# Rho parameters from odd and even chirality, thermal QCD sum rules

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Like the even chirality correlation functions of, say, the two vector currents, one can also consider odd chirality correlation functions to write thermal QCD sum rules. They contain fewer non-perturbative corrections, at least to the leading order. Here we write such a sum rule for the correlation function of vector and tensor "currents." The odd and even chirality sum rules are taken together to evaluate the effective parameters of the $\rho$ meson to second order in temperature.

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## I. INTRODUCTION

The vacuum QCD sum rules [1], when extended to finite temperature [2], provide a simple means to study the properties of the QCD medium. Below the critical temperature, they relate the temperature dependence of hadron parameters to the thermal average of local operators [3–5].

Generally the sum rules are written for correlation functions of two currents, which have the same chirality, e.g., of two vector or two axial vector currents. But one may also consider correlation functions of two quark bilinears of opposite chiralities [6]. Then their operator product expansions will be dominated, in each dimension, by the operators of odd chirality, the contributions of even chirality ones being highly suppressed by a factor of (small) quark mass in their coefficient functions. As a result the leading nonperturbative corrections will be given by a single operator, namely, the quark condensate; new odd chirality operators appear only as non-leading terms.

Here we write down the sum rule for the correlation function of the vector current and the tensor "current." Then both the present vector-tensor sum rule and the two vector-vector sum rules derived earlier get contributions from the same set of intermediate states with $J^{PG}=1^{-+}$. We consider the subtracted sum rules, obtained by subtracting out the vacuum sum rules from the corresponding thermal ones. The only intermediate states with significant contributions are then the non-resonant $2\pi$ state and the $\rho$-resonance. The three sum rules form a convenient set for evaluation [7].

Although generally valid, the sum rules appear simpler in the chiral limit. We evaluate these limiting sum rules at low temperature $T$ to the leading order, which is $T^2$ in this limit. The thermal sum rules are, in general, complicated by the presence of Lorentz non-scalar operators, in addition to the Lorentz scalars contributing to the vacuum sum rules [8]. But to order $T^2$ non-scalar operators cannot contribute to these sum rules. Thus to this order, the contributing operators are the same as at zero temperature, their vacuum expectation values being replaced by the thermal averages.

In Sec.II we describe the kinematics and present the result of operator product expansion. In Sec.III we saturate the sum rules with $2\pi$ and $\rho$ states. In Sec. IV the sum rules are evaluated for the parameters of the $\rho$ meson to order $T^2$ in the chiral limit. Finally in Sec. V we discuss the different aspects of the thermal sum rules and compare our evaluation with earlier ones.

## II. SUM RULES

We restrict here to the better known non-strange channels of unit isospin. The quark bilinears in this channel are

$$S^a(x),V^a_\mu(x),T^a_{\mu\nu}(x),A^a_\mu(x),P^a(x)$$
$$=\overline{q}(x)(1,\gamma_\mu,\sigma_{\mu\nu},\gamma_\mu\gamma_5,\gamma_5)\frac{\tau^a}{2}q(x),$$

$q(x)$ being the field of the $u$ and $d$ quark doublet and $\tau^a$ the Pauli matrices. Note that $S^a(x)$, $P^a(x)$ and $T^a_{\mu\nu}(x)$ are odd and $V^a_\mu(x)$ and $A^a_\mu(x)$ are even under $\gamma_5$ transformation on the quark field.

As an important example of odd chirality correlation functions, consider the thermal average of the time ordered ($T$) product of the vector current and the tensor "current,"

$$\mathcal{T}^{ab}_{\mu\alpha\beta}(q)=i\int d^4x e^{iq\cdot x}\langle T(V^a_\mu(x)T^b_{\alpha\beta}(\mathrm{o}))\rangle. \qquad (2.1)$$

Here the thermal average of an operator $O$ is denoted by $\langle O\rangle$,

$$\langle O\rangle=\mathrm{Tr}\, e^{-\beta H}O/\mathrm{Tr}\, e^{-\beta H},$$

where $H$ is the QCD Hamiltonian, $\beta$ is the inverse of the temperature $T$ and Tr denotes the trace over any complete set of states.

As usual, it is convenient for kinematics to restore Lorentz invariance by introducing the four-velocity $u_\mu$ of matter. Then we can build the Lorentz scalars, $\omega=u\cdot q$ and $\overline{q}=\sqrt{\omega^2-q^2}$, representing the time and the space components of $q_\mu$ in the matter rest frame, $u_\mu=(1,0,0,0)$ [9]. We now choose the three independent kinematic covariants as

$$P_{\mu\alpha\beta}=\eta_{\mu\alpha}q_\beta-\eta_{\mu\beta}q_\alpha,$$

$$Q_{\mu\alpha\beta}=q_\mu(q_\alpha u_\beta-q_\beta u_\alpha)-q^2(\eta_{\mu\alpha}u_\beta-\eta_{\mu\beta}u_\alpha),$$

 

$$R_{\mu\alpha\beta}=u_{\mu}(q_{\alpha}u_{\beta}-q_{\beta}u_{\alpha})-\omega(\eta_{\mu\alpha}u_{\beta}-\eta_{\mu\beta}u_{\alpha}). \quad (2.2)$$

The kinematic decomposition now reads

$$\mathcal{T}^{ab}_{\mu\alpha\beta}(q)=i\delta^{ab}(P_{\mu\alpha\beta}T_1+Q_{\mu\alpha\beta}T_2+R_{\mu\alpha\beta}T_3), \quad (2.3)$$

where the invariant amplitudes $T_{1,2,3}$ are functions of $\omega$ and $q^2$. In all computations, however, we shall revert back to the matter rest frame.

Only the amplitude $T_1$ survives at zero temperature. As we shall see below, at finite temperature the leading contributions (to order $T^2$) are also contained in $T_1$. So, working to the leading order, we have only to write the sum rule for this amplitude.

The advantage with odd chirality correlation functions becomes evident from an enumeration of local operators. The unit operator corresponds to the perturbative result. The non-perturbative power corrections begin with operators of dimension three and four. At dimension three, we have $\bar{q}q$ and $\bar{q}\not{u}q$ belonging to odd and even chirality respectively. (The operator $\bar{q}\not{u}q$ actually cannot contribute for zero chemical potential.) At dimension four, we have $\mathrm{tr}G_{\mu\nu}G^{\mu\nu}$, $u^{\mu}\Theta^{f}_{\mu\nu}u^{\nu}$ and $u^{\mu}\Theta^{g}_{\mu\nu}u^{\nu}$. Here the gauge field strength $G_{\mu\nu}=g(\lambda^a/2)G^{a}_{\mu\nu}$, $\lambda^a$ $(a=1,\ldots,8)$ are the $SU(3)$ Gell-Mann matrices and $g$ is the QCD coupling constant. The operators $\Theta^{f,g}_{\mu\nu}$ are the energy momentum tensors for the quarks and gluons. (Note that in the matter rest frame, the latter two are just the energy densities. Also the operator $\bar{q}u\cdot Dq$ where $D_{\mu}$ is the covariant derivative, is of odd chirality, but it can be reduced to $\hat{m}\bar{q}\not{u}q$ by using the equation of motion for the quark field.) All these operators of dimension four are of even chirality. Thus up to dimension four, only the operator $\bar{q}q$ can contribute significantly to the power correction for an odd chirality correlation function. At dimension five, there is only one Lorentz scalar operator, $\bar{q}\sigma^{\mu\nu}G_{\mu\nu}q$. In addition there are several Lorentz non-scalar operators contributing at finite temperature [10].

There is a simple configuration space approach [11] to the operator product expansion giving the Wilson coefficients of all operators, both scalars and non-scalars [12,13]. Using this method and restricting to scalar operators, we get, for the product under consideration,

$$TV^{a}_{\mu}(x)T^{b}_{\alpha\beta}(0)\rightarrow\delta^{ab}(\eta_{\alpha\mu}x_{\beta}-\eta_{\beta\mu}x_{\alpha})\left\{\frac{3\hat{m}}{2\pi^4}\frac{1}{(x^2-i\epsilon)^3}\mathbf{1}+\frac{1}{2\pi^2}\frac{1}{(x^2-i\epsilon)^2}\bar{u}u+\frac{1}{24\pi^2}\frac{1}{(x^2-i\epsilon)}O_5+\cdots\right\}, \quad (2.4)$$

where $O_5$ is the dimension five operator, $O_5=\bar{u}\sigma_{\mu\nu}G^{\mu\nu}u$, and the dots represent operators of still higher dimensions. We have assumed $SU(2)$ flavor symmetry; $\hat{m}$ is the degenerate mass of $u$ and $d$ quarks and $\bar{u}u=\bar{d}d=\frac{1}{2}\bar{q}q$. The Fourier transform gives for space-like momenta $(Q^2=-q^2\geqslant 0)$,

$$\mathcal{T}^{ab}_{\mu\alpha\beta}(q)\rightarrow i\delta^{ab}(\eta_{\mu\alpha}q_{\beta}-\eta_{\mu\beta}q_{\alpha})\left\{-\frac{3\hat{m}}{8\pi^2}\log(Q^2/\mu^2)-\frac{1}{Q^2}\langle\bar{u}u\rangle+\frac{1}{3(Q^2)^2}\langle O_5\rangle+\cdots\right\}. \quad (2.5)$$

Here $\mu$ $(\simeq 1$ GeV) is the renormalization scale.

We wish to include the renormalization effects on the operators $T^{a}_{\mu\nu}$ and $\bar{u}u$. The coefficient $C(Q^2/\mu^2,g(\mu))$ of $\bar{u}u$ satisfies [14]

$$\left(\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}+\gamma_1-\gamma_2\right)C(Q^2/\mu^2,g(\mu))=0, \quad (2.6)$$

where $\beta(g)=-b[g^3/(4\pi)^2]$, $b=9$. The anomalous dimensions $\gamma_1$ and $\gamma_2$ of the operators $T^{a}_{\mu\nu}$ and $\bar{u}u$ can be easily calculated to give $\gamma_1=d_1[g^2/(4\pi)^2]$ and $\gamma_2=d_2[g^2/(4\pi)^2]$ with $d_1=8/3$ and $d_2=-8$. The solution to Eq. (2.6) may be written as

$$C(Q^2/\mu^2,g(\mu))=a(Q^2)C(1,\bar{g}(Q^2)),$$

where $C(1,\bar{g}(Q^2))$ is the lowest order result obtained above and

$$a(Q^2)=\left(\frac{\log(Q^2/\Lambda^2)}{\log(\mu^2/\Lambda^2)}\right)^{(d_1-d_2)/2b}.$$

The strong interaction scale is $\Lambda\simeq 200$ MeV.

The other element needed for the sum rule is the spectral representation for the correlation function. The $T$ product at finite temperature has the so-called Landau representation in the variable $q_0$ at fixed $|\vec{q}|$ [15], which is the finite temperature extension of the Low representation for the vacuum amplitude [16]. At points on the imaginary axis $(q_0^2=-Q_0^2, Q_0^2>0)$, the representation for the invariant amplitudes are, up to subtractions, given by

$$T_{1,2,3}(q_0^2,|\vec{q}|)=\int_0^{\infty}dq_0'^2\frac{N_{1,2,3}(q_0',|\vec{q}|)}{q_0'^2+Q_0^2}, \quad (2.7)$$

where

$$N_{1,2,3}(q_0,|\vec{q}|)=\pi^{-1}\mathrm{Im}T_{1,2,3}(q_0^2,|\vec{q}|)\tanh(\beta q_0/2).$$

### III. SATURATION

In the channel under consideration, the $\rho$-meson dominates the absorptive part. To find this contribution we write the matrix elements of the currents,

$$\langle 0|V_\mu^a|\rho^b(k)\rangle=\delta^{ab}F_\rho m_\rho\epsilon_\mu,$$

$$\langle 0|T_{\alpha\beta}^a|\rho^b(k)\rangle=i\delta^{ab}G_\rho(\epsilon_\alpha k_\beta-\epsilon_\beta k_\alpha). \quad (3.1)$$

Here $F_\rho$ and $G_\rho$ are two coupling constants and $m_\rho$ and $\epsilon_\alpha$ are the mass and the polarization vector of the $\rho$-meson. $F_\rho$ is measured by the electronic decay rate of $\rho^0$, $F_\rho=153$ MeV [17]. Though the value of $G_\rho$ is not available directly from experiment, it can be obtained from one of the U(6) symmetry relations for the wave functions of a quark-antiquark pair [18]. Defining the pion decay constant $F_\pi$ by

$$\langle 0|A_\mu^a|\pi^b(k)\rangle=i\delta^{ab}k_\mu F_\pi, \quad (3.2)$$

this relation is

$$G_\rho=\frac{1}{2}(F_\rho+F_\pi). \quad (3.3)$$

As $F_\pi=93$ MeV, we get $G_\rho=123$ MeV. This value of $G_\rho$ is also obtained from the QCD sum rules for the correlation function of two tensor ''currents'' [6]. Comparing Eqs. (3.1) with the matrix element of the $\rho$-meson field operator $\rho_\mu^a(x)$, $\langle 0|\rho_\mu^a|\rho^b\rangle=\delta^{ab}\epsilon_\mu$, we get the operator relations

$$V_\mu^a(x)=m_\rho F_\rho\rho_\mu^a(x), \quad T_{\alpha\beta}^a(x)=G_\rho(\partial_\alpha\rho_\beta^a-\partial_\beta\rho_\alpha^a).$$

The absorptive parts are now given essentially by that of the $\rho$-meson propagator at finite temperature. Working in the real time formulation of the thermal field theory [19], it is the 11-component of the $2\otimes 2$ matrix propagator. We get

$$N_1(q^2)=m_\rho F_\rho G_\rho\delta(q^2-m_\rho^2), \quad N_2(q^2)=N_3(q^2)=0. \quad (3.4)$$

This calculation must be interpreted as one in the effective field theory at finite temperature, where loop corrections make the parameters $m_\rho, F_\rho$ and $G_\rho$ temperature dependent. [At finite temperature, each of the particle-current coupling constants in Eqs. (3.1),(3.2) has, in general, different temperature dependence for the time and space components of the currents [20]. But this bifurcation takes place only in orders higher than $T^2$. So it does not concern us here.]

At lower energies there is the contribution of non-resonant two-pion state. Although small compared to the $\rho$-meson contribution, it describes the interaction of the currents with the pions in the heat bath and may assume importance in the difference sum rules we shall consider below. We find this absorptive part by writing the field theoretic expression for the pion loop at finite temperature. The pionic content $J_\mu^a(x)$ of the quark vector current $V_\mu^a(x)$ is given, to lowest order, by

$$V_\mu^a(x)\to J_\mu^a(x)=\epsilon^{abc}\phi^b(x)\partial_\mu\phi^c(x).$$

However the pionic content $S_{\alpha\beta}^a(x)$ of the quark tensor ''current'' $T_{\alpha\beta}^a(x)$ is not immediately known, as it is not a symmetry current. From its index structure it may be written as

$$T_{\alpha\beta}^a(x)\to S_{\alpha\beta}^a(x)=c\epsilon^{abc}\partial_\alpha\phi^b(x)\partial_\beta\phi^c(x),$$

where we determine the constant $c$ by comparing the two divergences,

$$\partial^\alpha T_{\alpha\beta}^a(x)=2\hat{m}V_\beta^a(x)+\cdots, \quad \partial^\alpha S_{\alpha\beta}^a(x)=cm_\pi^2 J_\beta^a(x)+\cdots,$$

the dots standing for higher derivative terms. Like $V_\mu^a(x)$ and $J_\mu^a(x)$, we can also identify $T_{\alpha\beta}^a(x)$ and $S_{\alpha\beta}^a(x)$, giving $c=2\hat{m}/m_\pi^2=-F_\pi^2/\langle 0|\bar{u}u|0\rangle$, on using the Gell-Mann, Oakes and Renner relation [21].

With the pionic version of $V_\mu^a(x)$ and $T_{\mu\nu}^a(x)$, it is simple to evaluate Eq. (2.1) to lowest order as

$$T_{\mu\alpha\beta}^{ab}(q)=-c\delta^{ab}\int\frac{d^4k}{(2\pi)^4}(2k-q)_\mu(q_\alpha k_\beta-q_\beta k_\alpha)\Delta_{11}(k)\Delta_{11}(k-q), \quad (3.5)$$

where $\Delta_{11}$ is the 11-component of the thermal pion propagator. Its absorptive part can be calculated in the same way as for the vector-vector correlation function [5]. As we need to write the sum rule for the amplitude $T_1$, we quote the results for this amplitude only. In the time-like region (superscript $+$),

$$N_1^+=\frac{cq^2}{128\pi^2}\int_{-v}^{v}dx(v^2(q^2)-x^2)\{1+2n((|\vec{q}|x+q_0)/2)\},$$

$$\text{for} \quad q^2\geqslant 4m_\pi^2, \quad (3.6)$$

while in the space-like region (superscript $-$),

$$N_1^-=\frac{cq^2}{128\pi^2}\int_{v}^{\infty}dx(v^2(q^2)-x^2)\{n((|\vec{q}|x-q_0)/2)-n((|\vec{q}|x+q_0)/2)\}, \quad \text{for} \quad q^2\leqslant 0. \quad (3.7)$$

Here we have defined the functions $v(z)=\sqrt{1-4m_\pi^2/z}$ and $n(z)=(e^{\beta z}-1)^{-1}$.

For $|\vec{q}|\to 0$, they reduce to simple expressions. In the time-like region,

$$N_1^+=\frac{cq_0^2v^3(q_0^2)}{96\pi^2}\{1+2n(q_0/2)\}. \quad (3.8)$$

In the space-like region, $q_o^2=\lambda|\vec{q}|^2$, with $0\leqslant\lambda\leqslant 1$. Thus in the limit $|\vec{q}|\to 0$, $N_1^-$ and its contribution to the spectral representation (2.7) are zero.

We can now write a spectral sum rule by equating at a space-like point the operator product expansion (2.5) to the spectral representation (2.7) with $\rho$ and $2\pi$-contributions. Taking Borel transform as usual to get rid of any subtraction constant and to improve convergence of the spectral integral, we arrive at the thermal QCD sum rule

$$m_\rho(T)F_\rho(T)G_\rho(T)e^{-m_\rho^2(T)/M^2} - \frac{F_\pi^2}{96\pi^2\langle 0|\bar{u}u|0\rangle}\int_{4m_\pi^2}^{\infty} ds s e^{-s/M^2} v^3(s)(1+2n(\sqrt{s}/2)) = \frac{3\hat{m}}{8\pi^2}M^2 - a(M^2)\langle\bar{u}u\rangle + \frac{1}{3M^2}\langle O_5\rangle. \tag{3.9}$$

## IV. EVALUATION

As already stated, we shall evaluate sum rules to order $T^2$ in the chiral limit. To avoid new symbols in this section, we let the old ones denote their respective chiral limits. It is easy to check that the integral in Eq. (3.9) is $0(T^4)$ and the odd chirality sum rule, after subtraction of the corresponding vacuum sum rule, simplifies to

$$m_\rho(T)F_\rho(T)G_\rho(T)e^{-m_\rho^2(T)/M^2} - m_\rho F_\rho G_\rho e^{-m_\rho^2/M^2} = -a(M^2)\overline{\langle\bar{u}u\rangle} + \frac{1}{3M^2}\overline{\langle O_5\rangle}. \tag{4.1}$$

Here the bar over the thermal average indicates subtraction of the corresponding vacuum expectation value.

For the sake of consistent evaluation in a closed framework, we augment this sum rule with the two even chirality sum rules for the vector-vector correlation function derived earlier [5]. In the chiral limit and omitting terms of order higher than $T^2$, they become

$$F_\rho^2(T)e^{-m_\rho^2(T)/M^2} - F_\rho^2 e^{-m_\rho^2/M^2} + I_T(M^2) = -\frac{\pi}{4}\frac{\overline{\langle O_6\rangle}}{M^4}, \tag{4.2}$$

$$F_\rho^2(T)m_\rho^2(T)e^{-m_\rho^2(T)/M^2} - F_\rho^2 m_\rho^2 e^{-m_\rho^2/M^2} = \frac{\pi}{2}\frac{\overline{\langle O_6\rangle}}{M^2}, \tag{4.3}$$

where

$$I_T(M^2) = \frac{1}{24\pi^2}\int_0^\infty ds(1+e^{-s/M^2})\frac{1}{e^{\sqrt{s}/2T}-1} = \frac{T^2}{9} + 0(T^4) \tag{4.4}$$

and $O_6$ is the four-quark operator, $O_6 = \alpha_s\bar{q}\gamma_\mu\gamma_5\lambda^a\tau^3 q\ \bar{q}\gamma^\mu\gamma_5\lambda^a\tau^3 q$ [22]. This operator was ignored in [5].

At low temperature the heat bath consists primarily of dilute pion gas. The thermal trace can then be approximated by the vacuum and the one pion state. The pion matrix element of the operator can be evaluated by using PCAC and soft pion technique [3]. One gets

$$\langle\bar{u}u\rangle = \langle 0|\bar{u}u|0\rangle\left(1-\frac{T^2}{8F_\pi^2}\right),$$

$$\langle O_5\rangle = \langle 0|O_5|0\rangle\left(1-\frac{T^2}{8F_\pi^2}\right),$$

$$\langle O_6\rangle = \langle 0|O_6|0\rangle\left(1-\frac{T^2}{3F_\pi^2}\right). \tag{4.5}$$

where the vacuum expectation values are all known [1,23–25],

$$\langle 0|\bar{u}u|0\rangle = -(225\ \text{MeV})^3,$$

$$\langle 0|O_5|0\rangle = m_o^2\langle 0|\bar{u}u|0\rangle,$$

$$\langle 0|O_6|0\rangle = 6.5\times 10^{-4}\ \text{GeV}^6, \tag{4.6}$$

with $m_o^2 = .8\ \text{GeV}^2$. If we write

$$m_\rho(T) = m_\rho\left(1 + a\frac{T^2}{F_\pi^2}\right),$$

$$F_\rho(T) = F_\rho\left(1 + b\frac{T^2}{F_\pi^2}\right),$$

$$G_\rho(T) = G_\rho\left(1 + c\frac{T^2}{F_\pi^2}\right), \tag{4.7}$$

and use the relation (3.3), the sum rules (4.1)–(4.3) predict the values of the constants *a*, *b* and *c* as

$$a = f(M^2)\left\{\frac{1}{18} - \frac{K_2}{4M^4}\left(1+\frac{2M^2}{m_\rho^2}\right)\right\}, \tag{4.8}$$

$$b = -f(M^2)\left\{\frac{1}{18}\left(1-\frac{m_\rho^2}{M^2}\right) + \frac{K_2}{4M^4}\left(1+\frac{m_\rho^2}{M^2}\right)\right\}, \tag{4.9}$$

$$c = f(M^2)\left\{\frac{m_\rho^2}{18M^2} + K_1\left(a(M^2) - \frac{m_0^2}{3M^2}\right) - \frac{K_2}{4M^4}\left(\frac{m_\rho^2}{M^2} - \frac{2M^2}{m_\rho^2} + 2\right)\right\}, \tag{4.10}$$

where

$$f(M^2) = (F_\pi/F_\rho)^2 e^{m_\rho^2/M^2},$$

$$K_1 = \frac{\langle 0|\bar{u}u|0\rangle}{4m_\rho F_\pi^2(1+F_\pi/F_\rho)},$$

$$K_2 = \frac{\pi}{6F_\pi^2}\langle 0|O_6|0\rangle.$$

TABLE I. Coefficients of $T^2/F_\pi^2$ in $m_\rho(T)$, $F_\rho(T)$ and $G_\rho(T)$ at different values of $M^2$.

| $M^2$ ($\mathrm{GeV}^2$) | $a$ | $b$ | $c$ |
|---|---|---|---|
| 0.8 | -.023 | -.034 | -.086 |
| 1.0 | -.009 | -.028 | -.098 |
| 1.2 | -.001 | -.025 | -.106 |
| 2.0 | .010 | -.022 | -.125 |
| 4.0 | .016 | -.021 | -.142 |

We recall that the parameters $F_\pi$, $m_\rho$ and $F_\rho$ in the above equations refer to their chiral limits. Their estimates in this limit are [26]

$$F_\pi = 86 \ \mathrm{MeV}, \quad m_\rho = 630 \ \mathrm{MeV}, \quad F_\rho = 142 \ \mathrm{MeV}.$$

Table I shows the evaluation of the constants $a$, $b$ and $c$ for some values of the Borel parameter $M^2$ over the range $0.8 \leqslant M^2 \leqslant 4$ in $\mathrm{GeV}^2$. It is seen that as $M^2$ increases, the values change rather slowly. Averaging the results over this range, we get

$$a \simeq .006, \quad b \simeq -.023, \quad c \simeq -.12. \tag{4.11}$$

Note that to within errors the value of $a$ is consistent with zero.

There are independent methods to demonstrate that masses do not shift to $O(T^2)$ in the chiral limit [27–29]. Thus the fact that the coefficient $a$ does not vanish identically raises the question if we have included all the relevant contributions in the sum rules. To answer this question, let us note that on the phenomenological side, the $O(T^2)$ terms can arise only from $2\pi$ and $\rho$ intermediate states. (Interaction of current with more pions in the heat bath leads to higher order terms in $T$.) As regards the operators, *all* Lorentz scalar operators, irrespective of their dimensions, contribute to $O(T^2)$. Of course, the higher the dimension, the more will its contribution be suppressed. Thus even the operators of still higher dimension, for example, eight can contribute to the coefficient $a$, however small. As a result, we do not expect our expression for $a$ to vanish identically in the sum rule method.

## V. DISCUSSION

We begin by recalling the earlier criticisms to finite temperature extension of the QCD sum rules [29] and how they are overcome in later works [3]. The analytic structure at finite temperature appears complicated if the trace in the correlation function (2.1) is expanded over the physical states, as each term will have discontinuities not only in $q^2$ but also in its Mandelstam variables. By contrast, the Landau spectral representation in $q_0$ at fixed $|\vec{q}|$ provides a simple way to deal with these singularities. Its absorbtive part is due to the usual threshold singularities along with those due to the interaction of current with the real particles of the heat bath. As can be inferred from the results in Sec. III, the latter singularities give rise to a short cut in the $q_0$ variable around the origin, even from the increasingly massive states. Thus these contributions are not suppressed by the spectral denominator (or by the exponential in the Borel version). But the thermal distribution function does provide the exponential suppression of the massive states at low temperature. On the operator expansion side, it is clearly the short distance expansion which is relevant for large $q_o$ at fixed $|\vec{q}|$, rather than the one on the light cone.

In this work we derive an odd chirality thermal QCD sum rule. More such sum rules can be written by considering other correlation functions of odd chirality, for example, of the axial vector and pseudo-scalar currents. Generally they bring in a different set of operator expectation values, but are fewer in number compared to those in the even chirality sum rules, at least to the leading order. It would be useful to consider both the even and the odd sets of sum rules for numerical evaluation.

The subtraction of the vacuum sum rule eliminates the continuum contribution beyond about 1 GeV, as it is practically independent of $T$ for $T \leqslant 150 \ \mathrm{MeV}$, say. It should thus be possible to extend the range of Borel parameter $M$ on the upper side. On the other hand, the subtraction also removes the $T$-independent coefficient of the unit operator. So the higher dimension operators, which contributed earlier only (power) corrections to the unit operator, are now assuming the leading role. This sensitivity to higher dimension operators may be reduced if $M$ is not extended to too low values. Our evaluation shows that the results are indeed approximately constant over a wide interval of $M^2$, if its lower end is chosen not too small.

The present work is a continuation of our earlier work [5]. There our main concern was to include all the operators up to dimension four with their renormalization effects in the vector-vector sum rules. But the omission of the scalar, four quark operator was not justified. For, while the dimension four operators are significant at higher temperatures near the critical point, at low temperatures it is only this dimension six operator which dominates, contributing to order $T^2$ in the chiral limit.

The most extensive earlier work is that of Hatsuda *et al.* [3]. They include contributions up to order $T^6$ from the operators and plot the $T$-dependence of $m_\rho(T)$ and $F_\rho(T)$ ( and also parameters of other resonances) but do not obtain the coefficients of $T^2, T^4$ etc. (Incidentally their evaluation is incomplete, because they include contributions of order $T^4$ and $T^6$ only when they are leading in some operators, but neglect non-leading contributions even to order $T^4$ in others, e.g., $\langle \bar{u}u \rangle$.)

Finally let us compare the sum rule approach to the two point function with its direct evaluation with the one pion state to get the temperature dependence of the resonance parameters [28,29]. The sum rules relate these observable parameters to elements of the theoretical structure of the QCD theory, namely the thermal average of a number of local operators, built out of quark and gluon fields. It would thus appear that this procedure is rather involved compared with the other method. However the situation is actually not so. In the entire scheme of sum rules in channels with different quantum numbers, the same set of operators keep appearing with different resonances. Also the sum rules are formulated in terms of physical values of particle parameters, which may, for simplicity, be reduced to the chiral limit. But the other method is inherently tied up to this unphysical limit.


[1] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979); for a collection of original papers and comments, see *Vacuum Structure and QCD Sum Rules*, edited by M. A. Shifman (North-Holland, Amsterdam, 1992); see also S. Narison, *QCD Spectral Sum Rules* (World Scientific, Singapore, 1989).

[2] A. I. Bochkarev and M. E. Shaposhnikov, Nucl. Phys. **B268**, 220 (1986).

[3] T. Hatsuda, Y. Koike, and S. H. Lee, Nucl. Phys. **B394**, 221 (1993). The earlier papers may be traced from this one.

[4] V. L. Eletsky and B. L. Ioffe, Phys. Rev. D **51**, 2371 (1995).

[5] S. Mallik and K. Mukherjee, Phys. Rev. D **58**, 096011 (1998).

[6] S. Mallik, Nucl. Phys. **B206**, 90 (1982).

[7] One might think of considering also the correlation function of two tensor currents. But the corresponding sum rules bring in the $J^{PG}=1^{++}$ resonance also.

[8] E. V. Shuryak, Rev. Mod. Phys. **65**, 1 (1993).

[9] H. A. Weldon, Phys. Rev. D **26**, 1394 (1982).

[10] X. Jin, T. D. Cohen, R. J. Furnstahl, and D. K. Griegel, Phys. Rev. C **47**, 2882 (1993).

[11] H. Fritzsch and H. Leutwyler, Phys. Rev. D **10**, 1624 (1974).

[12] W. Hubschmid and S. Mallik, Nucl. Phys. **B207**, 29 (1982).

[13] S. Mallik, Phys. Lett. B **416**, 373 (1998).

[14] See, for example, M.E. Peskin and D.V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, Reading, MA, 1995).

[15] L. D. Landau, Zh. Éksp. Teor. Fiz. **34**, 261; (1958) [Sov. Phys. JETP **7**, 182 (1958)]; reproduced in *Collected Works of L. D. Landau*, edited by D. Ter Haar (Pergamon, New York, 1965); see also A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971).

[16] F. E. Low, Phys. Rev. **97**, 1392 (1955).

[17] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).

[18] H. Leutwyler, Nucl. Phys. **B76**, 413 (1974).

[19] A. J. Niemi and G. W. Semenoff, Ann. Phys. (N.Y.) **152**, 105 (1984); see also N. P. Landsman and Ch. G. van Weert, Phys. Rep. **145**, 141 (1987).

[20] H. Leutwyler, Phys. Rev. D **49**, 3033 (1994); R. D. Pisarski and M. Tytgat, *ibid.* **54**, 2989 (1996); D. Toublan, *ibid.* **56**, 5629 (1997).

[21] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).

[22] The other scalar piece in $O_6$ does not contribute to order $T^2$.

[23] J. Gasser and H. Leutwyler, Phys. Lett. B **184**, 83 (1987); P. Gerber and H. Leutwyler, Nucl. Phys. **B321**, 387 (1989).

[24] V. M. Belyaev and B. L. Ioffe, Zh. Éksp. Teor. Fiz. **84**, 1236 (1983) [Sov. Phys. JETP **57**, 716 (1983)].

[25] A higher value has been suggested for $\langle 0|O_6|0\rangle$ in S. Narison, *QCD Spectral Sum Rules* (World Scientific, Singapore, 1989). Again the vacuum saturation value becomes smaller with the present value for $\langle 0|\overline{u}u|0\rangle$. Here we stick to the value originally obtained in [1].

[26] J. Bijnens, Phys. Rep. **265**, 369 (1996); J. Bijnens, P. Gosdzinsky, and P. Talavera, Nucl. Phys. **B501**, 495 (1997); G. Ecker, J. Gasser, H. Leutwyler, A. Pich, and E. De Rafael, Phys. Lett. B **223**, 425 (1989).

[27] H. Leutwyler and A. V. Smilga, Nucl. Phys. **B342**, 620 (1990).

[28] M. Dey, V. L. Eletsky, and B. L. Ioffe, Phys. Lett. B **252**, 620 (1990).

[29] V. L. Eletsky and B. L. Ioffe, Phys. Rev. D **47**, 3083 (1993).