Dynamics and perturbations in assisted chaotic inflation

Nemanja Kaloper

Department of Physics, Stanford University, Stanford, California 94305

Andrew R. Liddle*

Astrophysics Group, The Blackett Laboratory, Imperial College, London SW7 2BZ, United Kingdom
and Isaac Newton Institute, University of Cambridge, Cambridge CB3 0EH, United Kingdom
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On compactification from higher dimensions, a single free massive scalar field gives rise to a set of effective four-dimensional scalar fields, each with a different mass. These can cooperate to drive a period of inflation known as assisted inflation. We analyze the dynamics of the simplest implementation of this idea, paying particular attention to the decoupling of fields from the slow-roll regime as inflation proceeds. Unlike normal models of inflation, the dynamics does not become independent of the initial conditions at late times. In particular, we estimate the density perturbations obtained, which retain a memory of the initial conditions even though a homogeneous, spatially flat universe is generated.

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I. INTRODUCTION

It was recently pointed out by Liddle *et al.* [1] that, under certain circumstances, it is possible for a set of scalar fields to act cooperatively to drive a period of cosmological inflation, even if none of the individual fields are capable of so doing. Such behavior is known as *assisted inflation*, and in Ref. [1] investigation was made of the case of multiple fields moving in exponential potentials with no interactions between them. For uncoupled fields this behavior is generic; the physics is simply that the fields feel the downward force from their own potential gradient, but the collective friction from the whole set of fields through the expansion rate H. It was however perhaps unexpected that assisted inflation solutions would be late-time attractors, confounding the expectation that only the field with the shallowest potential would dominate at late times. The dynamics of assisted inflation have subsequently been investigated by several authors [2–5]; one important additional point is that direct interactions between the fields tend to inhibit assisted behavior.

A particularly simple implementation of assisted dynamics arises in higher-dimensional theories with large compact internal spaces, as pointed out by Kanti and Olive [3,6]. There have been many studies of higher-dimensional theories with compact internal spaces with size larger than the fundamental scale, where the size could vary in the wide range between a millimeter [7], over TeV^{-1} [8], up to only a few orders of magnitude larger than the 4D Planck length [9,10]. Some aspects of universe as a domain wall have been considered before [11]. Most recently, a possibility that the extra dimensions may even be in some sense infinite has been considered [12–14]. Cosmology in these theories may be very different from the conventional 4D theory [15–17]. A single fundamental scalar living in the bulk of the higher-dimensional theory can give rise, upon compactification, to a set of effective scalar fields corresponding to the Kaluza-Klein modes. Provided the extra dimensions are sufficiently large, this yields a large number of scalar fields with similar potentials, which can support assisted inflation. The main advantage of this type of inflation is that individual fields need never exceed the fundamental Planck scale, which may allow supergravity corrections to remain consistently small. Another possible benefit of this scheme is that provided the number of fields is sufficiently great, the fundamental self-couplings of the fields may be much greater than the usual requirement that they be of order 10^{-12} or less.

In this paper, we analyze the dynamics of a simple implementation of this idea, where the scalar fields are uncoupled but have different masses depending on the Kaluza-Klein winding [6]. As inflation proceeds, the energy scale decreases, which leads to a reduction in the number of scalar fields with mass less than the Hubble scale. Only such fields can slow-roll. Therefore, the number of fields participating in the assisted behavior decreases as inflation proceeds. We will also study the effect of this on the density perturbation spectrum. Our results will reveal a very interesting property of assisted chaotic dynamics. In contrast to the common inflationary models, assisted chaotic models permit some information about the initial conditions to be retained in the form of a soft dependence of the spectral index, and other postinflationary predictions, on the number of fields which contributed to inflation.

II. THE MODEL

Following Kanti and Olive [6], we consider a bulk theory of gravity in five dimensions, with a minimally coupled massive scalar field Ψ. The action is

$$S_5 = \int d^5x \sqrt{g_5}\left[\frac{R_5}{2\kappa_5^2} - \frac{1}{2}(\nabla\Psi)^2 - \frac{1}{2}m^2\Psi^2\right]. \quad (1)$$

The scalar field Ψ is some weakly coupled light bulk field, and we assume that its self-interactions are negligible com-

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pared to the mass term. Such fields could arise in some bulk supergravity theories. After stabilization of the compactified dimension, the 5D metric can be split as

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu + L^2 d\theta^2, \tag{2}$$

where L is the constant size of the fifth dimension. The relationship between the 5D and 4D parameters is easy to compute: the 4D Planck mass, defined by $m_{\text{Pl}}^2 = 1/G$, is given in terms of the 5D one as $m_{\text{Pl}}^2 = M^3 L$, where M is the 5D unification scale (the string scale). The wave function renormalization for the scalar field and its Kaluza-Klein siblings is $\Phi_j = \sqrt{L}\Psi_j$, where Φ_j is the projection of the jth Kaluza-Klein state with mass

$$m_j^2 = m^2 + \frac{j^2}{L^2}. \tag{3}$$

Of the infinite tower of the Kaluza-Klein states, only those which admit a description in the field theory limit should be included here. They are those states which are lighter than the fundamental scale of the theory M. The states which are heavier than M can be described consistently only in the full quantum gravity limit, and hence are left out. Then, from the mass formula Eq. (3), the total number of the light Kaluza-Klein states is $N_{\text{max}} = ML = m_{\text{Pl}}^2/M^2$. Note that consistency requires $m \lesssim M$, otherwise none of the fields should be kept in the field theory limit. Therefore the 4D reduced action is

$$S_{4,\text{eff}} = \int d^4x \sqrt{g}\left\{\frac{m_{\text{Pl}}^2 R}{16\pi} - \sum_{i \geqslant 0}\left[\frac{1}{2}(\nabla\Phi_i)^2 + \frac{1}{2}m_i^2\Phi_i^2\right]\right\}. \tag{4}$$

In this model the 4D scalar fields are coupled only through gravity, whereas if the original Ψ field were self-coupled the Φ_j fields would be interacting.

Before exhibiting the calculation, let us overview the picture of the evolution. This more or less follows that outlined by Kanti and Olive [6], though they concentrated mainly on the lightest fields, whose masses were taken to be nearly identical. Assuming compactification has occurred, then, as in the chaotic inflation paradigm [18–20], at early times the fields may range over a wide variety of values in different regions of space. In the regime where the fields do not exceed the 4D Planck scale, supergravity corrections are small. In the original assisted picture using exponential potentials [1], the assisted inflation solution was the unique late-time attractor, with all fields eventually participating. In this model the situation is rather different. First, there is no late-time inflationary solution as eventually all fields will settle in the minima. Second, there is no attractor behavior; as we will see the fields tend to diverge from one another. Nevertheless, there is a form of a transient assisted behavior, simply because there are many fields, and in particular the light ones are in a slow-roll regime where they evolve slowly and feel the collective friction of all the fields.

An important feature of this model is that the fields do not all have the same potential; they receive a contribution to their mass from the Kaluza-Klein winding j, and barring severe fine tuning this mass difference will lead to significant effects during evolution. This is because for each mode the slow-roll regime can only be achieved if the mass of the mode m_j is less than the Hubble parameter; otherwise the field will instead evolve quickly to its minimum, with the energy density redshifting as $1/a^3$ during the oscillations. As assisted inflation proceeds, the Hubble scale is decreasing, and so lighter and lighter fields enter the regime $m > H$ and end their assisted behavior. Consequently fewer and fewer fields are involved as time goes by.

Although there is no formal attractor behavior, nevertheless the collective effect of the fields may give desirable advantages over single-field chaotic inflation. The most prominent of these is that, due to the collective friction, inflation becomes possible when all the fields have values less than the Planck mass. This means that supergravity corrections to the potential are much less likely to destroy inflation than in the single-field case, where inflation is only possible for $\phi \gtrsim m_{\text{Pl}}$. In the assisted variant, due to the collective contribution to the friction, we will find that inflation is possible down to field values $\Phi \sim m_{\text{Pl}}/\sqrt{mL}$, where m is the mass of the zero mode and L the size of the extra dimensions. As long as $mL \gg 1$, this is well below the 4D Planck scale; slow-roll can occur for sub-Planckian values of fields, and its duration is prolonged. It is interesting to note that because $L = m_{\text{Pl}}^2/M^3$ [7], inflation ends at $\Phi \sim \sqrt{M/m}\,M$. Thus for a fixed fundamental scale M, since $m < M$, it is generally higher than M, but it lies lower the larger m is. So even for relatively heavy fields, if they start out at $\Phi \sim m_{\text{Pl}}$, there may be many e-foldings of inflation provided $M < m_{\text{Pl}}$.

III. INFLATIONARY DYNAMICS

We now consider the dynamics in detail. Assuming a spatially flat homogeneous cosmology, the equations of motion for the assisted mass-driven chaotic inflation are

$$3H^2 = \frac{4\pi}{m_{\text{Pl}}^2}\sum_{j=0}^{N(t)}(\dot{\Phi}_j^2 + m_j^2\Phi_j^2), \tag{5}$$

$$\ddot{\Phi}_j + 3H\dot{\Phi}_j + m_j^2\Phi_j = 0. \tag{6}$$

Although in principle the sum in the Friedmann equation goes over all the fields up to $j = N_{\text{max}}$, following the above discussion the only fields which will contribute in practice are those which are in the slow-roll regime. Therefore we can take the sum to only include those Kaluza-Klein states whose mass is smaller than the Hubble parameter. We indicate this number by $N(t)$, and it is time-dependent because H is.

We are assuming initial conditions where all the fields are displaced from their minima. We will experiment with different choices. While one could adopt the weak condition that the total energy density is below the Planck scale, we prefer to take the more stringent condition that supergravity corrections to the scalar field evolution can be neglected. For the five-dimensional modes, this condition is $\Psi_j(t_{\text{initial}})$
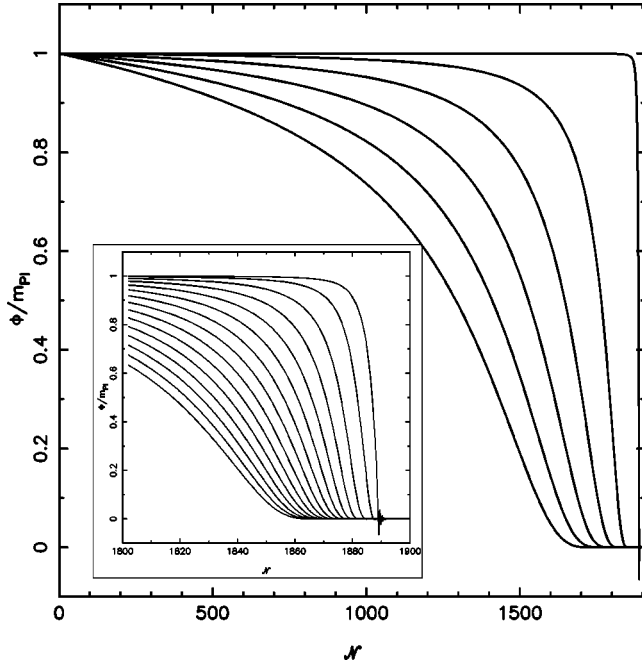
FIG. 1. This shows the result of a 300 field simulation, with $m=10^{-4}\,m_{\rm Pl}$ and $L=5000/m_{\rm Pl}$. The initial condition for each field was $\Phi_j=m_{\rm Pl}$, and evolution is shown as a function of $\mathcal{N}\equiv\ln a$. The main panel shows the fields with $j=0$, 25, 50, 75, 100 and 125, with the more massive fields decoupling first. The inset shows the lowest 15 fields at the end of inflation. Note that these fields are in the process of decoupling during the last 50 e-foldings.

$\lesssim M^{3/2}$, and after projecting to effective 4D degrees of freedom this translates to $\Phi_j(t_{\rm initial})\lesssim m_{\rm Pl}$. With a single field, this does not leave sufficient space for a prolonged slow-roll regime, but with multiple fields slow-roll can continue until $\Phi_j\ll m_{\rm Pl}$ [6].

A. Numerical solutions

The system of equations described in Eqs. (5) and (6) is readily solved numerically, and we first describe the results of some simulations to establish the general picture, before going on to describe some analytical approximations which can be used. For a given choice of the parameters m and L, and a choice of initial conditions used, one has to decide how many scalar fields need to be evolved. In general the more massive ones will always swiftly become negligible, and so long as we restrict ourselves to demanding an accurate description of only the observably accessible last 50 or so e-foldings of inflation, we need not necessarily simulate the entire $N_{\rm max}=m_{\rm Pl}^2/M^2$ fields.

Figures 1 and 2 show two separate simulations of the same model ($m=10^{-4}m_{\rm Pl}$ and $L=5000/m_{\rm Pl}$) with different initial conditions. With this L, $N_{\rm max}=(L\,m_{\rm Pl})^{2/3}\simeq 300$ so that many fields are included in the simulations. The fields are shown as a function of the number of e-foldings of inflation $\mathcal{N}\equiv a$ (here given as the number from the start of the simulation, rather than from the end of inflation). In one simulation all fields start with the same initial value, while in the other they have a spread of initial values. The main panels
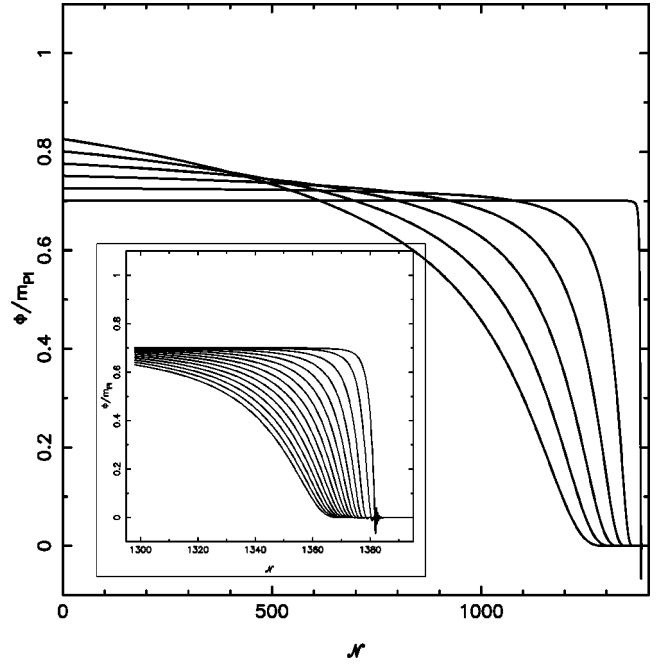

FIG. 2. As Fig. 1, with the same model parameters but now with different initial conditions for the fields.

show some fields up to quite large values of j, while the insets show just the fifteen lightest fields. Note that the total number of e-foldings is comfortably above the 70 or so required to solve the horizon and flatness problems, and recall that it is only a few e-foldings, centered around about 50 e-foldings from the end of inflation, that can be directly probed by structure formation.

We see that the behavior is indeed that outlined earlier. There is no evidence of a late-time attractor, but the fields do take a substantial time to evolve to the minimum and hence collectively drive inflation. In particular, we see from the insets that at least the lightest 15 fields are still dynamically relevant when structure-forming perturbations were imprinted. We also note that after decoupling most fields simply asymptote into $\Phi_j=0$; the other fields contribute enough friction that the heavier ones remain overdamped right to the minimum. Only the lightest few fields undergo oscillations when they reach the minimum.

One vital result is to notice that, unlike usual inflation, the initial conditions do not become irrelevant at late times, because in our region of the Universe the fields have evolved from particular points in field space which determines their time of decoupling. Studying Figs. 1 and 2, we see that the evolution of the fields is not identical even during only the last 50 e-foldings. The late-time behavior, and hence derived quantities such as the density perturbation spectrum, will depend not only on the underlying model parameters m and L, but also on the particular initial conditions for the fields pertaining to our region. This gives a significant reduction in the usefulness of observations in directly constraining the inflationary potential.

With these numerical results in mind, we now explore this further using the slow-roll approximation.

B. Analytical approximations

At a given time t all the fields whose mass is smaller than H are in the regime where the collective friction dominates over the acceleration, and hence are supporting inflation (perhaps with the exception of some which by chance started very near the origin). The mass of the heaviest contributing field is given by Eq. (3), with $j=N(t)$. If we choose an initial condition that all the fields were approximately the same, one may suspect that by time t their values may have become significantly different. In fact, we can give an accurate estimate of this spreading, and its effect on dynamics, as follows. The slow-roll version of Eq. (6), obtained by dropping the second derivative term, implies [6]

$$\frac{\Phi_j(t)}{\Phi_j(\text{initial})}=\left[\frac{\Phi_0(t)}{\Phi_0(\text{initial})}\right]^{m_j^2/m_0^2} \qquad (7)$$

where index 0 refers to the zero mode, of mass m. The initial values for all modes $\Phi_j(\text{initial})=\Phi_j(0)$ are taken to be of the order of m_{Pl}, as we have discussed above. However for the sake of generality we will retain them as an arbitrary set of input parameters, in order to study their influence on the dynamics. Then, we can approximate the jth mode by

$$\Phi_j(t)=\Phi_j(0)\exp\left[-\left(1+\frac{j^2}{m^2L^2}\right)\sigma(t)\right] \qquad (8)$$

where the field σ is the logarithm of the zero mode, $\sigma=-\ln[\Phi_0(t)/\Phi_0(0)]$ and the minimum of the potential corresponds to $\sigma\to\infty$.

Then, assuming that there are many light fields in the slow-roll regime so that we can replace the sum in Eq. (5) by an integral $\int_0^{N(t)}dj$, and ignoring their kinetic terms, after some simple algebra we find that

$$H^2=\frac{4\pi}{3m_{\text{Pl}}^2}\frac{m^3L}{\sqrt{2\sigma}}e^{-2\sigma}\int_0^{\sqrt{2\sigma}N(t)/mL}dy\,f^2(y)\left(1+\frac{y^2}{2\sigma}\right)e^{-y^2}. \qquad (9)$$

Here we have defined the function $f(y)$ by $f(y)=\Phi_{mLy/\sqrt{2\sigma}}(0)$, since we can certainly view the initial distribution of the Kaluza-Klein fields as a function of their mass. In practice, in most cases the upper limit of the integral can be taken to be of order unity. Indeed, at about 50 e-foldings before the end of inflation, the heaviest modes which are still in the slow-roll regime require $N(t)\sim HL$. On the other hand, by this time the field σ will typically be of order unity. In fact, even if $\sqrt{2\sigma}N(t)\gg mL$, the integrand falls rapidly to zero, as $\exp(-y^2)$, which effectively cuts off the contributions to the integral to only those values for which $\sqrt{2\sigma}N(t)\lesssim mL$, as long as the function $f(y)$ grows slower than $\exp(y^2)$. But this is a very natural assumption in the assisted inflation context: initially all the fields which contribute to the collective attractor should have similar values, and in fact the heavier fields should be lower along the potential well. Therefore the function $f(y)$ should be bounded by $f(y)\lesssim f(0)$. Hence $f^2(y)$ in the integrand can be approximated by a polynomial, $f^2(y)\sim\Phi_0^2(0)+p(y)$, which

changes slowly compared to $\exp(-y^2)$. So for as long as inflation is proceeding with many fields still in the slow-roll regime, we can safely replace the control parameter $\sqrt{2\sigma}N(t)/mL$ by a number $\simeq 1$ in the upper limit of integration.

On the other hand, for a certain subspace of the phase space of the theory, the quantity $\sqrt{2\sigma}N(t)/mL$ can be smaller than unity. Typically this happens for the choice of parameters in the Lagrangian where the zero mode is heavy, and so most of its Kaluza-Klein siblings are decoupled. Therefore although generically assisted dynamics leads to many more e-foldings than the minimum of 60, for sufficiently large mass m, the total number of e-foldings can be small. In this case the appropriate approximation for evaluating Eq. (9) is to take $2\sigma N(t)/mL\ll 1$, which we label as short assisted inflation. We will return to this case below. Here we must underscore that due to the strong nonlinear nature of the dynamics, we need to treat the approximations in a floating manner. To decide which approximation is applicable at $\mathcal{N}$ e-foldings before the end of inflation, we must check the value of the control parameter $\sqrt{2\sigma}N(t)/mL$, and choose the relevant formulas for the spectra of perturbations as it dictates.

1. Long assisted inflation

Armed with the above, in the case of long assisted inflation we can estimate the integral by using the error function $\text{erf}[x]=(2/\sqrt{\pi})\int_0^x dy\,e^{-y^2}$. Since, as we said, the main contribution comes from the range of values for which $x\lesssim 1$, we can use $\int_0^1 dy\,e^{-y^2}\sim\sqrt{2}c$, where c is a number of order unity. The precise value of c is not of immediate consequence here, and we will keep it as a free parameter for now. This gives

$$H^2\simeq\frac{4\pi c}{3}\frac{\Phi_0^2(0)}{m_{\text{Pl}}^2}\frac{m^3L}{\sqrt{\sigma}}e^{-2\sigma}. \qquad (10)$$

Using the definition of σ, we can now rewrite this equation in terms of the zero mode $\Phi_0(t)$, finding

$$H\simeq\sqrt{\frac{4\pi}{3}}\frac{(cmL)^{1/2}}{m_{\text{Pl}}\ln^{1/4}[\Phi_0(0)/\Phi_0(t)]}m\Phi_0(t). \qquad (11)$$

This equation permits us to replace the collection of fields by the zero mode Φ_0. Note the logarithmic dependence of the Hubble parameter on the initial value of the zero mode field Φ_0. This takes into account the decoupling of the heavy modes, which fall out of the slow-roll regime as $\Phi_0(t)$ rolls towards the minimum. Also note that because of the strong cutoff in the integration in Eq. (9) effected by $\exp(-y^2)$, the dynamics is sensitive only to the average of the initial values of the Kaluza-Klein fields $\Phi_0(0)$, and not to its dispersion. Hence, we can indeed safely assume that all the fields were initially essentially the same.

Therefore to the lowest order we can rewrite the equations of motion, using the slow-roll approximation, as

$$3H^2 = 4\pi \frac{cmL}{m_{\rm Pl}^2 \ln^{1/2}[\Phi_0(0)/\Phi_0(t)]} m^2 \Phi_0^2(t), \qquad (12)$$

$$3H\dot{\Phi}_0(t) + m^2\Phi_0(t) = 0. \qquad (13)$$

Since the field Φ_0 is in slow-roll as long as $m \leq H$, it is now easy to see that inflation continues as long as

$$\Phi_0 \geq \sqrt{\frac{3}{4\pi}} \frac{m_{\rm Pl}}{\sqrt{cmL}} \ln^{1/4}\left(\sqrt{\frac{4\pi cmL}{3}} \frac{\Phi_0(0)}{m_{\rm Pl}}\right). \qquad (14)$$

This equation is illustrative, since now we indeed see, as we have mentioned above, that in most cases during inflation

$$\sqrt{2}\sigma \frac{N}{mL} \simeq \sqrt{2}\sigma \frac{H}{m} \geq \sqrt{2}\ln^{1/2}(\Phi_0(0)/\Phi_0) \geq 1, \qquad (15)$$

except during the first few e-foldings immediately after the start. One should bear in mind that the ''first few'' e-foldings could in principle be enough to solve all the usual cosmological problems. However, this will not be the case for the most allowed values of the zero mode parameters. Let us define the quantity

$$\alpha \equiv \frac{cmL}{\ln^{1/2}[\Phi_0(0)/\Phi_0(t)]}, \qquad (16)$$

which is a slowly varying function of time through the time dependence of Φ_0. Now, we can combine Eqs. (12) and (13), and after some straightforward algebra obtain

$$\frac{d\mathcal{N}}{d\Phi_0} = 4\pi\alpha \frac{\Phi_0}{m_{\rm Pl}^2}, \qquad (17)$$

where $\mathcal{N} = \ln(a(\text{final})/a)$ is the number of e-foldings of inflation which occur after the field reaches Φ_0 [not to be confused with $N(t)$, the number of slow-rolling fields]. It is straightforward to integrate this equation: we find

$$\mathcal{N} = (2\pi)^{3/2} c \, mL \frac{\Phi_0^2(0)}{m_{\rm Pl}^2} \left\{ 1 - \text{erf}\left[\sqrt{2} \ln^{1/2}\left(\frac{\Phi_0(0)}{\Phi_0}\right) \right] \right\}. \qquad (18)$$

Since initially $\Phi_0(0) \sim m_{\rm Pl}$, this equation shows that the total number of e-foldings is $\mathcal{N}(\text{total}) \sim (2\pi)^{3/2} c \, mL$. Further, it can be seen that at a time considerably before the end of inflation, the number of e-foldings left to leading order scales as $\mathcal{O}(\Phi_0^2)$. Indeed, ignoring the variation of the denominator with Φ_0 in Eq. (17), it follows that at $\mathcal{N} \gg 1$ e-foldings before the end of inflation we can approximate Eq. (18) with

$$\mathcal{N} \simeq 2\pi\alpha \frac{\Phi_0^2}{m_{\rm Pl}^2} + \mathcal{O}\left(\frac{\Phi_0^3}{m_{\rm Pl}^3}\right). \qquad (19)$$

This formula is familiar from the usual chaotic inflation, except for the factor α which comes from the collective dynamics. In fact, this formula gives an accurate approximation

for the relationship between $\mathcal{N}$ and Φ_0 sufficiently far before the end of inflation, and we will use it hereafter.

2. Short assisted inflation

Let us now return to the case of short assisted inflation. As we will see later it is helpful to place an upper bound on the mass of the zero mode m which leads to significant assisted behavior. In this instance, the integral Eq. (9) is better approximated by

$$H^2 = \frac{4\pi m^2}{3m_{\rm Pl}^2} N\Phi_0^2\left(1 + \frac{N^2}{3m^2 L^2}\right). \qquad (20)$$

Since $N \sim HL$, using Eq. (20) we find

$$N \simeq \frac{4\pi m^2 L^2}{3m_{\rm Pl}^2} \Phi_0^2\left(1 + \frac{N^2}{3m^2 L^2}\right). \qquad (21)$$

Now, although the second term on the right-hand side (RHS) of this equation dominates, we will approximate N by the first term on the RHS, since using the second term in the subsequent framework would lead to an overestimation. By using the first term, the computation remains confined in the realm of perturbation theory, where errors are controllable. Thus $N \simeq 4\pi m^2 L^2 \Phi_0^2/3m_{\rm Pl}^2$. Therefore,

$$H^2 = \frac{16\pi^2 m^4 L^2}{9m_{\rm Pl}^4} \Phi_0^4\left(1 + \frac{16\pi^2 m^2 L^2 \Phi_0^4}{9m_{\rm Pl}^4}\right). \qquad (22)$$

Next, we will approximate the Hubble parameter H by retaining the second term on the RHS of this equation, since it clearly dominates over the first, while the perturbation theory is still valid using it. Hence taking the square root of Eq. (22) we find

$$H = \frac{16\pi^2 m^3 L^2}{9m_{\rm Pl}^4} \Phi_0^4. \qquad (23)$$

Clearly since inflation lasts as long as $m \leq H$, during it $\Phi_0 \geq \sqrt{3/4\pi} \, m_{\rm Pl}/\sqrt{mL}$, which is in good agreement with Eq. (14). We note that the authors of Ref. [6] treat assisted dynamics only in this regime where $\sqrt{\sigma} N/mL \ll 1$, as is evident from their equations of motion. However, their approximation for the Hubble parameter consists of retaining only the linear term in Eq. (20). The treatment here provides a more precise approximation. Then using Eq. (6), we obtain

$$\frac{d\mathcal{N}}{d\Phi_0} = \frac{256\pi^4 m^4 L^4}{27m_{\rm Pl}^8} \Phi_0^7. \qquad (24)$$

Its solution can be found immediately: it is

$$\mathcal{N} = \frac{32\pi^4 m^4 L^4}{27m_{\rm Pl}^8} \Phi_0^8. \qquad (25)$$

This equation replaces Eq. (19) when m is large, or equivalently when the parameter α given in Eq. (16) is small. We

will consider this case in more detail in the next section. Note that the approximations in this subsection give a good description of the assisted dynamics for the times when the control parameter $\sqrt{2}\sigma N(t)/mL$ is small, initially or by subsequent dynamics.

The previous analysis shows that the dissipation due to the decoupling of the massive modes is the dominant correction to the dynamics of the model. To estimate it, we need to establish how rapidly the modes decouple in the course of evolution. This can always be done by using the rule $N \sim HL$ and the solutions presented so far. In general, although it is clear that the details are complicated, we can nevertheless obtain a good approximation for the decoupling rate as

$$\frac{dN}{N} = \frac{d\mathcal{N}}{\gamma\mathcal{N}}, \tag{26}$$

where γ measures the decoupling rate, and depends on the parameters m and L and the initial conditions. In general it is a slowly-changing function, with the value within an order of magnitude or two of unity. Note, that the usual single-field chaotic inflation corresponds to the limit $\gamma \to \infty$. This is sufficient for our purposes here.

IV. PERTURBATIONS

A. Density perturbations

Now we compute the density contrast, using the notation of Ref. [21]. We will do this approximately, by using the usual formula as applied to the lightest scalar field. Unfortunately this ignores the effects of perturbations in the other fields, but in the absence of exact analytical solutions it is unclear how to include them. Ideally, one would follow the approach of Malik and Wands [2], in which a new set of fields is defined such that the linear perturbations in all but one give no first-order contribution to the perturbation in the total density, but this requires knowing the full solutions in advance. We hope to return to the question of a complete computation of the adiabatic density contrast in a later work.

With the above caveats, the power spectrum $\mathcal{N}$ e-foldings before the end of inflation is estimated as

$$\delta_H(k) = \frac{1}{5\pi}\frac{H^2}{\dot{\Phi}_0} = \sqrt{\frac{64\pi}{75}}\frac{(cmL)^{3/2}}{\ln^{3/4}[\Phi_0(0)/\Phi_0]}\frac{m\Phi_0^2}{m_{\rm Pl}^3}$$

$$= \sqrt{\frac{16}{75\pi}}\sqrt{\alpha}\frac{m}{m_{\rm Pl}}\mathcal{N}. \tag{27}$$

As always with inflation, the overall amplitude can be adjusted to match the value observed by COBE, $\delta_H(k_{\rm hor}) \simeq 2\times10^{-5}$, by suitable choice of the mass parameter. Assuming that the present Hubble radius equaled the Hubble radius 50 e-foldings from the end of inflation, this requires

$$\alpha_{50} = 2.3\times10^{-12}\frac{m_{\rm Pl}^2}{m^2}, \tag{28}$$

where α_{50} is the value of α at the relevant epoch. The model has three input parameters, m, L, and $\Phi_0(0)$, though the dependence on the last is weak.

Another bound on the parameters can be obtained from considering the initial energy density. Here we will at first ignore the precise aspects of the nonlinear assisted dynamics in order to estimate the range of allowed values of the zero mode mass. We will then reconsider some of the resulting inequalities with more precision, since they will provide the criteria for choosing the relevant approximation for computing the spectral properties. By the usual arguments in theories with large extra dimensions [7], the total energy density in 5D cannot exceed M^5. After the extra dimensions are stabilized, this places an upper bound on the projected energy density in 4D: $\rho \lesssim M^5L = M^2m_{\rm Pl}^2$. On the one hand, the initial energy density of the collective inflaton is $\rho \sim m^3Lm_{\rm Pl}^2$ by Eq. (9). This and the upper bound give us $m^3L \lesssim M^2$. On the other hand, by the definition of α in Eq. (16), ignoring the logarithm in the denominator, we can see that Eq. (28) implies $m^3L \sim 2.3\times10^{-12}m_{\rm Pl}^2$. Hence combining the inflationary and compactification constraints, we find the lower bound on the fundamental scale M:

$$M \gtrsim 1.5\times10^{-6}m_{\rm Pl}. \tag{29}$$

However, using Eq. (28) again, and noting that $m^3L = m_{\rm Pl}^2m^3/M^3$, we find that $m = 1.32\times10^{-4}M$. Hence, combining this and inequality (29), we obtain a lower bound for the mass of the zero mode m:

$$m \gtrsim 2\times10^{-10}m_{\rm Pl}. \tag{30}$$

Therefore we see that the assisted models of inflation could be a phenomenologically viable scenario of inflation only in theories with a unification scale $\gtrsim 10^{13}$ GeV, which is still considerably larger than the electroweak scale. Otherwise, assisted chaotic inflation would not produce the density contrast in the Cosmic Background Explorer (COBE) range. This agrees with the conclusion that inflation after stabilization of extra dimensions could give density contrast as measured by COBE only if the unification scale is high [15,16].

From this we find that the parameter α_{50} is bounded from above: combining Eqs. (28) and (30) we obtain

$$\alpha_{50} \lesssim 5.75\times10^{13}. \tag{31}$$

Since $\ln[\Phi_0(0)/\Phi_0] \lesssim \ln(2\pi\alpha/\mathcal{N})/2 \sim 15$, the approximation made above in ignoring the logarithm was justified. Finally from the fact that the total number of e-foldings is $\mathcal{N}(\text{total}) \sim (2\pi)^{3/2}mL$, we see that $\mathcal{N}(\text{total}) \lesssim 3\times10^8$, implying that generically it can be quite large.

The most useful predictions that can then be made are of the shape of the spectrum, which is independent of the normalization. As compared to single-field chaotic inflation, the density contrast in assisted chaotic inflation has additional dependence on the inflaton via the factor α, which will cause a deviation of the shape of the spectrum.

We begin by calculating the spectral index n of the spectrum. This is defined by

$$n - 1 = \frac{d \ln \delta_H^2(k)}{d \ln k}, \tag{32}$$

where in the slow-roll approximation $d \ln k \simeq -d\mathcal{N}$. The interesting thing is that, unlike the usual case, this scale dependence receives two contributions. The first is the usual one coming from the $\mathcal{N}$ term, and the second arises from the $\mathcal{N}$ dependence of α.

If initially we assume α is constant, we immediately find, using Eq. (19), the usual result for single-field quadratic chaotic inflation, namely

$$n = 1 - \frac{2}{\mathcal{N}}. \tag{33}$$

We can also compute the scale dependence of the spectral index

$$\frac{dn}{d \ln k} \simeq -\frac{dn}{d\mathcal{N}} = -\frac{2}{\mathcal{N}^2}. \tag{34}$$

If we assume the present Hubble scale equaled the Hubble scale 50 e-foldings before the end of inflation, we have $n = 0.96$ and $dn/d \ln k = -8 \times 10^{-4}$. The Planck satellite is capable of distinguishing the former from unity [22], but not the latter from zero [23].

However, we need to include the speed up of the inflaton field as heavy Kaluza-Klein modes fall out of the slow-roll regime, through the α term. From Eqs. (19) and (27), we find

$$n = 1 - \frac{2}{\mathcal{N}} \left[1 + \frac{3}{4 \ln[(2\pi\alpha\Phi_0^2(0))/(m_{\text{Pl}}^2\mathcal{N})]} \right]. \tag{35}$$

Its scale dependence is given by the equation

$$\frac{dn}{d \ln k} = -\frac{2}{\mathcal{N}^2} \left[1 + \frac{5}{4 \ln[(2\pi\alpha\Phi_0^2(0))/(m_{\text{Pl}}^2\mathcal{N})]} \right.$$
$$\left. - \frac{3}{8 \ln^2[(2\pi\alpha\Phi_0^2(0))/(m_{\text{Pl}}^2\mathcal{N})]} \right]. \tag{36}$$

In the limit $\alpha \to \infty$, these expressions correctly reduce to Eqs. (33) and (34), as appropriate for single-field models. However, for a generic assisted model, we see that the predictions for the spectral index and its scale dependence are different from the usual single-field chaotic inflation. For example, if $\Phi_0(0) \sim m_{\text{Pl}}$ and the mass of the zero mode m is of order of $m \sim 10^{-7} m_{\text{Pl}}$, the parameter α_{50} is, using Eq. (28), $\alpha_{50} = 230$, and hence the spectral index and its gradient are $n = 0.95$ and $dn/d \ln k = -1.1 \times 10^{-3}$, respectively. Clearly, these numbers are sensitive to the mass m: the smaller it is, the more similar assisted chaotic inflation becomes to the usual single-field driven chaotic inflation.

A closer look at Eqs. (35) and (36) shows that the predictions depend on the initial condition $\Phi_0(0)$ through the parameter α. Hence the spectral index can vary significantly with the initial condition. The explicit dependence of n (and $dn/\ln k$) on the initial condition can be computed from the above formulas. The important observation is that since the initial value of the field $\Phi_0(0)$ appears only through the logarithm, if $\Phi_0(0)$ varies through its full range of admissible values, $10^{-4} m_{\text{Pl}} \leq \Phi_0(0) \leq m_{\text{Pl}}$, the logarithm changes by a factor of ~ 3. Hence, n may vary in the range $0.94 \leq n \leq 0.96$, at the 50 e-foldings. In a manner of speaking, the assisted chaotic inflation does not impart amnesia on the universe as efficiently as single-field chaotic inflation, and some of the information about the initial state of the universe much before the last 60 e-foldings is imprinted on the late epoch too.

We note that Eqs. (35) and (36) suggest that for the value of the mass m where $\alpha_{50} = 25/\pi$ there is a divergence. But this divergence is clearly completely spurious: it merely signifies that at the large values of the mass m the approximations which led to Eqs. (35) and (36) break down. Instead, there we need to resort to the approximation for short assisted inflation, discussed at the end of the previous section. Using Eqs. (23) and (25), it is then easy to derive the density contrast in this case:

$$\delta_H = \frac{4096\pi^5 m^6 L^6}{1215 m_{\text{Pl}}^{11}} \frac{m}{m_{\text{Pl}}} \Phi_0^{11} = 1.5\sqrt{mL}\, \frac{m}{m_{\text{Pl}}} \mathcal{N}^{11/8}. \tag{37}$$

Then, the COBE normalization condition gives

$$mL = 3.7 \times 10^{-15} \frac{m_{\text{Pl}}^2}{m^2}. \tag{38}$$

The spectral index is

$$n = 1 - \frac{11}{4\mathcal{N}}, \tag{39}$$

and its gradient is

$$\frac{dn}{d \ln k} = -\frac{11}{4\mathcal{N}^2}. \tag{40}$$

From the requirement that there are at least 60 e-foldings and Eqs. (25) and (38) we can deduce the upper bound on the mass m. Clearly, the maximal number of e-foldings will come from the largest initial condition, $\Phi_0(0) \simeq m_{\text{Pl}}$. Thus, $\mathcal{N} \approx 50$ gives $m \sim 10^{-7} m_{\text{Pl}}$, and the spectral index and its gradient are $n = 0.945$ and $dn/d \ln k = -1.1 \times 10^{-3}$. These numbers are in a very good agreement with the corresponding numbers for the smallest attainable parameter α_{50} discussed above. Therefore, since we limit the initial condition of the field $\Phi_0(0)$ to be below the 4D Planck scale m_{Pl}, the largest mass m which still leads to sufficient inflation is $m \sim 10^{-7} m_{\text{Pl}}$. In this case the assistance effect produces a different spectrum of perturbations from the usual single-field chaotic inflation with quadratic potential.

We now turn to specifying the criteria for selecting the relevant approximation on the basis of the values of m. Since the control parameter is

$$\mathcal{C} = \sqrt{2\sigma}\,\frac{N}{mL} \sim \sqrt{2\sigma}\,\frac{H}{m}, \tag{41}$$

using Eqs. (11) and (19), we can rewrite it as $\mathcal{C} \sim \sqrt{4\sigma\mathcal{N}/3}$. Hence at $\mathcal{N} = 50$ e-foldings, this gives $\mathcal{C}_{50} \sim 8\sqrt{\sigma_{50}}$, and using the relationship between σ_{50} and α_{50}, and $M^3L = m_{Pl}^2$, we finally find

$$\mathcal{C}_{50} \sim 3c \times 10^{11}\,\frac{m^3}{M^3}. \tag{42}$$

The parameter c is never larger than $\sqrt{\pi/8} \sim 0.6264\ldots$, and while it can be smaller, we see that the control parameter is really dominated by the ratio of the zero mode mass to the 5D Planck scale. This is a consequence of the higher-dimensional origin of the effective inflaton field, as we have discussed before. Therefore, we finally have a clear-cut criterion for selecting one of the two approximate descriptions discussed above: if $\mathcal{C}_{50} \gg 1$ we cannot ignore the nonlinearities and should use the approximations for the long assisted inflation, whereas for $\mathcal{C}_{50} < 1$ we can use the (improved) quasilinear approximations appropriate for the short assisted inflation. Clearly, there is a transition region in between, where neither of our approximations will be very accurate, and where the complete treatment of the evolution necessitates a numerical approach. However, this occurs on only a small part of the phase space, while the analytical approximations which we have developed cover most of the admissible parameter space.

B. Gravitational waves

The gravitational waves produced during inflation are determined entirely by the evolution of the Hubble parameter. Following the notation of Ref. [24], their amplitude is given by

$$A_G = \frac{2}{5\sqrt{\pi}}\,\frac{H}{m_{Pl}} = \sqrt{\frac{16}{75}}\,\frac{(cmL)^{1/2}}{\ln^{1/4}[\Phi_0(0)/\Phi_0]}\,\frac{m\Phi_0}{m_{Pl}^2}. \tag{43}$$

As the gravitational wave production depends only on the expansion rate $a(t)$, unlike the case of density perturbations the first expression for A_G is exact up to the slow-roll approximation.

The ratio of the gravitational to scalar perturbations is, using Eqs. (19), (27) and (43),

$$\frac{A_G}{\delta_H} = \frac{1}{2\sqrt{\pi}}\,\frac{m_{Pl}}{\alpha\Phi_0} = \frac{1}{\sqrt{2\alpha\mathcal{N}}}. \tag{44}$$

From the value of α_{50} from Eq. (28), we see that the predicted ratio of the gravitational to scalar perturbations at 50 e-foldings is

$$\frac{A_G}{\delta_H} = 6.6 \times 10^4\,\frac{m}{m_{Pl}}. \tag{45}$$

Hence, the precise value of the mass of the zero mode determines this ratio. Clearly the heavier field will produce more gravitational waves relative to the scalar density contrast. Using the bound Eq. (30), we find

$$\frac{A_G}{\delta_H} \geq 1.32 \times 10^{-5}. \tag{46}$$

V. CONCLUSIONS

We have considered dynamics of assisted chaotic inflation which can arise in theories with large internal dimensions after compactification. In this work, we have focused on a simple model based on a single massive scalar field. If the field lives in the bulk of the fundamental theory, then upon compactification on manifolds larger than the fundamental Planck scale, it will give rise to a tower of massive Kaluza-Klein states. Many of these Kaluza-Klein states will be lighter than the 5D Planck scale, and hence can be treated in the field theory limit, where they can contribute to assisted inflation. The model is in agreement with COBE constraints provided that the fundamental Planck scale is greater than 10^{13} GeV. The main aspects of the ensuing assisted dynamics are rather interesting. Instead of the appearance of an asymptotic attractor for the multitude of the scalar fields displaced from their respective minima, as in the original assisted inflation with exponential potentials [1], here the fields with a different mass never develop a completely coherent motion. Rather, the fields keep accelerating away from each other. If they are viewed as a collective mode, this means that there is a constant spreading of the collective mode throughout the evolution. However, the spreading is very small compared to the expansion rate of the universe. Indeed, since the effective Hubble parameter of the universe receives contributions from all fields in the slow-roll regime, it is larger, and hence gives a stronger resistance to acceleration of each field down its respective potential well. This in turn prolongs the slow-roll regime for each field, and leads to longer inflation overall.

Furthermore, a combination of the assisted behavior and the higher-dimensional origin of the theory lowers very significantly the value of the fields Φ where slow-roll ends, giving $\Phi_{end} \sim M^{3/2}/m^{1/2}$, rather than m_{Pl} as is usual for inflation with fields confined to 4D (here M is the fundamental Planck scale and m the zero mode mass). So instead of inflation terminating at the 4D Planck scale, it can last well below it, almost as low as the higher-dimensional Planck scale. Therefore, to drive a long inflation, the zero mode and all of its Kaluza-Klein siblings can start with values of order of m_{Pl}, at energy densities far below the 4D Planck scale, and with values in the regime where higher-loop supergravity corrections are much less likely to destroy inflation. This also means that there may be less fine-tuning in choosing the parameters of the theory. The higher-dimensional couplings of order unity upon compactification can naturally produce small couplings needed to satisfy the COBE constraints, and these numbers can be perturbatively stable.

A very interesting novel feature of the assisted chaotic dynamics is that inflationary predictions depend softly on the

initial conditions preceding the stage of inflation. Most common models of inflation with a small number of dynamical scalar fields exert complete amnesia on the universe, which forgets all about the initial state before inflation. This is seen as a typical consequence of cosmic no-hair theorems. However assisted chaotic inflation appears to be more forgiving. Rather than completely washing away all the information about the state preceding inflation, at the level of precision we have pursued here assisted dynamics gives a density contrast and spectral index which depend logarithmically on the initial value of the inflationary scalars. In fact, this effect could have been expected due to the collective nature of the inflaton. To the subleading order in approximations, the predictions of dynamics should recognize how many fields contributed to inflation. On the other hand, the number of fields and the initial value of the inflationary scalars are related. Indeed, if we start with fewer fields higher up the potential, we may produce the same number of *e*-foldings as if we had more fields initially closer to their minima. Hence while both scenarios give the same picture to leading order, they differ in the sub-leading order. Since it is at this level that the density perturbations are produced, clearly they will depend on the initial values of fields and their number. The COBE normalization permits one to eliminate the number of fields in favor of the initial value of fields. Hence one, but not both,

of these parameters can be removed from the results for density perturbations and the spectral index, which therefore must depend softly on the initial value of fields. In the language of no-hair theorems, this dependence is analogous to a kind of discrete cosmic ''hair.'' Since it is soft, it will not jeopardize the onset of inflation. However, it leads to the possibility of getting different inflationary spectra from theories with the same zero mode parameters in the Lagrangian, and hence reduces the usefulness of observations in constraining inflationary models.

It would be very interesting to study generation of density perturbations in assisted chaotic models beyond the sub-leading order of approximations which we pursued here. The presence of the multitude of scalars and the absence of a late-time stable attractor could lead to additional interesting sub-leading corrections to the density spectrum. If such corrections are within the observable region, they could lead to an interesting signature of additional dimensions of the world visible in our own sky.

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