Vacuum domain walls in D dimensions: Local and global space-time structure

Mirjam Cvetič*

Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104-6396

Jing Wang†

Theory Division, Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510

(Received 30 December 1999; published 26 May 2000)

We study local and global gravitational effects of $(D-2)$-brane configurations (domain walls) in the vacuum of D-dimensional space-time. We focus on infinitely thin vacuum domain walls with arbitrary cosmological constants on either side of the wall. In the comoving frame of the wall we derive a general metric *ansatz*, consistent with the homogeneity and isotropy of the space-time intrinsic to the wall, and employ Israel's matching conditions at the wall. The space-time, intrinsic to the wall, is that of a $(D-1)$-dimensional Freedman-Lemaître-Robertson-Walker universe (with $k=-1,0,1$) which has a (local) description as either anti–de Sitter, Minkowski or de Sitter space-time. For each of these geometries, we provide a systematic classification of the local and global space-time structure transverse to the walls, for those with both positive and negative tension; they fall into different classes according to the values of their energy density relative to that of the extreme (supersymmetric) configurations. We find that in any dimension D, both local and global space-time structure for each class of domain walls is *universal*. We also comment on the phenomenological implications of these walls in the special case of $D=5$.

PACS number(s): 04.20.Jb, 11.27.+d, 98.80.Cq

I. INTRODUCTION

Recent months have witnessed a resurgence in the study of domain walls with asymptotically anti–de Sitter (AdS) space-times. This renewed interest is motivated both from the point of view of AdS conformal field theory (CFT) correspondence, providing new insights into the study of renormalization group equation (RGE) flows (see, e.g., [1–11] and references therein) as well as from the phenomenological perspective, providing a possible resolution to hierarchy problem in the context of a world on a domain wall in $D=5$ asymptotically AdS space-times (see, e.g., [12–20] and references therein). This is an exiting period when formal theoretical developments drive phenomenological implications and vice versa.

A prerequisite for addressing physics implications of such configurations is a detailed understanding of their space-time structure. Earlier work on the subject concentrated on the domain walls in $D=4$ space-time dimensions, where either side of the wall was to be interpreted as that of our four-dimensional Universe. The first example of static domain walls between vacua with different cosmological constants were found is [21] as supersymmetric [Bogomol'nyi-Prasad-Sommerfield- (BPS)-saturated or extreme] walls interpolating between supersymmetric vacua of $D=4$, $N=1$ supergravity vacua. In [22] a classification of the possible supersymmetric walls has been given, and the global structure of the space-times induced by these walls has been explored in [23,24]. A subsequent systematic investigation [25] of the space-times of domain walls separating regions of non-positive cosmological constant in Einstein's theory of

gravitation, revealed a more general class of domain wall configurations.[1] (For a review, see [26].)

The purpose of the present work is to generalize the results of the study of vacuum domain walls in $D=4$ [25] to $(D-2)$-brane configurations in D dimensions, with $D=5$ being of special interest to the physics implications for the four-dimensional domain wall world (as well as of theoretical interest for RGE flows of strongly coupled gauge theories in four dimensions).

Following the work of [25] we derive the local and global properties of the space-times induced by vacuum domain walls in D dimensions between vacua of arbitrary cosmological constant. We start with the *ansatz* that the gravitational field inherits the boost symmetry of the source, but we assume nothing about the topology of the $(D-1)$-dimensional space-times parallel to the surface of the domain wall. The space-time intrinsic to the wall, are Freedman–Lemaître-Robertson-Walker (FLRW) universes describing locally $(D-1)$-dimensional anti–de Sitter (AdS_{D-1}), Minkowski (M_{D-1}) or de Sitter (dS_{D-1}) space-time. For each of these space-times internal to the wall, the space-time transverse to the wall can be classified according to the values of cosmological constants $\Lambda_{1,2}$ on either side of the wall and their relationship to the energy density of the wall σ.

An important result of the analysis is that the space-times of domain walls have the same universal structure in all D; the space-time intrinsic to the wall is that of

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[1]Space-time properties of non-static domain walls in $D=5$ were recently addressed in [27–34] and references therein. As the results of this paper demonstrate, the space-time structure of domain walls in D dimensions (including $D=4$) is completely parallel.

*E-mail address: cvetic@cvetic.hep.upenn.edu

†E-mail address: jingw@fnal.gov

$(D-1)$-dimensional FLRW universe, and the metric coefficient, specifying the space-time transverse to the wall, has the same form with parameters depending on $\Lambda_i/[(D-1)(D-3)]$.

The paper is organized as follows. In Sec. II we derive the line-element ansatz, by working in the comoving frame of the wall in D dimensions. In Sec. III we classify the domain wall solutions according to their energy density and the values of cosmological constants on either side. We present the discussion of the result in Sec. IV, including the implications for their non-extreme generalizations. In particular, we discuss the examples in $D=5$, which have been recently studied intensively.

II. LOCAL PROPERTIES OF DOMAIN WALL SPACE-TIMES

In this section we present the metric ansatz for vacuum domain walls for Einstein gravity in D dimensions; these walls, created from a scalar field source, separate vacuum space-times of zero, positive, and negative cosmological constants. We study in detail only infinitely thin domain walls, by employing Israel's formalism [35] of singular hypersurfaces.

A. Metric ansatz

We solve D-dimensional Einstein's gravitational field equations in the co-moving frame of the domain wall, by assuming the following symmetry of the metric ansatz [25]:

The spatial part of the metric intrinsic to the wall is *homogeneous* and *isotropic*.

The space-time section transverse to the wall is *static*.

The directions parallel to the wall are *boost invariant* in the strong sense.

Homogeneity and isotropy reduce the metric part, intrinsic to the wall, to be the spatial part of a $(D-1)$-dimensional FLRW metric [36] of the form

$$(ds_\parallel)^2 = R^2[(1-kr^2)^{-1}dr^2 + r^2 d\Omega_{D-3}^2], \qquad (2.1)$$

where R is independent of the radial coordinate r and the angular coordinates ϕ_i ($i=1,\ldots,D-3$) specifying the line element $d\Omega_{D-3}^2$ of $(D-3)$-sphere S^{D-3}. The scalar curvature of this surface is equal to $2k/R^2$.

The sign of k determines the wall geometry. $k=0$ defines a planar wall, in which case the metric (2.1) can be written in Cartesian coordinates $(ds_\parallel)^2 = R^2(\Sigma_{i=1}^{D-2} dx_i^2)$. $k>0$ corresponds to a spherical wall—closed bubble—with r and ϕ_i's being compact coordinates; through a coordinate transformation $r=k^{-1/2}\sin\theta$ and rescaling of R, the line element can be written as $(ds_\parallel)^2 = R^2(d\theta^2 + \sin^2\theta d\Omega_{D-3}^2) = R^2 d\Omega_{D-2}^2$. $k<0$ corresponds to the negative curvature, non-compact Gauss-Bólyai-Lobachevski surface; introducing $r=(-k)^{-1/2}\sinh\varrho$, with $\varrho>0$, and rescaling R yields $(ds_\parallel)^2 = R^2(d\varrho^2 + \sinh^2\varrho\, d\Omega_{D-3}^2)$.

The condition that the two-dimensional space-time (t,z) transverse to the wall be *static* (as observed in the *rest frame* of the wall) implies the following form of the transverse part of the metric:

$$(ds_\perp)^2 = A(z)(dt^2 - dz^2), \qquad (2.2)$$

where z is the spatial direction transverse to the wall, t is the *proper* time and $A(z)>0$. With $ds^2 \equiv (ds_\perp)^2 - (ds_\parallel)^2$ the metric takes the form:

$$ds^2 = A(z)(dt^2 - dz^2) - R(t,z)^2[(1-kr^2)^{-1}dr^2 \\ + r^2 d\Omega_{D-3}^2], \qquad (2.3)$$

in which $z\in\langle-\infty,\infty\rangle$, and the other coordinates are those of the FLRW cosmological model [36].

A straightforward calculation of the nonzero components of the Einstein tensor $G^\mu{}_\nu = \mathcal{R}^\mu{}_\nu - \frac{1}{2}\mathcal{R}^\alpha{}_\alpha g^\mu{}_\nu$ yields the result

$$G^t{}_t = \frac{D-2}{A}\left[-\frac{R''}{R}+\frac{HR'}{2R}\right] + \frac{(D-3)(D-2)}{2R^2}\left[k+\frac{\dot{R}^2-R'^2}{A}\right]$$

$$G^z{}_t = \frac{D-2}{A}\left[\frac{\dot{R}'}{R}-\frac{H\dot{R}}{2R}\right]$$

$$G^z{}_z = \frac{D-2}{A}\left[\frac{\ddot{R}}{R}-\frac{HR'}{2R}\right] + \frac{(D-3)(D-2)}{2R^2}\left[k+\frac{\dot{R}^2-R'^2}{A}\right]$$

$$G^r{}_r = G^{\phi_i}{}_{\phi_i} = \frac{3-D}{A}\left[\frac{R''}{R}-\frac{\ddot{R}}{R}\right] - \frac{H'}{2A} - \frac{(D-3)(D-4)}{2R^2}$$

$$\times\left[k+\frac{\dot{R}^2-R'^2}{A}\right] \qquad (2.4)$$

where $i=1\cdots D-3$, $\dot{R}\equiv\partial_t R(t,z)$, $R'\equiv\partial_z R(t,z)$ and $H\equiv\partial_z\ln A(z)$.

The symmetry of the matter source (as specified in the rest frame of the wall) implies that the energy-momentum tensor is static, with $T^z{}_t=0$ [no energy flow in the (t,z) plane] and $T^\mu{}_\mu = g_\mu(z)$ [$\mu=(t,z,r,\phi_i)$], where $g_\mu(z)$ are functions of z, only. Then Einstein's equations $G^\mu{}_\nu = \kappa_D T^\mu{}_\nu$ imply the following constraints on $R(t,z)$ and $A(z)$:[2]

$$\frac{\dot{R}'}{R}-\frac{H\dot{R}}{2R}=0, \qquad (2.5)$$

$$\frac{R''}{R}-\frac{HR'}{2R}=f_2(z),$$

$$\frac{R''}{R}-\frac{\ddot{R}}{R}=f_3(z),$$

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[2] κ_D is defined as $\kappa_D \equiv 8\pi G_D$, where G_D is Newton's constant in D dimensions. We define the Lagrangian density as $(1/\kappa_D)(-R/2 + \mathcal{L}_{matter})$.

$$\frac{k}{R^2}+\frac{\dot{R}^2-R'^2}{AR^2}=f_4(z),$$

where $f_{1,2,3,4}(z)$ are arbitrary functions of z.

The static metric ansatz $\dot{R}=0$ automatically satisfies Eqs. (2.5). For non-static metric, time integration of the first equation in Eqs. (2.5) yields the condition

$$R'=\frac{HR}{2}+f_1(z) \qquad (2.6)$$

with $f_1(z)$ an arbitrary function of z. Adding the z derivative of Eq. (2.6) to the second equation in Eqs. (2.5) yields a condition $f_1(z)'=[-H'/2+f_2(z)]R$, which holds for any t only if $f_1(z)=f_0$ is a constant and $f_2(z)=H'/2$.

The assumption of boost invariance along the surfaces of constant z implies that $f_0=0$ and, as a consequence, Eq. (2.6) is solved by $R^2(r,t)=A(z)S^2(t)$. [In the static case, i.e. $\dot{R}=0$, the same symmetry constraint also implies $R^2(z) \propto A(z)$.[3]]

Therefore, the metric ansatz takes the form

$$ds^2=A(z)\{dt^2-dz^2-S^2(t)[(1-kr^2)^{-1}dr^2+r^2d\Omega^2_{D-3}]\}, \qquad (2.7)$$

which has a universal form for any D dimensions, and thus the same structure as the one obtained in $D=4$ [25]. This result is due to the fact that in the wall comoving frame the homogeneity and isotropy of space-time internal to the wall severely restrict the form of the Einstein tensor to be that of Eq. (2.4); the static space-time transverse to the wall and the boost invariance along the wall further fixes the metric to be of the universal, D-independent form (2.7).

In the following subsection we solve the Einstein equations for $A(z)$ and $S(t)$, which specify the possible space-time structure away from the domain wall. We then employ the infinitely thin wall approximation [35] in order to determine the energy density of the wall in terms of the parameters in the metric.

B. Local space-time solutions

We now solve Einstein's equations for $S(t)$ and $A(z)$ of the metric ansatz (2.7). We consider thin domain walls interpolating between two maximally symmetric vacua of zero, positive, or negative cosmological constant.[4]

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[3]Namely, we assume that the gravitational field inherits the symmetry of the source; the directions parallel to the wall are boost invariant in the strong sense and thick walls will have the same $S(t)$ as found in the thin wall approximation and will asymptotically approach the thin wall result for $A(z)$. (See [25] for a more detailed discussion.)

[4]Maximally symmetric vacuum solutions are well known [36]; nevertheless, we summarize the results here for the comoving coordinate system of the wall. Note also that Israel's matching conditions are easily satisfied in this frame.

Plugging the metric ansatz (2.7), i.e. $R^2(t,z)=A(z)S^2(t)$, into the Einstein tensor (2.4), yields the following equations for $S(t)$ and $A(z)$:

$$\frac{\ddot{S}}{S}=q_0=\frac{\dot{S}^2}{S^2}+\frac{k}{S^2}, \qquad (2.8)$$

$$\frac{1}{4}\left(\frac{A'}{A}\right)^2=q_0-\frac{\Lambda}{(D-3)(D-1)}A, \qquad (2.9)$$

where we have assumed that away from the wall, the energy momentum tensor is given by $T^\mu_{\ \nu}=\Lambda\,\delta^\mu_{\ \nu}$ with Λ the cosmological constant on either side of the wall. q_0 is an integration constant satisfying the consistency constraint $\Lambda A(z) \leq (D-3)(D-1)q_0$.

Since the equation for $S(t)$ is independent of dimensionality, the space-time intrinsic to the wall is universal. Equation (2.9) for $A(z)$, i.e. the metric coefficient specifying the space-time transverse to the wall, is also of the same form as that obtained in $D=4$, except for the D-dependent coefficient in front of the cosmological constant. We choose to parametrize the cosmological constant as

$$\Lambda\equiv\pm(D-3)(D-1)\alpha^2\equiv\pm\Delta\,\alpha^2. \qquad (2.10)$$

Thus, in terms of the parameter α, Eq. (2.9) has a universal, D-independent form and thus yields the same solutions as the ones obtained in $D=4$ [25].

The fact that the domain wall space-time structure is universal was anticipated in [26]. Nevertheless, the result is intriguing; *both the local as well as the global space-time properties of domain walls [(D-2)-configurations] in diverse (D) dimensions are universal.* The study of the local and global domain wall space-times in $D=4$ [25] can therefore be extended in a straightforward way to the study of ($D-2$) configurations in D dimensions. (The special case of $D=5$ recently attracted much attention due to its phenomenological implications.)

For the sake of completeness, we shall now write down the explicit results for $S(t)$ and $A(z)$ [25]. We parametrize the curvature constant of the space-time internal to the wall (see discussion at the beginning of the previous subsection) as $k\in\{-\beta^2,0,\beta^2\}$. The solutions of $S(t)$ from Eq. (2.8) can be classified according to the sign of q_0:

$$q_0=-\beta^2:\quad S_-=\cos(\beta t)\quad k=-\beta^2, \qquad (2.11)$$

$$q_0=0:\quad S_0=\begin{cases} \beta t & k=-\beta^2, \\ 1 & k=0, \end{cases} \qquad (2.12)$$

$$q_0=\beta^2:\quad S_+=\begin{cases} \sinh(\beta t) & k=-\beta^2, \\ e^{\beta t} & k=0, \\ \cosh(\beta t) & k=\beta^2, \end{cases} \qquad (2.13)$$

where the subscripts $(-,0,+)$ refer to the respective sign of

q_0. [As a result of the time-translation and time-reversal invariance, the integration constants are adjusted to yield the canonical form for $S(t)$, and without loss of generality, $\beta \geq 0$ is chosen.] The form of the solutions $S_{-,0,+}$ [Eqs. (2.11)–(2.13)] implies that the space-time intrinsic to the wall is that of AdS_{D-1}, M_{D-1} and dS_{D-1}, respectively.

Solutions of Eq. (2.9), classified according to the sign of q_0, yield the following form of $A(z)$:

$$q_0 = -\beta^2: \quad A_- = \beta^2[\alpha\cos(\beta z + \vartheta)]^{-2} \quad \Lambda = -\Delta\alpha^2 \leq -\Delta\beta^2, \tag{2.14}$$

$$q_0 = 0: \quad A_0 = \begin{cases} (\alpha z \pm 1)^{-2} & \Lambda = -\Delta\alpha^2, \\ 1 & \Lambda = 0, \end{cases} \tag{2.15}$$

$$q_0 = \beta^2: \quad A_+ = \begin{cases} \beta^2[\alpha\sinh(\beta z - \beta z')]^{-2} & \Lambda = -\Delta\alpha^2, \\ e^{\pm 2\beta z} & \Lambda = 0, \\ \beta^2[\alpha\cosh(\beta z - \beta z'')]^{-2} & \Lambda = \Delta\alpha^2 \leq \Delta\beta^2. \end{cases} \tag{2.16}$$

Again, the subscripts $(-,0,+)$ for $A(z)$ refer to the sign of q_0. Without loss of generality we have moved the origin of the z axis to the position of the wall ($z_0 = 0$). The three integration constants ϑ, z', and z'' are determined by the requirement that $A(z_0 = 0) = 1$ which yields

$$\vartheta_\pm = \pm\arccos(\beta/\alpha), \tag{2.17}$$

$$\beta z'_\pm = \frac{1}{2}\ln\left[1 + \frac{2\beta^2}{\alpha^2} \pm \frac{2\beta}{\alpha^2}(\alpha^2 + \beta^2)^{1/2}\right], \tag{2.18}$$

$$\beta z''_\pm = \frac{1}{2}\ln\left[-1 + \frac{2\beta^2}{\alpha^2} \mp \frac{2\beta}{\alpha^2}(\beta^2 - \alpha^2)^{1/2}\right]. \tag{2.19}$$

The constants $\beta z'$ and $\beta z''$ satisfy $e^{2\beta(z'_+ + z'_-)} = e^{2\beta(z''_+ + z''_-)} = 1$ and $e^{2\beta z''_-} > 1 > e^{2\beta z''_+}$ and $e^{2\beta z'_-} \leq 1 \leq e^{2\beta z'_+}$ where, in the last case, the equality is obtained when $\beta = 0$—the extreme limit—is taken. [As one can see from Eq. (2.19), there is no extreme limit ($\beta \to 0$) in the de Sitter case.]

C. Israel's matching conditions and surface energy density of the domain walls

In the thin wall approximation, the energy-momentum tensor and the Einstein tensor have δ-function singularities at the wall. In Israel's formalism [35] for singular layers the metric tensor has a discontinuity in its first order derivatives in the direction transverse to the wall, and the Lanczos tensor $S^i{}_j$, which is the surface energy-momentum tensor of the wall located at z_0, is related to the discontinuity of the extrinsic curvature $K^i{}_j$ in the following way:

$$\kappa_D S^i{}_j = -[K^i{}_j]^- + \delta^i{}_j[K]^-. \tag{2.20}$$

The square brackets $[\ ]^-$ are defined as $[\Omega]^- \equiv \lim_{\epsilon \to 0}[\Omega(z_0 + \epsilon) - \Omega(z_0 - \epsilon)]$ and $x^i \in \{t, r, \phi_i\}$ ($i = 1, \ldots, D-3$) are the coordinates parallel to the wall. $K^i{}_j$ is given by the covariant derivative of the space-like unit normal n^μ of the wall's hyper-space-time. In a normalized

coordinate system where $g_{\hat{z}\hat{z}} = -1$, the extrinsic curvature can be written as $K_{ij} = (-\zeta/2)g_{ij,\hat{z}}$ where $\zeta = \pm 1$ signifies the inherent sign ambiguity of the unit normal n^μ. Hence, in a comoving coordinate frame, where we have chosen $A(z_0 = 0) = 1$, the Lanczos tensor can be written as

$$\kappa_D S^i{}_j = -\delta^i{}_j[\zeta H]^-_{z=0}. \tag{2.21}$$

The energy density of the wall $\sigma \equiv S^t{}_t$, which is equal to the wall's tension $\tau \equiv S^r{}_r = S^{\phi_i}{}_{\phi_i}$, is given by

$$\kappa_D \sigma = -[\zeta H]^-_{z=0}. \tag{2.22}$$

Applying Israel's formalism to the local vacuum solutions specified by Eqs. (2.11)–(2.16), we find the surface energy density σ and tension $\tau = \sigma$ to be of the form

$$\kappa_D \sigma = 2\zeta_1 h_1\left(q_0 - \frac{\Lambda_1}{(D-3)(D-1)}\right)^{1/2}$$
$$- 2\zeta_2 h_2\left(q_0 - \frac{\Lambda_2}{(D-3)(D-1)}\right)^{1/2}, \tag{2.23}$$

where the first (second) contribution to the energy density comes from $z < 0$ ($z > 0$) side of the wall. We choose, without loss of generality, to orient the z coordinate so that the vacuum of lowest energy will be placed on the $z < 0$ side (i.e. $\Lambda_1 < \Lambda_2$).

In Eq. (2.23), in addition to the ambiguity in the sign of the unit normal n^μ ($\zeta_i = \pm 1$), there is another sign ambiguity, $h_i = \pm 1$, in taking the square root of Eq. (2.9). A kink-like solution for the wall, i.e. the scalar interpolating between the extrema of the potential, implies $\zeta_1 = \zeta_2 = 1$. In addition, we take $h_i = 1$ if $A_i(z)$ is an increasing function of z and $h_i = -1$ if $A_i(z)$ is decreasing.

The domain wall solutions fall into two categories: those with positive energy density (corresponding to the infinitely thin wall limit of a kink solution, interpolating between the *minima* of the potential) and those with negative energy density which correspond to choosing the reversed values of h_i.

Examples of negative tension walls are encountered in gauged supergravity theories (see, e.g., [10]) as a consequence of a kink solution interpolating between *maxima* of the potential.

For a given value of q_0 the walls can be classified according to the choice of h_i into the following classes [the notations are chosen to be compatible with earlier classifications [10,22] of extreme wall ($q_0=0$) solutions]:

Type I walls: a special case of Type II walls with $\Lambda_2 = 0$, and $q_0 = 0$.

Type II walls: positive-tension walls with $h_1 = -h_2 = 1$.

Type III walls: positive-tension walls with $h_1 = h_2 = 1$.

Type III$'$ walls: negative-tension walls with $h_1 = h_2 = -1$, and $\sigma_{III'} = -\sigma_{III}$.

Type IV walls: negative-tension walls with $h_1 = -h_2 = -1$, and $\sigma_{IV} = -\sigma_{II}$.

Type V walls: a special case of type IV walls with $\Lambda_2 = 0$, $q_0 = 0$, and $\sigma_V = -\sigma_I$.

The global and local space-times of positive tension walls with $q_0 = 0$ (and $S_0 = 1$) as well as $q_0 > 0$ [and $S_+ = \cosh(\beta t)$] were extensively studied in [25]. In [25], the $q_0 < 0$ [$S(t) = \cos(\beta t)$] examples were not further studied, in part due to the geodesic incompleteness of the space-time description of the AdS$_2$ space-time transverse to the wall. Nevertheless, these are proper local solutions deserving further study (see also [27]). On the other hand the negative tension walls are of interest in the study of AdS-CFT correspondence and thus deserve further investigation.

In the following section we provide a systematic classification of the (local and global) space-time structure of the possible domain wall solutions.

III. CLASSIFICATION OF THE DOMAIN WALL SOLUTIONS

We shall classify the solutions according to the values of the parameter q_0. The metric, intrinsic to the wall and specified by $S(t)$, is locally related to standard coordinates of M$_{D-1}$, AdS$_{D-1}$, or dS$_{D-1}$ space-time for $q_0 = 0$, $q_0 < 0$ or $q_0 > 0$, respectively. Within each class we then discuss the space-time structure transverse to the wall as determined by the metric conformal factor $A(z)$; its structure is governed by the energy density (2.23) and its relationship to the cosmological constants on either side of the wall.

A. Walls with ($q_0 = 0$): Extreme walls

The $q_0 = 0$ solutions, known as extreme domain walls [23], exist for $\Lambda_{1,2} \leq 0$. [The cosmological constant is defined as $\Lambda_{1,2} \equiv -(D-3)(D-1)\alpha_{1,2}^2$.]

Since the wall is homogeneous, isotropic and boost invariant, the spatial curvature of constant z sections is not unambiguously defined; there is no preferred frame in the $(D-1)$-dimensional space-time of the wall. The two S_0 solutions (2.12)—the Milne type solution with $S = \beta t$ and $k = -\beta^2$ and the inertial Minkowski solution with $S = 1$ and $k = 0$—both describe M$_{D-1}$ space-time. The two solutions are related by a coordinate transformation [37] that does not involve the transverse coordinate z and therefore describe

locally equivalent space-times.

The $S = 1$ solution is the *only* wall which represents a noncompact *planar* ($k = 0$) and static wall. These walls could be realized as supersymmetric bosonic configurations. Examples of supersymmetric domain walls in $D = 4$, $N = 1$ supergravity coupled to chiral matter superfields were first found and studied in [21], and recent examples within $D = 5$ gauged supergravities were given in [10].

The physically distinct solutions [with $S(t) = 1$] correspond to two sets of solutions for $A(z)$ in Eq. (2.15), with the asymptotic AdS$_D$ and M$_D$ space-times described by *horospherical* and Cartesian coordinates, respectively.

Positive energy solutions can be classified as the following three types [22], according to the relationship of the energy density of the wall σ to $\alpha_{1,2}$:

Type I: planar walls with $\kappa_D \sigma_{ext,I} = 2\alpha_1$, interpolate between M$_D$ and AdS$_D$ where on the latter side the metric conformal factor $A(z)$ decreases and reaches Cauchy horizon at $z \to -\infty$. These walls saturate a D-dimensional analogue of the Coleman-deLuccia [38] bound.

Type II: planar walls with $\kappa_D \sigma_{ext,II} = 2(\alpha_1 + \alpha_2)$ interpolate between two AdS$_D$ regions, in which $A(z)$ decreases (repulsive gravity) away from either side of the wall. [The special case with a Z_2 symmetry ($\alpha_1 = \alpha_2$) in $D = 5$ gives rise to the Randall-Sundrum scenario with one positive tension brane [13].] $z = \pm\infty$ correspond to the Cauchy AdS horizons. The geodesic extensions were studied extensively in [23–25] and bear striking similarities to the global space-times of extreme charged black holes.

Type III: planar walls with $\kappa_D \sigma_{ext,III} = 2(\alpha_1 - \alpha_2)$ interpolate between two AdS$_D$ spaces with different cosmological constants; the conformal factor goes to infinity on $z > 0$ side of the wall, while decreases on the other side. The singularity in $A(z)$ at a finite value of z represents the time-like boundary of the AdS$_D$ space-time. Again $z \to -\infty$ corresponds to the Cauchy horizon.

On the other hand, the walls with negative energy density fall into the following classes [10]:

Type III$'$: planar walls with $\sigma_{ext,III'} = -2(\alpha_1 - \alpha_2)$ have the space-time structure that is a mirror image of that of type III walls; the conformal factor goes to infinity on $z < 0$ side of the wall, on $z > 0$ side the conformal factor decreases, the AdS Cauchy horizon is at $z \to \infty$.

Type IV: planar walls with $\sigma_{ext,IV} = -2(\alpha_1 + \alpha_2)$ interpolate between two AdS$_D$ regions, in which $A(z)$ increases on either side of the wall, reaching the AdS boundaries at a finite value of z on either side. [Those are typical wall solutions encountered in gauged supergravity theories (see, e.g., [10]), and are of interest in the study of RGE flows in the context of the AdS-CFT correspondence.]

Type V: planar walls with $\sigma_{ext,V} = -2\alpha_1$, interpolating between M$_D$ and AdS$_D$; on the AdS side $A(z)$ increases away from the wall, approaching the boundary of the AdS at a finite value of z.

The behavior of the conformal factor $A(z)$ for each class of solutions can be easily seen from specific examples shown in Fig. 1. Type I–V, type II–IV and type III and III$'$ can be
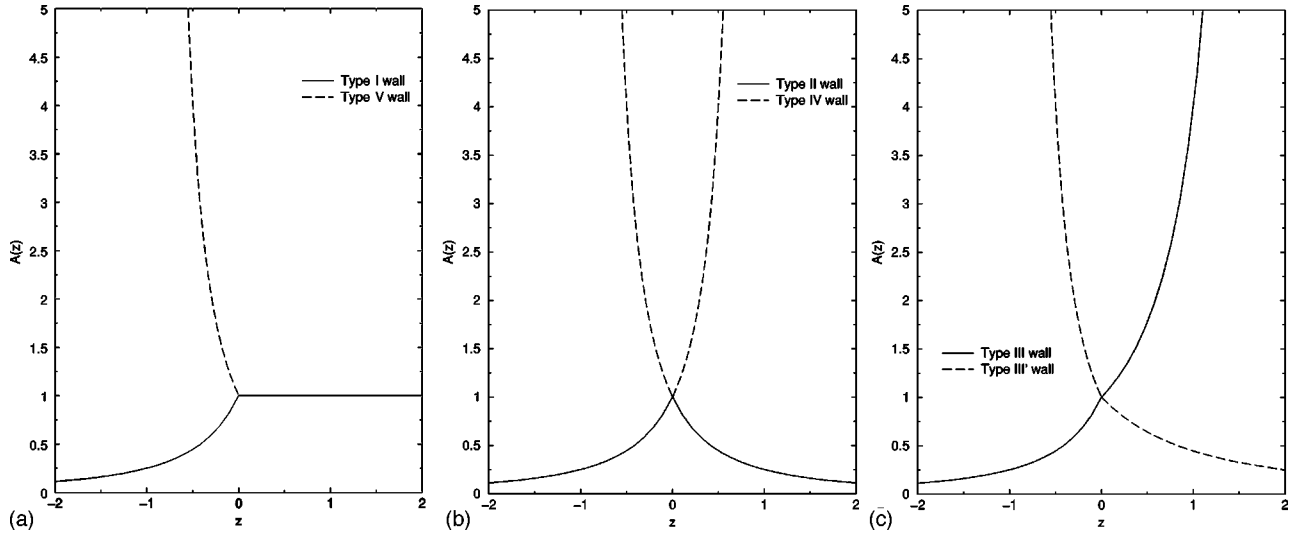
FIG. 1. The metric coefficients $A(z)$ are plotted as a function of transverse coordinate z, for dual pairs of extreme ($q_0=0$) domain walls. (a) denotes type-I–type-V pairs with the cosmological constant parameters $\alpha_1=1$ and $\alpha_2=0$, (b) shows type-II–type-IV pairs with $\alpha_1=\alpha_2=1$ and (c) shows type-III–type-III$'$ pairs with $\alpha_1=1$ and $\alpha_2=1/2$.

viewed as ''dual.'' Namely, the energy densities of these walls have opposite signs and the space-time patches of the AdS_D are complementary; i.e., AdS Cauchy horizons in one case are replaced by the boundaries of AdS space-times in another and vice versa. Note that in this sense the extreme type II walls, which provide a realization of the Randall-Sundrum scenario in $D=5$, and type IV walls, which are generically encountered in the study of the AdS-CFT correspondence, provide the *complementary* domains of the AdS space-time. We also note that within field-theoretic framework, such as gauged supergravity theories, a realization of (finite) negative tension domain walls and the issues of their stability require further study.

B. Walls with $q_0>0$

$q_0=\beta^2$ solutions with the form of S_+ in Eq. (2.13) describe dS_{D-1} parallel to the wall. The topology of dS_{D-1} is $\mathbf{R}$(time)$\times\mathbf{S}^{D-2}$(space). dS_{D-1} represents a hyperboloid embedded in a flat D-dimensional Minkowski space-time [24], and the three possible spatial curvatures ($k=-\beta^2,0,\beta^2$) correspond to three different choices of constant time slices of this hyperboloid. However, only the positive curvature solution with $S(t)=\cosh(\beta t)$ yields the complete covering of dS_{D-1} [24,25]; we will mainly focus on this class of solutions, which have a topology of the ''*expanding bubbles.*'' A possible way to create such expanding bubbles is via instantons of Euclidean gravity.

The walls can be classified according to their energy density (2.23) into the following classes:

Type II walls with

$$\kappa_D\sigma_{non,II}=2(\pm\alpha_1^2+\beta^2)^{1/2}+2(\pm\alpha_2^2+\beta^2)^{1/2}\geqslant\kappa_D\sigma_{ext,II}$$

(3.1)

are non-extreme; i.e., their energy density is above that of their extreme counterparts. Here $\Lambda_i\equiv\mp(D-3)(D-1)\alpha_i^2$.

Note that the domain walls in this class involve positive, zero and negative cosmological constants, and the minus sign in front of α_i^2 corresponds to a *positive* Λ_i. Furthermore, in the de Sitter case, $\alpha_i^2\leqslant\beta^2$ is required. There are 6 possible configurations of type II wall interpolating between different space-times, which are shown as 6 examples in Fig. 2.

Configurations with geodesically complete space-time internal to the wall $[S(t)=\cosh(\beta t)]$ describe *expanding bubbles with two insides* [25]. Namely, because the radius R_b of the curvature of concentric shells at distance z is proportional to $A^{1/2}(z)S(t)$, R_b, at fixed t, decreases as $A(z)$ decreases with increasing $|z|$, and therefore either side of the wall corresponds to an *inside* region of the bubble. In addition, since $S(t)$ increases with t, the bubble is *expanding* to an asymptotic observer on either side of the wall. (One possible origin of a creation of such configurations is via instantons of Euclidean gravity-quantum cosmology [39].)

Since $A(z)$ decreases with increasing $|z|$, gravity is repulsive on either side of the wall and $z\to\pm\infty$ corresponds to (cosmological) horizons. The geodesic extensions are studied in [25], and bear striking similarities to non-extreme charged black holes where time-like singularities are replaced by wall boundaries.

The walls with $\Lambda_{1,2}\leqslant0$ are generalizations of the extreme type II and type I walls with $\beta=0$ to $\beta>0$. The walls with Λ_1 or $\Lambda_2>0$ and $\Lambda_1=\Lambda_2=0$ do not have an extreme limit ($\beta\to0$). The latter class of M_D-M_D type II domain walls in $D=4$ [and $S(t)=e^{\beta t}$] was studied in [40]. dS-dS type II walls are unstable, since false vacuum decay walls (dS-dS Type III walls) are dynamically preferred [41,42], except for the case of $\Lambda_1=\Lambda_2>0$.

Type III walls with $q_0>0$ have an energy density lower than that of their extreme wall counterparts,

$$\kappa_D\sigma_{ultra,III}=2(\pm\alpha_1^2+\beta^2)^{1/2}-2(\pm\alpha_2^2+\beta^2)^{1/2}\leqslant\kappa_D\sigma_{ext,III},$$

(3.2)

and are referred to as ultra-extreme domain walls. The solutions with asymptotic dS_D space-times require $\alpha_i^2 \leq \beta^2$, and consequently they do not have an extreme limit. Specific examples of 5 possible configurations of the type III walls are shown in Fig. 3.

Configurations [with $S(t) = \cosh(\beta t)$] are *false vacuum decay bubbles* [43,42]. Namely, the radius $R_b \propto A^{1/2}(z)S(t)$ decreases (increases) for $z<0$ ($z>0$), and thus corresponds to the *inside* (*outside*) region the bubble. The solutions which only involve M_D or AdS_D are more like ordinary bubbles compared with the type II walls because as t increases the expanding bubble eventually sweeps out the space-time on the $z>0$ side.

On the other hand, on the de Sitter side of the wall, the metric function turns around at point z_{crit} and decreases beyond z_{crit}. Hence, beyond z_{crit}, the inside of the bubble becomes an outside. $z \to \infty$ corresponds to cosmological horizons that can be reached by test particles with energy larger than E_{crit}. In $D=4$ the non-negative cosmological constant domain walls of this type were extensively studied in [41,42].

$D=4$ false vacuum decay bubbles with non-positive cosmological constants were studied in [38,25]. The inside of the bubble ($z \leq 0$) has the same space-time structure (with cosmological horizons at $z \to -\infty$), just as the type II non-extreme walls ($q_0>0$). The outside ($z \geq 0$) of the bubble has
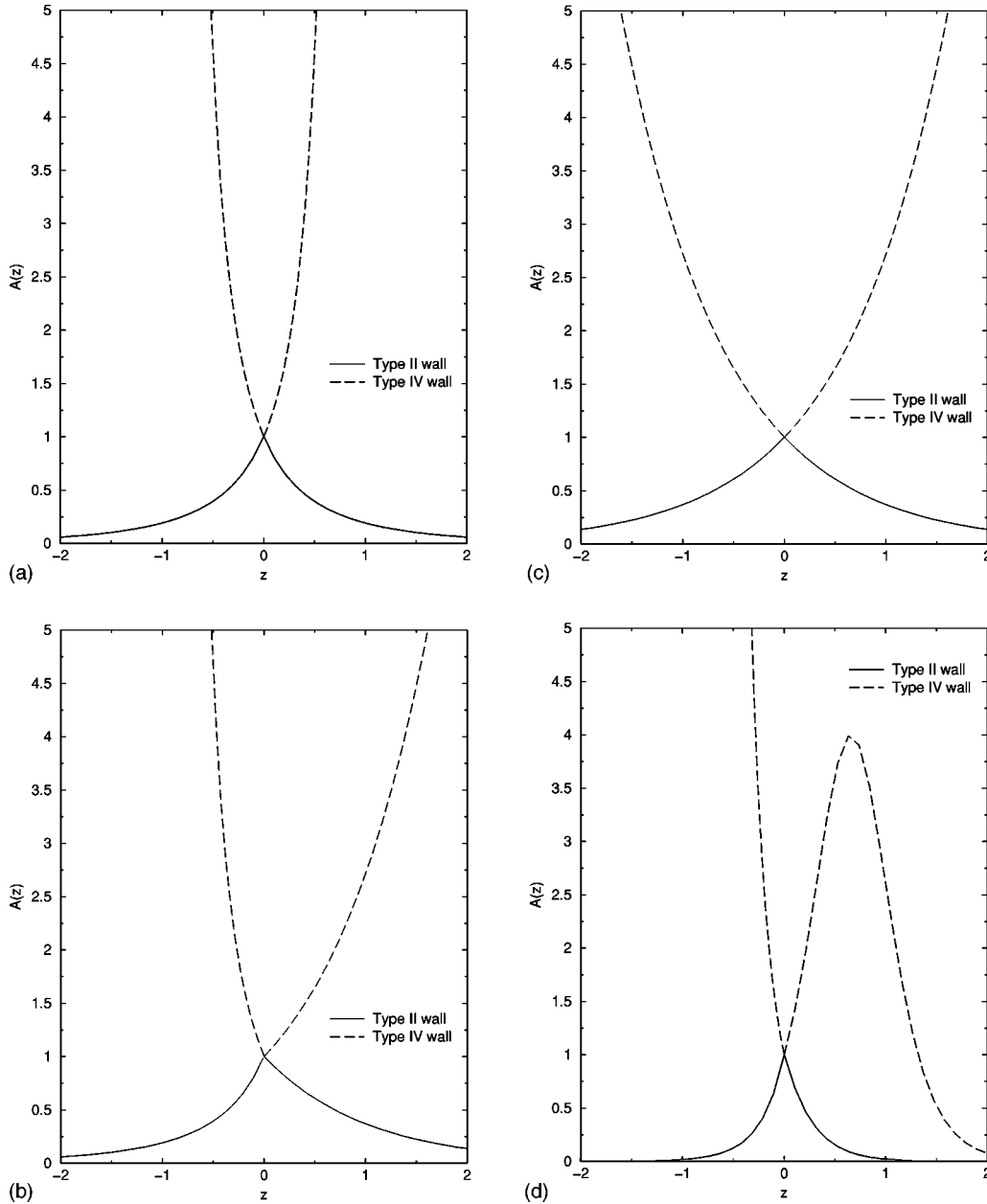

FIG. 2. The metric function $A(z)$ of type II and type IV walls in the case of ($q_0>0$). There are six configurations. (a) represents a AdS-AdS wall with $\alpha_1=\alpha_2=1$ and $\beta=1/2$, (b) represents a AdS-M wall with $\alpha_1=1$, $\alpha_2=0$ and $\beta=1/2$, (c) represents a M-M wall with $\beta=1/2$, (d) represents a AdS-dS wall with $\alpha_1=\alpha_2=1$ and $\beta=2$, (e) is a M-dS wall with $\alpha_1=0$, $\alpha_2=1$ and $\beta=2$ and (f) represents a dS-dS wall with $\alpha_1=\alpha_2=1$ and $\beta=2$.
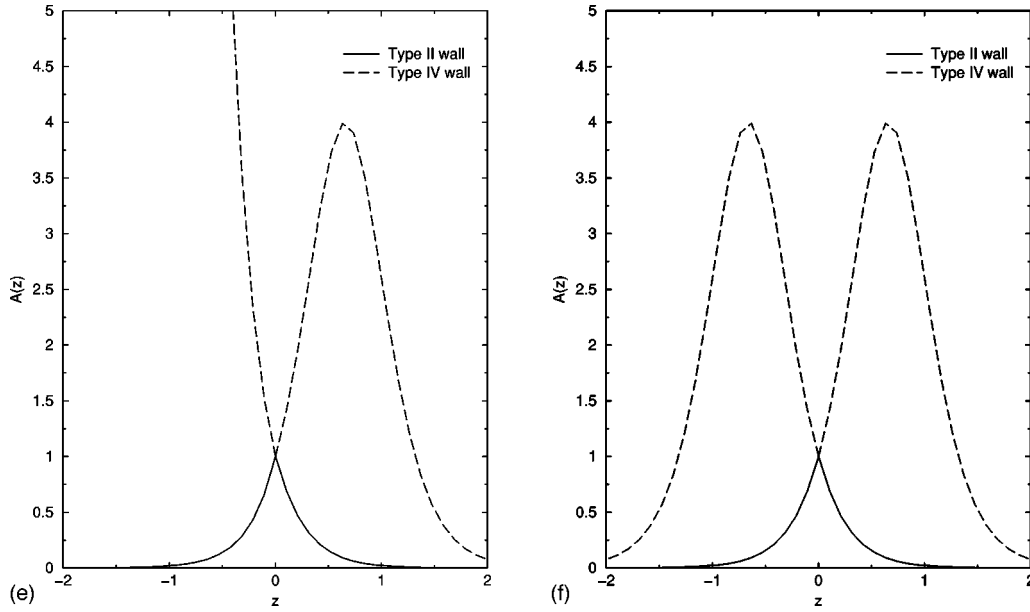
FIG. 2. (*Continued.*)

no horizons and on the M_D side of the wall $z \to \infty$ corresponds to the boundary of the space-time, while on the AdS side the affine boundary is at some finite value of z.

Type III$'$ have the energy density: $\sigma_{non,III'} = -\sigma_{ultra,III} \geq \sigma_{III,ext}$, which is above the corresponding extreme ($q_0 = 0$) counterparts. Their space-time is also *complementary* to that of the type III walls (see Fig. 3). These are ''false vacuum decay bubbles'' with the *larger* cosmological constant side ($z \geq 0$) sweeping out the vacuum with the smaller cosmological constant ($z \leq 0$), in most of the cases, except the dS-dS wall as shown in Fig. 3(e). In the latter case, the metric function $A(z)$ becomes a decreasing function of z for $z < z_{crit}$ such that the inside of the bubble becomes an outside, and $z \to -\infty$ is a cosmological horizon.

These configurations resemble the dynamics of an ''upside down world'' and an actual realization of such negative tension configurations within a field theoretical framework is needed.

Type IV walls with $\sigma_{ultra,IV} = -\sigma_{non,II} \leq \sigma_{ext,IV}$ are ultra-extreme negative tension ''*expanding bubbles with two outsides.*'' For non-positive cosmological constants this is the ''apocalypse world'' where on either side of the wall an asymptotic observer will be eventually hit by the bubble. Namely, the conformal factor $A(z)$ increases on either side of the wall reaching the boundary of the space-time, which is $z = \pm \infty$ for M_D or a finite value of z for AdS$_D$; thus either side corresponds to the outside of the wall. Since $S(t)$ grows with t, these are expanding bubbles which always hit an asymptotic observer.

C. Walls with $q_0 < 0$

For the case $q_0 = -\beta^2$ the solutions exist only for the negative-cosmological constant vacua that satisfy $\alpha_i^2 = -\Lambda_i/(D-1)(D-3) \geq \beta^2$. The space-time internal to the wall is described by a unique solution with Eqs. (2.11) and (2.14); the space-time intrinsic to the wall is AdS$_{D-1}$. Note

that with $S_-(t) = \cos(\beta t)$ [Eq. (2.11)], the region $-\pi/(2\beta) \leq t \leq \pi/(2\beta)$ describes only a patch of the AdS$_{D-1}$ space-time, and $t = \pm \pi/(2\beta)$ corresponds to an apparent coordinate singularity (see, e.g., [44]). Also, since the radius of the curvature transverse to the wall $R_b \propto |S(t)|$, the wall is expanding for $-\pi/(2\beta) \leq t \leq 0$ and then shrinking for $0 \leq t \leq \pi/(2\beta)$.

These walls could be viewed as a generalization of their extreme counterparts (with $q_0 = \beta = 0$) to $q_0 < 0$. They can also be viewed as an analytic continuation of AdS-AdS domain walls with $q_0 = \beta^2 > 0$ to imaginary β. In this sense the AdS-AdS walls with $q_0 > 0$ and $q_0 < 0$ are ''dual'' to each other, and extreme AdS-AdS walls with $q_0 = 0$ provide a dividing line between the two classes of solutions.

The walls with $q_0 < 0$ can be classified according to their energy densities relative to their extreme counterparts as:

Type II walls are ultra-extreme walls with $\kappa_D \sigma_{ultra,II} = 2(\alpha_1^2 - \beta^2)^{1/2} + 2(\alpha_2^2 - \beta^2)^{1/2} \leq \kappa_D \sigma_{ext,II}$. The space-time transverse to either side of the wall has repulsive gravity near the wall; i.e., $A(z)$ decreases away from the wall until the critical point $\beta z_{crit} + \vartheta_{\pm} \equiv 0$. Beyond z_{crit}, $A(z)$ increases with increasing $|z|$, reaching the affine boundary of the AdS space at $\beta z + \vartheta_{\pm} = \pi/2$. Therefore, these walls exhibit repulsive gravity only in the region close to the wall. Eventually, geodesics cross into a region of attractive gravity, with only null geodesics reaching the AdS boundary. Interestingly, there are no cosmological horizons. [Note the conformal factor $A(z)$ has a complementary behavior relative to that of $q_0 > 0$ dS-dS wall.] The behavior of $A(z)$ across the wall is shown in Fig. 4.

Type III walls are non-extreme walls with $\kappa_D \sigma_{non,III} = 2(\alpha_1^2 - \beta^2)^{1/2} - 2(\alpha_2^2 - \beta^2)^{1/2} \geq \sigma_{ext,III}$. On the $z < 0$ side the gravity is again first attractive and then repulsive, the point $\beta z + \vartheta_+ = -\pi/2$ corresponding to the AdS boundary. On the other hand, on the $z > 0$ side of the wall, gravity is attractive with the AdS boundary taking place at $\beta z + \vartheta_- = \pi/2$.

Type III$'$ walls are ultra-extreme walls with $\kappa_D \sigma_{ultra,III'}$ $= -2(\alpha_1^2 - \beta^2)^{1/2} + 2(\alpha_2^2 - \beta^2)^{1/2} \lesssim \kappa_D \sigma_{ext,III'}$. Its space-time structure in the transverse direction is a mirror image of the type III $q_0 < 0$ walls.

Type IV walls are ultra-extreme walls with $\kappa_D \sigma_{non,IV}$ $= -2(\alpha_1^2 - \beta^2)^{1/2} - 2(\alpha_2^2 - \beta^2)^{1/2} \gtrsim \kappa_D \sigma_{ext,IV}$, with attractive gravity on either-side of the wall until $\beta z + \vartheta_\pm = \pm \pi/2$, the AdS boundary.
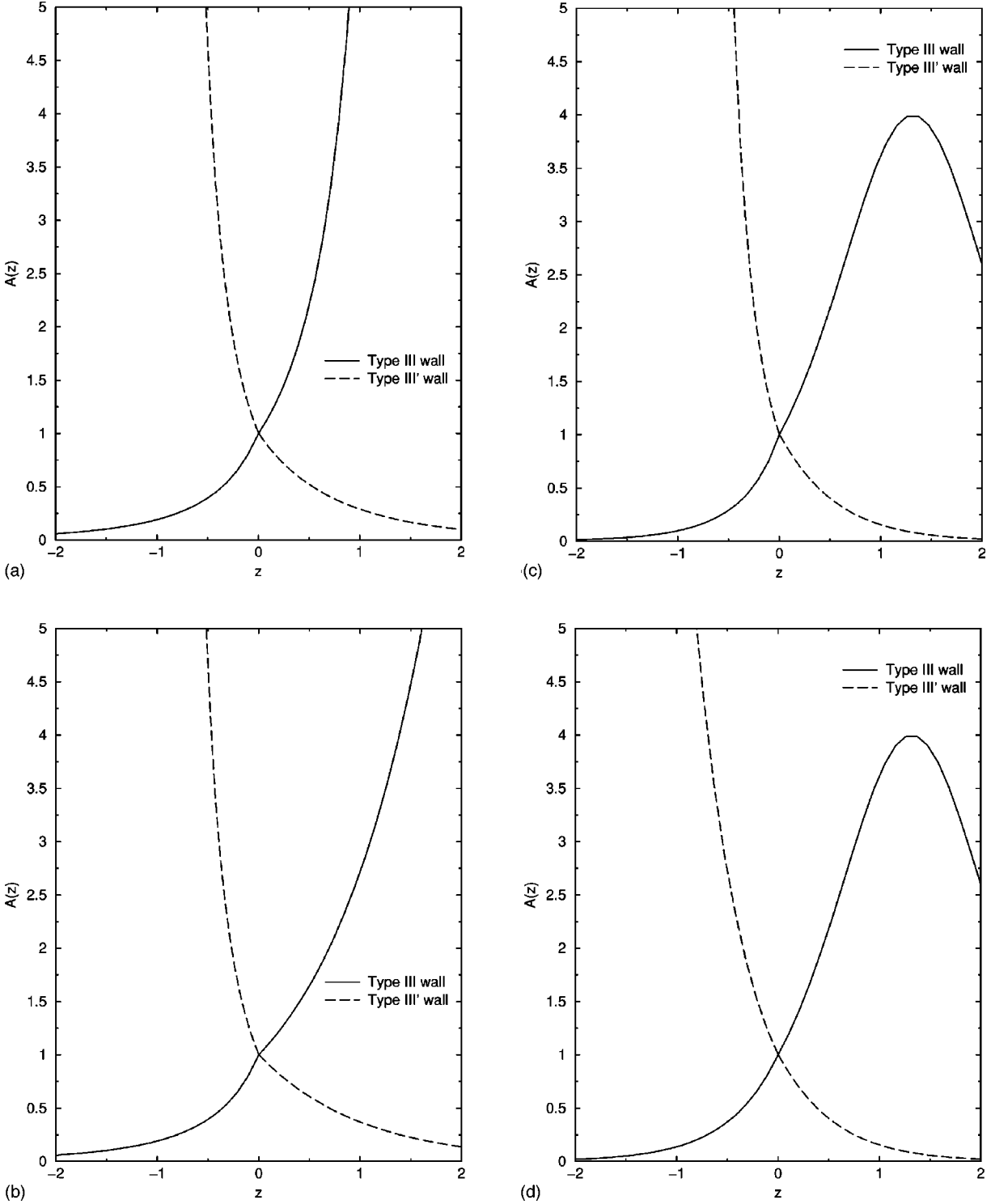
FIG. 3. The metric function $A(z)$ of type III and type III$'$ walls in the case of $(q_0 > 0)$. There are five configurations. (a) represents a AdS-AdS wall with $\alpha_1 = 1$, $\alpha_2 = 1/2$ and $\beta = 1/2$, (b) represents a AdS-M wall with $\alpha_1 = 1$, $\alpha_2 = 0$ and $\beta = 1/2$, (c) represents a AdS-dS wall with $\alpha_1 = 1$, $\alpha_2 = 1/2$ and $\beta = 1$, (d) is a M-dS wall with $\alpha_1 = 0$, $\alpha_2 = 1/2$ and $\beta = 1$ and (e) represents a dS-dS wall with $\alpha_1 = 1/4$, $\alpha_2 = 1/2$ and $\beta = 1$.
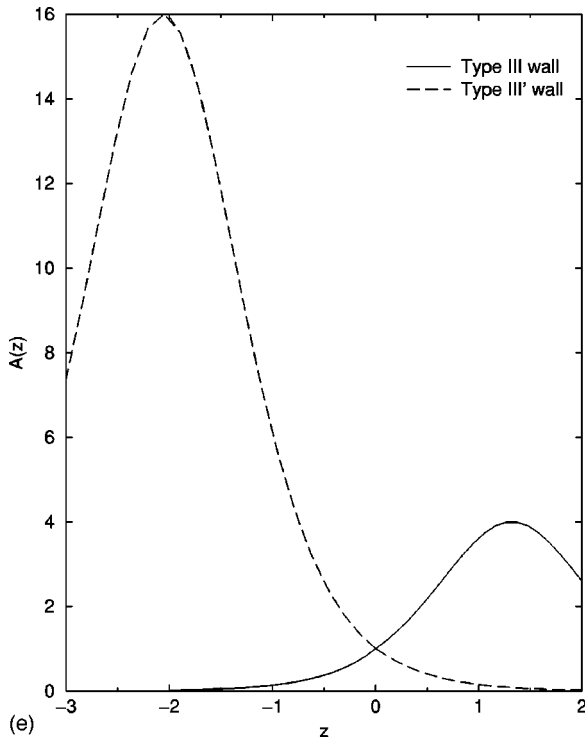
FIG. 3. (*Continued.*)

It would be very interesting to investigate further the global space-time properties of these configurations, the issues of their dynamic stability, as well as their field theoretic embedding.

IV. DISCUSSION

We have provided a systematic analysis of the space-time structure in the background of infinitely thin vacuum domain walls $[(D-2)$ configurations] in D-dimensional general relativity. We have shown that the homogeneity and isotropy of the space-time intrinsic to the wall strongly constrains the nature of the space-time (both intrinsic and transverse to the well) and that this space-time structure is universal for all D dimensions. The analysis also revealed an inherent connection between the global and local space-time structure of the wall and the value of the wall tension relative to the cosmological constants on either side of the wall.

The solutions fall into three classes according to the value of the ''non-extremality parameter'' q_0: $q_0=0$, $q_0>0$, and $q_0<0$. Within each class, depending on whether gravity is repulsive or attractive near either side of the wall, the walls can have positive tension solutions (type I, II, III walls) and negative tension solutions (type III$'$, IV, V walls) whose space-times transverse to the wall display complementary properties. In this sense type I–V, II–IV and III–III$'$ walls can be viewed as dual. (In particular, type II walls provide a set-up for Randall-Sundrum scenario in $D=5$ with repulsive gravity on either side of the wall.)

$q_0=0$ solutions are planar, static configurations. The precise tuning of their energy density to cancel the value of cosmological constant is ensured by supersymmetry. Such walls exist only for non-positive cosmological constants.
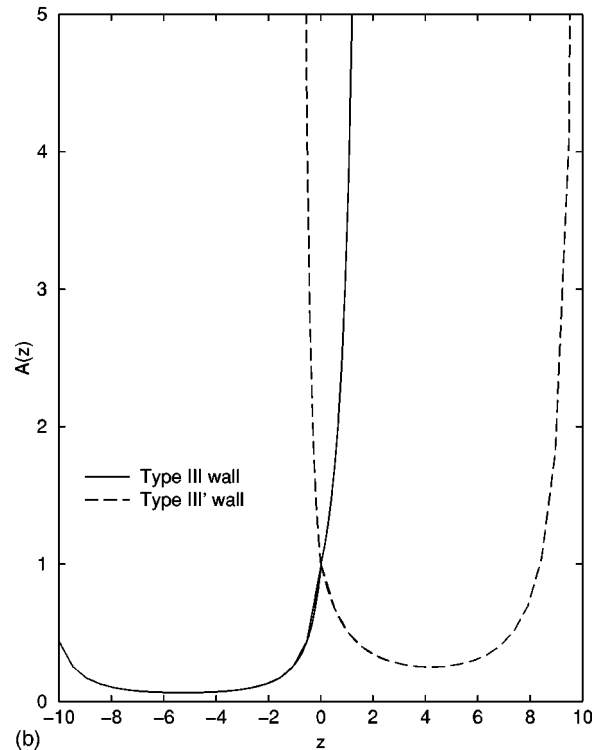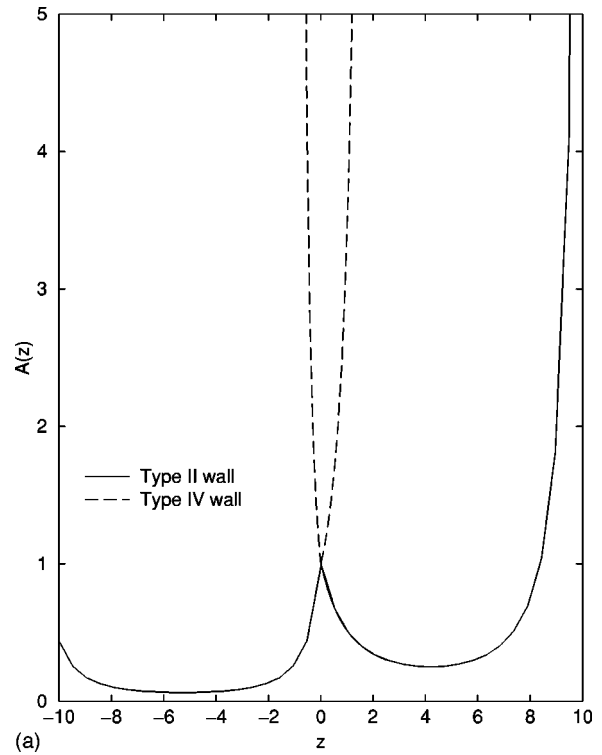

FIG. 4. The metric function $A(z)$ of type II–IV and type III–III$'$ walls in the case of $(q_0<0)$. They are both AdS-AdS walls with parameter $\alpha_1=1$, $\alpha_2=1/2$ and $\beta=1/4$.

Solutions with positive non-extremality parameter $(q_0=\beta^2>0)$ are expanding ''bubbles'' with the space-time internal to the wall corresponding to the expanding de Sitter (dS_{D-1}) FLRW universe. In particular, type II walls are ex-

panding bubbles with *two insides* and thus ''safe walls,'' type III and III′ walls are bubbles with *one inside and one outside* and which sweep out one side of the wall through ''false-vacuum'' decay, while type IV walls are expanding bubbles with *two outsides* and thus sweep out the vacuum on either side of the wall. These solutions exist both for positive and negative values of the cosmological constants.

Solutions with $q_0 = -\beta^2 < 0$ describe an anti–de Sitter (AdS$_{D-1}$) FLRW universe internal to the wall. However, the coordinates describe only a patch of the AdS space-time with the coordinate singularities at $t = \pm \pi/(2\beta)$. These walls have solutions only for the negative values of cosmological constants, and do not have cosmological horizons in directions transverse to the wall. (Their energy density is complementary to that of walls with $q_0 > 0$.) Further investigations of the geodesics extensions and their global structure is needed.

While the work provides a classification of vacuum domain wall space-times, we did not address in detail the dynamic issues such as their stability or the nature of their creation; nor did we elaborate on a field-theoretic embedding of such domain walls. Let us mention again that AdS-AdS type II walls with $q_0 > 0$ may be realized via quantum cosmology [39] and that type III walls are Euclidean bounce solutions of false-vacuum decay bubbles. As for field-theoretic realization, positive tension extreme walls could be realized as bosonic configurations in supersymmetric theories, corresponding to a kink solution interpolating between two supersymmetric minima. However, it is expected that negative tension walls are unstable due to the appearance of a ghost mode.[5] The gauged supergravity solutions tend to provide a framework for negative tension extreme wall solutions; i.e., the kink solution interpolates between *supersymmetric maxima*. This issue requires further study and it may have a resolution in the string theory context (see also, e.g., [11]).

The domain wall solutions studied in this paper can be stacked-up in the transverse z direction, thus provide a solution for an array of parallel walls. In particular, if D-dimensional space-time has possible vacuum solutions with cosmological constants $\Lambda_{1,2,\ldots n}$, then one can superimpose in z direction different types of domain walls interpolating between these vacua; this may yield interesting possibilities with phenomenological implications. However, the field-theoretic embedding of such multi-wall setups may be difficult; the multi-kink solutions are supposed to interpolate continuously between (isolated) extrema of the potential and the desired solution may not exist.

Let us consider specific examples with static (extreme) walls in $D=5$. Extreme type II walls provide a setup for Randall-Sundrum scenario with one positive tension brane in D=5 with repulsive gravity on either side of the wall [13]. The scenario with one positive tension brane and one nega-

tive tension brane [12], can be realized as a special (Z_2-symmetric) periodic array of type II and type IV extreme walls. On the other hand, the realization of such an array within field theory may be hard to realize and it should clearly involve more than one scalar. The example of [15] is a superposition of type II and type III walls. It could be realized with a scalar field that interpolates between two supersymmetric AdS minima with large enough potential barrier which yields a type II wall ($\kappa_D \sigma = \alpha_1 + \alpha_2$), and the third deeper minimum with a potential barrier insufficiently large yields a type III wall ($\kappa_D \sigma = \alpha_3 - \alpha_2$). (Note, however, that in spite of its positive energy-density the type III walls are inherently unstable; for a nonzero extremality parameter $q_0 > 0$ they turn into false vacuum decay bubbles, sweeping out the space on one side of this wall.) Another interesting possibility is a superposition of the two extreme type I walls [45], which can be realized via a single kink and anti-kink that interpolate between anti–de Sitter and Minkowski supersymmetric minima.

The non-static type II walls in AdS$_5$ both in the case of $q_0 > 0$ and $q_0 < 0$ are those studied in [27,32–34]. Another intriguing possibility may be a superposition of these solutions with $q_0 > 0$ (or $q_0 < 0$), where the conformal factors can again be matched from one wall to another. Note, however, that the non-static nature of these solutions may involve pathologies of space-times such as bubbles of false vacuum decays, and require further investigations.

We would like to conclude with a few remarks regarding the nature of the non-extremality parameter $q_0 = \beta^2 > 0$ within the cosmological context. (For related ideas implemented in the context of AdS-CFT correspondence, see [46].) Extreme domain walls ($q_0 = 0$) are static due to the ''miracle of supersymmetry.'' Thus, in a cosmological context, at zero temperature ($T=0$), domain walls between supersymmetric vacua remain static. On the other hand, at finite temperature $T>0$, supersymmetry is broken, and thus the domain walls are those with non-zero $q_0 = \beta^2$. Namely temperature corrections to the scalar potentials are $\propto T^2$, thus modifying the energy density of the wall $\sigma = \sigma_{ext} + \mathcal{O}(T^2)$. Clearly, since the leading corrections to σ are of $\mathcal{O}(q_0 = \beta^2)$, the result implies that $q_0 \propto T^2$ (or $\beta \propto T$). In particular, the static extreme type II [type III] domain wall (at $T=0$) becomes a non-extreme type II [type III] solution (at $T>0$) which is the expanding de Sitter FLRW bubble with two [one] insides. Thus the positive cosmological constant intrinsic to the wall as well as the rate of expansion of the bubble are proportional to $\beta \propto T$. Thus as the universe cools the expansion rate and the cosmological constant on the wall decrease.

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[5]We would like to thank R. Sundrum for a discussion on this point.

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