Semileptonic and nonleptonic B_c decays

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We make predictions for the exclusive semileptonic and nonleptonic decay widths of the B_c meson. We evaluate the B_c semileptonic form factors for different decay channels in a relativistic model, and use factorization to obtain the nonleptonic decay widths.

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The recent discovery of the B_c meson by the Collider Detector at Fermilab (CDF) Collaboration [1] attracted a great deal of attention. The B_c meson is very interesting because it carries nonvanishing flavor quantum numbers, and lies below the threshold of the BD decay. Therefore, it can only decay through weak interactions which makes this doubly heavy meson useful for studying the weak decays of heavy flavors. The B_c production mechanisms, spectroscopy, and decays have been analyzed using different approaches (see Ref. [2] for a review).

In a previous paper [3] we used a relativistic model [4] based on the Bethe-Salpeter equation (BSE) to evaluate the spectrum of the B_c meson. No free parameters were used to fit the B_c spectrum. Instead, all the model parameters had been fixed in previous investigations of other meson spectra. We also evaluated the decay constant of the B_c meson, and the inclusive decay widths of the c quark and the $\overline{b}$ quark together with the annihilation width. Our results agree very well with the CDF results of the B_c mass and lifetime. We have presented these results with two covariant reductions of the BSE and observed little dependence on the choice of the reduction especially in the heavy flavor sector.

In this paper we evaluate the exclusive semileptonic $B_c \to P(V) e \nu$ and two-body nonleptonic $B_c \to PP, PV, VV$ decay widths, where $P(V)$ denotes a pseudoscalar (vector) meson. We use our model to calculate the semileptonic form factors for different decay channels. We then use factorization to obtain the nonleptonic decay widths. We will utilize primarily a single reduction since this investigation uses BSE results from the heavy flavor sector.

The BSE provides an appealing starting point to describe hadrons as relativistic bound states of quarks. The BSE for a bound state may be written in momentum space in the form

$$G^{-1}(P,p)\psi(P,p) = \int \frac{1}{(2\pi)^4} V(P,p-p')\psi(P,p')d^4p', \tag{1}$$

where P is the four-momentum of the bound state, and p is the relative four-momentum of the constituents. The BSE has three elements, the interaction kernel (V) and the propagator (G), which we provide as input, and the amplitude (ψ) obtained by solving the equation. We also solve for the energy, which is contained in the propagator.

Different approaches have been developed to make the four-dimensional BSE more tractable and physically appealing. These include the instantaneous approximation (IA) and quasipotential equations (QPEs) [5]. In the IA, the interaction kernel is taken to be independent of the relative energy. In QPEs, the two particle propagator is modified in a way which keeps covariance and reduces the four-dimensional BSE to a three-dimensional equation. Of course, there is considerable freedom in carrying out this reduction.

Earlier, we have used two reductions of the QPE to study the meson spectrum [4]. These reductions correspond to different choices of the two-particle propagator used to reduce the problem into three dimensions. We refer to these reductions as A and B. Reduction A corresponds to a spinor form of the Thompson equation [6] and reduction B corresponds to a new QPE introduced in Ref. [7]. These two covariant reductions are chosen because they are shown to give good fits to the meson spectrum. In both reductions, we assume the interaction kernel to consist of a one-gluon exchange interaction V_{OGE} in the ladder approximation and a phenomenological, long range scalar confinement potential V_{CON} given in the form

$$V_{OGE} + V_{CON} = -\frac{4}{3}\alpha_s \frac{\gamma_\mu \otimes \gamma_\mu}{(p-p')^2} + \sigma \lim_{\mu \to 0} \frac{\partial^2}{\partial \mu^2} \frac{\mathbf{1} \otimes \mathbf{1}}{-(p-p')^2 + \mu^2}. \tag{2}$$

Here, α_s is the strong coupling, which is weighted by the meson color factor of $\frac{4}{3}$, and the string tension σ is the strength of the confining part of the interaction. We adopt a

TABLE I. Spectrum of B_c mesons in different channels (GeV/c^2).

State	Our work Reduction A	Our work Reduction B	Eichten and Quigg [8] BT potential	Gershtein *et al.* [9] Martin potential	Gershtein *et al.* [9] BT potential
$1\,{}^1S_0$	6.356	6.380	6.264	6.253	6.246
$1\,{}^3S_1$	6.397	6.415	6.337	6.317	6.337
$1\,{}^3P_0$	6.673	6.692	6.700	6.683	6.700
$1\,{}^3P_2$	6.751	6.773	6.747	6.743	6.747
$1\,{}^1P_1$	6.752	6.777		6.729	6.736
$2\,{}^1S_0$	6.888	6.874	6.856	6.867	6.856
$2\,{}^3S_1$	6.910	6.891	6.899	6.902	6.899
$1\,{}^3D_1$	6.984	6.955	7.012	7.008	7.012

scalar Lorentz structure V_{CON} as discussed in [4]. In our formulation of the BSE there is a total of seven parameters: four masses $m_u=m_d$, m_s, m_c, m_b; the string tension σ; and two other parameters used to govern the running of the strong coupling constant. We varied these parameters to get the best fit for a list of known mesons as described in [4].

In our subsequent work [3] on the B_c meson, we evaluated the B_c spectrum without changing the parameter values mentioned above (see Table I below) and compared our results with those of Eichten and Quigg [8] using a Buchmuller-Tye (BT) potential and Gershtein *et al.* [9] using both a Martin potential and BT potential. The first row of Table I should be compared with the experimental result [1] of 6.40 ± 0.39 (stat.) ±0.13 (syst.) GeV/c^2. We have also evaluated the inclusive c-quark and $\bar{b}$-quark decay lifetimes [3] and obtained a B_c lifetime of 0.46–0.47 ps, in good agreement with the experimental B_c lifetime of $0.46^{+0.18}_{-0.16}$ (stat.) ±0.03 (syst.) ps [1].

We now turn our attention to exclusive decays. The B_c exclusive semileptonic and nonleptonic decays have been discussed in the literature [9–11]. The effective Hamiltonian for the semileptonic decays has the standard current-current form, and is given by

$$H_W=\frac{G_F}{\sqrt{2}}V_{Qq}\bar{q}\gamma_\mu(1-\gamma_5)Q\bar{\nu}\gamma^\mu(1-\gamma_5)l. \tag{3}$$

The leptonic current is completely known and the matrix element of the vector (V_μ) and the axial vector (A_μ) hadronic currents between the meson states are represented in terms of form factors which are defined by [considering the channel $B_c\to B_s(B_s^*)$]

$$\langle B_s(P')|V_\mu|B_c(P)\rangle=f_+(P+P')_\mu+f_-(P-P')_\mu,$$

$$\langle B_s^*(P',\varepsilon)|V_\mu|B_c(P)\rangle=ig\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}(P+P')^\alpha(P-P')^\beta,$$

$$\langle B_s^*(P',\varepsilon)|A_\mu|B_c(P)\rangle=f\varepsilon_\mu^*+(\varepsilon^*.P)[a_+(P+P')_\mu+a_-(P-P')_\mu]. \tag{4}$$

f_+, f_-, g, f, a_+, and a_- are Lorentz invariant form factors which are scalar functions of the momentum transfer $q^2=(P-P')^2$ where P and P' are the four-momenta of the B_c and B_s (B_s^*) mesons, respectively.

In our formalism, the mesons are taken as bound states of a quark and an antiquark. We construct the meson states as [12]

$$|M(\mathbf{P_M},J,m_J)\rangle=\sqrt{2M}\int d^3\mathbf{p}\langle Lm_LSm_S|Jm_J\rangle \times\langle sm_s\bar{s}\bar{m}_s|Sm_S\rangle\Phi_{Lm_L}(\mathbf{p}) \times\left|\bar{q}\left(\frac{m_{\bar{q}}}{M_{q\bar{q}}}\mathbf{P}_M-\mathbf{p},m_{\bar{s}}\right)\right\rangle \times\left|q\left(\frac{m_q}{M_{q\bar{q}}}\mathbf{P}_M+\mathbf{p},m_s\right)\right\rangle, \tag{5}$$

where the quark states are given by

$$|q(\mathbf{p},m_s)\rangle=\sqrt{\frac{(E_q+m_q)}{2m_q}}\begin{pmatrix}\chi^{m_s}\\ \dfrac{\sigma\cdot\mathbf{p}}{(E_q+m_q)}\chi^{m_s}\end{pmatrix},$$

$$M_{q\bar{q}}=m_q+m_{\bar{q}},\qquad E_q=\sqrt{m_q^2+\mathbf{p}^2}. \tag{6}$$

In the above equations M is the meson mass. The meson and the constituent quark states satisfy the normalization conditions

$$\langle M(\mathbf{P}'_\mathbf{M},J',m'_J)|M(\mathbf{P}_M,J,m_J)\rangle =2E\,\delta^3(\mathbf{P}'_\mathbf{M}-\mathbf{P}_M)\,\delta_{J',J}\,\delta_{m'_J,m_J}, \tag{7}$$

$$\langle q(\mathbf{p}',m'_s)|q(\mathbf{p},m_s)\rangle =\frac{E_q}{m_q}\,\delta^3(\mathbf{p}'-\mathbf{p})\,\delta_{m'_s,m_s}. \tag{8}$$

The wave functions Φ_{Lm_L} appearing in Eq. (5) for the mesons are calculated by solving reductions of the Bethe-Salpeter equation [4]. We have applied this formalism to

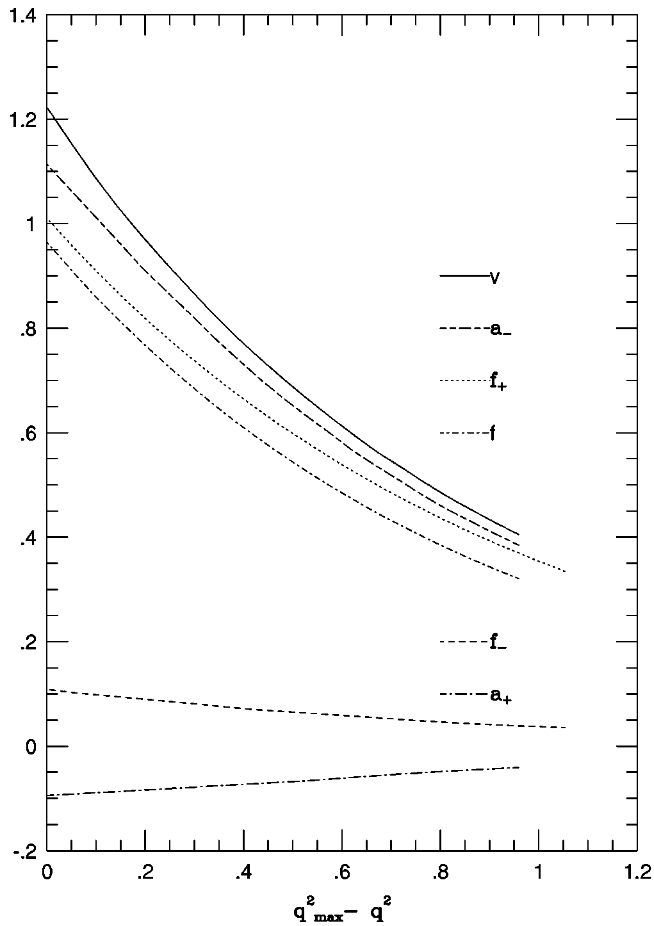

FIG. 1. The semileptonic form factors for $B_c \to B_s(B_s^*)$.

evaluate the semileptonic form factors of the B to D and D^* mesons and showed that our results [13] are consistent with heavy quark effective theory (HQET). We use wave fuctions from reduction B as we did in our previous work on B decays [13].

The values of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements we use in this paper are $V_{ud}=0.974$, $V_{us}=0.2196$, $V_{ub}=0.0033$, $V_{cd}=0.224$, $V_{cs}=0.974$, $V_{cb}=0.0395$ [14].

In Fig. 1 we show the semileptonic form factors for $B_c \to B_s(B_s^*)$ and in Table II we show the exclusive semileptonic decay widths to different pseudoscalar and vector final states $[B_c^+ \to P(V)e^+\nu]$. We also compare our results with those of [11]. While we use the QPE approach to solve the bound state problem as we discussed before, the authors of Ref. [11] have used a nonrelativistic IA to establish a relation between the Bethe-Salpeter amplitude and the Schrodinger wave function. They also invoke a nonrelativistic IA for the entire matrix element. While we use a one-gluon exchange interaction and a phenomenological long range scalar confinement potential, the authors of Ref. [11] did not provide sufficient information of their QCD-inspired potential or of other observables to permit a more detailed comparison of the two approaches. In general, however, one expects the differences to be larger when the bound state has high momentum components in our QPE approach.

TABLE II. Exclusive semileptonic $B_c^+ \to P(V)e^+\nu$ decay widths in 10^{-6} eV.

	Process	Decay width This work	Decay width Chang and Chen [11]
$\bar{b}$ decay	$B_c^+ \to \eta_c e^+\nu$	11.1	14.2
	$B_c^+ \to J/\psi e^+\nu$	30.2	34.4
	$B_c^+ \to D^0 e^+\nu$	0.049	0.094
	$B_c^+ \to D^{*0} e^+\nu$	0.192	0.269
c decay	$B_c^+ \to B_s^0 e^+\nu$	14.3	26.6
	$B_c^+ \to B_s^{*0} e^+\nu$	50.4	44.0
	$B_c^+ \to B^0 e^+\nu$	1.14	2.30
	$B_c^+ \to B^{*0} e^+\nu$	3.53	3.32

We notice that the semileptonic form factors shown in Fig. 1 are qualitatively similar to the $B \to D(D^*)$ ones [13]. However, one cannot use flavor symmetry to relate the initial B_c and the final meson (η_c or B_s, for example) states. Simply, B_c, η_c, and B_s have different sizes and flavor symmetry is absent in B_c decays as discussed in [15].

For nonleptonic decays, the effective Hamiltonian (considering the $B_c^+ \to B_s \pi^+$ channel) may be written as

$$H_W = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* [c_1(\mu) O_1 + c_2(\mu) O_2], \tag{9}$$

where

$$O_1 = (\bar{u}_i d_i)_{V-A} (\bar{s}_j c_j)_{V-A},$$
$$O_2 = (\bar{u}_i d_j)_{V-A} (\bar{s}_j c_i)_{V-A}, \tag{10}$$

with $(i,j=1,2,3)$ denoting color indices and $V-A$ referring to $\gamma_\mu(1-\gamma_5)$. $c_1(\mu)$ and $c_2(\mu)$ are short distance Wilson coefficients computed at the scale μ. By factorizing matrix elements of the four-quark operator contained in the effective Hamiltonian of Eq. (9), one can distinguish three classes of decays [16]. The first class (class I) contains those decays in which only a charged meson can be generated directly from a color-singlet current, as in $B_c^+ \to B_s \pi^+$. A second class of transitions (class II) consists of those decays in which the meson generated directly from the current is neutral, such as the π^0 meson in the decay $B_c^+ \to B^+ \pi^0$. Class I decay amplitudes are proportional to a_1; class II decay amplitudes are proportional to a_2 where

$$a_1 = c_1(\mu) + \xi c_2(\mu),$$
$$a_2 = c_2(\mu) + \xi c_1(\mu), \tag{11}$$

and $\xi = 1/N_c$, where N_c is the number of quark colors, and μ is the scale at which factorization is assumed to be relevant. For the third class (class III) the a_1 and a_2 amplitudes interfere. Although the QCD factors a_1 and a_2 have been calculated beyond the leading logarithmic approximation [17], we will follow the prevailing convention of theoretical predic-

TABLE III. Exclusive nonleptonic decay widths of the B_c meson in 10^{-6} eV. $\overline{b}$-quark decays with c quark spectator. The authors of Ref. [11] did not report the widths of some of the channels because it was thought, prior to the experimental discovery of the B_c meson, that these channels will be kinematically closed.

Class	Process	Decay width This work	Decay width Chang and Chen [11]
I	$B_c^+ \to \eta_c \pi^+$	$a_1^2 1.59$	$a_1^2 2.07$
	$B_c^+ \to \eta_c \rho^+$	$a_1^2 3.74$	$a_1^2 5.48$
	$B_c^+ \to J/\psi \pi^+$	$a_1^2 1.22$	$a_1^2 1.97$
	$B_c^+ \to J/\psi \rho^+$	$a_1^2 3.48$	$a_1^2 5.95$
	$B_c^+ \to \eta_c K^+$	$a_1^2 0.119$	$a_1^2 0.161$
	$B_c^+ \to \eta_c K^{*+}$	$a_1^2 0.200$	$a_1^2 0.286$
	$B_c^+ \to J/\psi K^+$	$a_1^2 0.090$	$a_1^2 0.152$
	$B_c \to J/\psi K^{*+}$	$a_1^2 0.197$	$a_1^2 0.324$
II	$B_c^+ \to D^+ \overline{D^0}$	$a_2^2 0.633$	$a_2^2 0.664$
	$B_c^+ \to D^+ \overline{D^{*0}}$	$a_2^2 0.762$	$a_2^2 0.695$
	$B_c^+ \to D^{*+} \overline{D^0}$	$a_2^2 0.289$	$a_2^2 0.653$
	$B_c^+ \to D^{*+} \overline{D^{*0}}$	$a_2^2 0.854$	$a_2^2 1.080$
	$B_c^+ \to D_s^+ \overline{D^0}$	$a_2^2 0.0415$	$a_2^2 0.0340$
	$B_c^+ \to D_s^+ \overline{D^{*0}}$	$a_2^2 0.0495$	$a_2^2 0.0354$
	$B_c^+ \to D_s^{*+} \overline{D^0}$	$a_2^2 0.0201$	$a_2^2 0.0334$
	$B_c^+ \to D_s^{*+} \overline{D^{*0}}$	$a_2^2 0.0597$	$a_2^2 0.0564$
III	$B_c^+ \to \eta_c D_s^+$	$(a_1 2.16 + a_2 2.57)^2$	$(a_1 1.13 + a_2 1.98)^2$
	$B_c^+ \to \eta_c D_s^{*+}$	$(a_1 2.03 + a_2 2.16)^2$	$(a_1 1.04 + a_2 1.90)^2$
	$B_c^+ \to J/\psi D_s^+$	$(a_1 1.62 + a_2 1.72)^2$	$(a_1 1.02 + a_2 1.95)^2$
	$B_c^+ \to J/\psi D_s^{*+}$	$(a_1 3.13 + a_2 3.67)^2$	
	$B_c^+ \to \eta_c D^+$	$(a_1 0.485 + a_2 0.528)^2$	$(a_1 0.193 + a_2 0.440)^2$
	$B_c^+ \to \eta_c D^{*+}$	$(a_1 0.466 + a_2 0.452)^2$	$(a_1 0.181 + a_2 0.430)^2$
	$B_c^+ \to J/\psi D^+$	$(a_1 0.372 + a_2 0.338)^2$	$(a_1 0.177 + a_2 0.442)^2$
	$B_c^+ \to J/\psi D^{*+}$	$(a_1 0.686 + a_2 0.732)^2$	

tions and express our results in terms of them. As an example the $B_c^+ \to B_s \pi^+$ amplitude takes the form

$$A(B_c^+ \to B_s \pi^+) = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* a_1(\mu) \langle \pi^+ | (\overline{u_i} d_i)_{V-A} | 0 \rangle \times \langle B_s | (\overline{s_j} c_j)_{V-A} | B_c \rangle. \quad (12)$$

The validity of the factorization approximation is hard to quantify. However, there is the argument that the amplitude for energetic weak decays is dominated by its factorizable part [16]. Bjorken [18] gave the intuitive ''color transparency argument'' that a directly generated quark-antiquark pair carrying a large momentum will hadronize far from the remaining quarks and will have almost no interaction with them. Therefore, one may speculate that factorization will work better for class I $\overline{b}$ quark decays of B_c.

The matrix elements $\langle B_s | (\overline{s_j} c_j)_{V-A} | B_c \rangle$ in Eq. (12) have already been evaluated in semileptonic decays of the B_c meson in terms of form factors, while the other matrix element ($\langle \pi^+ | (\overline{u_i} d_i)_{V-A} | 0 \rangle$) is related to the decay constant of the relevant meson. The weak decay constants f_P and f_V for pseudoscalar and vector mesons are defined by

$$\langle 0 | J_\mu | P(p) \rangle = i f_P p_\mu,$$

$$\langle 0 | J_\mu | V(p) \rangle = M_V f_V \varepsilon_\mu, \quad (13)$$

where P and V are pseudoscalar and vector states, respectively, and $J_\mu = V_\mu - A_\mu$ is the weak current (V_μ and A_μ are the vector and axial vector currents). The decay constants can be expressed in terms of the wave functions of the relevant mesons and are given by [19]

$$f_i = \sqrt{\frac{12}{M}} \int_0^\infty \frac{p^2 dp}{2\pi^3} \sqrt{\frac{(m_q + E_q)(m_{\overline{q}} + E_{\overline{q}})}{4 E_q E_{\overline{q}}}} F_i(p), \quad (14)$$

$$F_P(p) = \left[1 - \frac{p^2}{(m_q + E_q)(m_{\overline{q}} + E_{\overline{q}})} \right] \psi_P(p), \quad (15)$$

$$F_V(p) = \left[1 - \frac{p^2}{3(m_q + E_q)(m_{\overline{q}} + E_{\overline{q}})} \right] \psi_V(p), \quad (16)$$

TABLE IV. Exclusive nonleptonic decay widths of the B_c meson in 10^{-6} eV. c quark decays with $\bar{b}$ quark spectator.

Class	Process	Decay width This work	Decay width Chang and Chen [11]
I	$B_c^+ \to B_s^0 \pi^+$	$a_1^2 15.8$	$a_1^2 58.4$
	$B_c^+ \to B_s^0 \rho^+$	$a_1^2 39.2$	$a_1^2 44.8$
	$B_c^+ \to B_s^{*0} \pi^+$	$a_1^2 12.5$	$a_1^2 51.6$
	$B_c^+ \to B_s^{*0} \rho^+$	$a_1^2 171.$	$a_1^2 150.$
	$B_c^+ \to B_s^0 K^+$	$a_1^2 1.70$	$a_1^2 4.20$
	$B_c^+ \to B_s^{*0} K^+$	$a_1^2 1.34$	$a_1^2 2.96$
	$B_c^+ \to B_s^0 K^{*+}$	$a_1^2 1.06$	
	$B_c^+ \to B_s^{*0} K^{*+}$	$a_1^2 11.6$	
	$B_c^+ \to B^0 \pi^+$	$a_1^2 1.03$	$a_1^2 3.30$
	$B_c^+ \to B^0 \rho^+$	$a_1^2 2.81$	$a_1^2 5.97$
	$B_c^+ \to B^{*0} \pi^+$	$a_1^2 0.77$	$a_1^2 2.90$
	$B_c^+ \to B^{*0} \rho^+$	$a_1^2 9.01$	$a_1^2 11.9$
	$B_c^+ \to B^0 K^+$	$a_1^2 0.105$	$a_1^2 0.255$
	$B_c^+ \to B^0 K^{*+}$	$a_1^2 0.125$	$a_1^2 0.180$
	$B_c^+ \to B^{*0} K^+$	$a_1^2 0.064$	$a_1^2 0.195$
	$B_c^+ \to B^{*0} K^{*+}$	$a_1^2 0.665$	$a_1^2 0.374$
II	$B_c^+ \to B^+ \overline{K^0}$	$a_2^2 39.1$	$a_2^2 96.5$
	$B_c^+ \to B^+ \overline{K^{*0}}$	$a_2^2 46.8$	$a_2^2 68.2$
	$B_c^+ \to B^{*+} \overline{K^0}$	$a_2^2 24.0$	$a_2^2 73.3$
	$B_c^+ \to B^{*+} \overline{K^{*0}}$	$a_2^2 247$	$a_2^2 141$
	$B_c^+ \to B^+ \pi^0$	$a_2^2 0.51$	$a_2^2 1.65$
	$B_c^+ \to B^+ \rho^0$	$a_2^2 1.40$	$a_2^2 2.98$
	$B_c^+ \to B^{*+} \pi^0$	$a_2^2 0.38$	$a_2^2 1.45$
	$B_c^+ \to B^{*+} \rho^0$	$a_2^2 4.50$	$a_2^2 5.96$

where $\psi_{P(V)}$ are the momentum wave functions of the pseudoscalar (vector) mesons.

We have previously applied this formalism to evaluate the decay constants and the nonleptonic decays of the B mesons [20]. The values of the decay constants we use in this paper are $f_\pi = 0.130$ GeV, $f_\rho = 0.208$ GeV, $f_K = 0.159$ GeV, $f_{K*} = 0.214$ GeV, $f_D = 0.209$ GeV, $f_{D*} = 0.237$ GeV, $f_{D_s} = 0.213$ GeV, $f_{D_s^*} = 0.242$ GeV, $f_{\eta_c} = 0.400$ GeV, $f_{J/\psi} = 0.400$ GeV. These values are the available experimental ones [14]. Otherwise we use our values reported in [20]. These values of the decay constants are similar to those used by other authors [9–11].

In Table III we compare our results for the exclusive nonleptonic $B_c \to PP, PV, VV$ decay widths of different channels where the $\bar{b}$ quark decays with those of [11], while in Table IV, we make the same comparison for the case of c quark decays.

At first glance, our decay widths in Table III are generally smaller than those of Ref. [11] by 20–40 %. However, this is not a uniform trend as our $B_c^+ \to D^+ \overline{D^{*0}}$ is 10% larger than that of Ref. [11]. If we furthermore compare total lifetimes for B_c, we find that our lifetime (0.46 ps) is longer compared to Ref. [11] (0.40 ps) which is consistent with the dominant trends seen in the comparisons of the exclusive channels. Both theoretical lifetimes are well within current experimental uncertainties. Thus, experimental results for a set of exclusive channels could resolve between these two sets of theoretical predictions. Table IV displays even greater range of differences between our model and that of Ref. [11].

In conclusion, we have systematically evaluated the decay widths of the exclusive semileptonic channels $B_c \to P(V) e \nu$ and the exclusive two-body nonleptonic decays $B_c \to PP$, PV, and VV assuming that either c or $\bar{b}$ quark inside the B_c meson is a spectator quark and using our relativistic model [4]. In general, our predicted widths are smaller than those reported in Ref. [11] but there are exceptions to this trend. The variations between the theoretical predictions are wide enough so that experimental results should be able to discern between the models.

We note that the dominant decays are those when the b quark inside the B_c meson behaves as a spectator quark and a vector meson is produced in the final state. In fact, $B_c^+ \to B_s^{*0} e^+ \nu$ is the dominant decay among all the semileptonic channels (see Table II) and $B_c \to B_s^{*0} \rho^+$ becomes the dominant among all the two-body nonleptonic decays (see Table IV). Although these decays are suppressed by phase space, they are CKM favored.

Finally we point out that the ratio

$$\frac{\Gamma(B_c^+ \to V\rho^+)}{d\Gamma(B_c^+ \to Ve^+\nu)/dt|_{t=m_\rho^2}} = 6\pi^2 |V_{ud}|^2 a_1^2 f_\rho^2, \qquad (17)$$

with $V = B_s^{*0}, J/\psi$ will be a good experimental test for the numerical value of the coefficient a_1 of QCD [16].

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