Flavor changing effects in theories with a heavy Z' boson with family nonuniversal couplings

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There are theoretical and phenomenological motivations that there may exist additional heavy Z' bosons with family nonuniversal couplings. Flavor mixing in the quark and lepton sectors will then lead to flavor changing couplings of the heavy Z', and also of the ordinary Z when Z-Z' mixing is included. The general formalism of such effects is described, and applications are made to a variety of flavor changing and *CP*-violating tree and loop processes. Results are described for three specific cases motivated by a specific heterotic string model and by phenomenological considerations, including cases in which all three families have different couplings, and those in which the first two families, but not the third, have the same couplings. Even within a specific theory the results are model dependent because of unknown quark and lepton mixing matrices. However, assuming that typical mixings are comparable to the Cabibbo-Kobayashi-Maskawa matrix, processes such as coherent μ-e conversion in a muonic atom, K^0-$\overline{K^0}$ and B-$\overline{B}$ mixing, ϵ, and ϵ'/ϵ lead to significant constraints on Z' bosons in the theoretically and phenomenologically motivated range $M_{Z'} \sim 1$ TeV.

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I. INTRODUCTION

Additional heavy neutral Z' gauge bosons are amongst the best motivated extensions of the standard model (SM), or its supersymmetric extension [minimal supersymmetric standard model (MSSM)] [1]. In particular, they often occur in grand unified theories (GUTs), superstring theories, and theories with large extra dimensions [2]. In traditional GUTs, the scale of the Z' mass is arbitrary. However, in perturbative heterotic string models with supergravity mediated supersymmetry breaking, the U(1)′ and electroweak breaking are both driven by a radiative mechanism, with their scales set by the soft supersymmetry breaking parameters, implying that the Z' mass should be less than around a TeV [3]. (The breaking can be at a larger intermediate scale if it is associated with a *D*-flat direction [4].) Furthermore, the extra symmetry can forbid an elementary μ term, while allowing an effective μ and $B\mu$ to be generated at the U(1)′ breaking scale, providing a solution to the μ problem without introducing cosmological problems [5,3]. An extra U(1)′ provides an analogous solution to the μ problem in models of gauge-mediated supersymmetry breaking [6,7]. An extra U(1)′ gauge symmetry does not by itself spoil the successes of gauge coupling unification.

There are stringent limits on the mass of an extra Z' from the nonobservation of direct production followed by decays into e^+e^- or $\mu^+\mu^-$ by the Collider Detector at Fermilab (CDF) [8], while indirect constraints from precision data also limit the Z' mass (weak neutral current processes and the CERN e^+e^- collider LEP II) and severely constrain the Z-Z' mixing angle θ (Z pole) [9]. These limits are model dependent, but are typically in the range $M_{Z'} > O(500)$ GeV and $|\theta| <$ few$\times 10^{-3}$ for standard GUT models. Recently, it has been argued [10,11] that both Z-pole data and atomic parity violation are much better described if the SM or the MSSM is extended by an additional heavy Z'.

There is thus both theoretical and experimental motivation for an additional Z', most likely in the range 500 GeV to 1 TeV. If true, there should be a good chance to observe it at RunII at the Fermilab Tevatron, and certainly at the CERN Large Hadron Collider (LHC). Also, in this mass range, it should be possible to carry out significant diagnostic probes of the Z' couplings at the LHC and at a future NLC [12], which would complement those from the precision experiments [10]. The existence of a heavy Z' would also suggest a spectrum of sparticles considerably different than most versions of the MSSM [13].

Most studies have assumed that the Z' gauge couplings are family universal [14], so that they remain diagonal even in the presence of fermion flavor mixing by the Glashow-Iliopoulos-Maiani (GIM) mechanism. However, in string models it is possible to have family-nonuniversal Z' couplings, because of different constructions of the different families. For instance, the consequences of a model by Chaudhuri, Hockney, and Lykken [15] have been extensively analyzed in [16], where it was shown that most *F* and *D*-flat directions involve an additional U(1)′ broken at the TeV scale. In this case, the third generation quark couplings are different from the first two families, and all three lepton generations have different couplings. Other aspects of the model are not realistic, but nevertheless this provides a motivation to consider nonuniversal couplings. Similarly, possible anomalies in the Z-pole $b\overline{b}$ asymmetries [17] suggest that the data are better fitted with a nonuniversal Z' [10].

Family-nonuniversal Z' couplings necessarily lead to flavor-changing (nondiagonal) Z' couplings, and possibly to new *CP*-violating effects, when quark and lepton flavor mixing are taken into account.[1] This will also imply flavor vio-

[1]Models with an extra Z' often include additional exotic fermions [i.e., with nonstandard SU(2) assignments] as well. Mixing of ordinary and exotic fermions could lead to flavor changing Z' and Z couplings even in the absence of family nonuniversal charges, and to nonuniversal W couplings [18]. We do not consider such effects in this paper.

lating Z couplings if there is Z-Z' mixing. Thus, flavor changing neutral current (FCNC) and CP-violating effects should be considered an additional constraint, consequence, or possibly diagnostic probe of an extra Z', and conversely as another motivation to search for FCNC effects.

Even for a model in which the Z' couplings are specified (prior to flavor mixing), the predictions for FCNC are still model dependent, because they depend on the individual unitary transformations for the left (L) and right (R) chiral u quarks, d quarks, charged leptons, and neutrinos which diagonalize their respective mass matrices. However, only the combination of L matrices for the u and d occurring in the Cabibbo-Kobayashi-Maskawa (CKM) matrix is known experimentally (with weak constraints on the leptonic analog from neutrino oscillations), so one cannot make definitive predictions. We will present our results for arbitrary mixings, and then illustrate assuming that all of the mixing matrices are comparable to the CKM matrix.[2]

In the next section we discuss the general formalism and introduce our notation. In Sec. III we discuss Z' contributions to flavor changing processes forbidden in the standard model and to new contributions to SM processes, and we use experimental results to constrain the Z' couplings. Specific examples of a Z' with flavor changing couplings are discussed in Sec. IV. Models in which all three families have different Z' charges in the gauge eigenstate basis are strongly constrained by experimental results, and even models in which the first two families have the same couplings, but not the third, can yield flavor changing rates above experimental limits unless they are suppressed by small mixing elements. Finally, in Sec. V we summarize our results and present our conclusions.

II. FORMALISM

We will use the formalism developed in Ref. [19] and generalize it to the case of flavor violating Z' couplings. In the basis in which all the fields are gauge eigenstates the neutral current Lagrangian is given by

$$\mathcal{L}_{\rm NC} = -eJ^{\mu}_{\rm em}A_{\mu} - g_1 J^{(1)\mu} Z^0_{1,\mu} - g_2 J^{(2)\mu} Z^0_{2,\mu}, \qquad (1)$$

where Z^0_1 is the SU(2)×U(1) neutral gauge boson, Z^0_2 the new gauge boson associated with an additional Abelian gauge symmetry, and the currents are

$$J^{(1)}_{\mu} = \sum_i \bar{\psi}_i \gamma_{\mu} [\epsilon_L(i) P_L + \epsilon_R(i) P_R] \psi_i, \qquad (2)$$

$$J^{(2)}_{\mu} = \sum_{i,j} \bar{\psi}_i \gamma_{\mu} [\epsilon^{(2)}_{\psi_{L_{ij}}} P_L + \epsilon^{(2)}_{\psi_{R_{ij}}} P_R] \psi_j, \qquad (3)$$

where the sum extends over all quarks and leptons $\psi_{i,j}$ and $P_{R,L} = (1 \pm \gamma_5)/2$. $\epsilon^{(2)}_{\psi_{R,L_{ij}}}$ denote the chiral couplings of the new gauge boson, and the standard model chiral couplings are

$$\epsilon_R(i) = -\sin^2\theta_W Q_i, \quad \epsilon_L(i) = t^i_3 - \sin^2\theta_W Q_i, \qquad (4)$$

where t^i_3 and Q_i are the third component of the weak isospin and the electric charge of fermion i, respectively. $g_1 = g/\cos\theta_W = e/\sin\theta_W$ and g_2 are the gauge couplings of the two U(1) factors.

Flavor changing effects (FCNCs) immediately arise if the $\epsilon^{(2)}$ are nondiagonal matrices. If the Z^0_2 couplings are diagonal but nonuniversal, flavor changing couplings are induced by fermion mixing. The fermion Yukawa matrices h_{ψ} in the weak eigenstate basis can be diagonalized by unitary matrices $V^{\psi}_{R,L}$

$$h_{\psi,\rm diag} = V^{\psi}_R h_{\psi} V^{\psi^{\dagger}}_L, \qquad (5)$$

where the CKM matrix is given by the combination

$$V_{\rm CKM} = V^u_L V^{d\dagger}_L. \qquad (6)$$

Hence, the chiral Z^0_2 couplings in the fermion mass eigenstate basis read

$$B^{\psi_L}_{ij} \equiv (V^{\psi}_L \epsilon^{(2)}_{\psi_L} V^{\psi^{\dagger}}_L)_{ij} \quad \text{and} \quad B^{\psi_R}_{ij} \equiv (V^{\psi}_R \epsilon^{(2)}_{\psi_R} V^{\psi^{\dagger}}_R)_{ij}. \qquad (7)$$

Further, Z-Z' mixing is induced by electroweak symmetry breaking, implying that $Z^0_{1,2}$ are related to mass eigenstates by an orthogonal transformation. Hence, the couplings of the massive gauge boson mass eigenstates $Z^{(i)}$ are

$$\mathcal{L}^Z_{\rm NC} = -g_1 \left[\cos\theta J^{(1)\mu} + \frac{g_2}{g_1}\sin\theta J^{(2)\mu}\right] Z^{(1)}_{\mu} - g_1 \left[\frac{g_2}{g_1}\cos\theta J^{(2)\mu} - \sin\theta J^{(1)\mu}\right] Z^{(2)}_{\mu}, \qquad (8)$$

where θ is the Z-Z' mixing angle. The standard model weak neutral current $J^{(1)\mu}$ is given in Eq. (2), and $J^{(2)\mu}$ has a form analogous to Eq. (3), with the $\epsilon^{(2)}_{\psi_{R,L}}$ replaced by the couplings $B^{\psi_{R,L}}$ from Eq. (7).

We have neglected kinetic mixing [20], since it only amounts to a redefinition of the unknown Z' couplings.[3] At low energies, the effective four-fermion interactions are then given by

$$-\mathcal{L}_{\rm eff} = \frac{4G_F}{\sqrt{2}} \sum_{\psi,\chi} (\rho_{\rm eff} J^{(1)^2} + 2wJ^{(1)}J^{(2)} + yJ^{(2)^2}) \qquad (9)$$

[2]For a complete theory the U(1)′ charges would constrain the possible flavor mixings. However, such relations would be much more specific than the general issues considered here.

[3]Kinetic mixing allows the redefined Z^0_2 charges to have a component of weak hypercharge, which would otherwise not be allowed. This is irrelevant for the purposes of this paper.

$$= \frac{4G_F}{\sqrt{2}} \sum_{\psi,\chi} \sum_{i,j,k,l} [C_{kl}^{ij} Q_{kl}^{ij} + \tilde{C}_{kl}^{ij} \tilde{Q}_{kl}^{ij} + D_{kl}^{ij} O_{kl}^{ij} + \tilde{D}_{kl}^{ij} \tilde{O}_{kl}^{ij}], \tag{10}$$

with the local operators[4]

$$\begin{aligned} Q_{kl}^{ij} &= (\bar{\psi}_i \gamma^\mu P_L \psi_j)(\bar{\chi}_k \gamma_\mu P_L \chi_l), \\ \tilde{Q}_{kl}^{ij} &= (\bar{\psi}_i \gamma^\mu P_R \psi_j)(\bar{\chi}_k \gamma_\mu P_R \chi_l), \\ O_{kl}^{ij} &= (\bar{\psi}_i \gamma^\mu P_L \psi_j)(\bar{\chi}_k \gamma_\mu P_R \chi_l), \\ \tilde{O}_{kl}^{ij} &= (\bar{\psi}_i \gamma^\mu P_R \psi_j)(\bar{\chi}_k \gamma_\mu P_L \chi_l). \end{aligned} \tag{11}$$

ψ and χ represent classes of fermions with the same SM quantum numbers, i.e., u, d, e^-, and ν, while i,j,k,l are family indices. The coefficients are

$$\begin{aligned} C_{kl}^{ij} &= \rho_{\rm eff} \delta_{ij} \delta_{kl} \epsilon_L(i) \epsilon_L(k) + w \delta_{ij} \epsilon_L(\psi_i) B_{kl}^{\chi_L} \\ &\quad + w \delta_{kl} \epsilon_L(\chi_l) B_{ij}^{\psi_L} + y B_{ij}^{\psi_L} B_{kl}^{\chi_L}, \end{aligned} \tag{12}$$

$$\begin{aligned} \tilde{C}_{kl}^{ij} &= \rho_{\rm eff} \delta_{ij} \delta_{kl} \epsilon_R(i) \epsilon_R(k) + w \delta_{ij} \epsilon_R(\psi_i) B_{kl}^{\chi_R} \\ &\quad + w \delta_{kl} \epsilon_R(\chi_l) B_{ij}^{\psi_R} + y B_{ij}^{\psi_R} B_{kl}^{\chi_R}, \end{aligned} \tag{13}$$

$$\begin{aligned} D_{kl}^{ij} &= \rho_{\rm eff} \delta_{ij} \delta_{kl} \epsilon_L(i) \epsilon_R(k) + w \delta_{ij} \epsilon_L(\psi_i) B_{kl}^{\chi_R} \\ &\quad + w \delta_{kl} \epsilon_R(\chi_l) B_{ij}^{\psi_L} + y B_{ij}^{\psi_L} B_{kl}^{\chi_R}, \end{aligned} \tag{14}$$

$$\begin{aligned} \tilde{D}_{kl}^{ij} &= \rho_{\rm eff} \delta_{ij} \delta_{kl} \epsilon_R(i) \epsilon_L(k) + w \delta_{ij} \epsilon_R(\psi_i) B_{kl}^{\chi_L} \\ &\quad + w \delta_{kl} \epsilon_L(\chi_l) B_{ij}^{\psi_R} + y B_{ij}^{\psi_R} B_{kl}^{\chi_L}. \end{aligned} \tag{15}$$

The coefficients are given by

$$\rho_{\rm eff} = \rho_1 \cos^2\theta + \rho_2 \sin^2\theta, \quad \rho_i = \frac{M_W^2}{M_i^2 \cos^2\theta_W}, \tag{16}$$

$$w = \frac{g_2}{g_1} \sin\theta \cos\theta (\rho_1 - \rho_2), \tag{17}$$

$$y = \left(\frac{g_2}{g_1}\right)^2 (\rho_1 \sin^2\theta + \rho_2 \cos^2\theta), \tag{18}$$

where M_i are the masses of the neutral gauge boson mass eigenstates and θ_W is the electroweak mixing angle.

III. FLAVOR CHANGING PROCESSES

In this section we will discuss flavor violating processes forbidden in the SM and new contributions to SM processes. Experimental bounds or results on these processes (see Ref. [17]) can then be used to constrain the Z' couplings.

A. Z decays

Because of the Z-Z' mixing, Z couples to $J^{(2)}$. The decay width for a flavor changing Z decay at tree level is given by

$$\Gamma(Z \to \psi_i \bar{\psi}_j) = \frac{C G_F \rho_1 M_1^3}{3\sqrt{2}\pi} \left(\frac{g_2}{g_1}\right)^2 \sin^2\theta (|B_{ij}^{\psi_L}|^2 + |B_{ij}^{\psi_R}|^2), \tag{19}$$

where $C=1$ ($C=3$) is the color factor for leptons (quarks). Because of strong experimental constraints on the Z-Z' mixing angle θ, see Refs. [9,10], the $B^{\psi_{R,L}}$ cannot be strongly constrained from flavor violating Z decays.

B. Lepton decays

In the SM each lepton generation has a separately conserved lepton number, if one neglects small effects from nonvanishing neutrino masses and nonperturbative effects. The effective Lagrangian (10) gives rise to lepton family number violating processes, although the total lepton number is still conserved.

Consider first the decay of a charged lepton l_j into three different charged leptons l_i, l_k, and $\bar{l}_l$. At tree level, the decay width is

$$\begin{aligned} \Gamma(l_j \to l_i l_k \bar{l}_l) &= \frac{G_F^2 m_{l_j}^5}{48\pi^3} (|C_{l_k l_l}^{l_i l_j} + C_{l_i l_l}^{l_k l_j}|^2 + |\tilde{C}_{l_k l_l}^{l_i l_j} + \tilde{C}_{l_i l_l}^{l_k l_j}|^2 \\ &\quad + |D_{l_k l_l}^{l_i l_j}|^2 + |D_{l_i l_l}^{l_k l_j}|^2 + |\tilde{D}_{l_k l_l}^{l_i l_j}|^2 + |\tilde{D}_{l_i l_l}^{l_k l_j}|^2), \end{aligned} \tag{20}$$

where we have neglected the masses of the final state leptons. If two leptons in the final state are equal ($i=k$), taking permutations of the external fermion lines into account yields [21]

$$\begin{aligned} \Gamma(l_j \to l_i l_i \bar{l}_l) &= \frac{G_F^2 m_{l_j}^5}{48\pi^3} (2|C_{l_i l_l}^{l_i l_j}|^2 + 2|\tilde{C}_{l_i l_l}^{l_i l_j}|^2 \\ &\quad + |D_{l_i l_l}^{l_i l_j}|^2 + |\tilde{D}_{l_i l_l}^{l_i l_j}|^2). \end{aligned} \tag{21}$$

Since such processes are free of hadronic uncertainties and well constrained experimentally [17,21,22], they yield strong constraints on the leptonic couplings of a Z'.[5]

The strongest constraint on the Z'-μ-e coupling, however, comes from coherent μ-e conversion in a muonic atom [24]. The branching fraction for this process, i.e., the ratio of the coherent μ-e conversion rate to the μ capture rate for a nucleus of atomic number Z and neutron number N is given by [25,21]

[4]These operators are not all independent. For couplings of four fermions of the same type, $\psi=\chi$, e.g., four charged leptons, one has $Q_{kl}^{ij} = Q_{ij}^{kl}$, $\tilde{Q}_{kl}^{ij} = \tilde{Q}_{ij}^{kl}$, and $O_{kl}^{ij} = \tilde{O}_{ij}^{kl}$.

[5]In Ref. [23] these processes were considered in the case of vanishing Z-Z' mixing, Z' couplings of the V-A form, and assuming that the Z' has no diagonal couplings.

$$B(\mu^- N \to e^- N) = \frac{G_F^2 \alpha^3 m_\mu^5}{2\pi^2 \Gamma_{\rm capt}} \frac{Z_{\rm eff}^4}{Z} |F_P|^2 (|B_{12}^{l_L}|^2 + |B_{12}^{l_R}|^2) \times \left| w\left[\frac{1}{2}(Z-N) - 2Z \sin^2\theta_W\right] + y[(2Z+N)(B_{11}^{u_L} + B_{11}^{u_R}) + (Z+2N)(B_{11}^{d_L} + B_{11}^{d_R})]\right|^2, \quad (22)$$

where $\Gamma_{\rm capt}$ is the μ capture rate, $Z_{\rm eff}$ an effective atomic charge obtained by averaging the muon wave function over the nucleon, and F_P is a nuclear matrix element.

C. Radiative decays

Neutral current penguins give rise to radiative lepton decays. Neglecting the mass of the final state lepton, the decay width is

$$\Gamma(l_j \to l_i \gamma) = \frac{\alpha G_F^2 m_{l_j}^5}{8\pi^4} (|\xi_L^{l_i l_j}|^2 + |\xi_R^{l_i l_j}|^2), \quad (23)$$

where the dipole moment couplings of an on-shell photon to the chiral μ-e currents are

$$\xi_L^{l_i l_j} = \frac{1}{m_{l_j}} \sum_k m_{l_k} D_{l_i l_k}^{l_k l_j} = \frac{y}{m_{l_j}} (B^{l_R} m_l B^{l_L})_{ij} + w\epsilon_L(l_j) B_{ij}^{l_R}, \quad (24)$$

$$\xi_R^{l_i l_j} = \frac{1}{m_{l_j}} \sum_k m_{l_k} \tilde{D}_{l_i l_k}^{l_k l_j} = \frac{y}{m_{l_j}} (B^{l_L} m_l B^{l_R})_{ij} + w\epsilon_R(l_j) B_{ij}^{l_L}, \quad (25)$$

where m_l is the charged lepton mass matrix.

A similar result holds for the decay $b \to s\gamma$. Since the b-quark mass is much larger than the QCD scale Λ, long-range strong interaction effects are not expected to be important in the inclusive decay $B \to X_s \gamma$ [26]. Hence, the rate for this process is usually approximated by considering the ratio

$$R \equiv \frac{\Gamma(B \to X_s \gamma)}{\Gamma(B \to X_c e \bar{\nu}_e)} \approx \frac{\Gamma(b \to s\gamma)}{\Gamma(b \to c e \bar{\nu}_e)}. \quad (26)$$

Neglecting SM contributions, the contribution to R from the one-loop neutral current penguin diagrams is

$$R = \frac{8\alpha}{3\pi} |V_{cb}|^{-2} f^{-1}\left(\frac{m_c^2}{m_b^2}\right) (|\xi_L^{sb}|^2 + |\xi_R^{sb}|^2), \quad (27)$$

where f is the phase-space factor in the semileptonic b decay:

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x. \quad (28)$$

In analogy to Eqs. (24) and (25) the flavor violating effective couplings $\xi_{R,L}^{sb}$ are given by

TABLE I. Coefficients in the decay widths of pseudoscalar mesons P^0. Since K_L^0 is a linear combination of K^0 and $\bar{K}^0$, the $\beta_{R,L}$ for K_L^0 decays depend only on the real part of the Z'-d-s axial vector coupling $\mathrm{Re}(B_{12}^{d_R} - B_{12}^{d_L})$.

P^0	$\beta_L^{l_i l_j}$	$\beta_R^{l_i l_j}$
π^0	$D_{uu}^{l_i l_j} - C_{uu}^{l_i l_j} - D_{dd}^{l_i l_j} + C_{dd}^{l_i l_j}$	$\tilde{C}_{uu}^{l_i l_j} - \tilde{D}_{uu}^{l_i l_j} - \tilde{C}_{dd}^{l_i l_j} + \tilde{D}_{dd}^{l_i l_j}$
K_L^0	$D_{ds}^{l_i l_j} + D_{sd}^{l_i l_j} - C_{ds}^{l_i l_j} - C_{sd}^{l_i l_j}$	$-\tilde{D}_{ds}^{l_i l_j} - \tilde{D}_{sd}^{l_i l_j} + \tilde{C}_{ds}^{l_i l_j} + \tilde{C}_{sd}^{l_i l_j}$
D^0	$\sqrt{2}(D_{cu}^{l_i l_j} - C_{cu}^{l_i l_j})$	$\sqrt{2}(-\tilde{D}_{cu}^{l_i l_j} + \tilde{C}_{cu}^{l_i l_j})$
B^0	$\sqrt{2}(D_{bd}^{l_i l_j} - C_{bd}^{l_i l_j})$	$\sqrt{2}(-\tilde{D}_{bd}^{l_i l_j} + \tilde{C}_{bd}^{l_i l_j})$
B_s^0	$\sqrt{2}(D_{bs}^{l_i l_j} - C_{bs}^{l_i l_j})$	$\sqrt{2}(-\tilde{D}_{bs}^{l_i l_j} + \tilde{C}_{bs}^{l_i l_j})$

$$\xi_L^{sb} = \frac{1}{m_b} \sum_k m_{d_k} D_{sd_k}^{d_k b} = \frac{y}{m_b} (B^{d_R} m_d B^{d_L})_{23} + w\epsilon_L(b) B_{23}^{d_R}, \quad (29)$$

$$\xi_R^{sb} = \frac{1}{m_b} \sum_k m_{d_k} \tilde{D}_{sd_k}^{d_k b} = \frac{y}{m_b} (B^{d_L} m_d B^{d_R})_{23} + w\epsilon_R(b) B_{23}^{d_L}, \quad (30)$$

where d_k stands for the kth generation down-type quark and m_d is the diagonal mass matrix of down quarks.

D. Leptonic meson decays

Meson decays can be used to place limits on the Z' couplings to quarks. Consider the lepton family number violating decay of a neutral pseudoscalar meson P^0 into two charged leptons l_i and $\bar{l}_j$, with $i \neq j$. Due to the hierarchy of lepton masses, we can neglect the mass of the lighter lepton. Assuming that $m_{l_j} \ll m_{l_i}$ (the case $m_{l_i} \ll m_{l_j}$ can be obtained by exchanging the lepton indices i and j in the following) the decay width is

$$\Gamma(P^0 \to l_i \bar{l}_j) = 2 \frac{\Gamma(P^- \to l_i \bar{\nu}_i)}{|V_{kl}|^2} (|\beta_L^{l_i l_j}|^2 + |\beta_R^{l_i l_j}|^2), \quad (31)$$

where we have used isospin symmetry to relate the amplitude for $P^0 \to l_i \bar{l}_j$ to the amplitude for the SM decay $P^- \to l_i \bar{\nu}_i$, and V_{kl} is the element of the CKM matrix appearing in this SM process. The coefficients $\beta_{R,L}$ for the decays we have considered are given in Table I.

In the SM the decay of a pseudoscalar P^0 into a lepton and its antilepton is suppressed by the GIM mechanism and can only occur at one-loop level (see, e.g., Ref. [27] for a discussion of $K_L^0 \to l_i \bar{l}_i$ in the SM), whereas the Z' couplings allow tree-level contributions to such processes. Neglecting SM contributions, which are formally of higher order in the couplings, the decay width reads

$$\Gamma(P^0 \to l_i \bar{l}_i) = 4 \frac{\Gamma(P^- \to l_i \bar{\nu}_i)}{|V_{kl}|^2} \frac{m_P^3 \sqrt{m_P^2 - 4m_{l_i}^2}}{(m_P^2 - m_{l_i}^2)^2} |\beta_L^{l_i l_i} - \beta_R^{l_i l_i}|^2, \quad (32)$$

where the mass-dependent factor corrects for the different phase spaces in the decays of P^0 and P^-, and the couplings $\beta_{R,L}$ can again be found in Table I.

Similar formulas hold for semileptonic τ decays. For the process $\tau \to l_i \pi^0$ we have

$$\Gamma(\tau \to l_i \pi^0) = 2 \frac{\Gamma(\tau \to \nu_\tau \pi^-)}{|V_{ud}|^2} (|\beta_L^{l_i \tau}|^2 + |\beta_R^{l_i \tau}|^2), \quad (33)$$

where we have neglected the mass of the final state lepton and the $\beta_{R,L}$ are given in the first line of Table I, whereas for $\tau \to l_i K^0$ one finds

$$\Gamma(\tau \to l_i K^0) = 4 \frac{\Gamma(\tau \to \nu_\tau K^-)}{|V_{us}|^2} (|C_{ds}^{l_i \tau} - D_{ds}^{l_i \tau}|^2 + |\tilde{C}_{ds}^{l_i \tau} - \tilde{D}_{ds}^{l_i \tau}|^2). \quad (34)$$

Replacing the indices d and s yields the decay width for $\tau \to l_i \overline{K}^0$.

E. Semileptonic meson decays

All the processes discussed in the last section constrain only couplings of the form $D_{q_k q_l}^{l_i l_j} - C_{q_k q_l}^{l_i l_j}$, i.e., the axial vector couplings in the quark current. Limits on the corresponding vector couplings can be obtained by considering decays of P^0 into another pseudoscalar meson and two leptons.

Particularly interesting are lepton flavor conserving, *CP* violating contributions to decays $K_L^0 \to \pi^0 \overline{l} l$, since the branching ratios are expected to be small in the SM [27,28] and new limits from KTeV [29] allow the imaginary part of the Z'-d-s vector coupling to be constrained. Neglecting the electron mass but taking the μ-mass into account, the decay widths for the semileptonic K_L^0 decays considered are

$$\Gamma(K_L^0 \to e^+ e^- \pi^0) = 2 \frac{\Gamma(K^+ \to e^+ \nu_e \pi^0)}{|V_{us}|^2} (|\delta_L^{ee}|^2 + |\delta_R^{ee}|^2), \quad (35)$$

$$\Gamma(K_L^0 \to \mu^+ \mu^- \pi^0) = 2 \frac{\Gamma(K^+ \to \mu^+ \nu_\mu \pi^0)}{|V_{us}|^2} \times [0.57(|\delta_L^{\mu\mu}|^2 + |\delta_R^{\mu\mu}|^2) - 0.48\, \mathrm{Re}(\delta_L^{\mu\mu} \delta_R^{\mu\mu *})], \quad (36)$$

where the numerical coefficients in the last decay width arise due to the different phase spaces for the processes $K_L^0 \to \mu^+ \mu^- \pi^0$ and $K^+ \to \mu^+ \nu_\mu \pi^0$, and the couplings $\delta_{R,L}$ for these processes are given in Table II. We also considered the lepton flavor violating decay

$$\Gamma(K_L^0 \to \mu^+ e^- \pi^0) = 2 \frac{\Gamma(K^+ \to \mu^+ \nu_\mu \pi^0)}{|V_{us}|^2} (|\delta_L^{e\mu}|^2 + |\delta_R^{e\mu}|^2), \quad (37)$$

although experimental bounds on the Z'-μ-e coupling from coherent μ-e conversion imply that the branching ratio for this process is several orders of magnitude below the experimental bounds. Similarly, for semileptonic D^0 and B^0 decays one has

TABLE II. Coefficients for semileptonic meson decays. The $\delta_{R,L}$ for K_L^0 decays are proportional to the imaginary part of the Z'-d-s vector coupling $\mathrm{Im}(B_{12}^{d_R} + B_{12}^{d_L})$.

P	$\delta_L^{l_i l_j}$	$\delta_R^{l_i l_j}$
K_L^0	$D_{ds}^{l_i l_j} - D_{sd}^{l_i l_j} + C_{ds}^{l_i l_j} - C_{sd}^{l_i l_j}$	$\tilde{D}_{sd}^{l_i l_j} - \tilde{D}_{ds}^{l_i l_j} + \tilde{C}_{sd}^{l_i l_j} - \tilde{C}_{ds}^{l_i l_j}$
D^0	$\sqrt{2}(D_{cu}^{l_i l_j} + C_{cu}^{l_i l_j})$	$\sqrt{2}(\tilde{D}_{cu}^{l_i l_j} + \tilde{C}_{cu}^{l_i l_j})$
B^0	$\sqrt{2}(D_{bs}^{l_i l_j} + C_{bs}^{l_i l_j})$	$\sqrt{2}(\tilde{D}_{bs}^{l_i l_j} + \tilde{C}_{bs}^{l_i l_j})$

$$\Gamma(D^0 \to l_i \overline{l}_j \pi^0) = \frac{\Gamma(D^+ \to \overline{l}_j \nu_j \pi^0)}{|V_{cd}|^2} (|\delta_L^{l_i l_j}|^2 + |\delta_R^{l_i l_j}|^2), \quad (38)$$

$$\Gamma(B^0 \to l_i \overline{l}_j K^0) = \frac{\Gamma(B^+ \to \overline{l}_i \nu_i \overline{D}^0)}{|V_{cb}|^2} \frac{f(m_K^2/m_B^2)}{f(m_D^2/m_B^2)} \times (|\delta_L^{l_i l_j}|^2 + |\delta_R^{l_i l_j}|^2), \quad (39)$$

where the phase-space function f is given in Eq. (28).

F. Mass splittings and *CP* violation

The effective Lagrangian (10) also contributes to the mass splitting in a neutral pseudoscalar meson system. Again denoting the flavor eigenstates of a meson by P^0 and $\overline{P}^0$, the mass splitting Δm_P is given by

$$\Delta m_P = -2\, \mathrm{Re} \langle P^0 | \mathcal{L}_{\mathrm{eff}} | \overline{P}^0 \rangle. \quad (40)$$

The relevant hadronic matrix elements of the operators (11) have been determined in the vacuum insertion approximation using PCAC [30]. Hence, for a meson with the quark content $P^0 = \overline{q}_j q_i$ we obtain the following contribution to the mass splitting:

$$\Delta m_P = 4\sqrt{2} G_F m_P F_P^2 y \left\{ \frac{1}{3} \mathrm{Re}[(B_{ij}^{q_L})^2 + (B_{ij}^{q_R})^2] - \left[\frac{1}{2} + \frac{1}{3} \left(\frac{m_P}{m_{q_i} + m_{q_j}} \right)^2 \right] \mathrm{Re}(B_{ij}^{q_L} B_{ij}^{q_R}) \right\}, \quad (41)$$

where m_P and F_P are the mass and decay constant of the meson, respectively.

Further, phases in the Z' couplings $B_{ij}^{\psi_{R,L}}$ will contribute to *CP* violating processes. Limits on the imaginary parts of the s-d-Z' couplings can be placed by considering indirect *CP* violation ε_K in the neutral kaon system

$$|\varepsilon_K| = \frac{1}{2\sqrt{2}} \frac{\mathrm{Im}\langle K^0|\mathcal{L}_{\rm eff}|\overline{K}^0\rangle}{\mathrm{Re}\langle K^0|\mathcal{L}_{\rm eff}|\overline{K}^0\rangle} \tag{42}$$

$$= \frac{2G_F m_K F_K^2 y}{\Delta m_K} \left| \frac{1}{3}\mathrm{Im}[(B_{12}^{d_L})^2 + (B_{12}^{d_R})^2] - \left[\frac{1}{2} + \frac{1}{3}\left(\frac{m_P}{m_d + m_s}\right)^2\right] \mathrm{Im}(B_{12}^{d_L} B_{12}^{d_R}) \right|. \tag{43}$$

Direct *CP* violation ε' in the decays $K \to \pi\pi$ can be expressed in terms of the decay amplitudes $A_0 = A[K \to (\pi\pi)_0]$ and $A_2 = A[K \to (\pi\pi)_2]$, where the indices 0 and 2 denote the isospin of the final two pion state (see Ref. [26] for a review):

$$\varepsilon' = -\frac{1}{\sqrt{2}} \frac{\omega}{\mathrm{Re}\, A_0} \left(\mathrm{Im}\, A_0 - \frac{1}{\omega} \mathrm{Im}\, A_2 \right) e^{i\tilde{\phi}}, \tag{44}$$

where

$$\omega = \frac{\mathrm{Re}\, A_2}{\mathrm{Re}\, A_0}, \quad \tilde{\phi} = \frac{\pi}{2} + \delta_2 - \delta_0 . \tag{45}$$

The δ_I are the final state interaction phases. When using Eq. (44) to constrain physics beyond the standard model, it is common practice to take ω, $\mathrm{Re}\, A_0$, and $\tilde{\phi}$ from experiment,

$$\omega = 0.045, \quad \mathrm{Re}\, A_0 = 3.33 \times 10^{-7}\ \mathrm{GeV}, \quad \tilde{\phi} \approx \frac{\pi}{4}, \tag{46}$$

and consider new contributions to the imaginary parts of the amplitudes A_0 and A_2. This is due to the fact that the *CP* violating imaginary parts are dominated by short-distance effects and can be reliably determined by considering matrix elements of the effective Lagrangian (10). The hadronic matrix elements can be computed in the large N_c limit of chiral perturbation theory (see the Appendix), and one finds the following neutral current contribution to the real part of the ratio $\varepsilon'/\varepsilon_K$:

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon_K}\right) = 2 \times 10^3 w \left(\mathrm{Im}\, B_{21}^{d_L} + \frac{3}{2} \mathrm{Im}\, B_{21}^{d_R} \right) + 1.5 \times 10^3 y [(B_{11}^{u_L} - B_{11}^{d_L})(\mathrm{Im}\, B_{21}^{d_L} + 2\, \mathrm{Im}\, B_{21}^{d_R}) - (B_{11}^{u_R} - B_{11}^{d_R})(2\, \mathrm{Im}\, B_{21}^{d_L} + \mathrm{Im}\, B_{21}^{d_R})]. \tag{47}$$

G. Experimental constraints

Experimental limits or results on these processes can be used to constrain the flavor violating Z' couplings.[6] In the following we briefly discuss bounds coming from Z-Z' mixing contributions to these processes. The pure Z' contributions yield a multitude of bounds on products of Z' couplings which are less illuminating. In the examples that we discuss in the next section, these contributions are of the same order as the mixing contributions.

As already mentioned, the strongest bound on the Z'-μ-e coupling comes from the non-observation of coherent μ-e conversion by the Sindrum-II Collaboration [24]

$$w^2(|B_{12}^{l_L}|^2 + |B_{12}^{l_R}|^2) < 4 \times 10^{-14}, \tag{48}$$

while the decays $\tau \to 3e$ and $\tau \to 3\mu$ yield the strongest bounds on flavor violating τ couplings

$$w^2(|B_{13}^{l_L}|^2 + |B_{13}^{l_R}|^2) < 2 \times 10^{-5}, \quad w^2(|B_{23}^{l_L}|^2 + |B_{23}^{l_R}|^2) < 10^{-5}. \tag{49}$$

It is interesting to note that these constraints alone ensure that branching ratios for lepton flavor violating meson decays are below the experimental bounds, provided that the parameters w and y, given in Eqs. (17) and (18), are of the same order. (This holds in the most interesting case of a TeV scale Z' with small mixing, $\theta \lesssim 10^{-3}$.) For example, upper limits on the branching ratios for the processes $K_L \to \mu^{\pm} e^{\mp}$ from the BNL E871 Collaboration [31] and $K_L \to \pi^0 \mu^{\pm} e^{\mp}$ from KTeV [32] yield

$$y^2(|B_{12}^{l_L}|^2 + |B_{12}^{l_R}|^2)|\mathrm{Re}\, B_{12}^{d_R} - \mathrm{Re}\, B_{12}^{d_L}|^2 < 10^{-14}, \tag{50}$$

$$y^2(|B_{12}^{l_L}|^2 + |B_{12}^{l_R}|^2)|\mathrm{Im}\, B_{12}^{d_R} + \mathrm{Im}\, B_{12}^{d_L}|^2 < 2 \times 10^{-10}. \tag{51}$$

Hence, the experimental bounds on these processes would have to be improved by several orders of magnitude to yield interesting constraints on the real and imaginary parts of $B_{12}^{d_{R,L}}$. From Eqs. (48), (50), and (51) it is clear that lepton flavor violating meson decays cannot compete in constraining flavor nondiagonal Z' couplings, except in the limit $|w| \ll y$.

However, lepton flavor conserving meson decays can be used to constrain the Z' couplings to quarks, e.g., limits on $K_L \to \mu^+\mu^-$ [17] and $K_L \to \pi^0\mu^+\mu^-$ [29] give

$$w^2|\mathrm{Re}\, B_{12}^{d_R} - \mathrm{Re}\, B_{12}^{d_L}|^2 < 3 \times 10^{-11},$$

$$w^2|\mathrm{Im}\, B_{12}^{d_R} + \mathrm{Im}\, B_{12}^{d_L}|^2 < 5 \times 10^{-11}. \tag{52}$$

The most stringent bounds on the absolute values of the remaining nondiagonal Z' couplings to quarks then come from decays of D^0 and B^0 into a $\mu^+\mu^-$ pair [17] and from the process $B^0 \to K^0\mu^+\mu^-$ [33]:

$$w^2|B_{12}^{u_{R,L}}|^2 < 6 \times 10^{-4},$$

$$w^2|B_{13}^{d_{R,L}}|^2 < 10^{-5},$$

$$w^2|B_{23}^{d_{R,L}}|^2 < 3 \times 10^{-6}. \tag{53}$$

The top-quark couplings to a Z' cannot be constrained from these tree-level processes. In the future, studies of rare top

[6] Flavor diagonal Z' couplings can be constrained from fits to electroweak observables [9,10].

decays [34] and associated top-charm production [35] at the Tevatron, LHC, and a future e^+e^- linear collider will yield very useful constraints.

Further, experimental results on meson mass splittings[7] allow constraints on the real parts of the squared Z' couplings to quarks,

$$y|\mathrm{Re}[(B_{12}^{d_{R,L}})^2]|<10^{-8},\ \ y|\mathrm{Re}[(B_{13}^{d_{R,L}})^2]|<6\times10^{-8}, \tag{54}$$

$$y|\mathrm{Re}[(B_{23}^{d_{R,L}})^2]|<2\times10^{-6},\ \ y|\mathrm{Re}[(B_{12}^{u_{R,L}})^2]|<10^{-7}, \tag{55}$$

and *CP* violation in the Kaon system yields constraints on the imaginary part of the Z'-d-s coupling

$$y|\mathrm{Im}[(B_{12}^{d_{R,L}})^2]|<8\times10^{-11},\quad w|\mathrm{Im}\,B_{12}^{d_{R,L}}|<10^{-6}. \tag{56}$$

IV. MODELS

In the following we shall study concrete examples of extended Abelian gauge structures with flavor nonuniversal couplings, in order to see where such effects are most likely to be seen. Although we only discussed bounds coming from Z-Z' mixing contributions to FCNC processes in Sec. III G, we will also take pure Z' contributions into account here, since they are of the same order as mixing contributions in the models considered.

First we consider a perturbative heterotic superstring model, based on the free fermionic construction [15]. Such models have been studied in detail [16] and it was shown that they generically contain extended Abelian gauge structures and additional matter at the string scale. The running of a scalar mass square due to large Yukawa couplings then triggers the radiative breaking of the U(1)′, naturally giving a Z' in the TeV mass range.

The Z' couplings can be calculated and the fermion charges Q' can be found in Table III. In the quark sector, the first two generations have the same charges, i.e., in the fermion mass eigenstate basis only mixings of the third generation quarks induce flavor changing quark-couplings in Eq. (7). Nevertheless, all the B_{ij}^q are nonzero in general. The same holds true for right-handed leptons, but all three left-handed lepton generations have different Q' charges, which could give rise to strong flavor violating effects.

To study these FCNCs we have chosen a Z' mass of 1 TeV and a Z-Z' mixing angle $\theta=10^{-3}$. The Z' coupling strength, predicted from the string model, is $g_2=0.105$ [16]. Further, we have to specify the unknown fermion mixing matrices $V_{R,L}^{\psi}$. As an example, we will assume that they are equal to the CKM matrix.

In the charged lepton sector these couplings then predict rates for flavor violating processes which are six orders of magnitude above the experimental limits for coherent μ-e conversion, five orders of magnitude above the limit for the decay $\mu\to3e$, and of the same order as the recent experimental bound from the MEGA Collaboration [37] for the radiative decay $\mu\to e\gamma$. On the other hand, predictions for flavor violating τ decays are well below the experimental limits. This is due in part to the fact that these bounds are much less restrictive than for the muon, and in part to the assumed CKM mixing, where the 13 and 23 elements are rather small. Assuming larger mixing of third generation leptons, as suggested by the atmospheric neutrino data [38], would give flavor changing rates close to the experimental bounds, particularly for τ decays into three charged leptons.

For processes involving quarks, we obtain contributions of the same order as SM contributions for the B-$\overline{B}$ and B_s-$\overline{B_s}$ mass differences, and, assuming maximal *CP* violation, a contribution to ε_K which is of the same order as the measured value. Predictions for lepton flavor violating meson decays are well below the experimental bounds.

As we have seen, one obtains flavor violating rates above the experimental limits in the lepton sector if the first two generations have different Q' charges. As an example of a model in which the first two quark families also have different charges, we again consider the string motivated model; however, we set the charges to zero by hand for all of the first generation fermions. Then the rates for coherent μ-e conversion and $\mu\to3e$ are still too large by four and two orders of magnitude, respectively, and we find the same contributions as before to the B-$\overline{B}$ and B_s-$\overline{B_s}$ mass differences. In addition, however, we obtain contributions to the mass splitting in the K and D systems, which are larger than the

TABLE III. Fermion charges in the Z' models motivated from string theory and from precision electroweak data.

String model		EW fit model	
Multiplet	$100\,Q'$	Multiplet	$100\,Q'$
$\begin{pmatrix}t\\b\end{pmatrix}_L$	−71	$\begin{pmatrix}t\\b\end{pmatrix}_L$	+132
t_R	+133	t_R	+100
b_R	−136	b_R	+848
$\begin{pmatrix}u\\d\end{pmatrix}_L,\begin{pmatrix}c\\s\end{pmatrix}_L$	+68	$\begin{pmatrix}u\\d\end{pmatrix}_L,\begin{pmatrix}c\\s\end{pmatrix}_L$	−52
u_R,c_R	−6	u_R,c_R	+38
d_R,s_R	+3	d_R,s_R	+172
$\begin{pmatrix}\nu_\tau\\\tau\end{pmatrix}_L$	+74	$\begin{pmatrix}\nu_\tau\\\tau\end{pmatrix}_L$	−24
τ_R	−130	τ_R	+3
$\begin{pmatrix}\nu_\mu\\\mu\end{pmatrix}_L$	−65	$\begin{pmatrix}\nu_\mu\\\mu\end{pmatrix}_L$	−32
μ_R	+9	μ_R	−31
$\begin{pmatrix}\nu_e\\e\end{pmatrix}_L$	−204	$\begin{pmatrix}\nu_e\\e\end{pmatrix}_L$	−32
e_R	+9	e_R	−31

[7]The $B_s-\overline{B_s}$ mass difference has not been measured yet. We required that new contributions be smaller than the lower limit on the mass splitting, $\Delta m_{B_s^0}>14.3\,\mathrm{ps}^{-1}$ at 95% C.L. [36].

measured values by two orders of magnitude, and the predicted rates for lepton flavor violating and conserving K_L decays are well above experimental limits or results. Further, again assuming maximal *CP* violation, we have contributions to ε_K and $\mathrm{Re}(\varepsilon'/\varepsilon_K)$ which are too large by factors 6×10^5 and 20, respectively.

From these examples, we conclude that any TeV scale Z' would almost certainly have to have equal couplings to the first two families. However, there is still the possibility of different couplings for the third family.

As a final example we consider a flavor nonuniversal Z' that was recently shown to improve the fit to precision electroweak data [10]. Assuming that the first two fermion generations are flavor universal,[8] one can determine the Z' couplings from the fit. The central values found in Ref. [10] are reproduced in Table III. Since the Z' coupling of right-handed top quarks was not determined we have set $Q'_{t_R}=1$ for definiteness, although this coupling has only very little influence on the processes we discussed.

Since in this model the first two lepton generations have the same Z' couplings, the predicted rates for flavor violating μ decays are well below the experimental limits. Only for coherent μ-e conversion do we find a predicted rate of the same order as the experimental limit. However, we obtain contributions to the B-$\overline{B}$ and B_s-$\overline{B_s}$ mass differences which are too large by factors 7 and 40, respectively. Further, the predicted value for ε_K is larger than the measured value by a factor 20 and there is a contribution to ε' which is of the same order as the measured one. Finally, the branching ratios predicted for lepton flavor conserving decays $K_L\to\pi^0 l^+ l^-$ and $B^0\to K^0 l^+ l^-$ are only two orders of magnitude below the experimental bounds, i.e., further experimental progress on these processes could help to constrain this model. Thus, even with universal couplings for the first two families, mixing with the third family induces significant effects in the first two families, at least if the fermion mixing matrices for the charged leptons and the *d*-type quarks are comparable to the CKM matrix.

V. CONCLUSIONS

We conclude that additional Z' bosons with a TeV scale mass and family nonuniversal couplings are severely constrained by experimental results on flavor changing processes. The most stringent bounds come from muon decays, coherent μ-e conversion in muonic atoms, and from lepton flavor conserving processes in the neutral K system, i.e., from processes involving the coupling of a Z' to first and second generation fermions. Couplings to the third generation are less constrained, but future studies of rare top, bottom and τ decays will help to further constrain these models.

If the Z' couplings are diagonal but family nonuniversal in the gauge eigenstate basis, flavor changing couplings arise due to fermion mixing. In the examples we assumed that these unknown mixing matrices are comparable to the CKM matrix. If all three families have different couplings we find contributions to flavor changing processes involving the first two generations which are above experimental bounds by several orders of magnitude. We obtain particularly large contributions to coherent μ-e conversion, the decay $\mu\to3e$, meson mass splittings, K_L decays, and *CP* violation in the neutral K system.

Since couplings of third generation fermions are much less constrained and we assumed that the fermion mixing matrices have a structure similar to the CKM matrix, these problems can be alleviated by assuming that the first two families, but not the third, have the same $\mathrm{U}(1)'$ charges. Mixing with the third family still induces flavor changing effects involving the first two families, but they are suppressed since in the CKM matrix those mixings are small. In the model considered, the new contributions are too large for the B-$\overline{B}$ and B_s-$\overline{B_s}$ mass differences, and *CP* violation in the neutral K system. The experimental bounds for all other processes are respected.

All of the constraints are model dependent. In addition to the Z' mass, mixing with the Z, and charges, they are dependent on the mixing matrices for the left and right chiral quarks and leptons. In the standard model, the right chiral mixing matrices are unobservable, and only the combinations of left chiral matrices in the CKM matrix (6) and its leptonic analogue are observable. However, all of these matrices are in principle observable in the presence of nonuniversal Z' couplings. For example, the flavor changing effects in the B and K systems could be eliminated if the CKM mixing were due entirely to the u quark sector, i.e., $V_{\mathrm{CKM}}=V_L^u$, with $V_L^d=V_R^d=1$. Similarly, μ-e conversion and the decay $\mu\to3e$ would be absent at tree level if all leptonic mixing observable in neutrino oscillations originated in neutrino (rather than charged lepton) mixing, i.e., $V_L^e=V_R^e=1$. For models in which the first two families have the same couplings, these conditions could be relaxed so that $V_{L,R}^{d,e}$ mix the first two families only.

Much stricter bounds on these and similar models, including models with alternative assumptions concerning the fermion mixings, will be available once rare top decays have been studied at the Tevatron, LHC, and a future e^+e^- collider, and more stringent bounds on bottom and tau decays become available from existing *b*-factories and planned charm-τ factories. The rare top decays in particular would constrain the possibility that quark mixing is restricted to the u-c-t sector. Improvements in the sensitivity of searches for rare K_L and μ decays and μ-e conversion are also highly desirable.

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[8]Such models arise for example in the framework of E_6 models, if discrete symmetries which lead to small neutrino masses are imposed [39].

APPENDIX: MATRIX ELEMENTS FOR $K \to \pi\pi$

The evaluation of the hadronic matrix elements in the decay amplitudes A_0 and A_2 is clearly a nonperturbative problem. Several methods have been advocated, and it turns out that chiral perturbation theory in the large N_c limit, N_c being the number of colors, offers the best description of $K \to \pi\pi$ amplitudes [40,41].

We will denote the hadronic matrix elements of the operators by

$$\langle Q_{qq}^{sd} \rangle_I = \langle (2\pi)_I | Q_{qq}^{sd} | K^0 \rangle, \tag{A1}$$

where $I=0,2$ denotes the isospin of the two-pion state. Since only the pseudoscalar part of the effective Lagrangian contributes to the $K \to \pi\pi$ amplitudes, the matrix elements of $\tilde{Q}_{qq}^{sd}$ are given by those of Q_{qq}^{sd}, with an additional minus sign. Analogous formulas hold for the matrix elements of O_{qq}^{sd} and $\tilde{O}_{qq}^{sd}$. In the limit corresponding to the vacuum insertion approximation, chiral perturbation theory yields the following matrix elements:

$$\langle Q_{uu}^{sd} \rangle_0 = \frac{1}{3}\left(\frac{2}{N_c} - 1\right) X, \quad \langle Q_{uu}^{sd} \rangle_2 = \frac{\sqrt{2}}{3}\left(1 + \frac{1}{N_c}\right) X, \tag{A2}$$

$$\langle Q_{dd}^{sd} \rangle_0 = \frac{1}{3}\left(1 + \frac{1}{N_c}\right) X, \quad \langle Q_{dd}^{sd} \rangle_2 = -\frac{\sqrt{2}}{3}\left(1 + \frac{1}{N_c}\right) X, \tag{A3}$$

$$\langle O_{uu}^{sd} \rangle_0 = \frac{1}{3} X - \frac{1}{3N_c} Y, \quad \langle O_{uu}^{sd} \rangle_2 = -\frac{\sqrt{2}}{3} X - \frac{1}{3\sqrt{2}N_c} Y, \tag{A4}$$

$$\langle O_{dd}^{sd} \rangle_0 = -\frac{1}{3} X + \frac{1}{3N_c}\left(3\frac{F_K}{F_\pi} - 2\right) Y,$$

$$\langle O_{dd}^{sd} \rangle_2 = \frac{\sqrt{2}}{3} X + \frac{1}{3\sqrt{2}N_c} Y, \tag{A5}$$

where

$$X = \sqrt{\frac{3}{2}} F_\pi (m_K^2 - m_\pi^2)\left(1 + \frac{m_\pi^2}{\Lambda_\chi^2}\right),$$

$$Y = -\sqrt{\frac{3}{2}} F_\pi \left(\frac{m_K^2}{m_s}\right)^2 \left(1 + \frac{m_\pi^2}{\Lambda_\chi^2}\right), \tag{A6}$$

and Λ_χ is a parameter in the Lagrangian of chiral perturbation theory related to the ratio of the π and K decay constants

$$\frac{(m_K^2 - m_\pi^2)}{\Lambda_\chi^2} = \frac{F_K}{F_\pi} - 1. \tag{A7}$$

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