0.53	0.12	0.92	0.22	1.74
$B\to D^{*\prime} e\nu$	0.70	0.17	0.78	0.18	1.11
$\Sigma(B\to D^{(*)\prime} e\nu)$	1.23	0.29	1.70	0.40	1.37
$B_s\to D_s' e\nu$	0.66	0.16	1.18	0.28	1.80
$B_s\to D_s^{*\prime} e\nu$	0.86	0.20	0.95	0.22	1.10
$\Sigma(B_s\to D_s^{(*)\prime} e\nu)$	1.52	0.36	2.13	0.50	1.40

$\to D^{*\prime} e\nu$ after inclusion of first order $1/m_Q$ corrections. In the infinitely heavy quark mass limit we have for the ratio $\mathrm{Br}(B\to D' e\nu)/\mathrm{Br}(B\to D^{*\prime} e\nu)=0.75$, while the account of $1/m_Q$ corrections results in the considerable increase of this ratio to 1.22.

In Table III we also present the sum of the branching ratios over first radially excited states. Inclusion of $1/m_Q$ corrections results in approximately a 40% increase of this sum. We see that our model predicts that 0.40% of B meson decays go to the first radially excited D meson states. If we add this value to our prediction for B decays to the first orbitally excited states 1.45% [7], we get the value of 1.85%. This result means that approximately 2% of B decays should go to higher excitations.

In Figs. 5 and 6 we plot the electron spectra $(1/\Gamma_0)(d\Gamma/dy)$ for $B\to D' e\nu$ and $B\to D^{*\prime} e\nu$ decays. Here $y=2E_e/m_B$ is the rescaled lepton energy. These differential decay rates can be easily obtained from double differential decay rates (2), using the relation $y=1-rw-r\sqrt{w^2-1}\cos\theta$ and then integrating in w over $[(1-y)^2+r^2]/[2r(1-y)]<w<(1+r^2)/(2r)$. We present our results both in the heavy quark limit $m_Q\to\infty$ (dashed curves) and with the inclusion of first order $1/m_Q$ corrections (solid curves). From Fig. 6 we see that inclusion of $1/m_Q$ corrections significantly changes the shape of electron spectrum for $B\to D^{*\prime} e\nu$ decay. The maximum is considerably shifted to higher lepton energies.

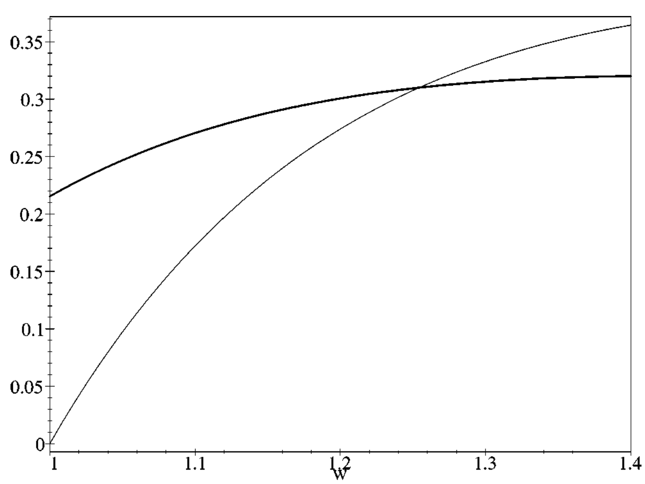

FIG. 3. Isgur-Wise functions $\xi^{(1)}(w)$ (solid curve) and $\tilde{\xi}_3(w)$ [bold curve, in units $(\bar{\Lambda}^{(1)}+\bar{\Lambda})/2$] for the $B\to D^{(*)\prime} e\nu$ decay.

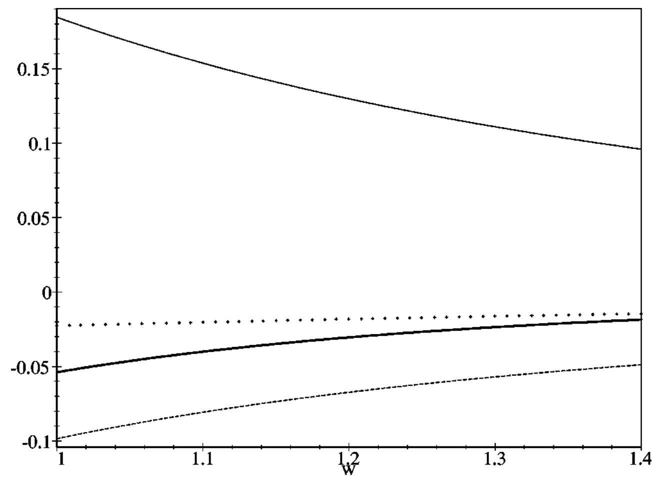

FIG. 4. Isgur-Wise functions $\tilde{\chi}_1(w)$ (solid curve), $\tilde{\chi}_2(w)$ (bold curve), $\tilde{\chi}_3(w)$ (dotted curve), and $\chi_b(w)$ (dashed curve) for the $B\to D^{(*)\prime} e\nu$ decay in units $(\bar{\Lambda}^{(1)}+\bar{\Lambda})/2$.

VI. CONCLUSIONS

In this paper we have carried out the heavy quark expansion for the decay matrix elements of weak currents between the B meson and radially excited $D^{(*)\prime}$ meson states up to first order. It is found that five additional functions of the product of velocities w are necessary to parametrize first order $1/m_Q$ corrections. One of these functions $\tilde{\xi}_3(w)$ arises from corrections to the weak current. The other four func-

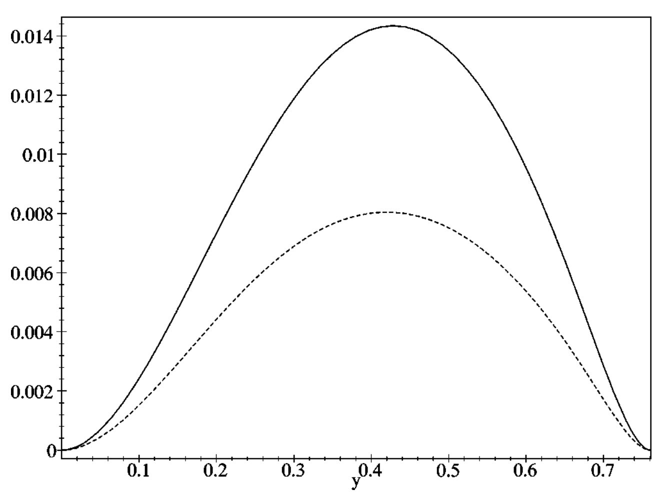

FIG. 5. Electron spectra $(1/\Gamma_0)(d\Gamma/dy)$ for the $B\to D' e\nu$ decay as a function of the rescaled lepton energy $y=2E_e/m_B$. Dashed curve shows the $m_Q\to\infty$ limit, solid curve includes first order $1/m_Q$ corrections.

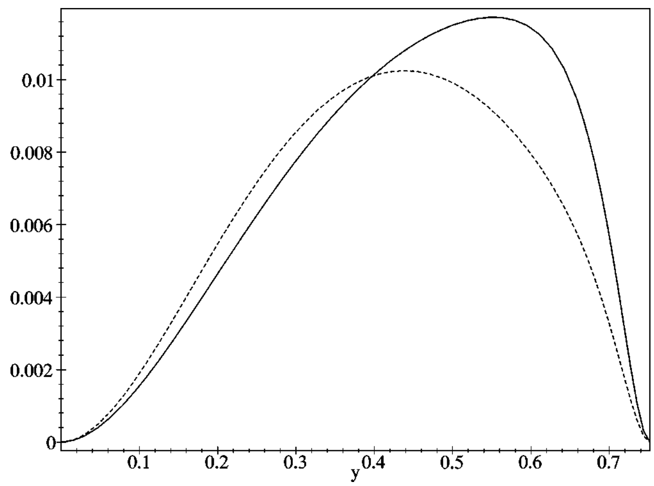

FIG. 6. Electron spectra $(1/\Gamma_0)(d\Gamma/dy)$ for the $B\to D^{*\prime}e\nu$ decay as a function of the rescaled lepton energy $y=2E_e/m_B$. Dashed curve shows the $m_Q\to\infty$ limit; solid curve includes first order $1/m_Q$ corrections.

tions $\chi_i(w)$ $(i=1,\dots,3,b)$ parametrize corrections to the HQET Lagrangian. The leading order function vanishes at the point of zero recoil $(w=1)$ of the final $D^{(*)\prime}$ meson due to the heavy quark symmetry and orthogonality of the radial parts of meson wave functions in the heavy quark limit. The contributions to the decay matrix elements coming from the corrections to the current also vanish at zero recoil for the same reason. Thus the only nonzero contributions to the weak decay matrix elements at $w=1$ come from the corrections to the Lagrangian $\tilde{\chi}_{1,3,b}$ [see Eq. (35)], since there is no condition requiring them to vanish at this point as in the case of *B* decays to ground state *D* mesons.

Then we apply the relativistic quark model for the consideration of semileptonic $B\to D^{(*)\prime}e\nu$ decays. It is found that our model correctly reproduces the structure of decay matrix elements found from the heavy quark symmetry analysis. This allows us to determine the leading and subleading Isgur-Wise functions for this transition. We find that both the relativistic transformation of the meson wave function from the rest frame to the moving one as well as the first order $1/m_Q$ corrections to meson wave functions give essential contributions to the subleading order functions. Thus, the account for corrections to the wave functions gives contributions to decay matrix elements which do not vanish at zero recoil. These contributions turn out to be rather small numerically. However, their role is considerably increased since the kinematical range for *B* decays to radially excited $D^{(*)\prime}$ mesons is small and thus the leading order contribution, vanishing at zero recoil, is suppressed. Another important contribution to decay rates comes from the terms $\varepsilon_Q(\overline{\Lambda}^{(1)}-\overline{\Lambda})\xi^{(1)}/(w-1)$ originating from the corrections to the current. These terms turn out to be numerically important. We find an interesting interplay of these two types of $1/m_Q$ corrections. They contribute to the $B\to D'e\nu$ decay rate with the same sign, but their contributions to the $B\to D^{*\prime}e\nu$ rate have opposite signs. As a result the former decay rate is substantially (1.75 times) increased by the inclusion of first order $1/m_Q$ corrections while there is only a slight (1.1 times) increase of the latter decay rate. This leads to the increase in the ratio $\mathrm{Br}(B\to D'e\nu)/\mathrm{Br}(B\to D^{*\prime}e\nu)$ from 0.75 in the heavy quark limit to 1.22, when first order $1/m_Q$ corrections are taken into account. Finally, we find that the semileptonic *B* decays to first radial excitations of *D* mesons acquire in total 0.4% of the *B* decay rate.

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