Electroweak penguin amplitudes and constraints on γ in charmless $B \to VP$ decays

Michael Gronau
Physics Department, Technion–Israel Institute of Technology, 32000 Haifa, Israel
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Electroweak penguin (EWP) amplitudes are studied model independently in B meson decays to charmless final states consisting of a vector meson (V) and a pseudoscalar meson (P). A set of SU(3) relations is derived between EWP contributions and tree amplitudes, in the approximation of retaining only the dominant EWP operators Q_9 and Q_{10}. Two applications are described for constraining the weak phase γ, in $B^{\pm} \to \rho^{\pm} K^0$ and $B^{\pm} \to \rho^0 K^{\pm}$ (or $B^{\pm} \to K^{*\pm} \pi^0$ and $B^{\pm} \to K^{*0} \pi^{\pm}$), and in $B^0 \to K^{*\pm} \pi^{\mp}$ and $B^{\pm} \to \phi K^{\pm}$. Theoretical uncertainties are discussed.

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I. INTRODUCTION

B meson decays to charmless final states open a window into new phenomena of CP violation [1], providing useful information about the Kobayashi-Maskawa phase $\gamma = \mathrm{Arg} V_{ub}^*$. Decays to states consisting of two light pseudoscalars ($B \to PP$), such as $B \to \pi\pi$ and $B \to K\pi$, have been for some time the subject of extensive studies. The amplitudes of these processes involve hadronic matrix elements of a low energy effective weak Hamiltonian between an initial B meson and two final pseudoscalar mesons. The weak Hamiltonian consists of the sum of three types of four quark operators, namely two $(V-A)(V-A)$ current-current operators ($Q_{1,2}$), four QCD penguin operators ($Q_{3,4,5,6}$), and four electroweak penguin (EWP) operators ($Q_{7,8,9,10}$) with different chiral structures. A major line of analysis [2] consists of model-independent studies of hadronic matrix elements of these operators, which do not rely on factorization-based models [3,4]. Approximate flavor SU(3) symmetry of strong interactions was employed [5–8] in order to describe these matrix elements in a graphical manner in terms of a relatively small number of amplitudes.

A useful simplification was achieved [9,10] by noting that in certain cases, such as in B decay to an isospin 3/2 $K\pi$ state, the *dominant* EWP amplitude is simply related by SU(3) to the corresponding current-current contribution, and does not introduce a new unknown quantity into the analysis. This simplification, obtained when retaining only the $(V-A)(V-A)$ EWP operators (Q_9 and Q_{10}), led to a promising way of measuring the weak phase γ [11,12].

A first SU(3) analysis of B mesons decays to a charmless vector meson (V) and a pseudoscalar meson (P), classifying contributions in terms of graphical SU(3) amplitudes, was presented in [13]. Several factorization-based calculations of these processes can be found in [3]. Measurements of $B \to VP$ decays were reported by the CLEO Collaboration working at the Cornell Electron Storage Ring (CESR) [14]. The experimental results were used recently [15] in order to identify dominant and subdominant interfering amplitudes in certain processes. Interference effects between these amplitudes seem to favor (but do not necessarily imply) a weak phase γ in the second quadrant of the unitarity triangle plot. A similar conclusion was drawn recently in factorization-based analyses [16]. Such values of γ are in sharp conflict with an overall CKM parameter analysis [17] based on very optimistic assumptions about theoretical uncertainties in hadronic parameters. A more conservative estimate of these errors [18] implies $\gamma \leqslant 90°$, based primarily on a lower limit for B_s-$\overline{B_s}$ mixing, $\Delta m_s > 14.3\ \mathrm{ps}^{-1}$ [19]. Using a somewhat wider range for the relevant SU(3) breaking parameter, $f_{B_s}\sqrt{B_{B_s}}/f_B\sqrt{B_B}$, we concluded recently [15] that values of γ slightly larger than 90° cannot be definitely excluded.

Two major assumptions were made in [15] in order to arrive at the indication that $\gamma > 90°$ within the framework of flavor SU(3). (1) The relative strong phase between penguin and tree amplitudes in $B^0 \to K^{*+}\pi^-$ was assumed to be smaller than 90°. (2) The magnitude of a color-favored EWP contribution in $B^+ \to \phi K^+$ was taken from factorization-based calculations [20,21], and color-suppressed EWP contributions were neglected. The first assumption is rather plausible, and can be justified on the basis of both perturbative [4,22] and statistical [23] estimates of final state phases. The second assumption is manifestly model-dependent. One would hope to be able to replace it by a model-independent study.

In order to study $B \to VP$ decays in a model-independent manner, we propose in this article to derive SU(3) relations between color-favored and color-suppressed EWP amplitudes, on the one hand, and current-current contributions on the other hand. The relations, obtained in the approximation of retaining only the dominant $(V-A)(V-A)$ EWP operators, eliminate eight of the hadronic parameters describing charmless $B \to VP$ decays in the SU(3) framework.

In Sec. II we recall the general SU(3) structure of the effective weak Hamiltonian, paying particular attention to its current-current part and its dominant electroweak term. We show that corresponding components of these operators, transforming as given SU(3) representations, are proportional to each other. In Sec. III we use this feature to study the SU(3) structure of $B \to VP$ amplitudes, and in Sec. IV we express EWP contributions in $B \to VP$ decays in terms of corresponding tree amplitudes.

Two applications are demonstrated in Sec. V for constraining the weak phase γ by charge-averaged ratios of rates in $B^{\pm} \to \rho^{\pm} K^0$ and $B^{\pm} \to \rho^0 K^{\pm}$ (or $B^{\pm} \to K^{*\pm}\pi^0$ and $B^{\pm} \to K^{*0}\pi^{\pm}$), and in $B^0 \to K^{*\pm}\pi^{\mp}$ and $B^{\pm} \to \phi K^{\pm}$. In the first

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case, no data exist at this time. In the second case, existing data may imply a lower limit on γ, provided that a better understanding is achieved for SU(3) breaking in QCD penguin amplitudes and for the effects of color- and Okubo-Zweig-Iizuka- (OZI-) suppressed amplitudes. We conclude in Sec. VI.

II. SU(3) STRUCTURE OF WEAK HAMILTONIAN

The low energy effective weak Hamiltonian governing B meson decays is given by [24]

$$\mathcal{H}=\frac{G_F}{\sqrt{2}}\sum_{q=d,s}\left(\sum_{q'=u,c}\lambda_{q'}^{(q)}[c_1(\bar{b}q')_{V-A}(\bar{q}'q)_{V-A}+c_2(\bar{b}q)_{V-A}(\bar{q}'q')_{V-A}]-\lambda_t^{(q)}\sum_{i=3}^{10}c_iQ_i^{(q)}\right),\tag{1}$$

where $\lambda_{q'}^{(q)}=V_{q'b}^{*}V_{q'q}$, $q=d,s$, $q'=u,c,t$, $\lambda_u^{(q)}+\lambda_c^{(q)}+\lambda_t^{(q)}=0$. The first terms, involving the coefficients c_1 and c_2 and describing both $\bar{b}\to\bar{q}u\bar{u}$ and $\bar{b}\to\bar{q}c\bar{c}$, will be referred to as "current-current" operators, while the other terms, involving c_i $i=3-10$, consist of four QCD penguin operators $(i=3-6)$ and four EWP operators $(i=7-10)$. The EWP operators with the dominant Wilson coefficients, Q_9 and Q_{10}, both have a $(V-A)(V-A)$ structure similar to the current-current term. Their flavor structure is

$$Q_9^{(q)}=\frac{3}{2}\left[(\bar{b}q)\left(\frac{2}{3}\bar{u}u-\frac{1}{3}\bar{d}d-\frac{1}{3}\bar{s}s+\frac{2}{3}\bar{c}c\right)\right],$$

$$Q_{10}^{(q)}=\frac{3}{2}\left[\frac{2}{3}(\bar{b}u)(\bar{u}q)-\frac{1}{3}(\bar{b}d)(\bar{d}q)-\frac{1}{3}(\bar{b}s)(\bar{s}q)+\frac{2}{3}(\bar{b}c)(\bar{c}q)\right].\tag{2}$$

All four-quark operators appearing in Eq. (1) are of the form $(\bar{b}q_1)(\bar{q}_2q_3)$ and can be written as a sum of $\overline{\mathbf{15}}$, $\mathbf{6}$ and $\overline{\mathbf{3}}$, into which the product $\overline{\mathbf{3}}\otimes\mathbf{3}\otimes\overline{\mathbf{3}}$ is decomposed [5,7,8]. The representation $\overline{\mathbf{3}}$ appears both symmetric $(\overline{\mathbf{3}}^{(s)})$, and antisymmetric $(\overline{\mathbf{3}}^{(a)})$ under the interchange of q_1 and q_3. Four-quark operators, belonging to each of these SU(3) representations and carrying given values of isospin, are listed in the appendix of [10].

The "tree" part of the current-current Hamiltonian, corresponding to the term $q'=u$ describing $\bar{b}\to\bar{q}u\bar{u}$ transitions, can be written as [10]

$$\frac{\sqrt{2}\mathcal{H}_T}{G_F}=-\lambda_u^{(s)}\left[\frac{c_1-c_2}{2}(\overline{\mathbf{3}}^{(a)}_{I=0}+\mathbf{6}_{I=1})+\frac{c_1+c_2}{2}\left(\overline{\mathbf{15}}_{I=1}+\frac{1}{\sqrt{2}}\overline{\mathbf{15}}_{I=0}-\frac{1}{\sqrt{2}}\overline{\mathbf{3}}^{(s)}_{I=0}\right)\right]-\lambda_u^{(d)}\left[\frac{c_1-c_2}{2}(\overline{\mathbf{3}}^{(a)}_{I=1/2}-\mathbf{6}_{I=1/2})+\frac{c_1+c_2}{2}\left(\frac{2}{\sqrt{3}}\overline{\mathbf{15}}_{I=3/2}+\frac{1}{\sqrt{6}}\overline{\mathbf{15}}_{I=1/2}-\frac{1}{\sqrt{2}}\overline{\mathbf{3}}^{(s)}_{I=1/2}\right)\right].\tag{3}$$

The dominant EWP term, excluding $\bar{b}\to\bar{q}c\bar{c}$ (to be referred to as the noncharming EWP operator), is

$$\frac{\sqrt{2}\mathcal{H}_{EWP}}{G_F}=-\frac{3\lambda_t^{(s)}}{2}\left[\frac{c_9-c_{10}}{2}\left(\frac{1}{3}\overline{\mathbf{3}}^{(a)}_{I=0}+\mathbf{6}_{I=1}\right)+\frac{c_9+c_{10}}{2}\left(-\overline{\mathbf{15}}_{I=1}-\frac{1}{\sqrt{2}}\overline{\mathbf{15}}_{I=0}-\frac{1}{3\sqrt{2}}\overline{\mathbf{3}}^{(s)}_{I=0}\right)\right]-\frac{3\lambda_t^{(d)}}{2}\left[\frac{c_9-c_{10}}{2}\left(\frac{1}{3}\overline{\mathbf{3}}^{(a)}_{I=1/2}-\mathbf{6}_{I=1/2}\right)+\frac{c_9+c_{10}}{2}\left(-\frac{2}{\sqrt{3}}\overline{\mathbf{15}}_{I=3/2}-\frac{1}{\sqrt{6}}\overline{\mathbf{15}}_{I=1/2}-\frac{1}{3\sqrt{2}}\overline{\mathbf{3}}^{(s)}_{I=1/2}\right)\right].\tag{4}$$

Equations (3) and (4) teach us something very important. *For a given strangeness-change, the* $\overline{\mathbf{15}}$ *and* $\mathbf{6}$ *components of the tree operator and the dominant EWP operator in the Hamiltonian are proportional to each other*:

$$\mathcal{H}_{EWP}^{(q)}(\overline{\mathbf{15}})=-\frac{3}{2}\frac{c_9+c_{10}}{c_1+c_2}\frac{\lambda_t^{(q)}}{\lambda_u^{(q)}}\mathcal{H}_T^{(q)}(\overline{\mathbf{15}}),\tag{5}$$

$$\mathcal{H}_{EWP}^{(q)}(\mathbf{6})=\frac{3}{2}\frac{c_9-c_{10}}{c_1-c_2}\frac{\lambda_t^{(q)}}{\lambda_u^{(q)}}\mathcal{H}_T^{(q)}(\mathbf{6}).\tag{6}$$

Here the superscripts $q=d,s$ denote strangeness-conserving and strangeness-changing transitions, respectively. The above two relations are unaffected by the inclusion of the current-current and EWP operators describing $\bar{b}\to\bar{q}c\bar{c}$ transitions, each of which transforms as an antitriplet.

A similar relation between the $\overline{\mathbf{3}}$ parts of $\mathcal{H}_{EWP}$ and $\mathcal{H}_T$ holds only when the two ratios of Wilson coefficients, $(c_9+c_{10})/(c_1+c_2)$ and $(c_9-c_{10})/(c_1-c_2)$, are equal. Indeed, these two ratios, which are approximately renormalization scale independent, are equal to a very good approximation [10]. At a scale $\mu=m_b$, they differ by less than 3% [24,25]

$$\frac{c_9+c_{10}}{c_1+c_2}=-1.139\alpha, \quad \frac{c_9-c_{10}}{c_1-c_2}=-1.107\alpha, \tag{7}$$

where $\alpha=1/129$. We will take the average of the two ratios and denote it by κ

$$\kappa\equiv\frac{c_9+c_{10}}{c_1+c_2}=\frac{c_9-c_{10}}{c_1-c_2}=-1.12\alpha. \tag{8}$$

In this approximation, we also have

$$\mathcal{H}^{(q)}_{EWP}(\overline{\mathbf{3}})=\frac{1}{2}\kappa\frac{\lambda_t^{(q)}}{\lambda_u^{(q)}}\mathcal{H}^{(q)}_T(\overline{\mathbf{3}}). \tag{9}$$

We note that this relation excludes the current-current and EWP $\bar{b}\to\bar{q}c\bar{c}$ operators which transform as an independent antitriplet. As mentioned, the two relations (5) and (6) are unaffected by the inclusion of these operators.

The operator relations (5), (6) and (9) lead to corresponding relations between tree and EWP amplitudes contributing to various processes. An interesting example [9,10] is $B\to(K\pi)_{I=3/2}$, in which one chooses the final $K\pi$ state to be in $I=3/2$. This S-wave state, which is symmetric under interchanging the two SU(3) octets, is pure **27**. The only SU(3) operator in the Hamiltonian which contributes to this transition is $\overline{\mathbf{15}}$ [5,7,8]. Consequently, the ratio of EWP and tree contributions in $B\to(K\pi)_{I=3/2}$ is given simply by $-(3/2)\kappa(\lambda_t^{(s)}/\lambda_u^{(s)})$. This feature was shown to have a useful implication when studying the weak phase γ in $B^+\to K\pi$ decays. In the next two sections we will study generalizations of this relation in $B\to VP$ decays.

III. SU(3) DECOMPOSITION OF $B\to VP$ AMPLITUDES

In Refs. [13,15] $B\to VP$ amplitudes were expressed in terms of reduced SU(3) amplitudes depicted in graphical form. For the most part, we will consider in this paper decay amplitudes into states involving two octet mesons, which consist of T_M (tree), C_M (color-suppressed), P_M (QCD-penguin), E_M (exchange), A_M (annihilation) and PA_M (penguin annihilation). The suffix $M=P$, V on the three amplitudes T, C and P denotes whether the spectator quark is included in a pseudoscalar or vector meson, respectively. In E_M, A_M and PA_M the suffix denotes the type of meson into which the outgoing quark q_3 enters in $\bar{b}q_1\to\bar{q}_2q_3$. In the last six amplitudes the spectator quark enters into the decay Hamiltonian. These contributions, which were neglected in [13,15], may be important in the presence of rescattering [26], and will not be neglected here.

We will use a somewhat different notation for the graphical amplitudes than in [13,15] by factoring out Cabibbo-Kobayashi-Maskawa (CKM) elements. We will also separate the charming penguin contributions [27], related to $\bar{b}\to\bar{q}c\bar{c}$, from the noncharming terms. Thus, for instance, a typical $\Delta S=1$ amplitude is given by

$$\begin{aligned}A(B^0\to\rho^-K^+)=&\lambda_u^{(s)}[-P_{uc,V}-T_V]\\&+\lambda_t^{(s)}[-P_{tc,V}+EW(B^0\to\rho^-K^+)\\&+EW_c(B^0\to\rho^-K^+)],\end{aligned} \tag{10}$$

where $P_{uc,V}=P_{u,V}-P_{c,V}$, $P_{tc,V}=P_{t,V}-P_{c,V}$. Both T_V and $P_{u,V}$ are contributions from the tree Hamiltonian (3), and will be referred to as tree amplitudes, in spite of the fact that $P_{u,V}$ may be depicted as a penguin diagram with an internal u quark. $P_{c,V}$ and EW_c originate in $\bar{b}\to\bar{c}q\bar{c}$ current-current and EWP operators, respectively, and will be referred to as charming penguin and charming EWP terms. Finally, $P_{t,V}$ and EW are contributions from QCD penguin operators and from the dominant noncharming EWP Hamiltonian (4), and will be referred to as noncharming penguin amplitudes. (One expects $|EW_c|\ll|EW|$.)

In previous analyses, EWP contributions [28] multiplying $\lambda_t^{(q)}$ were taken to be independent of the other terms. They were introduced through the substitution [29]

$$\begin{gathered}T_M\to t_M\equiv T_M+EW_M^C, \quad C_M\to c_M\equiv C_M+EW_M,\\ P_M\to p_M\equiv P_M-\frac{1}{3}EW_M^C.\end{gathered} \tag{11}$$

The color-favored (EW_M) and color-suppressed (EW_M^C) EWP amplitudes, in which the spectator enters the meson M, were considered to be independent of the other amplitudes. They are calculable in specific models based on factorization [3]. Four other EWP contributions [30], in which the spectator quark enters into the effective EWP Hamiltonian, were neglected. Such amplitudes can be enhanced by rescattering. Including these amplitudes introduces a total of eight additional unknown parameters into the SU(3) analysis.

Here we wish to use the approximate operator equalities (5), (6) and (9) in order to relate all eight EWP parameters to tree amplitudes. We will find relations between the dominant noncharming EWP contributions EW multiplying $\lambda_t^{(q)}$ and the tree amplitudes multiplying $\lambda_u^{(q)}$. This program, analogous to the study of EWP amplitudes in $B\to PP$ decays [10], can be carried out by expressing tree and EWP amplitudes in terms of reduced SU(3) matrix elements.

Counting the number of reduced matrix elements for B decays to two octet VP states, one finds [5] five amplitudes for SU(3) symmetric VP states

$$\langle\mathbf{1}\|\overline{\mathbf{3}}\|\mathbf{3}\rangle,\quad\langle\mathbf{8_s}\|\overline{\mathbf{3}}\|\mathbf{3}\rangle,\quad\langle\mathbf{8_s}\|\mathbf{6}\|\mathbf{3}\rangle,\quad\langle\mathbf{8_s}\|\overline{\mathbf{15}}\|\mathbf{3}\rangle,\quad\langle\mathbf{27}\|\overline{\mathbf{15}}\|\mathbf{3}\rangle, \tag{12}$$

and five matrix elements for antisymmetric states

$$\begin{gathered}\langle\mathbf{8_a}\|\overline{\mathbf{3}}\|\mathbf{3}\rangle,\quad\langle\mathbf{8_a}\|\mathbf{6}\|\mathbf{3}\rangle,\quad\langle\mathbf{10}\|\mathbf{6}\|\mathbf{3}\rangle,\\ \langle\mathbf{8_a}\|\overline{\mathbf{15}}\|\mathbf{3}\rangle,\quad\langle\overline{\mathbf{10}}\|\overline{\mathbf{15}}\|\mathbf{3}\rangle.\end{gathered} \tag{13}$$

The decomposition of $B\to VP$ amplitudes in terms of these reduced matrix elements, occurring both in tree and EWP contributions, is given in Table I for $\Delta S=1$ decays. The coefficients of the reduced matrix elements are tabulated

TABLE I. Decomposition of $B\to\rho K$ amplitudes into SU(3) reduced matrix elements (12) and (13). The coefficients for corresponding $B\to K^*\pi$ decays are the same for Eq. (12) and have opposite signs for Eq. (13).

Reduced amplitude	$B^+\to\rho^+K^0$	$B^+\to\rho^0K^+$	$B^0\to\rho^-K^+$	$B^0\to\rho^0K^0$
$\langle \mathbf{1}\Vert\overline{\mathbf{3}}\Vert\mathbf{3}\rangle$	0	0	0	0
$\langle \mathbf{8_s}\Vert\overline{\mathbf{3}}\Vert\mathbf{3}\rangle$	$-\sqrt{3/5}$	$\sqrt{3/10}$	$\sqrt{3/5}$	$-\sqrt{3/10}$
$\langle \mathbf{8_s}\Vert\mathbf{6}\Vert\mathbf{3}\rangle$	$1/\sqrt{5}$	$-1/\sqrt{10}$	$1/\sqrt{5}$	$-1/\sqrt{10}$
$\langle \mathbf{8_s}\Vert\overline{\mathbf{15}}\Vert\mathbf{3}\rangle$	$-3\sqrt{3}/5$	$3\sqrt{6}/10$	$-\sqrt{3}/5$	$\sqrt{6}/10$
$\langle \mathbf{27}\Vert\overline{\mathbf{15}}\Vert\mathbf{3}\rangle$	$2\sqrt{3}/5$	$4\sqrt{6}/5$	$4\sqrt{3}/5$	$3\sqrt{6}/5$
$\langle \mathbf{8_a}\Vert\overline{\mathbf{3}}\Vert\mathbf{3}\rangle$	$1/\sqrt{3}$	$-1/\sqrt{6}$	$-1/\sqrt{3}$	$1/\sqrt{6}$
$\langle \mathbf{8_a}\Vert\mathbf{6}\Vert\mathbf{3}\rangle$	$-1/3$	$\sqrt{2}/6$	$-1/3$	$\sqrt{2}/6$
$\langle \mathbf{10}\Vert\mathbf{6}\Vert\mathbf{3}\rangle$	$\sqrt{2}/3$	$2/3$	$\sqrt{2}/3$	$2/3$
$\langle \mathbf{8_a}\Vert\overline{\mathbf{15}}\Vert\mathbf{3}\rangle$	$\sqrt{3/5}$	$-\sqrt{3/10}$	$1/\sqrt{15}$	$-1/\sqrt{30}$
$\langle \overline{\mathbf{10}}\Vert\overline{\mathbf{15}}\Vert\mathbf{3}\rangle$	0	0	$2/\sqrt{3}$	$-\sqrt{2/3}$

for all four $B\to\rho K$ decay processes. The amplitudes for $B\to K^*\pi$ processes are obtained by interchanging the SU(3) flavors of the vector and pseudoscalar mesons. Consequently, the coefficients of the five symmetric elements (12) are the same as in the corresponding $B\to\rho K$ decays, whereas the coefficients of the five antisymmetric elements (13) change sign.

Expressions of the reduced elements (12) in terms of graphical tree amplitudes in $B\to PP$ were given in the appendix of [7]. They can be transcribed to the case of $B\to VP$ by defining combinations of amplitudes, $X_s\equiv(X_V+X_P)/2$, which are symmetric under interchanging the vector and pseudoscalar mesons [we define $(X+Y)_s\equiv X_s+Y_s$]:

$$\langle \mathbf{27}\Vert\overline{\mathbf{15}}\Vert\mathbf{3}\rangle=-\frac{1}{2\sqrt{3}}(T+C)_s, \tag{14}$$

$$\langle \mathbf{8_s}\Vert\overline{\mathbf{15}}\Vert\mathbf{3}\rangle=-\frac{1}{8\sqrt{3}}(T+C+5A+5E)_s, \tag{15}$$

$$\langle \mathbf{8_s}\Vert\mathbf{6}\Vert\mathbf{3}\rangle=-\frac{\sqrt{5}}{4}(T-C-A+E)_s, \tag{16}$$

$$\langle \mathbf{8_s}\Vert\overline{\mathbf{3}}\Vert\mathbf{3}\rangle=-\frac{1}{8}\sqrt{\frac{5}{3}}(3T+3A-C-E+8P_u)_s, \tag{17}$$

$$\langle \mathbf{1}\Vert\overline{\mathbf{3}}\Vert\mathbf{3}\rangle=\frac{1}{2\sqrt{3}}(3T-C+8E+8P_u+12PA_u)_s. \tag{18}$$

The set of six graphical amplitudes on the right-hand sides is over-complete. The physical processes involve only five linear combinations of these amplitudes. Similar relations are obtained for the amplitudes (13) in terms of antisymmetric combinations $X_a\equiv(X_V-X_P)/2$ [we define $(X+Y)_a\equiv X_a+Y_a$]:

$$\langle \overline{\mathbf{10}}\Vert\overline{\mathbf{15}}\Vert\mathbf{3}\rangle=\frac{1}{2\sqrt{3}}(T+C)_a, \tag{19}$$

$$\langle \mathbf{8_a}\Vert\overline{\mathbf{15}}\Vert\mathbf{3}\rangle=-\frac{1}{8}\sqrt{\frac{5}{3}}(T+C-3A-3E)_a, \tag{20}$$

$$\langle \mathbf{10}\Vert\mathbf{6}\Vert\mathbf{3}\rangle=-\frac{1}{\sqrt{2}}(T-C)_a, \tag{21}$$

$$\langle \mathbf{8_a}\Vert\mathbf{6}\Vert\mathbf{3}\rangle=-\frac{1}{4}(T-C+3A-3E)_a, \tag{22}$$

$$\langle \mathbf{8_a}\Vert\overline{\mathbf{3}}\Vert\mathbf{3}\rangle=\frac{\sqrt{3}}{8}(3T+3A-C-E+8P_u)_a. \tag{23}$$

Here the number of graphical amplitudes is identical to that of the reduced SU(3) matrix elements. The amplitude $PA_{u,a}$ vanishes, since the penguin annihilation graph leads to an SU(3) singlet state. By substituting the expressions of Eqs. (14)–(23) into Table I, it is straightforward to check that one obtains the appropriate graphical description of tree amplitudes for all $B\to PV$ decays, such as written directly for $B^0\to\rho^-K^+$ in Eq. (10).

IV. EWP IN TERMS OF TREE AMPLITUDES

The expressions (14)–(23), for tree amplitudes corresponding to given SU(3) representations, and the proportionality relations (5), (6) and (9), can be used with Table I in order to calculate EWP contributions to $B\to VP$ decays in terms of graphical tree amplitudes. The results for $\Delta S=1$ processes, multiplying $\lambda_t^{(s)}$, are summarized in Table II. Also included are expressions for the corresponding tree amplitudes multiplying $\lambda_u^{(s)}$. For comparison with $B\to PP$ decays, we give expressions for the amplitudes of $B^+\to K^0\pi^+$ and $B^+\to K^+\pi^0$. In Table III we list the graphical expansion of tree amplitudes in $\Delta S=0$ $B\to VP$ decays. For comparison with $B\to PP$, we also include the tree amplitude of $B^+\to\pi^+\pi^0$. EWP contributions in this process [10], as well as in several VP amplitudes involving T_P and T_V, are negligible.

Before discussing a few interesting relations between EWP and tree amplitudes following from Tables II and III, let us recall the relation between our present results and the traditional approach to EWP contributions. The graphical EWP amplitudes, which in the conventional approach are independent parameters, are given here in terms of graphical tree amplitudes. In the notation of [29], expanded in the case of rescattering effects to a set of eight graphical EWP amplitudes [30], one finds, for $M=P,V$,

$$EW_M=-\frac{3\kappa}{2}(T_M+P_{u,M'}),\quad M'\neq M, \tag{24}$$

$$EW_M^C=\frac{3\kappa}{2}(P_{u,M}-C_M), \tag{25}$$

TABLE II. Graphical EWP and tree amplitudes in $\Delta S=1$ $B\rightarrow VP$ decays. Amplitudes for $B^+\rightarrow K\pi$ are given for comparison.

Decay mode	Tree amplitude	EWP amplitude
$B^+\rightarrow\rho^+K^0$	$A_V+P_{u,V}$	$\frac{\kappa}{2}(C_V-2E_V+P_{u,V})$
$B^+\rightarrow\rho^0K^+$	$-\frac{1}{\sqrt{2}}(T_V+C_P+A_V+P_{u,V})$	$\frac{\kappa}{2\sqrt{2}}(3T_P+2C_V+2E_V-P_{u,V})$
$B^0\rightarrow\rho^-K^+$	$-(T_V+P_{u,V})$	$\frac{\kappa}{2}(2C_V-E_V-P_{u,V})$
$B^0\rightarrow\rho^0K^0$	$\frac{1}{\sqrt{2}}(-C_P+P_{u,V})$	$\frac{\kappa}{2\sqrt{2}}(3T_P+C_V+E_V+P_{u,V})$
$B^+\rightarrow K^{*0}\pi^+$	$A_P+P_{u,P}$	$\frac{\kappa}{2}(C_P-2E_P+P_{u,P})$
$B^+\rightarrow K^{*+}\pi^0$	$-\frac{1}{\sqrt{2}}(T_P+C_V+A_P+P_{u,P})$	$\frac{\kappa}{2\sqrt{2}}(3T_V+2C_P+2E_P-P_{u,P})$
$B^0\rightarrow K^{*+}\pi^-$	$-(T_P+P_{u,P})$	$\frac{\kappa}{2}(2C_P-E_P-P_{u,P})$
$B^0\rightarrow K^{*0}\pi^0$	$\frac{1}{\sqrt{2}}(-C_V+P_{u,P})$	$\frac{\kappa}{2\sqrt{2}}(3T_V+C_P+E_P+P_{u,P})$
$B^+\rightarrow K^0\pi^+$	$A+P_u$	$\frac{\kappa}{2}(C-2E+P_u)$
$B^+\rightarrow K^+\pi^0$	$-\frac{1}{\sqrt{2}}(T+C+A+P_u)$	$\frac{\kappa}{2\sqrt{2}}(3T+2C+2E-P_u)$

$$EWE_M=\frac{3\kappa}{2}(P_{u,M}-E_M), \tag{26}$$

$$EWA_M=\frac{3\kappa}{2}(PA_{u,M}-A_M). \tag{27}$$

The amplitudes EWA_M do not occur in the processes of Table II. They do occur in B_s decays.

Tables II and III imply a few SU(3) relations between EWP and tree amplitudes of corresponding $B\rightarrow VP$ decay processes, which are similar to the relation noted recently to hold in $B^+\rightarrow K\pi$ decays [9,10]. Starting with the latter case, and denoting EWP and tree contributions by EW and TR, respectively, we have

$$\begin{aligned} EW(K^0\pi^+)+\sqrt{2}EW(K^+\pi^0) &= -\frac{3\kappa}{2}[TR(K^0\pi^+)+\sqrt{2}TR(K^+\pi^0)] \\ &= -\frac{3\kappa}{\sqrt{2}\lambda_u^{(d)}}A(\pi^+\pi^0) \\ &= \frac{3\kappa}{2}(T+C). \end{aligned} \tag{28}$$

This relation follows directly from Eq. (5). The two states, $|K^0\pi^+\rangle+\sqrt{2}|K^+\pi^0\rangle=|I=3/2\rangle$ and $\sqrt{2}|\pi^+\pi^0\rangle_{\text{S-wave}}=|I=2\rangle$, are members of a **27** representation to which only the $\overline{\mathbf{15}}$ operator contributes. The corresponding relation in $B\rightarrow VP$ is

$$\begin{aligned} EW(\rho^+K^0)+\sqrt{2}EW(\rho^0K^+)+EW(K^{*0}\pi^+) &+\sqrt{2}EW(K^{*+}\pi^0) \\ &= -\frac{3\kappa}{\sqrt{2}\lambda_u^{(d)}}[A(\rho^+\pi^0)+A(\rho^0\pi^+)] \\ &= \frac{3\kappa}{2}(T_P+T_V+C_P+C_P). \end{aligned} \tag{29}$$

In this case the two SU(3)-symmetrized VP states, $|\rho^+K^0\rangle+\sqrt{2}|\rho^0K^+\rangle+|K^{*0}\pi^+\rangle+\sqrt{2}|K^{*+}\pi^0\rangle$ (isospin 3/2) and $\sqrt{2}(|\rho^+\pi^0\rangle+|\rho^0\pi^+\rangle)$ (isospin 2), belong to a **27** representation.

TABLE III. Graphical tree amplitudes in $\Delta S=0$ $B\to VP$ decays. Tree amplitude for $B^+\to\pi^+\pi^0$ is given for comparison.

Decay mode	Tree amplitude
$B^+\to\rho^+\pi^0$	$-\frac{1}{\sqrt{2}}(T_P+C_V+P_{u,P}-P_{u,V}+A_P-A_V)$
$B^+\to\rho^0\pi^+$	$-\frac{1}{\sqrt{2}}(T_V+C_P-P_{u,P}+P_{u,V}-A_P+A_V)$
$B^+\to K^{*+}\overline{K}^0$	$A_V+P_{u,V}$
$B^+\to\overline{K}^{*0}K^+$	$A_P+P_{u,P}$
$B^0\to\rho^-\pi^+$	$-(T_V+P_{u,V}+E_P+\frac{1}{2}PA_{u,P}+\frac{1}{2}PA_{u,V})$
$B^0\to\rho^+\pi^-$	$-(T_P+P_{u,P}+E_V+\frac{1}{2}PA_{u,P}+\frac{1}{2}PA_{u,V})$
$B^0\to\rho^0\pi^0$	$\frac{1}{2}(P_{u,P}+P_{u,V}-C_P-C_V+E_P+E_V+PA_{u,P}+PA_{u,V})$
$B^0\to K^{*+}K^-$	$-(E_V+\frac{1}{2}PA_{u,P}+\frac{1}{2}PA_{u,V})$
$B^0\to K^{*-}K^+$	$-(E_P+\frac{1}{2}PA_{u,P}+\frac{1}{2}PA_{u,V})$
$B^0\to K^{*0}\overline{K}^0$	$P_{u,V}+\frac{1}{2}PA_{u,P}+\frac{1}{2}PA_{u,V}$
$B^0\to\overline{K}^{*0}K^0$	$P_{u,P}+\frac{1}{2}PA_{u,P}+\frac{1}{2}PA_{u,V}$
$B^+\to\pi^+\pi^0$	$-\frac{1}{\sqrt{2}}(T+C)$

Two other relations can be obtained from Table II:

$$EW(\rho^+K^0)+\sqrt{2}EW(\rho^0K^+)$$
$$=-\frac{3\kappa}{2}[TR(K^{*0}\pi^+)+\sqrt{2}TR(K^{*+}\pi^0)]$$
$$=\frac{3\kappa}{2}(T_P+C_V), \tag{30}$$

$$EW(K^{*0}\pi^+)+\sqrt{2}EW(K^{*+}\pi^0)$$
$$=-\frac{3\kappa}{2}[TR(\rho^+K^0)+\sqrt{2}TR(\rho^0K^+)]$$
$$=\frac{3\kappa}{2}(T_V+C_P). \tag{31}$$

These relations can be understood in the following way. The two $I=3/2$ states, $|\rho^+K^0\rangle+\sqrt{2}|\rho^0K^+\rangle$ and $|K^{*0}\pi^+\rangle+\sqrt{2}|K^{*+}\pi^0\rangle$, form the sum and difference, respectively, of a **27** and a **10** representation. This can be easily verified in Table I. The **27** and **10** states obtain contributions only from $\overline{\mathbf{15}}$ and **6** operators, respectively. Equations (5) and (6), in which the proportionality constants have equal magnitudes and opposite signs, lead immediately to Eqs. (30) and (31). It is clear from these considerations that these relations hold also in the presence of current-current and EWP $\bar{b}\to\bar{q}c\bar{c}$ operators transforming as an antitriplet.

A useful approximation is obtained by neglecting in $B^+\to\rho\pi$ the rescattering amplitudes, $P_{u,M}+A_M$ [26]. The smallness of these terms can be tested in $B^+\to K^{*+}\overline{K}^0$ and $B^+\to\overline{K}^{*0}K^+$. In this approximation, one can also express the sums of EWP contributions in Eqs. (30) and (31) in terms of $B^+\to\rho\pi$ amplitudes, similar to Eq. (28),

$$EW(\rho^+K^0)+\sqrt{2}EW(\rho^0K^+)=-\frac{3\kappa}{\sqrt{2}\lambda_u^{(d)}}A(\rho^+\pi^0), \tag{32}$$

$$EW(K^{*0}\pi^+)+\sqrt{2}EW(K^{*+}\pi^0)=-\frac{3\kappa}{\sqrt{2}\lambda_u^{(d)}}A(\rho^0\pi^+). \tag{33}$$

These approximate relations are useful when studying charge-averaged ratios of rates for the processes on the left hand sides.

Relations of the form (30) and (31) are obeyed also by EWP and tree decay amplitudes of B^0 to ρ^-K^+, ρ^0K^0, $K^{*+}\pi^-$ and $K^{*0}\pi^0$. This is easy to understand. These contributions, as well as the entire decay amplitudes, which also contain dominant gluonic penguin terms, satisfy an isospin relation with corresponding B^+ decay amplitudes [31]:

$$A(B^+\to\rho^+K^0)+\sqrt{2}A(B^+\to\rho^0K^+)$$
$$=A(B^0\to\rho^-K^+)+\sqrt{2}A(B^0\to\rho^0K^0). \tag{34}$$

An analogous isospin equality holds for $B\to K^*\pi$ decay amplitudes. Finally, a relation similar to Eq. (29), between $\Delta S=1$ decays to $I=3/2$ on the one hand and $\Delta S=0$ decays to $I=2$ on the other hand, can be written by combining all seven neutral B decay amplitudes to ρK, $K^*\pi$ and $\rho\pi$ states:

$$EW(\rho^- K^+) + \sqrt{2}EW(\rho^0 K^0) + EW(K^{*+}\pi^-) + \sqrt{2}EW(K^{*0}\pi^0)$$
$$= -\frac{3\kappa}{2\lambda_u^{(d)}}[A(\rho^-\pi^+) + A(\rho^+\pi^-) + 2A(\rho^0\pi^0)]$$
$$= \frac{3\kappa}{2}(T_P + T_V + C_P + C_P). \quad (35)$$

V. APPLICATIONS

1. Resolving EWP in $B^+ \to K\pi$.

Let us first reiterate the manner in which Eq. (28) has been applied in order to obtain a model-independent constraint on γ from the charge-averaged ratio [9]

$$R_*^{-1}(K\pi) \equiv \frac{2[B(B^+ \to K^+\pi^0) + B(B^- \to K^-\pi^0)]}{B(B^+ \to K^0\pi^+) + B(B^- \to \overline{K}^0\pi^-)}. \quad (36)$$

Using our graphical notation for amplitudes, one has

$$\sqrt{2}A(B^+ \to K^+\pi^0) = -\lambda_u^{(s)}[T + C + P_{uc} + A] - \lambda_t^{(s)}[P_{tc} - \sqrt{2}EW(K^+\pi^0)],$$
$$A(B^+ \to K^0\pi^+) = \lambda_u^{(s)}[P_{uc} + A] + \lambda_t^{(s)}[P_{tc} + EW(K^0\pi^+)]. \quad (37)$$

The two electroweak penguin terms, containing also contributions from $\overline{b} \to \overline{q}c\overline{c}$ operators, satisfy Eq. (28). Substituting these expressions into Eq. (36), applying unitarity of the CKM matrix, and expanding in small quantities, one finds

$$R_*^{-1}(K\pi) = 1 - 2\epsilon\cos\phi(\cos\gamma - \delta_{EW}) + \mathcal{O}(\epsilon^2) + \mathcal{O}(\epsilon\epsilon_A) + \mathcal{O}(\epsilon_A^2), \quad (38)$$

where $\phi = \mathrm{Arg}([T+C]/[P_{tc} + EW(B^+ \to K^0\pi^+)])$.

The real and positive parameter δ_{EW} [9] stands for the ratio of EWP and tree contributions in the sum $A(B^+ \to K^0\pi^+) + \sqrt{2}A(B^+ \to K^+\pi^0)$, and is determined purely by Wilson coefficients and by a presently poorly known CKM factor [see Eq. (28)]

$$\delta_{EW} = -\frac{3\kappa}{2}\left|\frac{V_{cb}^* V_{cs}}{V_{ub}^* V_{us}}\right| = 0.65 \pm 0.15. \quad (39)$$

The quantity $\epsilon = [|V_{ub}^* V_{us}|/|V_{cb}^* V_{cs}|][|T+C|/|P_{tc} + EW(B^+ \to K^0\pi^+)|]$ is measurable from [32,33]

$$\epsilon = \sqrt{2}\frac{V_{us}}{V_{ud}}\frac{f_K}{f_\pi}\frac{|A(B^+ \to \pi^0\pi^+)|}{|A(B^+ \to K^0\pi^+)|} = 0.21 \pm 0.05. \quad (40)$$

ϵ_A denotes a small rescattering amplitude [26], which introduces a term $P_{uc} + A$ with weak phase γ into the $B^+ \to K^0\pi^+$ decay amplitude. Keeping the dominant term in Eq. (38) and neglecting smaller terms, one obtains the bound

$$|\cos\gamma - \delta_{EW}| \geq \frac{|1 - R_*^{-1}(K\pi)|}{2\epsilon}. \quad (41)$$

This bound can provide useful information about γ in case that a value different from one is measured for R_*^{-1}. Further information about the weak phase can be obtained by measuring separately B^+ and B^- decay rates [11]. Equation (28) plays a crucial role in these applications.

2. Generalization to $B^+ \to \rho K$ and $B^+ \to K^*\pi$.

Whereas Eq. (28) relates EWP and tree contributions in the same sum of two $B^+ \to K\pi$ amplitudes, the analogous Eqs. (30) and (31) relate EWP contributions in a sum of $B^+ \to \rho K$ amplitudes to tree contributions in another sum of $B \to K^*\pi$ amplitudes, and vice versa. This introduces some hadronic dependence in possible constraints on γ from these processes.

Consider, for instance, the charge-averaged ratio of rates for the processes $B^\pm \to \rho^\pm K^0$ and $B^\pm \to \rho^0 K^\pm$

$$R_*^{-1}(\rho K) \equiv \frac{2[B(B^+ \to \rho^0 K^+) + B(B^- \to \rho^0 K^-)]}{B(B^+ \to \rho^+ K^0) + B(B^- \to \rho^- \overline{K}^0)}. \quad (42)$$

Using Table II for graphical expressions of amplitudes, applying Eq. (30), and neglecting rescattering contributions $P_{uc,V} + A_V$, which affect the above ratio only by second order terms, as in Eq. (38), one has

$$\sqrt{2}A(B^+ \to \rho^0 K^+) = -|\lambda_u^{(s)}|[(T_V + C_P)e^{i\gamma} - (T_P + C_V)\delta_{EW}] - \lambda_t^{(s)}[P_{tc,V} + EW],$$
$$A(B^+ \to \rho^+ K^0) = \lambda_t^{(s)}[P_{tc,V} + EW],$$
$$EW \equiv EW(B^+ \to \rho^+ K^0). \quad (43)$$

We define two ratios of amplitudes

$$\epsilon_V e^{i\phi_V} = \frac{|V_{ub}^* V_{us}|}{|V_{cb}^* V_{cs}|}\frac{T_V + C_P}{P_{tc,V} + EW},$$
$$\epsilon_P e^{i\phi_P} = \frac{|V_{ub}^* V_{us}|}{|V_{cb}^* V_{cs}|}\frac{T_P + C_V}{P_{tc,V} + EW}, \quad (44)$$

the magnitudes of which are measured in $B^+ \to \rho^0\pi^+$ and $B^+ \to \rho^+\pi^0$, respectively (see Table III, where we neglect rescattering terms $P_{uc,M} + A_M$)

$$\epsilon_V = \sqrt{2}\frac{V_{us}}{V_{ud}}\frac{f_K}{f_\pi}\frac{|A(B^+ \to \rho^0\pi^+)|}{|A(B^+ \to \rho^+ K^0)|},$$
$$\epsilon_P = \sqrt{2}\frac{V_{us}}{V_{ud}}\frac{f_{K^*}}{f_\rho}\frac{|A(B^+ \to \rho^+\pi^0)|}{|A(B^+ \to \rho^+ K^0)|}. \quad (45)$$

We find

$$R_*^{-1}(\rho K)=1-2\epsilon_V\cos\phi_V\cos\gamma+2\epsilon_P\delta_{EW}\cos\phi_P. \quad (46)$$

This expression, which neglects higher order corrections, simplifies into the form (38) in the case $T_V+C_P=T_P+C_V$, or $A(B^+\to\rho^0\pi^+)=A(B^+\to\rho^+\pi^0)$. In general, this is not the case. Without making any assumption about the magnitudes of these amplitudes and their strong phases, one obtains the rather weak constraint

$$|\cos\gamma|\geqslant\frac{|1-R_*^{-1}(\rho K)|}{2\epsilon_V}-\delta_{EW}\left(\frac{\epsilon_P}{\epsilon_V}\right). \quad (47)$$

This bound is manifestly weaker than the constraint (41) obtained [9] for $B\to K\pi$. In order to exclude values of γ around 90° the right hand side must be positive. This does not only require that a value different from one is measured for $R_*^{-1}(\rho K)$, but also that $|A(B^+\to\rho^+\pi^0)|$ is considerably smaller than $|A(B^+\to\rho^0\pi^+)|$. A similar argument applies to the ratio $R_*^{-1}(K^*\pi)$ in $B^\pm\to K^*\pi$ decays. Since these two ratios have not yet been measured, no constraint on γ can be obtained at this time.

3. A lower bound on γ from $B^0\to K^{*+}\pi^-$ and $B^+\to\phi K^+$.

A plausible argument which favors $\cos\gamma<0$ was presented recently in Ref. [15], based primarily on recent CLEO data on $B^0\to K^{*+}\pi^-$ and $B^+\to\phi K^+$. In this argument, only the dominant amplitudes contributing to these processes, $P_{tc,P}$, EW_P and T_P, were taken into account, while smaller terms were neglected. The argument was based on certain model calculations of EWP amplitudes [20,21], which imply $EW_P\approx P_P/2$. This relation, obtained for certain values of a set of parameters, including the effective number of colors in a $1/N_c$ expansion, also assumes that the two amplitudes have equal strong phases. Here we would like to replace these model-dependent assumptions by our general SU(3) results, which relate electroweak penguin contributions to tree amplitudes rather than to gluonic penguin amplitudes. As in [15], we will keep only the dominant and subdominant terms.

The amplitude of $B^0\to K^{*+}\pi^-$ is

$$\begin{aligned}A(B^0\to K^{*+}\pi^-)=&-\lambda_u^{(s)}[T_P+P_{uc,P}]\\&-\lambda_t^{(s)}[P_{tc,P}-EW(K^{*+}\pi^-)\\&-EW_c(K^{*+}\pi^-)],\end{aligned} \quad (48)$$

where $EW(K^{*+}\pi^-)$ is given in Table II

$$EW(K^{*+}\pi^-)=\frac{\kappa}{2}(2C_P-E_P-P_{u,P}). \quad (49)$$

The amplitude of $B^+\to\phi K^+$ involves also the SU(3) singlet component of the ϕ. In a general SU(3) analysis, this component introduces three new reduced SU(3) amplitudes, of the $\overline{\mathbf{3}}$, **6** and $\overline{\mathbf{15}}$ operators, for the final octet state. These three amplitudes are described by three new graphs: A disconnected penguin diagram, S_P [15], in which a singlet $q\bar{q}$ pair is connected to the rest of the diagram by at least three gluons, and two ''hairpin'' diagrams, of annihilation (AS_P) and exchange (ES_P) types, in which the extra $q\bar{q}$ forms the singlet vector meson. Thus, one has

$$\begin{aligned}A(B^+\to\phi K^+)=&\lambda_u^{(s)}[A_P+P_{uc,P}+AS_P]\\&+\lambda_t^{(s)}[P_{tc,P}+S_P+EW(\phi K^+)\\&+EW_c(\phi K^+)],\end{aligned} \quad (50)$$

where, applying Eqs. (24)–(26),

$$\begin{aligned}EW(\phi K^+)&=-\frac{1}{3}(EW_P+EW_P^C-2EWE_P)\\&=\frac{\kappa}{2}(T_P+C_P-2E_P+P_{u,P}+P_{u,V}).\end{aligned} \quad (51)$$

We will assume, as usual [4,7,29], that T_P is larger than all other tree amplitudes and larger than the current-current amplitude associated with $\bar{b}\to\bar{q}c\bar{c}$. (Recall that the CKM coefficients are factored out.) Similarly, we will assume that $|EW|\geqslant|EW_c|$. The amplitude S_P will be neglected by virtue of the Okubo-Zweig-Iizuka (OZI) rule. We note that in the factorization approach [21], S_P is very sensitive to the number of colors N_c, and vanishes at $N_c=3$. The EWP contribution in $B^+\to\phi K^+$ is dominated by a term proportional to T_P, which is measured in $B^+\to\rho^+\pi^0$ and $B^0\to\rho^+\pi^-$ as discussed below.

Keeping only dominant and subdominant terms in each amplitude, one has

$$A(B^0\to K^{*+}\pi^-)=-\lambda_t^{(s)}P_{tc,P}-\lambda_u^{(s)}T_P, \quad (52)$$

$$A(B^+\to\phi K^+)=\lambda_t^{(s)}\left[P_{tc,P}+\frac{\kappa}{2}T_P\right]. \quad (53)$$

In this approximation, the two amplitudes, $P_{tc,P}$ and T_P, contribute with the same weak phase in $B^+\to\phi K^+$, and interfere with a relative weak phase $\pi-\gamma$ in $B^0\to K^{*+}\pi^-$. Defining

$$r\,e^{i\delta}=\frac{|\lambda_u^{(s)}|}{|\lambda_t^{(s)}|}\frac{T_P}{P_{tc,P}}\quad(r>0), \quad (54)$$

we have

$$A(B^0\to K^{*+}\pi^-)=-\lambda_t^{(s)}P_{tc,P}[1-r\,e^{i(\delta+\gamma)}], \quad (55)$$

$$A(B^+\to\phi K^+)=\lambda_t^{(s)}P_{tc,P}\left[1-\frac{1}{3}\delta_{EW}r\,e^{i\delta}\right], \quad (56)$$

where δ_{EW} is defined in Eq. (39).

In the limit of neglecting the tree amplitude, $r=0$, the rates of the two processes are seen to be equal. Experiments obtain 90% confidence level limits on the charge-averaged rates [14], $\mathcal{B}(B^0\to K^{*\pm}\pi^\mp)>12\times10^{-6}$ and $\mathcal{B}(B^\pm\to\phi K^\pm)<5.9\times10^{-6}$. This is evidence for a nonzero contribution of

T_P, namely $r \neq 0$. The ratio of charge-averaged rates satisfies, at 90% C.L. (we neglect the $B^+ - B^0$ lifetime difference),

$$\frac{|A(B^0 \to K^{*\pm}\pi^{\mp})|^2}{|A(B^\pm \to \phi K^\pm)|^2} = \frac{1 + r^2 - 2r\cos\delta\cos\gamma}{1 + (\delta_{EW}/3)^2 r^2 - (2/3)\,\delta_{EW}\, r\cos\delta} > 2.0. \tag{57}$$

In order to use this inequality for information about γ, one must include some input about r and δ, the relative magnitude and strong phase of tree and penguin amplitudes in $B^0 \to K^{*+}\pi^-$. A reasonable assumption, supported both by perturbative [4,22] and statistical [23] calculations, is that δ does not exceed 90°, i.e. $\cos\delta \geqslant 0$. A conservative assumption about r is $r \leqslant 1$. Making these two assumptions, one finds

$$\cos\gamma - \frac{2}{3}\delta_{EW} < \frac{-1 + r^2[1 - 2(\delta_{EW}/3)^2]}{2r}. \tag{58}$$

This implies $\gamma > 62°$ for $r=1$, and $\gamma > 105°$ for $r=0.5$, when δ_{EW} is taken in the range (39). Some very indirect evidence for $r < 0.55$ was presented in [15], relying on a nonzero value of T_P obtained from $B^0 \to \rho^\pm \pi^\mp$ and $B^+ \to \rho^0\pi^+/\omega\pi^+$. More direct information about r is required, and can be inferred from future rate measurements of $B^+ \to \rho^+\pi^0$ or $B^0 \to \rho^+\pi^-$ and $B^+ \to K^{*0}\pi^+$. These processes are dominated by T_P and $P_{tc,P}$, respectively (see Table III and [15]).

The bound on γ (58), which is based on the experimental limit (57), neglects smaller terms in the amplitudes (48) and (50), primarily the color-suppressed terms C_P in Eqs. (49) and (51) and the OZI-suppressed penguin amplitude S_P in Eq. (50). For $|C_P/T_P| = 0.1$ (0.2) [4], our limits move up or down by about 5° (10°), depending on whether the interference between C_P and T_P is destructive or constructive, respectively.

The above limits also assume [by SU(3)] equal gluonic penguin contributions in the two processes. An important question relevant to these bounds is the magnitude and sign of SU(3) breaking in penguin amplitudes. For instance, if the penguin amplitude in $B^+ \to \phi K^+$ is *smaller* by 30% than in $B^0 \to K^{*+}\pi^-$, then the above bounds are completely invalidated. On the other hand, the constraint becomes stronger if the penguin amplitude in $B^+ \to \phi K^+$ is *larger* than in $B^0 \to K^{*+}\pi^-$. This is the case in explicitly SU(3) breaking factorization-based calculations, in which the two amplitudes involve the products of corresponding vector meson decay constants and B-to-pseudoscalar form factors.

In the factorization approximation, SU(3) breaking factors in penguin and tree amplitudes occurring in Eqs. (52) and (53) are given by [21]

$$R_{SU(3)} = \frac{P_{tc,P}(B^+ \to \phi K^+)}{P_{tc,P}(B^0 \to K^{*+}\pi^-)} = \frac{T_P(B^+ \to \phi K^+)}{T_P(B^0 \to K^{*+}\pi^-)} \simeq \frac{f_\phi}{f_{K^*}} \frac{F_{BK}(m_\phi^2)}{F_{B\pi}(m_{K^*}^2)} \simeq 1.25. \tag{59}$$

Since this factor enhances the amplitude of $B^+ \to \phi K^+$ relative to that of $B^0 \to K^+\pi^-$, the bound on γ (57) becomes stronger by a factor $R_{SU(3)}^2$. This would imply, for instance, $\gamma > 80°$ if a value $r=1$ is measured in $B^+ \to \rho^+\pi^0$ and $B^+ \to K^{*0}\pi^+$, and stronger bounds if a lower value of r is measured. This, and the above comment on the possibility that $R_{SU(3)} < 1$, illustrate the sensitivity of these bounds to SU(3) breaking effects.

Although the present experimental inequality (57) (which may change with time) is already interesting, our above discussion shows that it would be premature at this point to translate this inequality into a realistic lower bound on γ. Further study is required of the following effects:

(a) SU(3) breaking in penguin amplitudes: are these amplitudes approximately factorizable?

(b) Magnitudes and strong phases of smaller terms, including color-suppressed tree and OZI-suppressed penguin amplitudes.

(c) An actual measurement of r, the ratio of tree-to-penguin amplitudes in $B^0 \to K^{*+}\pi^-$.

VI. CONCLUSION

We have studied EWP amplitudes in $B \to VP$ decays within the model independent framework of flavor SU(3). While retaining only contributions from the dominant $(V-A)(V-A)$ operators, Q_9 and Q_{10}, we were able to express these contributions in terms of tree amplitudes. This reduces considerably the number of hadronic parameters describing a large number of processes.

Two applications were demonstrated in attempting to constrain the weak phase γ. In $B^+ \to \rho^+ K^0$ and $B^+ \to \rho^0 K^+$ (or in $B^+ \to K^{*0}\pi^+$ and $B^+ \to K^{*+}\pi^0$) we studied a generalization of the method suggested in [9] for $B^+ \to K\pi$. We find that the constraint becomes weaker due to some dependence on hadronic matrix elements.

In a second application we reexamined the decays $B^0 \to K^{*+}\pi^-$ and $B^+ \to \phi K^+$, studied recently in [15], where EWP contributions were taken from model-calculations. We kept only the dominant and subdominant terms and assumed that the relevant strong phase does not exceed 90°. The present lower limit on the charge-averaged ratio of rates for these two processes leads to an interesting lower bound on γ, Eq. (58). The bound depends on r, the ratio of tree to penguin amplitudes in $B^0 \to K^{*+}\pi^-$, which can be measured in $B^{+,0} \to \rho^+\pi^{0,-}$ and $B^+ \to K^{*0}\pi^+$. Corrections from color-suppressed and OZI-suppressed terms are estimated to move the bound by about 10 degrees. A larger correction may be due to SU(3) breaking in penguin amplitudes. In case that SU(3) breaking decreases the penguin amplitude in $B^+ \to \phi K^+$ relative to the one in $B^0 \to K^{*+}\pi^-$, contrary to the prediction of factorization, the bound on γ may become considerably weaker. A proof of approximate factorization for penguin amplitudes in $B \to VP$ decays, which would strengthen the bound, is therefore of great importance.

Note added. Three months after the submission for publication of this paper a work appeared [34], in which flavor SU(3) symmetry (or, actually an extended nonet symmetry) was applied to charmless $B \to VP$ decays, in order to prove several relations between CP violating rate differences. This work did not make use of the symmetry relations between EWP and tree amplitudes studied in the present paper.

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