Filtering overpopulated isoscalar tensor states with mass relations

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Schwinger-type mass formulas are used to analyze glueball-meson mixing for isoscalar tensor mesons. In one solution, the $f_J(2220)$ is the physical glueball, and in the other the glueball is distributed over various states, with $f_2(1810)$ having the largest glueball component. Neither the $f_2(1565)$ nor the $f_J(1710)$ are among the physical states without assuming significant coupling to decay channels. The decay $f_2(1525)\rightarrow\pi\pi$ is consistent with experiment, and $f_J(2220)$ is neither narrow nor decays flavor democratically.

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The numerous isoscalar tensor states seen by experiments [1] resist simple meson interpretations, because of the predicted existence of the tensor glueball amongst these states [2]. Glueball-meson mixing can be substantial, as found for isoscalar scalar mesons [3].

In the absence of lattice QCD predictions, the complexity of glueball-meson mixing calls for an analysis with as few assumptions as possible about detailed dynamics. In this paper we present a mass matrix analysis of considerable, although not total, generality.

Although QCD is not expected to admit an exact mass matrix representation, and only effective mass matrix field theories are known to exist, Schwinger's original mass formula, derived from mass matrices, is in excellent agreement with experiment for the $J^{PC}=1^{--}$, 2^{++}, 2^{-+} and 3^{--} sectors [4]. The formula contains a primitive (bare) isoscalar meson $n\bar{n}\equiv(u\bar{u}+d\bar{d})/\sqrt{2}$, referring to the partner of the light quark isovector meson, with a primitive $s\bar{s}$ partner; and the two physical states, representing the primitive states after mixing. The analysis assumes lack of imaginary parts and that all quarks couple the same way in the mixing. The primitive $n\bar{n}$ meson mass is related to the isovector meson mass taken from experiment. As motivated in Ref. [4], the two masses are typically set equal. An additional relation, commonly the extraction of mixing angles from experiment, or the quark model relation $M_{n\bar{n}}+M_{s\bar{s}}=2M_{s\bar{n}}$, is needed to test, with knowledge of the masses of the two physical states, the validity of the Schwinger formula. We shall use equality of isoscalar and isovector meson masses in every nonet of this work, and sometimes apply the quark model relation.

For a single meson nonet, the Schwinger formula with lack of imaginary parts can be extended, without any additional assumptions, to include glueballs [4]. However, with multiple meson nonets this is not possible, and it is assumed that the mesons and glueball mix only via meson-glueball coupling, with no direct meson-meson coupling. This assumption follows from large N_c QCD.[1] The assumption already had the phenomenological success of providing an understanding of the decay pattern of the scalar glueball found in lattice QCD [3]. Ideal mixing of the mesons before they mix with the glueball is assumed, since the small experimental decay $f_2'(1525)\rightarrow\pi\pi$ supports this choice of basis.

The first excited tensor glueball is expected to lie above the experimentally accessed mass region [2], and its effect can hence safely be neglected. This means that we can restrict consideration to the low-lying mesons and one primitive glueball. After allowing these to mix, we shall derive Schwinger-type mass formulas. Mixing with hypothetical four-quark states is not taken into account.

The isovector mesons will be given labels such as P-wave, F-wave or hybrid meson, indicating the dominant component in a quark model interpretation of the state. However, the mass matrix analysis does not assume the P-wave, F-wave or hybrid meson nature of a state, nor that it is a pure quark model state.

I. MASSES AND VALENCE CONTENTS

The isovector tensor mesons should act as beacons for the mass scales of various nonets. Unfortunately, only the $a_2(1320)$, which we will take to fix the mass of the primitive $n\bar{n}$ ground state P-wave state, $1P$, is well established [1]. There is recent evidence for $a_2(1660)$ at 1660 ± 40 MeV or 1660 ± 15 MeV [5], and for an $a_2(1600-1700)$ [6]. The $a_2(1660)$ is taken to fix the mass of the primitive $n\bar{n}$ first radially excited P-wave state, $2P$.

There is also some recent evidence for isovector tensor states at 2060 ± 20 MeV and 1990^{+15}_{-30} MeV [7], signaling the $3P$ and $1F$ nonets. The $1F$ nonet is signposted by the $a_4(2050)$. There are recent indications from VES that the

[1]The matrix element between different $n\bar{n}$ states is zero because we chose a basis in which the isovector meson matrix is diagonal, and the primitive $n\bar{n}$'s are taken to be the isoscalar analogues of the isovector states. The $s\bar{s}$ states are taken to be heavier analogues of the light ones, so that the mixing between different $s\bar{s}$ states is zero.

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mass of the $a_4(2050)$ is $1944\pm8\pm50$ or 1950 ± 20 MeV [6]. We place the primitive $n\bar{n}$ $1F$ state at 1.94 GeV and the primitive $s\bar{s}$ state higher by twice the difference between the $K_4(2045)$ and VES' $a_4(2050)$ masses, i.e. at 2.15 GeV. We shall take the primitive $n\bar{n}$ $3P$ level to be at 2.05 GeV.

There is some evidence for an isovector tensor state at 2265 ± 20 MeV [7], signalling the $2F$ or $4P$ nonets. The presence of both nonets is indicated by an a_1 at 2340 ± 40 MeV [7], which can be 4P but not 2F; or an a_4 at 2300 ± 20 MeV [7], which can be $2F$ but not $4P$.

There are 13 isoscalar tensor mesons up to ~2.3 GeV listed by the Particle Data Group, with 6 well-established[2] [1]. One expects a glueball, and the $1P$, $2P$, $3P$ and $1F$ nonets in this mass region, yielding 9 states, and possibly $n\bar{n}$ $4P$ and $2F$ in addition, giving 11 states. There is hence an overpopulation of experimental isoscalar tensors, albeit not for the well-established ones.

Since the $1P$, $2P$, $3P$ and $1F$ nonets are expected below ~2.3 GeV, our analysis can safely be restricted to a 9×9 mass matrix. There is the possibility of the $4P$ and $2F$ mesons contaminating results at the upper end of our simulation, at ~2.3 GeV, which is also investigated.

The mixing of a glueball and n pairs of isoscalar mesons is described by the following mass matrix, which is diagonalized by the masses of $(2n+1)$ physical states:

$$\begin{pmatrix} G & g_1 & g_1\sqrt{2} & \cdot & \cdot & \cdot & g_n & g_n\sqrt{2} \\ g_1 & S_1 & 0 & \cdot & \cdot & \cdot & 0 & 0 \\ g_1\sqrt{2} & 0 & N_1 & \cdot & \cdot & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & & & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & & \cdot & \cdot \\ \cdot & \cdot & \cdot & & & \cdot & \cdot & \cdot \\ g_n & 0 & 0 & \cdot & \cdot & \cdot & S_n & 0 \\ g_n\sqrt{2} & 0 & 0 & \cdot & \cdot & \cdot & 0 & N_n \end{pmatrix} \Rightarrow \mathrm{diag}(h_1,h_2,\ldots,h_{2n+1}). \quad (1)$$

G, S and N stand for the masses of the primitive glueball, $s\bar{s}$ and $n\bar{n}$ mesons, respectively, the subscript indicating the number of the nonet the state belongs to. h_i stand for the masses of the physical states. g_i are the glueball-meson couplings that have dimensionality (mass).

Applying the techniques of Ref. [4], one can obtain $2n$ pairs of relations for the coupling in terms of the primitive and physical masses ($i=1,2,\ldots,n$):

$$g_i=\sqrt{-\frac{\prod_{j=1}^{2n+1}(S_i-h_j)}{\prod_{j=1}^{n}(S_i-N_j)\prod_{j=1 j\neq i}^{n}(S_i-S_j)}}=\sqrt{-\frac{\prod_{j=1}^{2n+1}(N_i-h_j)}{2\prod_{j=1}^{n}(N_i-S_j)\prod_{j=1 j\neq i}^{n}(N_i-N_j)}}. \quad (2)$$

Each pair of these relations represents a Schwinger-type mass formula. Hence, for $(2n+1)\times(2n+1)$ mass matrix (1) one has n Schwinger mass relations. These n formulas, together with the trace condition for the mass matrix (1),

$$G+S_1+N_1+S_2+N_2+\cdots+S_n+N_n=h_1+h_2+h_3+\cdots+h_{2n}+h_{2n+1}, \quad (3)$$

constitute $n+1$ mass relations (without couplings) for the mixing of a glueball and n meson nonets.

The strategy we pursue is as follows.

Since we take the $f_2(1275)$ and $f_2'(1525)$ as the established ground state $1P$ tensor mesons, we incorporate them by fixing $N=1.318$ for $1P$ [1]. We fix the following values of the primitive masses (in GeV): $N=1.66$ for $2P$, $N=1.94$, $S=2.15$ for $1F$ and $N=2.05$ for $3P$. We also take one of the physical states to have a mass in agreement with one of the glueball candidates $f_J(2220)$, $f_2(2150)$, $f_2(1950)$ and $f_J(1710)$ [2], and the other three physical states to have masses in agreement with three states among $f_2(1565)$, $f_2(1640)$, $f_J(1710)$, $f_2(1810)$, $f_2(1950)$, $f_2(2010)$, $f_2(2150)$, $f_J(2220)$, $f_2(2300)$ and $f_2(2340)$, excluding the state already chosen for the physical glueball.

We discard the possibility that $f_2(1420)$ exists. Although claimed by a number of old experiments in a variety of production processes, recent experiments do not confirm its existence. This is most vividly illustrated by its observation in (mostly) double Pomeron exchange in $pp\to p_f(\pi^+\pi^-)p_s$ at $\sqrt{s}=63$ GeV [2]. Recent examination of the *same* reaction

[2]Taking both $f_J(1710)$ and $f_J(2220)$ to have 2^{++} components.

does not see any evidence for $f_2(1420)$ [8].

We admit the following criteria for holding physical solutions and separating out non-physical ones:

(i) The mass of the primitive glueball satisfies $G \geqslant 2$ GeV [2].

(ii) In all the cases when a primitive $s\bar{s}$ mass is to be obtained, it is higher than the corresponding $n\bar{n}$ mass, and the $s\bar{s}-n\bar{n}$ mass splitting is consistent with the quark model motivated estimate 200 ± 50 MeV [3].

We find two different solutions, which are almost identical with respect to the physical masses. Particularly, the primitive glueball masses are in the range 2.0–2.1 GeV, consistent with models and lattice QCD [2]. The couplings are in the range 30–120 MeV for the various nonets. We find that the physical masses are insensitive to changes in the input, but that the valence content is more sensitive: especially for states at similar masses to where the parameters are changed, and for small valence components.

(i) For the first solution, the $f_J(2220)$ turns out to be the physical glueball.

(ii) For the second solution, the physical glueball is distributed, with $f_2(1810)$ containing the largest component.

Both solutions have the following similarities:

(i) The physical states are $f_2(1270)$, $f_2'(1525)$, $f_2(1640)$, $f_2(1810)$, $f_2(1950)$, $f_2(2010)$, $f_2(2150)$, $f_J(2220)$ and either the $f_2(2300)/f_2(2340)$ or $f_2(2340)$ in the first and second solutions, respectively.

(ii) The valence content has almost entirely the same signs between the various components, the only exception being different signs for the two dominant meson components in $f_2(1950)$.

(iii) The physical mesons have a substantial glueball content, contrary to naive expectations, with the exception of $f_2(1950)$ in the second solution. This would, for example, explain why $f_2(2010)$, $f_2(2300)$ and $f_2(2340)$ were observed in the OZI forbidden process $\pi p \to \phi\phi n$, and would suggest that several tensor mesons should be produced in glue-rich processes.

(iv) The $f_2(1270)$, $f_2'(1525)$ and $f_2(1640)$ are composed of more than 90% of the expected primitive state.

(v) For the $1P$ nonet, $S+N$ is found to be 2.87 GeV, consistent with $2M(K_2^*)=2.86\pm 0.01$ GeV [1], motivating the quark model relation.

The solutions differ as follows:

(i) In the first solution all physical mesons have one component which has a valence content of larger than 90%, i.e. the state is dominantly a specific primitive state. The second solution does not behave in this way.

(ii) For the first solution, the physical glueball has substantial valence content in all the meson states it couples to, contrary to the second solution.

(iii) For the second solution, both the $f_2(1950)$ and $f_2(2150)$ have large $n\bar{n}$ and $s\bar{s}$ components. Hence, both of these states will have many different decay modes, which is in excellent agreement with data. Although this feature is absent for these states for the first solution, where $f_2(1950)$ is mostly $n\bar{n}$ and $f_2(2150)$ mostly $s\bar{s}$, many of the observed decay modes arise from connected decay of both $u\bar{u}$ and $s\bar{s}$ components, so that this avenue to distinguish between solutions may not be definitive.

We never found the $f_2(1565)$ and $f_J(1710)$.

It is possible that $f_2(1565)$ and $f_2(1640)$ are aspects of the same state, which would remove one extra state. We have neglected the mixing of mesons with decay channels throughout, since it is believed to produce only tiny mass shifts [2]. We suggest that the mass of the $2P$ $n\bar{n}$ state found in our formalism is shifted downward by the $\rho\rho$ and $\omega\omega$ thresholds, which the $^3P_0$ model predicts it to couple strongly to [9].

$f_2(1565)$ decays to $\rho\rho$ and $\omega\omega$ and has an abnormally small branching ratio to $\pi\pi$ and $\eta\eta$ [1]. This, together with the nearness of $f_2(1565)$ to the $\rho\rho$ and $\omega\omega$ thresholds has lead to the alternative suggestion that $f_2(1565)$ is a $\rho\rho$ molecule [2].

$f_J(1710)$ has been suggested as a K^*K^* molecule [2]. However, it is not well established that a $J=2$ component exists. Recent evidence supports only the $J=0$ component [8].

Once the 9×9 mass matrix is fixed, one can easily add extra meson nonets to it. We add the primitive $n\bar{n}$ and $s\bar{s}$ masses of the $2F$ nonet at 2.3 and 2.5 GeV, respectively, to the mass matrices. Similarly, the primitive states of the $4P$ nonet are added at 2.35 and 2.55 GeV, to yield a 13×13 mass matrix. The physical states are required to be among *both* $f_2(2300)$ and $f_2(2340)$.

The result is a 13×13 ''counterpart'' to each 9×9 matrix, called solutions 1a and 2a. The primitive states in common have the same couplings and primitive masses, and similar valence content (with the same signs). The valence content of a given primitive state tends to decrease from the 9×9 counterpart, since the physical state is spread over more primitive states. Remarkably, the ratios of valence contents of the 13×13 solutions and their 9×9 counterparts remain extremely similar ($\sim 1\%$), except for the components in the 9×9 matrix which has similar mass to the new components being added. This means that for low-lying states, there is usually no need to extend the number of primitive components in order to study decay.

We also find two new solutions, called 1b and 2b, since they are respectively similar to the 9×9 solutions 1 and 2. They have, however, no 9×9 counterparts.

II. ILLUSTRATIVE DECAYS

The decay of a physical state to a specific final state is calculated by adding the decay amplitudes of all its primitive components, weighted by their valence content.

The decay amplitudes of the primitive components are calculated in the $^3P_0$ model, meaning that pair creation is with vacuum quantum numbers and decays proceed via a connected quark diagram. In the mass matrix analysis, specific quark model identifications were not assumed for the various components. However, to calculate decays, the quark model content is assumed.

$f_2'(1525)\to\pi\pi$: The Okubo-Zweig-Iizuka (OZI) rule forbidden decay $f_2'(1525)\to\pi\pi$ is well-known experimentally and is zero in models where the state has only one valence component. For the 13×13 solutions 1a, 1b, 2a and 2b we find $\Gamma(f_2'(1525)\to\pi\pi)=1.6(1.4),1.2(1.1),1.0\ (0.9),0.7(0.7)$ MeV, in two phase space conventions [2,9]. Solution 2b is in best agreement with the experimental value 0.60 ± 0.12 MeV [1]. The valence content of the $f_2'(1525)$ is such that its $1P$ and $2P$ $n\bar{n}$ components are in destructive interference (they have opposite signs), which results in the suppression of the $\pi\pi$ decay mode of this state. Furthermore, for components higher in mass than $2P$, the valence content has the same sign as the $2P$ component, leading to further suppression. To be specific, we illustrate this for solution 2b. $\Gamma(f_2'(1525)\to\pi\pi)=14$ and 4 MeV if only $1P$, and $1P$ and $2P$ components are included. In fact, the width remains above 1.5 MeV as long as not all of the $1P$, $2P$, $1F$ and $3P$ components are included. We have thus provided the first quantitative understanding of the process $f_2'(1525)\to\pi\pi$. This demonstrates that the techniques of both the mass matrix and $^3P_0$ decay analysis yield predictions consistent with experiment, motivating their continued use. Remarkably, the decay $f_2'(1525)\to\pi\pi$ can only be understood when at least four different components of $f_2'(1525)$ are included. It is also apparent that there is no need to postulate a non-connected decay mechanism, whereby primitive $s\bar{s}$ components would directly decay to $\pi\pi$. Because $f_2'(1525)$ is dominantly $s\bar{s}$, such processes must be small indeed.

We proceed to study the tensor glueball candidate $f_J(2220)$.

Evaluating the total width of $f_J(2220)$, with $J=2$, to $\pi\pi$ and $K\bar{K}$, for all the 13×13 solutions, and phase space conventions, we obtain 20–150 MeV. It is evident that the tiny total experimental width of 23^{+8}_{-7} MeV [1] cannot be sustained in our model. This is not necessarily in contradiction with current experiments, as it is possible to understand data on $f_J(2220)$ if one does not take it to be narrow [10].

It is often claimed that $f_J(2220)$ has a flavor democratic decay pattern expected for a pure glueball [2,3], whereby $R\equiv\Gamma(f_J(2220)\to\pi^+\pi^-)/\Gamma(f_J(2220)\to K^+K^-)=1$ without phase space included, and $R=1.7$ with phase space included. However, naive flavor factors give that a pure $n\bar{n}$ and $s\bar{s}$ state should have, without phase space included, $R=4,0$ respectively. Thus a mixture between $n\bar{n}$ and $s\bar{s}$ can also look flavor democratic. For the 13×13 solutions 1a, 1b, 2a and 2b we find $R=0.6,0.7,0.4,0.5$ respectively, independent of phase space conventions. Although these values of R do not represent flavor democratic decay, they are all consistent with experiment [1], which possesses large error bars.

When decays are to final S-wave mesons, one almost always finds that the decay amplitudes decrease sharply as the decaying component is progressively radially excited. The same is true as the decaying component is orbitally excited. This has the consequence that although a physical state may have a dominantly excited component, its decay mainly proceeds through a lower excited component. This means that naive quark model calculations that assign a single component to an excited state [9] might be completely unreliable. One would *a priori* expect this situation to be worst in J^{PC} sectors where low-lying glueballs are present, i.e. $J^{PC}=0^{++}$, 2^{++} and 0^{-+} [2]. We illustrate the phenomenon by analyzing $f_J(2220)\to\pi\pi$ and $K\bar{K}$ for the 13×13 solutions. Although $f_J(2220)$ is never dominantly $1P$, this contribution is always one of the largest ones. One tends to find that half of the width can be found by including only the $1P$ and $2P$ contributions, even though the state may be dominantly $1F$ and $3P$.

III. ABSTRACTED FEATURES

When we restrict to glueball-meson mixing, Schwinger-type mass formulas can be obtained. With some physical isovector and isoscalar masses known, these formulas can predict unknown masses and couplings. The utility of this new analysis technique was demonstrated in the tensor sector.

It has been shown that in order to understand the decay $f_2'(1525)\to\pi\pi$, one has to consider more than the $1P$ $n\bar{n}$ component. This implies that the use of a 2×2 mixing formula where the physical states are linear combinations of $n\bar{n}$ and $s\bar{s}$ components can be inadequate.

In our approach the physical glueball is *a priori* narrower than mesons due to the large glueball component, which is taken not to decay. However, as shown for the $f_J(2220)$, the physical glueball is not unusually narrow, because of the presence of significant meson components. Also, there is no reason to expect a flavor democratic decay pattern for the physical glueball.

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