Determining $|V_{ub}|$ from the sum rule for semileptonic B decay

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A precise determination of $|V_{ub}|$ can be obtained exploiting the sum rule for inclusive charmless semileptonic B-meson decays. The sum rule is derived on the basis of light-cone expansion and b-flavored quantum number conservation. The sum rule does not receive any perturbative QCD correction. In this determination of $|V_{ub}|$, there is no perturbative QCD uncertainty, while the dominant hadronic uncertainty is avoided. Moreover, this method is not only theoretically quite clean, but experimentally also very efficient in the discrimination between $B \rightarrow X_u l \nu$ signal and $B \rightarrow X_c l \nu$ background. The sum rule requires measuring the lepton pair spectrum. We analyze the lepton pair spectrum, including the leading perturbative and nonperturbative QCD corrections.

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I. INTRODUCTION

The measurement of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1] element $|V_{ub}|$ is currently one of the important goals of B physics. Standard model predictions employ the fundamental parameter $|V_{ub}|$ as input. As the experiments at the B factories are starting to take data, a precise determination of $|V_{ub}|$ is increasingly vital for testing the standard model. For example, the standard model prediction for the CP-violating asymmetry in $B \rightarrow J/\psi K_S$ decays depends on the value of $|V_{ub}|$. Evidence for the CP-violating asymmetry has already been provided by the Collider Detector at Fermilab (CDF) Collaboration [2]. As the dedicated experiments are expected to achieve much greater precision, an increase in the accuracy of $|V_{ub}|$ is highly desirable in order to test whether the complex phase of the CKM matrix is the only source of CP violation. On the other hand, despite empirical successes, the standard model is definitely not the final theory of particle physics. There remain many outstanding issues: electroweak symmetry breaking, fermion masses and mixing, CP violation, replication of families, etc. A precise determination of $|V_{ub}|$ is also an important link in pursuit of deeper principles to extend the standard model.

$|V_{ub}|$ can be determined from both inclusive and exclusive charmless semileptonic B meson decays. The precision of the determination is limited by experimental and theoretical difficulties. Charmless semileptonic B decays are a rare process. The main experimental difficulty in observing signals from $b \rightarrow u l \nu$ processes is the very large background due to $b \rightarrow c l \nu$ whose branching fraction is two orders of magnitude larger. On the theoretical side, calculations are needed in order to relate the measured quantity to $|V_{ub}|$ to extract a value from the data. Strong interaction effects complicate the calculations, causing theoretical uncertainties. The uncertainties result not just from nonperturbative QCD which is notoriously difficult to calculate, but also from perturbative QCD although in priciple it is calculable in perturbation theory. With the developments in nonperturbative techniques, the uncertainty from perturbative QCD due to the truncation of perturbation series in practical calculations can be comparable to that from nonperturbative QCD, indicating

that these two sources of uncertainty can be equally important.

Two approaches have been developed for a QCD treatment of inclusive decays of heavy hadrons. One called the heavy quark expansion approach [3] is based on the operator product expansion. One called the light-cone approach [4–8] is based on the light-cone expansion, using the notion and method of deep inelastic lepton-hadron scattering. In both approaches the heavy quark effective theory [9] is exploited as a limit of QCD. There is important distinction between the two approaches. The former is a short-distance expansion in local operators of increasing dimension, while the latter is a non-local light-cone expansion in matrix elements of increasing twist. Moreover, the heavy quark expansion approach invokes quark-hadron duality, which assumes rates evaluated at the parton level to be equal to observable rates summed over a sufficient number of hadronic channels. Quark-hadron duality cannot be exact [10]. The heavy quark expansion approach has to use quark kinematics and cannot account for the rate due to the extension of phase space from the parton level to the hadron level. The rate missing [11] is just a manifestation of duality violation. In contrast, the light-cone approach does not rely on the assumption of quark-hadron duality. Rates are evaluated in the light-cone approach using physical phase space at the hadron level. The light-cone approach was criticized in Refs. [12]. However, it has been shown [11] that that criticism is mistaken. The validity of the light-cone approach has been tested [6] experimentally on the lepton energy spectrum in inclusive semileptonic decays of B mesons. It is important to test theoretical approaches experimentally in various ways.

To overcome the aforementioned difficulties in the determination of $|V_{ub}|$, we proposed [13] a method of extracting $|V_{ub}|$ from inclusive charmless semileptonic decays of B mesons. We proposed to measure the lepton pair spectrum, i.e., the decay distribution of the kinematic variable $\xi_u = (q^0 + |\mathbf{q}|)/M_B$ in the B-meson rest frame, where q is the momentum transfer to the lepton pair and M_B denotes the B meson mass. Because most of $B \rightarrow X_u l \nu$ ($l = e$ or μ) events have a value of ξ_u beyond the threshold allowed for $B \rightarrow X_c l \nu$ decay, $\xi_u > 1 - M_D/M_B = 0.65$ with M_D being the D meson mass, this kinematic requirement provides a powerful tool

for background suppression. If nothing else, the decay distribution of ξ_u is of direct interest from an experimental standpoint. The sum rule can then be used to extract $|V_{ub}|$ from the weighted integral of the measured ξ_u spectrum. The sum rule follows from the light-cone expansion for inclusive charmless semileptonic decays of B mesons and b-flavored quantum number conservation. The sum rule is thus independent of phenomenological models. Moreover, the sum rule does not receive any perturbative QCD correction. Therefore, this method is not only experimentally very efficient, but theoretically also quite clean, allowing a precise determination of $|V_{ub}|$ with a minimal overall (experimental and theoretical) error.

The sum rule requires measuring the ξ_u spectrum in inclusive charmless semileptonic B meson decays, weighted with ξ_u^{-5}. The purpose of the present paper is to refine the analysis of [13]. We calculate the perturbative QCD correction to the ξ_u spectrum to order α_s. We investigate in detail how gluon radiation and hadronic bound state effects affect the shape of the ξ_u spectrum.

The rest of the paper is organized as follows. In Sec. II the sum rule for inclusive charmless semileptonic decays of B mesons is briefly described. We discuss how to extract $|V_{ub}|$ from experiment exploiting the sum rule. In Sec. III we study the ξ_u spectrum, including both the leading perturbative and nonperturbative QCD corrections. The exploration is extended to the weighted ξ_u spectrum in Sec. IV. Finally in Sec. V we present our conclusions.

II. SUM RULE

The sum rule for the decay distribution of ξ_u in inclusive charmless semileptonic decays of B mesons has been derived [13] from a nonperturbative treatment of QCD based on light-cone expansion. The large B meson mass sets a hard energy scale in the reactions, so that the light-cone expansion is applicable [4–8] to inclusive B decays that are dominated by light-cone singularities, just as deep inelastic scattering. For inclusive charmless semileptonic decays of B mesons, 96% of phase space has $q^2 \geq 1$ GeV^2. This dominance of the high-q^2 region in the whole phase space $0 \leq q^2 \leq M_B^2$ renders the light-cone expansion especially reasonable. The theoretical expression for the weighted integral of the distribution of ξ_u,

$$S \equiv \int_0^1 d\xi_u \frac{1}{\xi_u^5} \frac{d\Gamma}{d\xi_u} (B \rightarrow X_u l \nu), \tag{1}$$

is very clean. In the leading-twist approximation of QCD, it was found [13] that

$$S = |V_{ub}|^2 \frac{G_F^2 M_B^5}{192\pi^3} \langle B|\bar{b}\gamma^\mu b|B\rangle \frac{P_\mu}{2M_B^2}, \tag{2}$$

where P is the momentum of the decaying B meson. The masses of leptons, the u quark and the π meson are neglected. Because b-flavored quantum number conservation under strong interactions implies

$$\langle B|\bar{b}\gamma^\mu b|B\rangle = 2P^\mu, \tag{3}$$

the leading-twist contribution of nonperturbative QCD to the observable S in Eq. (2) is precisely calculable from first principles. The result is the sum rule [13]

$$S \equiv \int_0^1 d\xi_u \frac{1}{\xi_u^5} \frac{d\Gamma}{d\xi_u} (B \rightarrow X_u l \nu) = |V_{ub}|^2 \frac{G_F^2 M_B^5}{192\pi^3}. \tag{4}$$

The conserved current $b\gamma^\mu b$ is not subjected to renormalization and the corresponding anomalous dimension vanishes thanks to the Ward-Takahashi identity. Hence we recognize that the sum rule (4) does not receive any perturbative QCD correction.

It might seem surprising that one is able to calculate S so precisely in QCD when the experimentalist measures hadrons, and it is well known that hadrons cannot be understood completely within perturbation theory. There is a physical way to make clear that we really do not need to know anything about how hadrons are formed in order to predict S in inclusive B decays. The observable S measures something fundamental—namely the b-flavored quantum number carried by the B meson, and is basically determined by the underlying global symmetry of QCD. It is therefore insensitive to hadronic bound state effects.

A measurement of the observable quantity S in inclusive charmless semileptonic decays of B mesons will lead to a determination of $|V_{ub}|$ by use of the sum rule (4) which contains no additional parameter. This determination of $|V_{ub}|$ is free of perturbative QCD uncertainties. Meanwhile, the dominant hadronic uncertainty is avoided. Higher-twist corrections of nonperturbative QCD to the sum rule (4) are suppressed by a power of $\Lambda_{\text{QCD}}^2/M_B^2$. Once the observable S is measured, then Eq. (4) yields $|V_{ub}|$ with small theoretical uncertainty. Moreover, this method is not only theoretically quite clean, but experimentally also very efficient in the discrimination between $b \rightarrow u$ signal and $b \rightarrow c$ background, as we shall discuss.

III. THE ξ_u SPECTRUM

In this section we describe the theoretical prediction for the ξ_u spectrum. We explore step by step perturbative and nonperturbative QCD effects on the ξ_u spectrum. We take into account first the perturbative QCD correction to order α_s and then the leading nonperturbative QCD correction.

A. Leading perturbative QCD correction

Ignoring QCD corrections, the tree-level ξ_u spectrum in the free quark decay $b \rightarrow u l \nu$ in the b-quark rest frame takes the form

$$\frac{1}{\Gamma_0} \frac{d\Gamma_0}{d\xi_u} (b \rightarrow u l \nu) = \delta\left(\xi_u - \frac{m_b}{M_B}\right), \tag{5}$$

with

$$\Gamma_0 = \frac{G_F^2 m_b^5 |V_{ub}|^2}{192\pi^3}, \qquad (6)$$

where m_b represents the b-quark mass. The resulting spectrum is a discrete line at $\xi_u = m_b/M_B$. This is simply a consequence of kinematics that fixes ξ_u to the single value m_b/M_B, no other values of ξ_u are kinematically allowed in $b \to u l \nu$ decays. This is also the case for $b \to u l \nu$ processes with virtual gluon emission. Therefore, the ξ_u spectrum in free quark decays is still a discrete line at $\xi_u = m_b/M_B$, shown in Fig. 1, even if virtual gluon emission occurs. However, the above kinematic relation no longer holds for free b-quark decays with gluon bremsstrahlung, and hence the spectrum expands downward below the parton-level end point $\xi_u = m_b/M_B$.

Including both virtual gluon emission and gluon bremsstrahlung, the differential decay rate as a function of ξ_u in the
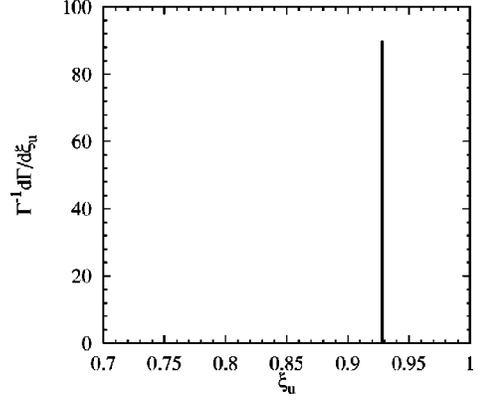
FIG. 1. The ξ_u spectrum in the tree level and virtual gluon exchange processes $b \to u l \nu$. The b-quark mass $m_b = 4.9$ GeV is used.

b-quark rest frame is calculated to order α_s to be

$$\frac{1}{\Gamma_0} \frac{d\Gamma}{d\xi_u}(b \to u l \nu) = \delta\left(\xi_u - \frac{m_b}{M_B}\right)\left[1 - \frac{2\alpha_s}{3\pi}\left(\pi^2 + \frac{13}{72}\right)\right] + \frac{2\alpha_s}{3\pi}\frac{M_B}{m_b}\left[-2\left(\frac{\ln r}{r}\right)_+ - \frac{13}{3}\left(\frac{1}{r}\right)_+ + \frac{79}{9} + \frac{407}{36}r - \frac{367}{12}r^2\right.$$
$$\left. + \frac{59}{3}r^3 - \frac{50}{9}r^4 + \frac{11}{12}r^5 - \frac{7}{36}r^6 + \left(-\frac{2}{3} + \frac{23}{3}r + 3r^2 - \frac{8}{3}r^3\right)\ln r + 2r^2(-3 + 2r)\ln^2 r\right], \qquad (7)$$

where $r = 1 - M_B \xi_u/m_b$ varying in the range $0 \leq r \leq 1$, corresponding to the kinematic range $0 \leq \xi_u \leq m_b/M_B$ at the parton level of quarks and gluons. The distribution $[g(r)]_+$ is defined to coincide with the function $g(r)$ for all values of r greater than 0, and to have a singularity at $r = 0$ such that the integral of this distribution with any smooth function $t(r)$ gives

$$\int_0^1 dr\, [g(r)]_+ t(r) = \int_0^1 dr\, g(r)[t(r) - t(0)]. \qquad (8)$$

The result for the perturbative ξ_u spectrum with the QCD radiative correction[1] to order α_s is shown in Fig. 2. The perturbative ξ_u spectrum is singular at its end point $\xi_u = m_b/M_B$ due to infrared divergences. We observe that gluon bremsstrahlung generates a small tail below $\xi_u = m_b/M_B$.

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[1]The sum rule (4) does not receive any perturbative QCD correction because it results from the conserved current. The numerical calculation by integrating Eq. (7) with the factor ξ_u^{-5} has not reached the absolute zero but yielded a tiny correction (about 0.5%). We have checked that this is only a numerical artifact due to the limited accuracy of numerical integration by computer, especially given the rapid oscillation of the integrand near the end point $\xi_u = 0$.

Integrating Eq. (7) over ξ_u yields the total perturbative decay rate

$$\Gamma(b \to u l \nu) = \Gamma_0 \left[1 - \frac{2\alpha_s}{3\pi}\left(\pi^2 - \frac{25}{4}\right)\right]. \qquad (9)$$

This agrees with the well-known result obtained in [14].

B. Leading nonperturbative QCD correction

To calculate the real physical decay distribution in inclusive decays $B \to X_u l \nu$, perturbative QCD alone is not suffi-
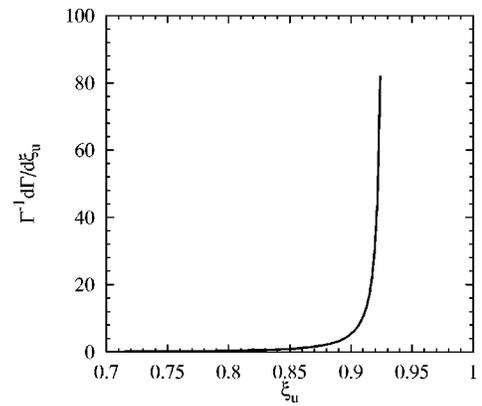
FIG. 2. The perturbative ξ_u spectrum with the QCD radiative correction to order α_s, obtained from input values $m_b = 4.9$ GeV and $\alpha_s = 0.2$.

cient. We must also account for hadronic bound state effects due to the confinement of the b quark inside the B meson. We now consider nonperturbative QCD effects on the ξ_u spectrum. In the framework of the light-cone expansion, the leading nonperturbative QCD effect is incorporated in the b-quark distribution function [8]

$$f(\xi) = \frac{1}{4\pi} \int \frac{d(y \cdot P)}{y \cdot P} e^{i\xi y \cdot P} \langle B | \bar{b}(0) \slashed{y}$$

$$\times \mathcal{P} \exp \left[i g_s \int_y^0 dz^\mu A_\mu(z) \right] b(y) | B \rangle |_{y^2=0}, \quad (10)$$

where $\mathcal{P}$ denotes path ordering. The distribution function $f(\xi)$ has a simple physical interpretation: It is the probability of finding a b-quark with momentum ξP inside the B meson with momentum P. The real physical spectrum is then obtained from a convolution of the hard perturbative spectrum with the soft nonperturbative distribution function:

$$\frac{d\Gamma}{d\xi_u}(B \rightarrow X_u l \nu) = \int_{\xi_u}^1 d\xi f(\xi) \frac{d\Gamma}{d\xi_u}(b \rightarrow u l \nu, p_b = \xi P), \quad (11)$$

where the b-quark momentum p_b in the perturbative spectrum is replaced by ξP. The analytic result for the perturbative spectrum to order α_s is given in Eq. (7). The interplay between nonperturbative and perturbative QCD effects has been accounted for since confinement implies that free quarks are not asymptotic states of the theory and the separation of perturbative and nonperturbative effects cannot be done in a clear-cut way.

Equation (11) demonstrates that the physical ξ_u spectrum depends on the distribution function $f(\xi)$. The definition of it, Eq. (10), involving the B-meson matrix element of the non-local b-quark operators separated along the light cone, makes clear that the distribution function is a nonperturbative quantity. Although a complete calculation of the distribution function in QCD is impossible at present due to our ignorance of nonperturbative QCD, some basic properties of it are known [8]. The distribution function is universal in the sense that the same distribution function also summarizes the leading nonperturbative QCD contribution in inclusive radiative B decays $B \rightarrow X_s \gamma$. It is gauge invariant and obeys positivity. It has a support between 0 and 1 and is exactly normalized to unity because of b-flavored quantum number conservation.[2] It contains the free quark decay as a limiting case with $f(\xi) = \delta(\xi - m_b/M_B)$. In the free quark limit, Eq. (11) consistently reproduces the free quark spectrum.

In addition, the mean μ and the variance σ^2 of the distribution function were deduced [8] using operator product expansion and heavy quark effective theory (HQET) [9]:

$$\mu \equiv \int_0^1 d\xi \, \xi f(\xi) = \frac{m_b}{M_B} \left(1 + \frac{5E_b}{3} \right), \quad (12)$$

$$\sigma^2 \equiv \int_0^1 d\xi \, (\xi - \mu)^2 f(\xi) = \left(\frac{m_b}{M_B} \right)^2 \left[\frac{2K_b}{3} - \left(\frac{5E_b}{3} \right)^2 \right], \quad (13)$$

where $E_b = K_b + G_b$ and K_b and G_b are the dimensionless HQET parameters of order $(\Lambda_{QCD}/m_b)^2$, which are often referred to by the alternate names $\lambda_1 = -2m_b^2 K_b$ and $\lambda_2 = -2m_b^2 G_b/3$. The parameter λ_2 can be extracted from the $B^* - B$ mass splitting: $\lambda_2 = (M_{B*}^2 - M_B^2)/4 = 0.12$ GeV^2. The parameter λ_1 suffers from large uncertainty.

The mean value and variance of the distribution function characterize the location of the ''center of mass'' of the distribution function and the square of its width, respectively. They specify the primary shape of the distribution function. From Eqs. (12) and (13) we know that the distribution function is sharply peaked around $\xi = \mu \approx m_b/M_B$ close to 1 and its width of order Λ_{QCD}/M_B is narrow, suggesting that the distribution function is close to the delta function form in the free quark limit.

Nonperturbative QCD methods such as lattice simulation and QCD sum rules could help determine further the functional form of the distribution function. The distribution function can also be extracted directly from experiments of inclusive semileptonic [13] or radiative [8] decays of B mesons. The universality of the distribution function implies great predictive power: Once the distribution function is measured from one process, it can be used to make predictions in all other processes in a model-independent manner. Since these are as yet not done, we perform the calculations using the parametrization [5] of the distribution function

$$f(\xi) = N \frac{\xi(1-\xi)^\alpha}{[(\xi-a)^2 + b^2]^\beta} \theta(\xi) \theta(1-\xi), \quad (14)$$

where α, β, a, and b are four parameters and N is the normalization constant. The parametrization (14) respects all the known properties of the distribution function, in particular the strong constraints of the sum rules (12) and (13).

The ξ_u spectrum can be calculated using Eqs. (11), (7) and (14). Including both the leading nonperturbative and perturbative QCD corrections, we obtain the ξ_u spectrum shown in Fig. 3, using $\alpha_s = 0.2$, $\alpha = \beta = 1$, $a = 0.9548$ and $b = 0.005444$. Here both values of α and β for the four-parameter distribution function (14) are preset to be 1, but in general they need not be integers. The values of a and b are then inferred from the sum rules (12) and (13) using $m_b = 4.9$ GeV and $\lambda_1 = -0.5$ GeV^2, giving the mean $\mu = 0.93$ and the variance $\sigma^2 = 0.006$ for the distribution function.

Bound-state effects lead to the extension of phase space from the parton level to the hadron level, also stretch the spectrum downward below m_b/M_B, and are solely responsible for populating the spectrum upward in the gap between the parton-level end point $\xi_u = m_b/M_B$ and the hadron-level end point $\xi_u = 1$. The interplay between nonperturbative and perturbative QCD effects eliminates the singularity at the end point of the perturbative spectrum, so that the physical spectrum shows a smooth behavior over the entire range of

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[2]The same conservation law results in the sum rule (4).
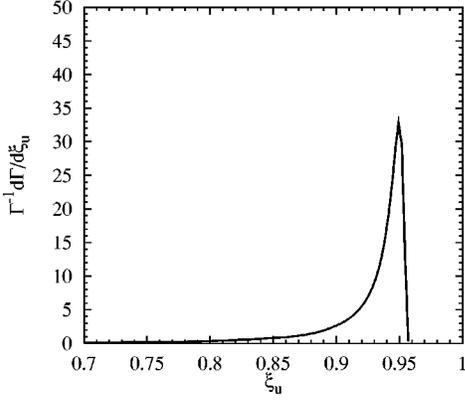
FIG. 3. The ξ_u spectrum including both the leading nonperturbative and perturbative QCD corrections.

ξ_u, $0 \leq \xi_u \leq 1$, as in Fig. 3. Therefore, nonperturbative QCD effects play a crucial role in shaping the ξ_u spectrum.

By integrating Eq. (11) over ξ_u, we obtain the fraction of $B \rightarrow X_u l \nu$ events above the charm threshold allowed for the predominant $B \rightarrow X_c l \nu$ decays, defined as

$$R \equiv \frac{1}{\Gamma(B \rightarrow X_u l \nu)} \int_{1-M_D/M_B}^{1} d\xi_u \frac{d\Gamma}{d\xi_u} (B \rightarrow X_u l \nu). \quad (15)$$

We find that $R = 79\%$ for the spectrum shown in Fig. 3. This result refines our previous result [13]. The earlier calculation without QCD radiative corrections found $R = 99\%$. Gluon bremsstrahlung streches the ξ_u spectrum downward, and hence gives rise to a decrease of the fraction of $B \rightarrow X_u l \nu$ events above the charm threshold. Nevertheless, most of $B \rightarrow X_u l \nu$ events remain above the charm threshold. Thus the kinematic cut on the observable quantity ξ_u is very efficient in disentangling $B \rightarrow X_u l \nu$ signal from $B \rightarrow X_c l \nu$ background. This efficiency can be explained by the uniqueness of the ξ_u spectrum: The ξ_u spectrum stemming from the quark-level and virtual gluon exchange processes would only concentrate at $\xi_u = m_b/M_B$, shown in Fig. 1, solely on kinematic grounds, and gluon bremsstrahlung and hadronic bound state effects smear the spectrum about this point, but most of the decay rate remains at large values of ξ_u, as revealed by Figs. 2 and 3.

The fraction R of course depends on forms of the distribution function. However, we find that R is relatively insensitive to forms of the distribution function once the mean and variance of it, which are known from HQET, given by Eqs. (12) and (13), are kept fixed. Therefore, the above calculation of R can be considered as a typical estimate of the fraction of $B \rightarrow X_u l \nu$ events above the charm threshold.

IV. THE WEIGHTED ξ_u SPECTRUM

The weighted spectrum $\xi_u^{-5} d\Gamma(B \rightarrow X_u l \nu)/d\xi_u$ is more directly relevant for the measurement of the observable S in inclusive charmless semileptonic decays of B mesons. Measurements of S can be obtained from an extrapolation of the weighted spectrum measured above the charm threshold to the full phase space available in $B \rightarrow X_u l \nu$ decays. While the
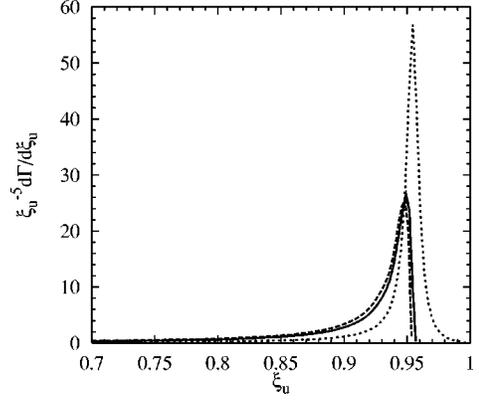
FIG. 4. The weighted ξ_u spectrum without (dotted curve) and with (solid curve: $\alpha_s = 0.2$; dashed curve: $\alpha_s = 0.27$) the QCD radiative correction. The four parameters for the distribution function are taken to be $\alpha = \beta = 1$, $a = 0.9548$, and $b = 0.005444$. All the spectra are normalized to have unit area.

normalization of the weighted spectrum given by the sum rule (4) does not depend on the b-quark distribution function $f(\xi)$, thus being model-independent, the shape of the weighted spectrum does. In this section we perform a detailed analysis of the shape of the weighted spectrum, using Eqs. (11), (7) and (14).

We first explore the impact of gluon radiation on the shape of the weighted ξ_u spectrum. We show in Fig. 4 the weighted ξ_u spectram without and with the QCD radiative correction. It is evident from Fig. 4 that the shape of the weighted ξ_u spectrum receives a significant correction due to gluon radiation. However, the shape of the spectrum appears to be insensitive to the value of the strong coupling α_s, varied within a reasonable range.

We investigate next the sensitivity of the spectrum to the form of the distribution function, which would reflect the impact of hadronic bound state effects. For this purpose we choose to use two very different forms [15] for the distribution function, albeit having the same mean value and variance. The calculated weighted spectra are shown in Fig. 5, taking into account the QCD radiative correction to order α_s. The weighted spectrum exhibits a strong dependence on the form of the distribution function, even though the mean value and variance of the distribution functions are the same.

Generally, since the quark-level processes, exclusive of gluon bremsstrahlung, generate a discrete line, the shape of the ξ_u spectrum directly reflects the inner long-distance dynamics of the reaction. This argument elucidates the strong variation of the weighted spectrum with the form of the distribution function, illustrated in Fig. 5.

In fact, ignoring QCD radiative corrections, one obtains from Eqs. (11) and (5) the weighted spectrum in the B rest frame

$$\frac{1}{\xi_u^5} \frac{d\Gamma}{d\xi_u} (B \rightarrow X_u l \nu) = \frac{G_F^2 M_B^5 |V_{ub}|^2}{192 \pi^3} f(\xi_u). \quad (16)$$

It makes clear that the resulting weighted spectrum is directly proportional to the distribution function. In other
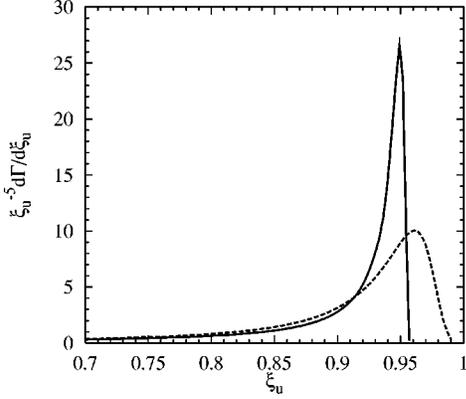
FIG. 5. Comparison of the weighted ξ_u spectra in two forms of the distribution function. The solid curve corresponds to the four-parameter distribution function (14) with $\alpha = \beta = 1$, $a = 0.9548$, and $b = 0.005444$, while the dashed curve corresponds to the four-parameter distribution function (14) with $\alpha = \beta = 2$, $a = 0.9864$, and $b = 0.02557$. These two forms of the distribution function have the same mean value $\mu = 0.93$ and variance $\sigma^2 = 0.006$. The strong coupling is taken to be $\alpha_s = 0.2$. Both spectra are normalized to have unit area.

words, the weighted ξ_u spectrum is most sensitive to the distribution function. This salient feature renders the weighted spectrum ideally suited for the direct extraction of the distribution function from experiment. After an observed spectrum $\xi_u^{-5} d\Gamma(B \to X_u l\nu)/d\xi_u$ is radiatively corrected, it is nothing but the b-quark distribution function if higher-twist terms are neglected.

Extrapolation of the weighted ξ_u spectrum measured above the charm threshold to the full ξ_u range requires a theoretical calculation of the rate ratio

$$W \equiv \frac{1}{S} \int_{1-M_D/M_B}^{1} d\xi_u \frac{1}{\xi_u^5} \frac{d\Gamma}{d\xi_u} (B \to X_u l\nu), \quad (17)$$

where S is defined in Eq. (1). We calculate the ratio W using Eqs. (4), (11), (7) and (14), and the theoretical uncertainties are estimated as follows:

We study the variation of W with respect to the mean value and the variance of the distribution function setting $\alpha = \beta = 1$ in Eq. (14). Actually, this amounts to the study of the ratio W as functions of m_b and λ_1, since, essentially, the mean value of the distribution function is determined by the b-quark mass and its variance is determined by λ_1 according to the sum rules in Eqs. (12) and (13). At present, the estimated values of the b-quark mass and λ_1 vary in the ranges

$$m_b = 4.9 \pm 0.15 \text{ GeV}, \quad (18)$$

$$\lambda_1 = -(0.5 \pm 0.2) \text{ GeV}^2. \quad (19)$$

The variation of m_b leads to an uncertainty of 8% in W if other parameters are kept fixed. A small uncertainty of 3% in W results from the variation of λ_1. In other words, the ratio W displays a strong dependence on the mean value of the distribution function of the b quark inside the B meson, but is insensitive to the variance of the distribution function.

We examine the further sensitivity of the rate ratio W to the form of the distribution function when keeping the mean value and variance of it fixed, by varying the values of the two additional parameters α and β in the parametrization (14). We estimate that the variation of W is 3% if the form of the distribution function is changed but with the same mean value and variance.

We estimate the uncertainty due to the truncation of the perturbative series in Eq. (7) by varying the renormalization scale between $m_b/2$ and $2m_b$. We find that an uncertainty of 8% in the ratio W stems from the renormalization scale dependence.

This analysis implies that at present the theoretical error in the calculation of W has two main sources: the value of m_b (or equivalently, the mean value of the distribution function) and the renormalization scale dependence. Finally, adding all the uncertainties in quadrature we arrive at

$$W = (76 \pm 9)\% \quad (20)$$

with an error of 12%.

V. CONCLUSIONS

We have studied the ξ_u spectrum in inclusive charmless semileptonic decays of B mesons. The perturbative QCD correction to the spectrum is calculated to order α_s. The leading nonperturbative QCD effect is calculated using light-cone expansion and heavy quark effective theory. The ξ_u spectrum is unique in that the tree-level and virtual gluon exchange processes $b \to u l\nu$ at the parton level generate a trivial ξ_u spectrum—a discrete line at $\xi_u = m_b/M_B \approx 0.93$ in the b-quark rest frame, which is well above the charm threshold $\xi_u = 1 - M_D/M_B = 0.65$. Gluon bremsstrahlung results in a small tail in the lepton pair spectrum below m_b/M_B. Bound-state effects lead to the extension of phase space from the parton level to the hadron level, also stretch the spectrum downward below m_b/M_B, and are solely responsible for populating the spectrum upward in the gap between the parton-level endpoint $\xi_u = m_b/M_B$ and the hadron-level end point $\xi_u = 1$, smoothing out the singularity in the perturbative spectrum. As a result of these two distinct effects, the lepton pair spectrum spreads over the entire physical range $0 \leq \xi_u \leq 1$. Still, we find that about 80% of the lepton pair spectrum in $B \to X_u l\nu$ lies above the charm threshold, $\xi_u > 1 - M_D/M_B$. This kinematic cut is most efficient in the suppression of the background of B semileptonic decays into charmed particles.

The observable S can be measured by the extrapolation of the weighted spectrum $\xi_u^{-5} d\Gamma(B \to X_u l\nu)/d\xi_u$ measured above the charm threshold to the entire phase space. Gluon bremsstrahlung and hadronic bound state effects strongly affect the shape of the weighted ξ_u spectrum. However, the shape of the weighted ξ_u spectrum is insensitive to the value of the strong coupling α_s, varied in a reasonable range. The overall picture appears to be that the weighted ξ_u spectrum is peaked towards larger values of ξ_u with a narrow width. The contribution below $\xi_u = 0.65$ is moderate and relatively insensitive to forms of the distribution function. This suggests

that extrapolating the weighted ξ_u spectrum down to low ξ_u would not introduce a considerable uncertainty in the value of S. Quantitatively, our analysis determines the rate ratio for the extrapolation of the weighted ξ_u spectrum to be $W = (76 \pm 9)\%$ with an error of 12% at present.

Another interesting use of the weighted ξ_u spectrum is its utility for directly extracting the b-quark distribution function. The universality of the distribution function implies that the distribution function extracted from inclusive semileptonic B decays can be used to make predictions in inclusive radiative decays $B \to X_s \gamma$ in a model-independent manner and vice versa. In particular, a measurement of the distribution function from the photon energy spectrum in $B \to X_s \gamma$ [8] would be very useful to improve the measurement of S from extrapolation of the weighted ξ_u spectrum.

An improved knowledge of the form of the distribution function is important for an error reduction in extrapolation. More precisely, if by direct measurements or theoretical studies the uncertainty in the mean value of the distribution function can be improved from currently 3% to 1.5% in the future, the resulting uncertainty in the ratio W for the extrapolation would decrease from 8% to 3%. A calculation of the $O(\alpha_s^2)$ perturbative QCD correction to the ξ_u spectrum is also important for improving the accuracy of the extrapolation.

The physical observable quantity S is connected with $|V_{ub}|$ via the sum rule (4) for semileptonic B decay in the leading-twist approximation of QCD. The sum rule is a fundamental, model-independent prediction of QCD. Note that the sum rule does not rely on the heavy quark effective theory. No arbitrary parameter other than $|V_{ub}|$ enters the sum rule. The dominant hadronic uncertainty is avoided in the sum rule. As important, there is no perturbative QCD

modification of this sum rule, so that the potential source of theoretical uncertainty associated with perturbative QCD calculations is totally averted.

The kinematic cut on ξ_u, $\xi_u > 1 - M_D/M_B$, and the semileptonic B decay sum rule, Eq. (4), offer an outstanding opportunity for the precise determination of $|V_{ub}|$ from the observable S. We wish to emphasize that this method is both exceptionally clean theoretically and very efficient experimentally in background suppression.

There remain two types of theoretical uncertainties in the determination of $|V_{ub}|$. First, higher-twist (or power suppressed) corrections to the sum rule cause a theoretical error of order $\Lambda_{\rm QCD}^2/M_B^2 \sim 1\%$ on $|V_{ub}|$. A quantitative study of higher-twist effects could further reduce this small theoretical uncertainty associated with simply extracting the value of $|V_{ub}|$ from the measured value of S. Second, the extrapolation of the weighted ξ_u spectrum to low ξ_u gives rise to a systematic error for the measurement of S. The status of this additional theoretical uncertainty associated with extrapolation, as well as how to improve it, have been discussed above.

Eventually, the error on $|V_{ub}|$ determined by this method would mainly depend on how well the observable S can be measured. To measure S experimentally one needs to be able to reconstruct the neutrino. This poses a challenge to experiment. Given the unique potential of determining $|V_{ub}|$, we would urge our experimental colleagues to examine the feasibility of the method.

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