Field-to-particle transition based on the zero-brane approach to quantization of multiscalar field theories and its application for Jackiw-Teitelboim gravity

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The field-to-particle transition formalism based on the effective zero-brane action approach is generalized for arbitrary multiscalar fields. As a fruitful example, by virtue of this method we derive the nonminimal particle action for the Jackiw-Teitelboim (JT) gravity at a fixed gauge in the vicinity of the black hole solution as an instanton-dilaton doublet. When quantizing it as the theory with higher derivatives, it is shown that the appearing quantum equation has an SU(2) dynamical symmetry group realizing the exact spin-coordinate correspondence. Finally, we calculate the quantum corrections to the mass of the JT black hole.

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I. INTRODUCTION

The first and foremost aim of this paper is to develop the classical and quantum field-to-particle transition formalism for multiscalar field theory in two-dimensional (2D) flat spacetime. Below we will call this formalism ''zero-brane'' in the sense of a ''nonminimal point particle'' rather than in the sense of supersymmetric string or brane theories. The study of the field-to-particle transition formalism as such also lies within the well-known dream program of constructing a theory which would not contain matter as an external postulated entity but would consider fields as sources of matter and particles as special field configurations. Such a program was inspired probably first by Lorentz and Poincaré and since that time much effort was made to bring it to reality; one may recall the Einstein, Klein, and Heisenberg attempts, but the discovery of fermionic fields and the success of the standard model decreased interest in such theories. Nevertheless, the problem of the origin of matter still remains open and important, especially in what concerns the theoretical explanation of the fundamental properties of observable particles. Nowadays, it seems possible to realize this program for boson fields (below we will show that it is easily possible for multiscalar field in two dimensions) but it is yet unclear how to obtain fermionic matter. There is some hope that it can be done through supersymmetry but the generalization of the proposed approach on higher-dimensional theories encounters severe mathematical troubles.

But thinking about the physical relevance of the presented approach one should not exclude the second possibility. Namely, as we will demonstrate below the special field solutions indeed can be correctly regarded as particles which are sufficiently *point-like* ones (despite the presence in action of nonminimal terms depending on curvature, etc.). On the other hand, there exist extended models of particles which suggest that pointness is no more than scale approximation [1]. Thus, one can question the (justified) choice between ''nonminimal point-particle'' and ''extended particle'' paradigmes. We cannot answer this question yet, we just point out that this paper is devoted to the former point of view.

Then, by way of an example, we will apply the field-particle approach to the particular 2D theory of gravity admitting at certain parametrization the correspondence to some scalar field theory acting on flat spacetime. The Jackiw-Teitelboim (JT) dilaton gravity discovered in 1984 [2] can be obtained also as a dimensional reduction of the 3D Bañados-Teitelboim-Zanelli black hole [3,4] and spherically symmetric solution of 4D dilaton Einstein-Maxwell gravity used as a model of the evaporation process of a near-extremal black hole [5]. In spite of the fact that the JT solution is locally diffeomorphic to the DeSitter space, it has all the global attributes of a black hole. Besides, it is simple enough to obtain the main results in a nonperturbative way that seems to be important for highly nonlinear general relativity.

The wide literature devoted to classical and quantum aspects of the theory (see Refs. [6–8], and references therein), is concerned mainly with standard methods of study, whereas it is clear that black holes are extended objects and thus should be correctly considered within the framework of brane theory where there is no rigid fixation of spatial symmetry. The quantum aspects were studied mainly in connection with group features of JT dilaton gravity in general, whereas we will quantize the theory in the vicinity of a certain nontrivial vacuum induced by a static solution emphasizing the corrections to mass spectrum. Thus, our purpose is to study the 2D dilaton gravity in the neighborhood of the classical and quantum Jackiw-Teitelboim black hole within the frameworks of the brane approach, which consists of constructing the effective action where the nonminimal terms (first of all, depending on the world-volume curvature) are induced by field fluctuations. Then the effective action evidently arises after nonlinear reparametrization of an initial theory when excluding zero-field oscillations.

The paper is arranged as follows. In Sec. II we study the JT solution as a soliton-dilaton (more correctly, instanton-dilaton) doublet and its properties at the classical level. In Sec. III we generalize the approach [9] for arbitrary multiscalar fields and apply it for JT dilaton gravity in the fixed-gauge (flat-spacetime) formulation. Minimizing the action

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with respect to field fluctuations, we remove zero modes and obtain the point-particle action with nonminimal terms corresponding to this theory. Section IV is devoted to quantization of the action as a constrained theory with higher derivatives. In result we obtain the Schrödinger wave equation describing wave function and mass spectrum of a point particle with curvature and apply them to the JT black hole. Then we calculate the zeroth and first excited levels to get the mass of the quantum JT black hole with quantum corrections. Conclusions are made in Sec. V.

II. JACKIW-TEITELBOIM GRAVITY

Consider the action of the Jackiw-Teitelboim dilaton gravity

$$S_{JT}[\tau,g]=\frac{1}{2G}\int d^2x\sqrt{-g}\,\tau(R+2m^2), \tag{2.1}$$

where G is the gravitational coupling constant, dimensionless in the 2D case. Extremizing this action with respect to metric and dilaton field variations we obtain the following equations of motion:

$$R+2m^2=0, \tag{2.2}$$

$$(\nabla_\mu\nabla_\nu-m^2g_{\mu\nu})\tau=0. \tag{2.3}$$

Further, if one performs the parametrization of a metric

$$ds^2=-\sin^2(u/2)dt^2+\cos^2(u/2)dx^2, \tag{2.4}$$

and puts the metric ansatz into Eqs. (2.1)–(2.3), they can be rewritten [6], respectively, as

$$S_{JT}[\tau,u]=\frac{1}{2G}\int d^2x\,\tau(\Delta u-m^2\sin u), \tag{2.5}$$

$$\Delta u-m^2\sin u=0, \tag{2.6}$$

$$(\Delta-m^2\cos u)\tau=0, \tag{2.7}$$

where Δ is the flat Euclidean Laplacian, $\partial_t^2+\partial_x^2$.

If we wish to choose from the solutions of Eq. (2.6) only the one-instanton ones, we have the following instanton-dilaton pair:

$$u^{(s)}(x,t)=4\arctan\exp(m\rho), \tag{2.8}$$

$$\tau^{(s)}(x,t)=-C_1\,\mathrm{sech}(m\rho)+C_2[\sinh(m\rho)+m\rho\;\mathrm{sech}(m\rho)], \tag{2.9}$$

where C_i are arbitrary constants, $\rho=\gamma(x-vt)$, $1/\gamma=\sqrt{1+v^2}$. Then the metric (2.4) after the transformation $\{x,t\}\rightarrow\{R,T\}$, such that

$$dT=v\sqrt{\frac{m}{2G\gamma M}}\left[dt-\frac{v/\gamma}{\mathrm{cosech}^2(m\rho)-v^2}d\rho\right],$$

$$R=\frac{1}{v}\sqrt{\frac{2GM}{\gamma m^3}}\,\mathrm{sech}(m\rho),$$

where M is an arbitrary constant, can be rewritten in the explicit form representing the JT black hole solution

$$ds^2=-\left(m^2R^2-\frac{2G\gamma M}{m}\right)dT^2+\left(m^2R^2-\frac{2G\gamma M}{m}\right)^{-1}dR^2, \tag{2.10}$$

having the following energy and event horizon:

$$E_{\mathrm{BH}}=\gamma M,\quad R_{\mathrm{BH}}=\sqrt{\frac{2G\gamma M}{m^3}}. \tag{2.11}$$

Our purpose now will be to take into account the field fluctuations in the neighborhood of this solution and to construct the effective action of the JT black hole as a zero brane. Before we go further, one should develop a general approach.

III. EFFECTIVE ACTION

In this section we will construct the nonlinear effective action of an arbitrary multiscalar 2D theory in the vicinity of a solitary-wave solution, and then apply it for JT dilaton gravity. In fact, here we will describe the procedure of the correct transition from field to particle degrees of freedom. Indeed, despite that the solitary-wave solution resembles a particle both at the classical and quantum levels, it yet remains to be a *field solution* with an infinite number of field degrees of freedom whereas a true particle has a finite number of degrees of freedom. Therefore, we are obliged to correctly handle this circumstance (and several others), otherwise deep contradictions may appear.

A. General formalism

Let us consider the action describing N scalar fields

$$S[\varphi]=\int L(\varphi)d^2x, \tag{3.1}$$

$$L(\varphi)=\frac{1}{2}\sum_{a=1}^{N}(\partial_m\varphi_a)(\partial^m\varphi_a)-U(\varphi). \tag{3.2}$$

The corresponding equations of motion are

$$\partial^m\partial_m\varphi_a+U_a(\varphi)=0, \tag{3.3}$$

where we defined

$$U_a(\varphi)=\frac{\partial U(\varphi)}{\partial\varphi_a},\quad U_{ab}(\varphi)=\frac{\partial^2U(\varphi)}{\partial\varphi_a\partial\varphi_b}.$$

Suppose we have a solution in the class of solitary waves

$$\varphi_a^{(s)}(\rho)=\varphi_a^{(s)}[\gamma(x-vt)], \quad \gamma=1/\sqrt{1-v^2}, \tag{3.4}$$

having the localized energy density

$$\varepsilon(\varphi)=\sum_a \frac{\partial L(\varphi)}{\partial(\partial_0\varphi_a)}\partial_0\varphi_a - L(\varphi), \tag{3.5}$$

and finite mass integral

$$\mu=\int_{-\infty}^{+\infty}\varepsilon(\varphi^{(s)})d\rho=-\int_{-\infty}^{+\infty}L(\varphi^{(s)})d\rho<\infty, \tag{3.6}$$

coinciding with the total energy up to the Lorentz factor γ.

Let us change the set of the collective coordinates $\{\sigma_0=s,\ \sigma_1=\rho\}$ such that

$$x^m=x^m(s)+e_{(1)}^m(s)\rho, \quad \varphi_a(x,t)=\tilde{\varphi}_a(\sigma), \tag{3.7}$$

where $x^m(s)$ turn out to be the coordinates of a (1+1)-dimensional point particle and $e_{(1)}^m(s)$ is the unit spacelike vector orthogonal to the world line. Hence, the action (3.1) can be rewritten in new coordinates as

$$S[\tilde{\varphi}]=\int L(\tilde{\varphi})\,\Delta\, d^2\sigma,$$
$$L(\tilde{\varphi})=\frac{1}{2}\sum_a\left[\frac{(\partial_s\tilde{\varphi}_a)^2}{\Delta^2}-(\partial_\rho\tilde{\varphi}_a)^2\right]-U(\tilde{\varphi}), \tag{3.8}$$

where

$$\Delta=\det\left\|\frac{\partial x^m}{\partial\sigma^k}\right\|=\sqrt{\dot{x}^2}(1-\rho k),$$

and k is the curvature of a particle world line

$$k=\frac{\varepsilon_{mn}\dot{x}^m\ddot{x}^n}{(\sqrt{\dot{x}^2})^3}, \tag{3.9}$$

where ε_{mn} is the unit antisymmetric tensor. This new action contains the N-redundant degrees of freedom which eventually lead to appearance of the so-called ''zero modes.'' To eliminate them we must constrain the model by means of the condition of vanishing of the functional derivative with respect to field fluctuations about a chosen static solution. In result we will obtain the required effective action.

So, the fluctuations of the fields $\tilde{\varphi}_a(\sigma)$ in the neighborhood of the static solution $\varphi_a^{(s)}(\rho)$ are given by the expression

$$\tilde{\varphi}_a(\sigma)=\varphi_a^{(s)}(\rho)+\delta\varphi_a(\sigma). \tag{3.10}$$

Substituting them into Eq. (3.8) and considering the static equations of motion (3.3) for $\varphi_a^{(s)}(\rho)$ we have

$$S[\delta\varphi]=\int d^2\sigma\Bigg\{\Delta\Bigg[L(\varphi^{(s)})+\frac{1}{2}\sum_a\Bigg(\frac{(\partial_s\delta\varphi_a)^2}{\Delta^2}-(\partial_\rho\delta\varphi_a)^2-\sum_b U_{ab}(\varphi^{(s)})\delta\varphi_a\delta\varphi_b\Bigg)\Bigg]-k\sqrt{\dot{x}^2}\sum_a\varphi_a^{(s)\prime}\delta\varphi_a+O(\delta\varphi^3)\Bigg\}+\{\text{surf. terms}\}, \tag{3.11}$$

$$L(\varphi^{(s)})=-\frac{1}{2}\sum_a\varphi_a^{(s)\prime 2}-U(\varphi^{(s)}),$$

where prime means the derivative with respect to ρ. Extremizing this action with respect to $\delta\varphi_a$, one can obtain the system of equations in partial derivatives for field fluctuations:

$$(\partial_s\Delta^{-1}\partial_s-\partial_\rho\Delta\partial_\rho)\delta\varphi_a+\Delta\sum_b U_{ab}(\varphi^{(s)})\delta\varphi_b+\varphi_a^{(s)\prime}k\sqrt{\dot{x}^2}=O(\delta\varphi^2), \tag{3.12}$$

which has to be the constraint removing the redundant degrees of freedom. Supposing $\delta\varphi_a(s,\rho)=k(s)f_a(\rho)$, in the linear approximations $\rho k\ll 1$ (which naturally guarantees also the smoothness of a world line at $\rho\to 0$) and $O(\delta\varphi^2)=0$, we obtain the system of three ordinary derivative equations

$$\frac{1}{\sqrt{\dot{x}^2}}\frac{d}{ds}\frac{1}{\sqrt{\dot{x}^2}}\frac{dk}{ds}+ck=0, \tag{3.13}$$

$$-f_a''+\sum_b[U_{ab}(\varphi^{(s)})-c\,\delta_{ab}]f_b+\varphi_a^{(s)\prime}=0, \tag{3.14}$$

where c is the constant of separation. Searching for a solution of the last subsystem in the form

$$f_a=g_a+\frac{1}{c}\varphi^{(s)\prime}, \tag{3.15}$$

we obtain the homogeneous system

$$-g_a''+\sum_b(U_{ab}(\varphi^{(s)})-c\,\delta_{ab})g_b=0. \tag{3.16}$$

Strictly speaking, the explicit form of $g_a(\rho)$ is not significant for us, because we always can suppose integration constants to be zero, thus restricting ourselves by the special solution. Nevertheless, the homogeneous system should be considered as the eigenvalue problem for c (see below).

Substituting the found functions $\delta\varphi_a=kf_a$ back into the action (3.11), we can rewrite it in the explicit zero-brane form

$$S_{\rm eff} = S_{\rm eff}^{\rm (class)} + S_{\rm eff}^{\rm (fluct)} = -\int ds \sqrt{\dot{x}^2}(\mu + \alpha k^2), \quad (3.17)$$

describing a point particle with curvature, where μ was defined in Eq. (3.6), and

$$\alpha = \frac{1}{2}\sum_a \int_{-\infty}^{\infty} f_a \varphi_a^{(s)\,\prime} d\rho + \frac{1}{2}\sum_a \int_{-\infty}^{+\infty} (f_a f_a^{\prime})^{\prime} d\rho. \quad (3.18)$$

Further, contracting Eq. (3.3) with $\varphi_a^{(s)\,\prime}$, we obtain the expression

$$\sum_a [\varphi_a^{(s)\,\prime\prime} - U_a(\varphi^{(s)})]\varphi_a^{(s)\,\prime} = 0, \quad (3.19)$$

which can be rewritten as

$$\sum_a \varphi_a^{(s)\,\prime 2} = 2U(\varphi^{(s)}(\rho)). \quad (3.20)$$

Considering Eqs. (3.5), (3.6), (3.15), (3.19), and (3.20), the expression for α can be written in the simple form [for simplicity here we suppose the same eigenvalues, $c_a \equiv c$, otherwise the first integral in Eq. (3.18) cannot be reduced to the integral (3.6) and should be evaluated separately]

$$\alpha = \frac{\mu}{2c} + \frac{1}{2c^2}\int_{-\infty}^{+\infty} U^{\prime\prime}(\varphi^{(s)}(\rho))d\rho, \quad (3.21)$$

where the second term can be integrated as a full derivative. Therefore, even if it is non-zero,[1] we always can remove it by means of including into the surface terms the action (3.11) or adding an appropriate counterterm to the action (3.8):

$$S^{\rm (reg)}[\tilde{\varphi}] = S[\tilde{\varphi}] - \frac{1}{2c^2}\int_{-\infty}^{+\infty} d^2\sigma \Delta k^2 U^{\prime\prime}(\rho).$$

Thus, we obtain the final form of the effective zero-brane action of the theory

$$S_{\rm eff} = -\mu \int ds \sqrt{\dot{x}^2}\left(1 + \frac{1}{2c}k^2\right). \quad (3.22)$$

It is straightforward to derive the corresponding equation of motion in the Frenet basis

$$\frac{1}{\sqrt{\dot{x}^2}}\frac{d}{ds}\frac{1}{\sqrt{\dot{x}^2}}\frac{dk}{ds} + \left(c - \frac{1}{2}k^2\right)k = 0, \quad (3.23)$$

hence, one can see that Eq. (3.13) was nothing but this equation in the linear approximation $k \ll 1$, as was expected.

Thus, the only problem which yet demands resolving is the determination of eigenvalue c. It turns out to be the Stourm-Liouville problem for the system (3.16) under some chosen boundary conditions. If one supposes, for instance, the finiteness of g at infinity then the c spectrum turns out to be discrete. Moreover, it often happens that c has only one or two admissible values.[2]

Be that as it may, the exact value of c is necessary hence the system (3.16) should be resolved as exactly as possible. Let us consider it more closely. The main problem there is that the functions g_a are mixed between equations. To separate them, let us recall that there exist $N-1$ orbit equations, whose varying resolves the separation problem. We consider this for the case $N=2$, i.e., for a biscalar theory, all the more so it will be helpful when applying for JT gravity.

Considering Eq. (3.15), the varying of a single orbit equation yields

$$\frac{\delta\varphi_2}{\delta\varphi_1} = \frac{\varphi_2^{(s)\,\prime}}{\varphi_1^{(s)\,\prime}} = \frac{g_2}{g_1}, \quad (3.24)$$

hence the system (3.16), $N=2$, can be separated into the two independent equations

$$-g_a^{\prime\prime} + \left(\frac{\varphi_a^{(s)\,\prime\prime\prime}}{\varphi_a^{(s)\,\prime}} - c\right)g_a = 0, \quad (3.25)$$

if one uses $\varphi_a^{\prime\prime\prime} = \Sigma_b U_{ab}(\varphi)\varphi_b^{\prime}$. In this form it is much easier to resolve the eigenvalue problem. Therefore, the two independent parameters for the action (3.22), μ and c, can be determined immediately by virtue of Eqs. (3.6) and (3.25).

Finally, it should be pointed out that the developed method can be generalized both in the qualitative direction (considering it for the sigma-model, Yang-Mills and spinor Lagrangians [17]) and toward the increasing of spatial dimensions.

B. Application for JT black hole

For further studies it is convenient to perform the Wick rotation and to work in terms of solitons and Lorentzian time rather than in terms of instantons and Euclidean time, all the more so the main results of the previous subsection are independent of v. Omitting topological surface terms, we will consider instead the action (2.5) and its Lorentzian analog

$$S_{JT}[\tau, u] = \frac{1}{2G}\int d^2x(\partial_m \tau \partial^m u - m^2 \tau \sin u). \quad (3.26)$$

The soliton-dilaton doublet (2.8), (2.9) has the localized energy density (3.5)

$$\varepsilon(x,t) = \frac{2m^2}{G}\frac{C_1 \tanh(m\rho) + C_2[1 - m\rho \tanh(m\rho)]}{\cosh^2(m\rho)},$$

[1] It identically vanishes when $|\varphi_a^{(s)}(\rho)| \leq O(1)$ at infinity.

[2] For instance, in the works [9] (one-scalar φ^4 theory) or [10] (φ^3 and Liouville model), where the special cases of this formalism were used, c has the form βm^2 where β is a single positive half-integer or integer; the cases with $c<0$ does not have, as a rule, independent physical sense, because at quantization they either can be interpreted in terms of antiparticles or appear to be unphysical at all.

and can be interpreted as the relativistic point particle with the energy

$$E_{\text{class}} = \int_{-\infty}^{+\infty} \varepsilon(x,t)\, dx \equiv \gamma\mu = \frac{2C_2\gamma m}{G}, \tag{3.27}$$

i.e., the integral (3.6) is finite and coincides with the energy (2.11)

$$\mu = M, \tag{3.28}$$

if one redefines C_2.

The action (3.26) always can be linearly rearranged in the form (3.1), (3.2), if we introduce fields φ_a such that

$$2\theta u = \varphi_1 - i\varphi_2, \quad \tau/\theta = \varphi_1 + i\varphi_2, \tag{3.29}$$

where θ is an arbitrary real constant which (similarly to C_1) will not affect on final results. We will suppose the final zero-brane action (3.22). The flat spacetime coordinates x^μ Eq. (3.7) should be understood in the sense that we substituted the initial curved spacetime for a flat one with the effective potential, but the meaning of the collective coordinates ρ and s remains unchanged because it describes internal structure and hence is independent of whether we are working in curved or flat space.

Therefore, the main task now is to specify the parameters of the action (3.22) for our case. We have μ already determined by Eq. (3.26), and the eigenvalue c remains to be the only unknown parameter for Eq. (3.22). For Eqs. (3.25) we will require the traditional boundary conditions

$$g_a(+\infty) - g_a(-\infty) = O(1), \tag{3.30}$$

whereas, provided Eqs. (2.8), (2.9), (3.27), the system (3.25) has the form

$$-g_a'' + (K_a - c)g_a = 0, \tag{3.31}$$

where

$$K_1 = \frac{m^2}{\cosh^2(m\rho)} \frac{A_1[\cosh^2(m\rho) - 2] + C_2\cosh^5(m\rho) + A_2[6 - \cosh^2(m\rho)]}{\cosh^2(m\rho)[4\theta^2 + C_2 + C_2\cosh(m\rho)] - A_2},$$

$$K_2 = K_1|_{C_i \to -C_i},$$

$$A_1 = \cosh(m\rho)(4\theta^2 + 3C_2), \quad A_2 = \sinh(m\rho)(C_1 + C_2 m\rho),$$

hence it is clear that

$$K_a(0) = -m^2, \quad K_a(-\infty) = K_a(+\infty) = m^2. \tag{3.32}$$

This eigenvalue equation is evidently hard to solve exactly, hence we use the method of the approximing potential which would have the main properties of K_a especially those presented by Eq. (3.32). Besides, we will consider the equation for K_1 only because both potentials have approximately the same behavior.

Thus, omitting the index we will assume the following eigenvalue equation:

$$-g'' + m^2\left(1 - \frac{2}{\cosh^2(m\rho)}\right)g - cg = 0, \tag{3.33}$$

instead of Eq. (3.31). Its potential has the main features of K_a but appears to be exactly solvable: according to the proven theorem (see the Appendix), the only admissible nonzero c is

$$c = m^2. \tag{3.34}$$

This result is confirmed also by the quasiclassical approximation. Indeed, the necessary condition of convergence of the phase integral

$$\oint p\, d\rho = 2\int_{-\infty}^{+\infty} \sqrt{K_a - c}\; d\rho$$

appears to be the following one:

$$K_a(\pm\infty) - c = 0,$$

which yields Eq. (3.34) again.

Therefore, the effective zero-brane action of the dilaton gravity about the Jackiw-Teitelboim black hole with fluctuational corrections is

$$S_{\text{eff}} = -\mu\int ds\,\sqrt{\dot{x}^2}\left(1 + \frac{k^2}{2m^2}\right), \quad \mu = M = \frac{2C_2 m}{G}. \tag{3.35}$$

In the next section we will quantize it to obtain the quantum corrections to the mass of the fixed-gauge JT black hole.

IV. QUANTIZATION

In the previous section we obtained a classical effective action for the model in question. Thus, to quantize it we must consecutively construct the Hamiltonian structure of dynamics of the point particle with curvature [11–13].

A. General formalism

From the brane action (3.22) and definition of the world-line curvature one can see that we have the theory with higher derivatives [12,13]. Hence, below we will treat the coordinates and momenta as the canonically independent coordinates of the phase space. Besides, the Hessian matrix constructed from the derivatives with respect to accelerations,

$$M_{ab}=\left\| \frac{\partial^2 L_{\text{eff}}}{\partial \ddot{x}^a \partial \ddot{x}^b} \right\|,$$

appears to be singular that signalizes the presence of the constraints for phase coordinates of the theory.

As was mentioned, the phase space consists of the two pairs of canonical variables:

$$x_m, \quad p_m=\frac{\partial L_{\text{eff}}}{\partial q^m}-\dot{\Pi}_m, \tag{4.1}$$

$$q_m=\dot{x}_m, \quad \Pi_m=\frac{\partial L_{\text{eff}}}{\partial \dot{q}^m}, \tag{4.2}$$

hence we have

$$p^n=-e^n_{(0)}\mu\left[1-\frac{1}{2c}\right]+\frac{\mu}{c}\frac{e^n_{(1)}}{\sqrt{q^2}}\dot{k}, \tag{4.3}$$

$$\Pi^n=-\frac{\mu}{c}\frac{e^n_{(1)}}{\sqrt{q^2}}k, \tag{4.4}$$

where the components of the Frenet basis are

$$e^m_{(0)}=\frac{\dot{x}^m}{\sqrt{\dot{x}^2}}, \quad e^m_{(1)}=-\frac{1}{\sqrt{\dot{x}^2}}\frac{\dot{e}^m_{(0)}}{k}.$$

There exist two primary constraints of the first kind

$$\Phi_1=\Pi^m q_m\approx 0, \tag{4.5}$$

$$\Phi_2=p^m q_m+\sqrt{q^2}\left[\mu+\frac{c}{2\mu}q^2\Pi^2\right]\approx 0, \tag{4.6}$$

in addition, we should add the proper time gauge condition,

$$G=\sqrt{q^2}-1\approx 0, \tag{4.7}$$

to remove the nonphysical gauge degree of freedom. Then, when introducing the new variables,

$$\rho=\sqrt{q^2}, \quad v=\text{arctanh}(p_{(1)}/p_{(0)}), \tag{4.8}$$

the constraints can be rewritten in the form

$$\Phi_1=\rho\Pi_\rho,$$

$$\Phi_2=\rho\left[-\sqrt{p^2}\cosh v+\mu-\frac{c}{2\mu}(\Pi_v^2-\rho^2\Pi_\rho^2)\right], \tag{4.9}$$

$$G=\rho-1.$$

Hence, finally, we obtain the constraint

$$\Phi_2=-\sqrt{p^2}\cosh v+\mu-\frac{c}{2\mu}\Pi_v^2\approx 0, \tag{4.10}$$

which in the quantum theory ($\Pi_v=-i\partial/\partial v$) yields

$$\hat{\Phi}_2|\Psi\rangle=0.$$

As was shown in Ref. [9] (see also Ref. [12]), the constraint Φ_2 on the quantum level admits several coordinate representations that, generally speaking, lead to different nonequivalent theories, therefore, the choice between the different forms of $\hat{\Phi}_2$ should be based on the physical relevance. Then the physically admissible equation determining quantum dynamics of the quantum kink and bell particles has the form

$$[\hat{H}-\varepsilon]\Psi(\zeta)=0, \tag{4.11}$$

$$\hat{H}=-\frac{d^2}{d\zeta^2}+\frac{B^2}{4}\sinh^2\zeta-B\left(S+\frac{1}{2}\right)\cosh\zeta, \tag{4.12}$$

where

$$\zeta=v/2, \quad \sqrt{p^2}=\mathcal{M},$$

$$B=8\sqrt{\frac{\mu\mathcal{M}}{c}}, \tag{4.13}$$

$$\varepsilon=\frac{8\mu^2}{c}\left(1-\frac{\mathcal{M}}{\mu}\right),$$

and $S=0$ in our case.

As was established [14,15], SU(2) has to be the dynamical symmetry group for this Hamiltonian which can be rewritten in the form of the spin Hamiltonian

$$\hat{H}=-S_z^2-BS_x, \tag{4.14}$$

where the spin operators,

$$S_x=S\cosh\zeta-\frac{B}{2}\sinh^2\zeta-\sinh\zeta\frac{d}{d\zeta},$$

$$S_y=i\left\{-S\sinh\zeta+\frac{B}{2}\sinh\zeta\cosh\zeta+\cosh\zeta\frac{d}{d\zeta}\right\}, \tag{4.15}$$

$$S_z=\frac{B}{2}\sinh\zeta+\frac{d}{d\zeta},$$

satisfy the commutation relations

$$[S_i, S_j] = i\epsilon_{ijk} S_k.$$

In addition,

$$S_x^2 + S_y^2 + S_z^2 \equiv S(S+1).$$

In this connection it should be noted that though the reformulation of some interactions concerning the coordinate degrees of freedom in terms of spin variables is widely used (e.g., in the theories with the Heisenberg Hamiltonian, see Ref. [16]), it has to be just the physical approximation as a rule, whereas in our case the spin-coordinate correspondence is exact.

Further, at $S \geqslant 0$ there exists an irreducible $(2S+1)$-dimensional subspace of the representation space of the su(2) Lie algebra, which is invariant with respect to these operators. Determining eigenvalues and eigenvectors of the spin Hamiltonian in the matrix representation which is realized in this subspace, one can prove that the solution of Eq. (4.11) is the function

$$\Psi(\zeta) = \exp\left(-\frac{B}{2}\cosh\zeta\right) \sum_{\sigma=-S}^{S} \frac{c_\sigma}{\sqrt{(S-\sigma)!(S+\sigma)!}} \times \exp(\sigma\zeta), \tag{4.16}$$

where the coefficients c_σ are the solutions of the system of linear equations

$$(\varepsilon + \sigma^2)c_\sigma + \frac{B}{2}[\sqrt{(S-\sigma)(S+\sigma+1)}\, c_{\sigma+1} + \sqrt{(S+\sigma)(S-\sigma+1)}\, c_{\sigma-1}] = 0,$$

$$c_{S+1} = c_{-S-1} = 0, \quad \sigma = -S, \quad -S+1, \dots, S.$$

However, it should be noted that these expressions give only the finite number of exact solutions which is equal to the dimensionality of the invariant subspace (this is the so-called, quasiexactly solvable system). Therefore, for the spin $S=0$ we can find only the ground-state wave function and eigenvalue

$$\Psi_0(\zeta) = C_1 \exp\left(-\frac{B}{2}\cosh\zeta\right), \quad \varepsilon_0 = 0. \tag{4.17}$$

Hence, we obtain that the ground-state mass of the quantum particle with curvature coincides with the classical one,

$$\mathcal{M}_0 = \mu, \tag{4.18}$$

as was expected.

Further, in the absence of exact wave functions for more excited levels one can find the first (small) quantum correction to the mass in the approximation of the quantum harmonic oscillator. It is easy to see that at $B \geqslant 1$ the (effective) potential

$$V(\zeta) = \left(\frac{B}{2}\right)^2 \sinh^2\zeta - \frac{B}{2}\cosh\zeta \tag{4.19}$$

has the single minimum

$$V_{\min} = -B/2 \ \text{ at } \ \zeta_{\min} = 0.$$

Then, following the $\hbar$-expansion technique, we shift the origin of coordinates in the point of minimum (to satisfy $\varepsilon = \varepsilon_0 = 0$ in the absence of quantum oscillations), and expand V in the Taylor series to second order near the origin thus reducing the model to the oscillator of the unit mass, energy $\varepsilon/2$, and oscillation frequency

$$\omega = \frac{1}{2}\sqrt{B(B-1)}.$$

Therefore, the quantization rules yield the discrete spectrum

$$\varepsilon = \sqrt{B(B-1)}(n+1/2) + O(\hbar^2), \quad n = 0,1,2, \dots, \tag{4.20}$$

and the first quantum correction to particle masses will be determined by the lower energy of oscillations:

$$\varepsilon = \frac{1}{2}\sqrt{B(B-1)} + O(\hbar^2), \tag{4.21}$$

which gives the algebraic equation for $\mathcal{M}$ as a function of m and μ.

We can easily resolve it in the approximation

$$B \gg 1 \Leftrightarrow c/\mu^2 \to 0, \tag{4.22}$$

which is admissible for the major physical cases, and obtain

$$\varepsilon = \frac{B}{2} + O(\hbar^2 c/\mu^2), \tag{4.23}$$

which, after considering Eqs. (4.13) and (4.18), yields

$$(\mathcal{M} - \mu)^2 = \frac{c\mathcal{M}}{4\mu} + O(\hbar^2 c/\mu^2). \tag{4.24}$$

Then one can seek for the mass in the form $\mathcal{M} = \mu + \delta$ ($\delta \ll \mu$), and finally we obtain the mass of a particle with curvature (3.22) with first-order quantum corrections

$$\mathcal{M} = \mu \pm \frac{\sqrt{c}}{2} + O(\hbar^2 c/\mu^2). \tag{4.25}$$

The nature of the justified choice of the root sign before the second term is not so clear as it seems for a first look, because there exist two historically interfering arguments. The first (physical) one is: if we apply this formalism for the one-scalar φ^4 model [9] and compare the result with that obtained in other ways [18], we should suppose the sign ''+.'' However, the second, mathematical, counterargument is as follows: the known exact spectra of the operators with the QES potentials like Eq. (4.11) are split, as a rule by virtue of radicals, hence the signs ''±'' can approximately represent such a bifurcation and thus should be unharmed. If it is really so, quantum fluctuations should divide the classically unified particle with curvature into several subtypes with respect to mass.

Finally, comparing the first term (4.25) and the estimate (4.22), one can see that the obtained spectrum is nonperturbative and cannot be derived by virtue of the Taylor series with respect to $1/\mu$.

B. Mass of quantum JT black hole

Thus, considering Eqs. (3.35) and (4.25), the mass of the quantum JT black hole as a soliton-dilaton boson in the first approximation is

$$\mathcal{M} = M \pm m/2 + O(m^2/M^2), \tag{4.26}$$

therefore, the approximation (4.22) has to be justified in this case. The problem of obtaining further corrections turns out to be reduced to the mathematically standard Stourm-Liouville problem for the Razavi potential, all the more so the latter is well-like on the whole axis and hence admits only the bound states with a discrete spectrum.

Finally, it should be noted that we quantized the reduced theory (3.26) rather than complete dilaton gravity because in the general case the latter has two first-class constraints which were removed by a fixed metric gauge. In addition, unlike the previous works we quantized the theory about the static solution rather than in the neigborhood of the trivial vacuum, and were interested first of all in obtaining the mass spectrum. The question of the construction of corresponding formalism for dilaton gravity in the general case remains open because it requires consistent generalization of the field-to-particle transition procedure for fields in curved spacetime.

V. CONCLUSION

Let us enumerate the main items studied. It was shown that the Jackiw-Teitelboim dilaton gravity can be reduced to biscalar theory admitting the doublet consisting of instanton and dilaton components, which can be interpreted as a massive quantum particle. Further, considering field fluctuations in the neighborhood of the JT black hole, the action for the JT field doublet as a nonminimal point particle with curvature was ruled out thereby we generalized the procedure of obtaining brane actions for the multiscalar case. From the fact that the (1+1)-dimensional dilaton gravity yields the effective action for the JT black hole as a spatially zero-dimensional brane (nonminimal point particle), we can conclude that the ordinary 4D black hole (in the case of arbitrary symmetry and field fluctuations in a neighborhood) could be consecutively described within the framework of a five-dimensional field theory.

When quantizing this action as the constrained theory with higher derivatives, it was shown that the resulting Schrödinger equation is the special case of that with the Razavi potential having the SU(2) dynamical symmetry group in the ground state. Finally, we found the first quantum correction to the mass of the Jackiw-Teitelboim black hole which could not be calculated by means of the perturbation theory.

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APPENDIX: EIGENVALUE THEOREM

Theorem. The bound-state singular Stourm-Liouville problem

$$-f''(u) + (1 - 2\,\mathrm{sech}^2 u) f(u) - c f(u) = 0, \tag{A1}$$

$$f(+\infty) = f(-\infty) = O(1), \tag{A2}$$

has only the two sets of eigenfunctions and eigenvalues

$$f_0 = K_0 \mathrm{sech}\, u, \quad c_0 = 0,$$

$$f_1 = K_1 \tanh u, \quad c_1 = 1,$$

where K_i are arbitrary integration constants.

Proof. Performing the change $z = \cosh^2 u$, we rewrite the conditions of the theorem in the form

$$2z(z-1) f_{zz} + (2z-1) f_z - \left(\frac{\tilde{c}}{2} - \frac{1}{z} \right) f = 0, \tag{A3}$$

$$f(1) = 0, \quad f(+\infty) = O(1), \tag{A4}$$

where $\tilde{c} = 1 - c$. The general integral of Eq. (A3) can be expressed in terms of the hypergeometric functions

$$f = \frac{C_1}{\sqrt{z}} F\left(\frac{-1-\sqrt{\tilde{c}}}{2}, \frac{-1+\sqrt{\tilde{c}}}{2}, -\frac{1}{2}; z \right) + C_2 z F\left(1 - \frac{\sqrt{\tilde{c}}}{2}, \frac{1+\sqrt{\tilde{c}}}{2}, \frac{5}{2}; z \right).$$

Using the asymptotics of the hypergeometric functions in the neighborhood $z=1$, it is straightforward to derive that the first of the conditions (A4) will be satisfied if we suppose

$$\frac{1}{C_1} f^{(\mathrm{reg})} = \frac{1}{\sqrt{z}} F\left(-1 - \frac{\sqrt{\tilde{c}}}{2}, -1 + \frac{\sqrt{\tilde{c}}}{2}, -\frac{3}{2}; z \right) - C^{(\mathrm{reg})} z F\left(\frac{3-\sqrt{\tilde{c}}}{2}, \frac{3+\sqrt{\tilde{c}}}{2}, \frac{7}{2}; z \right), \tag{A5}$$

where

$$C^{(\mathrm{reg})}=\sqrt{\tilde{c}}(\tilde{c}-1)\tan\left(\frac{\pi\sqrt{\tilde{c}}}{2}\right).$$

Further, to specify the parameters at which this function satisfies the second condition (A4), we should consider the asymptotical behavior of $f^{(\mathrm{reg})}$ near infinity. We have

$$\frac{1}{C_1}f^{(\mathrm{reg})}(z\to\infty)=\frac{2\breve{\gamma}}{\pi^{3/2}}(-1)^{1+\sqrt{\tilde{c}}/2}\tan\left(\frac{\pi\sqrt{\tilde{c}}}{2}\right) \times\sin\left(\frac{\pi\sqrt{\tilde{c}}}{2}\right)z^{\sqrt{\tilde{c}}/2}[1+O(1/z)], \qquad (A6)$$

where

$$\breve{\gamma}=\Gamma(\sqrt{\tilde{c}})[i\sqrt{\tilde{c}}(\tilde{c}-1)\Gamma(-1/2-\sqrt{\tilde{c}}/2)\Gamma(\sqrt{\tilde{c}}/2) -8\,\Gamma(1-\sqrt{\tilde{c}}/2)\Gamma(3/2-\sqrt{\tilde{c}}/2)].$$

From this expression it can easily be seen that $f^{(\mathrm{reg})}$ diverges at infinity everywhere except perhaps the points

$$\tilde{c}=(2n)^2=0,4,16,\dots, \quad \text{and} \quad \tilde{c}=1,$$

which demand individual consideration. From Eq. (A3) we have

$$f_{\tilde{c}=0}=C_1\sqrt{1-\frac{1}{z}}+C_2\left[i-\sqrt{1-\frac{1}{z}}\arcsin\sqrt{z}\right],$$

$$f_{\tilde{c}=1}=\frac{C_1}{\sqrt{z}}+C_2\left[\sqrt{1-\frac{1}{z}}-i\frac{\arcsin\sqrt{z}}{\sqrt{z}}\right],$$

$$\tilde{f}_{\tilde{c}=4}=C_1\sqrt{1-\frac{1}{z}}(2z+1)+C_2 z,$$

$$f_{\tilde{c}=16}=C_1\sqrt{1-\frac{1}{z}}(24z^2-8z-1)+C_2 z(1-6z/5),$$

and so on. By induction it is clear that at $\tilde{c}\geqslant 4$ there are no C_i at which f would satisfy the requirements (A2).

[1] Such models are very popular in general relativity because there it is possible to take into account the quantum nature through the considering of features of some phenomenological ‘‘macroscopical’’ (quasi)matter and to work with collective degrees of freedom using simplest symmetries. For some historical review and modern examples, see, K. G. Zloshchastiev, Class. Quantum Grav. **16**, 1737 (1999); Gen. Relativ. Gravit. **31**, 1821 (1999); Int. J. Mod. Phys. D **8**, 165 (1999).

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