Angular distribution and *CP* asymmetries in the decays $\bar{B}\to K^-\pi^+e^-e^+$ and $\bar{B}\to\pi^-\pi^+e^-e^+$

Frank Krüger*
Centro de Física das Interacções Fundamentais (CFIF), Departamento de Física, Instituto Superior Técnico, Av. Rovisco Pais, 1049-001 Lisboa, Portugal

Lalit M. Sehgal†
Institut für Theoretische Physik (E), RWTH Aachen, D-52056 Aachen, Germany

Nita Sinha‡ and Rahul Sinha§
Institute of Mathematical Sciences, Taramani, Chennai 600113, India
(Received 19 July 1999; published 11 May 2000)

The short-distance Hamiltonian describing $b\to s(d)e^-e^+$ in the standard model is used to obtain the decay spectrum of $\bar{B}\to K^-\pi^+e^-e^+$ and $\bar{B}\to\pi^-\pi^+e^-e^+$, assuming the $K\pi$ and $\pi\pi$ systems to be the decay products of K^* and ρ respectively. The specific features calculated are (i) the angular distribution of K^- (or π^-) in the $K^-\pi^+$ (or $\pi^-\pi^+$) center-of-mass (c.m.) frame, (ii) the angular distribution of e^- in the e^-e^+ c.m. frame, and (iii) the correlation between the meson and lepton planes. We also derive *CP*-violating observables obtained by combining the above decays with the conjugate processes $B\to K^+\pi^-e^-e^+$ and $B\to\pi^-\pi^+e^-e^+$.

PACS number(s): 13.20.He, 11.30.Er

I. INTRODUCTION

The purpose of this paper is to investigate the angular distribution of the decays $\bar{B}\to K^-\pi^+e^-e^+$ and $\bar{B}\to\pi^-\pi^+e^-e^+$, when the $K\pi$ and $\pi\pi$ systems are the decay products of K^* and ρ respectively. The aim is to derive the detailed consequences of the effective short-distance Hamiltonian describing the four-fermion interaction $b\to s(d)e^-e^+$. This work may be regarded as an extension of previous studies of the exclusive processes $\bar{B}\to K^*(\rho)e^-e^+$[1,2] that were limited to the kinematical variables s_l (the invariant mass of the lepton pair) and $\cos\theta_l$ (the angular distribution of l^- in the l^-l^+ c.m. system). The additional information we provide is the distribution in $\cos\theta_P$, where θ_P is the angle of the K^- (π^-) in the $K^-\pi^+$ ($\pi^-\pi^+$) c.m. frame, and the dependence on the angle ϕ between the e^-e^+ and $K^-\pi^+$ or $\pi^-\pi^+$ planes. This information is sensitive to the polarization state of the vector meson K^* (ρ), and thus provides a new probe of the effective Hamiltonian.

An important aspect of the calculation is the prediction of *CP*-violating observables that can be obtained by combining information from B and $\bar{B}$ decays. These observables probe a term in the Hamiltonian proportional to $\mathrm{Im}\,(V_{ub}V_{us}^*/V_{tb}V_{ts}^*)$ (in the case of $B\to K^*$) and $\mathrm{Im}\,(V_{ub}V_{ud}^*/V_{tb}V_{td}^*)$ (in the case of $B\to\rho$). While the numerical estimates of these asymmetries in the standard model turn out to be small, the formalism we present is also applicable to more general Hamiltonians transcending the standard model. The dependence on the variable ϕ can be an especially useful probe of *CP* violation, as has been demonstrated in the analogous case of $K_L\to\pi^+\pi^-e^+e^-$[3,4].

II. MATRIX ELEMENT

We are concerned with the matrix element of the decay $\bar{B}(p)\to P(k_1)P'(k_2)l^+(q_1)l^-(q_2)$, where $PP'=K^-\pi^+$ or $\pi^-\pi^+$ and $l=e,\mu$. Introducing the linear combinations [5,6]

$$k=k_1+k_2,\ K=k_1-k_2,\ q=q_1+q_2,\ Q=q_1-q_2, \tag{2.1}$$

the short-distance Hamiltonian for $b\to fl^+l^-$ ($f=s$ or d) is [2,7,8]

$$H_{\rm eff}=\frac{G_F\alpha}{\sqrt{2}\pi}V_{tb}V_{tf}^*\Big\{c_9^{\rm eff}(\bar{f}\gamma_\mu P_L b)\bar{l}\gamma^\mu l + c_{10}(\bar{f}\gamma_\mu P_L b)\bar{l}\gamma^\mu\gamma^5 l - 2c_7^{\rm eff}\bar{f}i\sigma_{\mu\nu}\frac{q^\nu}{q^2}(m_bP_R+m_fP_L)b\bar{l}\gamma^\mu l\Big\}, \tag{2.2}$$

where $P_{L,R}=(1\mp\gamma_5)/2$ and

$$c_7^{\rm eff}=-0.315,\ c_{10}=-4.642, \tag{2.3}$$

$$c_9^{\rm eff}=c_9+(3c_1+c_2)\{g(m_c,s_l)+\lambda_u[g(m_c,s_l)-g(m_u,s_l)]\}+\cdots$$
$$=4.224+0.361[(1+\lambda_u)g(m_c,s_l)-\lambda_u g(m_u,s_l)]+\cdots, \tag{2.4}$$

with

*Email address: krueger@gtae3.ist.utl.pt
†Email address: sehgal@physik.rwth-aachen.de
‡Email address: nita@imsc.ernet.in
§Email address: sinha@imsc.ernet.in

$$\lambda_u \equiv \frac{V_{ub}V_{uf}^*}{V_{tb}V_{tf}^*}, \tag{2.5}$$

$$g(m_i,s_l) = -\frac{8}{9}\ln(m_i/m_b) + \frac{8}{27} + \frac{4}{9}y_i - \frac{2}{9}(2+y_i)\sqrt{|1-y_i|} \times \left\{ \Theta(1-y_i)\left(\ln\left(\frac{1+\sqrt{1-y_i}}{1-\sqrt{1-y_i}}\right) - i\pi\right) + \Theta(y_i-1)2\arctan\frac{1}{\sqrt{y_i-1}}\right\}, \tag{2.6}$$

where $s_l \equiv q^2$ and $y_i = 4m_i^2/s_l$. The ellipses in the above represent numerically insignificant terms involving the Wilson coefficients $c_3, \ldots, c_6$ (see, e.g., Ref. [2]). The corresponding matrix element is

$$\mathcal{M}_{\mathrm{SD}} = \frac{G_F\alpha}{\sqrt{2}\pi} V_{tb}V_{tf}^* \left\{ \left[(c_9^{\mathrm{eff}} - c_{10})\langle P(k_1)P'(k_2)|\bar{f}\gamma_\mu P_L b|\bar{B}(p)\rangle - \frac{2c_7^{\mathrm{eff}}}{q^2}\langle P(k_1)P'(k_2)|\bar{f}i\sigma_{\mu\nu}q^\nu(m_bP_R + m_fP_L)b|\bar{B}(p)\rangle \right] \bar{l}\gamma^\mu P_L l + [c_{10} \to -c_{10}]\bar{l}\gamma^\mu P_R l \right\}. \tag{2.7}$$

Observe that the coefficient c_9^{eff} contains both a weak phase (associated with the imaginary part of λ_u) and a strong phase [connected with the imaginary part of $g(m_i,s_l)$], thus opening the way to CP-violating asymmetries between B and $\bar{B}$ decays.

The hadronic part of the matrix element, describing the transition $\bar{B} \to PP'$, can be written in terms of $B \to V$ form factors ($V = K^*$ or ρ)

$$\langle P(k_1)P'(k_2)|\bar{f}\gamma_\mu P_L b|\bar{B}(p)\rangle = -D_V(k^2)\left\{ i\epsilon_{\mu\nu\alpha\beta}K^\nu p^\alpha k^\beta g(q^2) + \frac{1}{2}K_\mu f(q^2) + k_\mu\left[(q\cdot W)a_+(q^2) - \frac{1}{2}\xi f(q^2)\right] + \cdots \right\}, \tag{2.8}$$

with the convention $\epsilon_{0123} = +1$,

$$\langle P(k_1)P'(k_2)|\bar{f}i\sigma_{\mu\nu}q^\nu P_{R,L} b|\bar{B}(p)\rangle = D_V(k^2)\left\{ -i\epsilon_{\mu\nu\alpha\beta}K^\nu p^\alpha k^\beta g_+(q^2) \mp \frac{1}{2}K_\mu[g_+(q^2)\Delta + q^2 g_-(q^2)] \pm k_\mu\left\{(q\cdot W)\left[g_+(q^2) + \frac{1}{2}q^2 h(q^2)\right] + \frac{1}{2}\xi[g_+(q^2)\Delta + q^2 g_-(q^2)]\right\} + \cdots \right\}, \tag{2.9}$$

where the ellipses denote terms proportional to q_μ which may be dropped in the case of massless leptons and

$$\Delta = (M_B^2 - M_V^2),\ W^\mu = K^\mu - \xi k^\mu,\ \xi = \frac{M_P^2 - M_{P'}^2}{k^2}. \tag{2.10}$$

Throughout this paper, we neglect the lepton mass, and assume the above form factors to be real, the absorptive parts due to real $c\bar{c}$ and $u\bar{u}$ states having been included in the functions $g(m_c,s_l)$ and $g(m_u,s_l)$ appearing in the short-distance coefficient c_9^{eff} [Eq. (2.4)]. Furthermore, we will limit ourselves to neutral B mesons, i.e. $\bar{B} \equiv \bar{B}^0$.

The function $D_V(k^2)$ appearing in Eqs. (2.8) and (2.9) is defined via

$$|D_V(k^2)|^2 = \frac{48\pi^2}{\beta^3 M_V^2}\delta(k^2 - M_V^2), \tag{2.11}$$

where we have used a narrow-width approximation, and

$$\beta = \frac{\lambda^{1/2}(k^2, M_P^2, M_{P'}^2)}{k^2}, \tag{2.12}$$

with the triangle function

$$\lambda(a,b,c) = a^2 + b^2 + c^2 - 2(ab + bc + ac). \tag{2.13}$$

We adopt the $B \to K^*$ and $B \to \rho$ form factors g, f, h, $g_\pm$, a_+ given by Melikhov and Nikitin [9] (see Appendix A). The matrix element (2.7) can then be written compactly as

$$\mathcal{M}_{\mathrm{SD}} = \frac{G_F\alpha}{\sqrt{2}\pi}V_{tb}V_{tf}^*\{(i\epsilon_{\mu\nu\alpha\beta}K^\nu k^\alpha q^\beta x_L + K_\mu y_L + k_\mu z_L)\bar{l}\gamma^\mu P_L l + (L \to R)\}, \tag{2.14}$$

where

$$x_{L,R}=D_V(k^2)\left\{(c_9^{\rm eff}\mp c_{10})g(q^2)-\frac{2c_7^{\rm eff}}{q^2}(m_b+m_f)g_+(q^2)\right\}, \tag{2.15}$$

$$y_{L,R}=-\frac{1}{2}D_V(k^2)\left\{(c_9^{\rm eff}\mp c_{10})f(q^2)-\frac{2c_7^{\rm eff}}{q^2}(m_b-m_f)\times[g_+(q^2)\Delta+q^2g_-(q^2)]\right\}, \tag{2.16}$$

$$z_{L,R}=(q\cdot W)z'_{L,R}-\xi y_{L,R}, \tag{2.17}$$

with

$$z'_{L,R}=-D_V(k^2)\left\{(c_9^{\rm eff}\mp c_{10})a_+(q^2)+\frac{2c_7^{\rm eff}}{q^2}(m_b-m_f)\times\left[g_+(q^2)+\frac{1}{2}q^2h(q^2)\right]\right\}. \tag{2.18}$$

The matrix element for the corresponding antiparticle channel can be obtained from Eq. (2.14) by means of *CPT* invariance. In fact, we have

$$\overline{\mathcal{M}}_{\rm SD}=\mathcal{M}_{\rm SD}(x_{L,R}\to-\overline{x}_{L,R};y_{L,R}\to\overline{y}_{L,R};z_{L,R}\to\overline{z}_{L,R};Q\to-Q), \tag{2.19a}$$

and

$$\overline{\mathcal{M}}_{\rm SD}=\mathcal{M}_{\rm SD}(x_{L,R}\to-\overline{x}_{L,R};y_{L,R}\to\overline{y}_{L,R};z_{L,R}\to\overline{z}_{L,R};Q\to-Q,K\to-K), \tag{2.19b}$$

for the conjugate processes $B\to K^+\pi^-e^-e^+$ and $B\to\pi^-\pi^+e^-e^+$ respectively, where Q and K are as in Eq. (2.1), and $\overline{x},\overline{y},\overline{z}$ are related to x,y,z by

$$\phi_{\rm w}\to-\phi_{\rm w},\ \ \delta\to\delta. \tag{2.20}$$

Here $\phi_{\rm w}$ and δ denote the weak and strong phases respectively that appear in the matrix element $\mathcal{M}_{\rm SD}$.

III. DIFFERENTIAL DECAY RATE

Squaring the matrix element (2.14) and summing over spins, we obtain for the decay $\overline{B}\to PP'l^-l^+$ [10]

$$\begin{aligned}|\mathcal{M}_{\rm SD}(\overline{B}\to PP'l^-l^+)|^2=\frac{G_F^2\alpha^2}{2\pi^2}|V_{tb}V_{tf}^*|^2\{&2\,{\rm Re}\,(y_L^*z_L+y_R^*z_R)[(k\cdot q)(q\cdot K)-(k\cdot Q)(K\cdot Q)-s_l(k\cdot K)]\\&+2\,{\rm Re}\,(x_L^*z_L-x_R^*z_R)[(k\cdot q)(k\cdot Q)(q\cdot K)+s_ls_P(K\cdot Q)-(k\cdot q)^2(K\cdot Q)\\&-s_l(k\cdot K)(k\cdot Q)]+2\,{\rm Re}\,(x_Ry_R^*-x_Ly_L^*)[s_lK^2(k\cdot Q)-(k\cdot Q)(q\cdot K)^2\\&+(k\cdot q)(q\cdot K)(K\cdot Q)-s_l(k\cdot K)(K\cdot Q)]+(x_L^2+x_R^2)[-2(k\cdot q)(k\cdot Q)(q\cdot K)(K\cdot Q)\\&-s_ls_P(K\cdot Q)^2+(k\cdot q)^2(K\cdot Q)^2-s_lK^2(k\cdot Q)^2+(k\cdot Q)^2(q\cdot K)^2\\&+2s_l(k\cdot K)(k\cdot Q)(K\cdot Q)]+(y_L^2+y_R^2)[-s_lK^2+(q\cdot K)^2-(K\cdot Q)^2]\\&+(z_L^2+z_R^2)[-s_ls_P+(k\cdot q)^2-(k\cdot Q)^2]+2\epsilon_{\mu\nu\alpha\beta}k^\mu K^\nu q^\alpha Q^\beta[(K\cdot Q){\rm Im}\,(x_Ly_L^*+x_Ry_R^*)\\&+{\rm Im}\,(y_R^*z_R-y_L^*z_L)-(k\cdot Q){\rm Im}\,(x_L^*z_L+x_R^*z_R)]\},\end{aligned} \tag{3.1}$$

with $s_P\equiv k^2$. Introducing the shorthand notation

$$X=[(k\cdot q)^2-s_ls_P]^{1/2}=\frac{1}{2}\lambda^{1/2}(M_B^2,s_l,s_P), \tag{3.2}$$

we find that $(m_l=0)$

$$k\cdot K=M_P^2-M_{P'}^2=\xi s_P, \tag{3.3a}$$

$$k\cdot q=\frac{1}{2}(M_B^2-s_l-s_P), \tag{3.3b}$$

$$k\cdot Q=X\cos\theta_l, \tag{3.3c}$$

$$q\cdot K=\beta X\cos\theta_P+\xi(k\cdot q), \tag{3.3d}$$

$$q\cdot W=\beta X\cos\theta_P, \tag{3.3e}$$

$$K\cdot Q=\xi(k\cdot Q)+\beta[(k\cdot q)\cos\theta_l\cos\theta_P-(s_ls_P)^{1/2}\sin\theta_l\sin\theta_P\cos\phi], \tag{3.3f}$$

$$\epsilon_{\mu\nu\alpha\beta}k^\mu K^\nu q^\alpha Q^\beta=-(s_ls_P)^{1/2}\beta X\sin\theta_l\sin\theta_P\sin\phi, \tag{3.3g}$$

$$K^2=2(M_P^2+M_{P'}^2)-s_P=(\xi^2-\beta^2)s_P,\ \ Q^2=-s_l. \tag{3.3h}$$

To write the differential decay rate in compact form, we define the auxiliary functions

$$F_{1L,R}=\beta[X^2 z'_{L,R}+(k\cdot q)y_{L,R}], \tag{3.4}$$

$$F_{2L,R}=\beta(s_l s_P)^{1/2}y_{L,R}, \tag{3.5}$$

$$F_{3L,R}=\beta X(s_l s_P)^{1/2}x_{L,R}, \tag{3.6}$$

so that

$$d^5\Gamma=\frac{G_F^2\alpha^2}{2^{16}\pi^8 M_B^3}\times|V_{tb}V_{tf}^*|^2\beta X I(s_l,s_P,\theta_l,\theta_P,\phi)\times ds_l\,ds_P\,d\cos\theta_l\,d\cos\theta_P\,d\phi, \tag{3.7}$$

with

$$I=I_1+I_2\cos 2\theta_l+I_3\sin^2\theta_l\cos 2\phi+I_4\sin 2\theta_l\cos\phi+I_5\sin\theta_l\cos\phi+I_6\cos\theta_l+I_7\sin\theta_l\sin\phi+I_8\sin 2\theta_l\sin\phi+I_9\sin^2\theta_l\sin 2\phi. \tag{3.8}$$

The functions $I_1,\ldots,I_9$ have the following form (see Appendix B for details):

$$I_1=\left[\frac{3}{2}(|F_{3L}|^2+|F_{2L}|^2)\sin^2\theta_P+|F_{1L}|^2\cos^2\theta_P\right]+(L\to R), \tag{3.9a}$$

$$I_2=\left[\frac{1}{2}(|F_{3L}|^2+|F_{2L}|^2)\sin^2\theta_P-|F_{1L}|^2\cos^2\theta_P\right]+(L\to R), \tag{3.9b}$$

$$I_3=(|F_{3L}|^2-|F_{2L}|^2)\sin^2\theta_P+(L\to R), \tag{3.9c}$$

$$I_4=\mathrm{Re}(F_{1L}F_{2L}^*)\sin 2\theta_P+(L\to R), \tag{3.9d}$$

$$I_5=2\ \mathrm{Re}(F_{1L}F_{3L}^*)\sin 2\theta_P-(L\to R), \tag{3.9e}$$

$$I_6=4\ \mathrm{Re}(F_{2L}F_{3L}^*)\sin^2\theta_P-(L\to R), \tag{3.9f}$$

$$I_7=2\ \mathrm{Im}(F_{1L}F_{2L}^*)\sin 2\theta_P-(L\to R), \tag{3.9g}$$

$$I_8=\mathrm{Im}(F_{1L}F_{3L}^*)\sin 2\theta_P+(L\to R), \tag{3.9h}$$

$$I_9=-2\ \mathrm{Im}(F_{2L}F_{3L}^*)\sin^2\theta_P+(L\to R). \tag{3.9i}$$

The physical region of phase space is defined through

$$0\leqslant s_l\leqslant(M_B-\sqrt{s_P})^2,\ (M_P+M_{P'})^2\leqslant s_P\leqslant M_B^2, \tag{3.10a}$$

$$0\leqslant\phi\leqslant 2\pi,\ -1\leqslant\cos\theta_P\leqslant 1,\ -1\leqslant\cos\theta_l\leqslant 1. \tag{3.10b}$$

Note that θ_l is the angle of the l^- in the l^-l^+ c.m. frame; θ_P is the angle of π^- (or K^-) in the $\pi^-\pi^+$ (or $K^-\pi^+$) system; ϕ is the angle between $\mathbf{p}_{\pi^-}\times\mathbf{p}_{\pi^+}$ and $\mathbf{p}_{l^-}\times\mathbf{p}_{l^+}$ (in the case of the $\pi^-\pi^+$ final state) and between $\mathbf{p}_{K^-}\times\mathbf{p}_{\pi^+}$ and $\mathbf{p}_{l^-}\times\mathbf{p}_{l^+}$ (in the case of the $K^-\pi^+$ final state), $\mathbf{p}_i$ denoting the 3-momentum vectors of the corresponding particles in the $\bar{B}$ rest frame. The z-axis is chosen along the total momentum vector of the PP' system in the $\bar{B}$ rest frame. This definition will also be retained in the case of the antiparticle reactions $B\to\pi^-\pi^+e^-e^+$ and $B\to K^+\pi^-e^-e^+$ (with $\mathbf{p}_{K^+}$ replacing $\mathbf{p}_{K^-}$ in the latter case).

The amplitudes $F_{iL,R}$ $(i=1,2,3)$ defined in Eqs. (3.4)–(3.6) are closely related to the transversity amplitudes $A_0,A_\parallel,A_\perp$ sometimes used in connection with the angular distribution of the four-body final state arising from decays of the form $B\to V_1V_2$ [11]. The differential decay rate in this alternative notation is again given by Eqs. (3.7)–(3.9), with the functions $F_{iL,R}$ written in terms of $A_0,A_\parallel,A_\perp$ as follows:

$$F_{1L,R}=\frac{A_{0L,R}}{N}, \tag{3.11a}$$

$$F_{2L,R}=\frac{A_{\parallel L,R}}{N\sqrt{2}}, \tag{3.11b}$$

$$F_{3L,R}=\frac{A_{\perp L,R}}{N\sqrt{2}}, \tag{3.11c}$$

where the normalization factor

$$N=\frac{1}{3}\left[\frac{G_F^2\alpha^2}{2^{11}\pi^7 M_B^3}|V_{tb}V_{tf}^*|^2\beta X\right]^{1/2}, \tag{3.12}$$

has been chosen in such a way that $\Gamma=|A_0|^2+|A_\parallel|^2+|A_\perp|^2$.

IV. ANGULAR DISTRIBUTIONS

We derive from the differential decay rate, Eq. (3.7), one-dimensional angular distributions of interest, namely $d\Gamma/d\cos\theta_P$, $d\Gamma/d\cos\theta_l$, and $d\Gamma/d\phi$. These distributions, as well as the observables calculated in Sec. V, depend on different combinations of the short-distance coefficients $c_7^{\rm eff}$, $c_9^{\rm eff}$, c_{10} and the form factors g, f, h, $g_\pm$, a_+.

A. Decay rate as a function of cos θ_P

Integrating $d^5\Gamma$ over the variables s_l, s_P, $\cos\theta_l$, and ϕ, we obtain

$$\frac{d\Gamma}{d\cos\theta_P}=\frac{G_F^2 M_B^5\alpha^2}{2^{14}\pi^7}|V_{tb}V_{tf}^*|^2\left(J_1-\frac{1}{3}J_2\right), \tag{4.1}$$

with $(i=1,2)$

TABLE I. Values of the functions $J_i^{s,c}$ [Eq. (4.2)], using $\rho=0.19$ and $\eta=0.35$ [12].

	J_1^s	J_1^c	J_2^s	J_2^c
$\bar{B}\to\pi^-\pi^+e^-e^+$	76.2	76.8	25.4	-76.8
$\bar{B}\to K^-\pi^+e^-e^+$	261.4	278.8	87.2	-278.8

TABLE II. Estimate of the functions K_i according to Eq. (4.5).

	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9
$\bar{B}\to\pi^-\pi^+e^-e^+$	152.8	-17.3	-14.1	0	0	-61.8	0	0	-0.05
$\bar{B}\to K^-\pi^+e^-e^+$	534.4	-69.6	-64.6	0	0	-179.0	0	0	-0.4

$$J_i \equiv J_i^s \sin^2\theta_P + J_i^c \cos^2\theta_P\,,\quad J_i^{s,c} = \frac{1}{M_B^8}\int I_i^{s,c}\beta X ds_l\, ds_P\,, \tag{4.2}$$

where $I_i^{s,c}$ stand for the coefficients of $\sin^2\theta_P$ and $\cos^2\theta_P$ in the expressions for $I_{1,2}$ given in Eqs. (B1) and (B2) of the Appendix. The absence of a term odd in $\cos\theta_P$ is connected with the fact that the PP' system is in a pure $L=1$ state. As a consequence, the forward-backward (FB) asymmetry in the PP' system, defined as

$$A_{\rm FB}^P = \frac{\int_0^1 \frac{d\Gamma}{d\cos\theta_P} d\cos\theta_P - \int_{-1}^0 \frac{d\Gamma}{d\cos\theta_P} d\cos\theta_P}{\int_0^1 \frac{d\Gamma}{d\cos\theta_P} d\cos\theta_P + \int_{-1}^0 \frac{d\Gamma}{d\cos\theta_P} d\cos\theta_P}, \tag{4.3}$$

vanishes. The predictions for $J_i^{s,c}$ are contained in Table I.

B. Decay rate as a function of cos θ_l

Integration of $d^5\Gamma$ over s_l, s_P, $\cos\theta_P$, and ϕ yields

$$\frac{d\Gamma}{d\cos\theta_l} = \frac{G_F^2 M_B^5 \alpha^2}{2^{15}\pi^7}|V_{tb}V_{tf}^*|^2 (K_1 + K_2 \cos 2\theta_l + K_6 \cos\theta_l), \tag{4.4}$$

where

$$K_i = \frac{1}{M_B^8}\int I_i \beta X ds_l\, ds_P d\cos\theta_P\,. \tag{4.5}$$

Our results for the parameters K_i are tabulated in Table II. Observe that K_4, K_5, K_7, and K_8 vanish in our model. The FB asymmetry of l^- in the l^-l^+ c.m. system is

$$A_{\rm FB} = \frac{K_6/2}{K_1 - K_2/3} = \begin{cases} -0.19 \text{ for } \pi^-\pi^+, \\ -0.16 \text{ for } K^-\pi^+. \end{cases} \tag{4.6}$$

This result reproduces the FB asymmetry of the lepton calculated in previous analyses of the exclusive channels $\bar{B}\to K^*(\rho)e^-e^+$ [1,2].

C. Decay rate as a function of ϕ

Finally, the distribution in the angle ϕ between the lepton and meson planes, after integration over other variables, takes the simple form

$$\frac{d\Gamma}{d\phi} = \frac{G_F^2 M_B^5 \alpha^2}{2^{15}\pi^8}|V_{tb}V_{tf}^*|^2\left[\left(K_1 - \frac{1}{3}K_2\right) + \frac{2}{3}(K_3 \cos 2\phi + K_9 \sin 2\phi)\right], \tag{4.7}$$

with the K_i's shown in Table II.

V. *CP*-VIOLATING OBSERVABLES

We now focus on *CP*-violating observables that can be constructed by combining information on $\bar{B}$ and B decays: namely,

$$\bar{B}\to K^-\pi^+e^-e^+ \text{ and } B\to K^+\pi^-e^-e^+, \tag{5.1a}$$

or

$$\bar{B}\to \pi^-\pi^+e^-e^+ \text{ and } B\to \pi^-\pi^+e^-e^+. \tag{5.1b}$$

With the definition of $\cos\theta_P$, $\cos\theta_l$, and ϕ given after Eq. (3.10), the differential decay rate for $\bar{B}$ and B decays is

$$d^5\Gamma|_{\bar{B}} = \mathcal{N}\; I(s_l, s_P, \theta_l, \theta_P, \phi)\beta X d\text{PS}, \tag{5.2a}$$

$$d^5\Gamma|_{B} = \mathcal{N}\; \overline{I}(s_l, s_P, \theta_l, \theta_P, \phi)\beta X d\text{PS}, \tag{5.2b}$$

where $\mathcal{N}$ is a normalization factor

$$\mathcal{N} = \frac{G_F^2\alpha^2}{2^{16}\pi^8 M_B^3}|V_{tb}V_{tf}^*|^2, \tag{5.3}$$

and dPS represents the phase-space element

$$d\text{PS} = ds_l\, ds_P\, d\cos\theta_l\, d\cos\theta_P\, d\phi. \tag{5.4}$$

The function I is given in Eq. (3.8), whereas the function $\overline{I}$ is obtained from I by the substitution

$$I_{1,\dots,6}(x_{L,R}; y_{L,R}; z_{L,R}) \to I_{1,\dots,6}(\overline{x}_{L,R}; \overline{y}_{L,R}; \overline{z}_{L,R}) \equiv \overline{I}_{1,\dots,6}, \tag{5.5a}$$

$$I_{7,8,9}(x_{L,R}; y_{L,R}; z_{L,R}) \to -I_{7,8,9}(\overline{x}_{L,R}; \overline{y}_{L,R}; \overline{z}_{L,R}) \equiv -\overline{I}_{7,8,9}, \tag{5.5b}$$

where $\overline{x}$, $\overline{y}$, $\overline{z}$ are defined via Eq. (2.20).

We now define the sum and the difference of the differential spectra $d^5\Gamma|_{\overline{B}}$ and $d^5\Gamma|_B$ as follows:

$$d\Gamma_{\text{diff}}=\mathcal{N}[(I_1-\overline{I}_1)+(I_2-\overline{I}_2)\cos 2\theta_l+(I_3-\overline{I}_3)\sin^2\theta_l\cos 2\phi+(I_4-\overline{I}_4)\sin 2\theta_l\cos\phi+(I_5-\overline{I}_5)\sin\theta_l\cos\phi$$
$$+(I_6-\overline{I}_6)\cos\theta_l+(I_7+\overline{I}_7)\sin\theta_l\sin\phi+(I_8+\overline{I}_8)\sin 2\theta_l\sin\phi+(I_9+\overline{I}_9)\sin^2\theta_l\sin 2\phi]\beta X d\text{PS}, \tag{5.6}$$

$$d\Gamma_{\text{sum}}=\mathcal{N}[(I_1+\overline{I}_1)+(I_2+\overline{I}_2)\cos 2\theta_l+(I_3+\overline{I}_3)\sin^2\theta_l\cos 2\phi+(I_4+\overline{I}_4)\sin 2\theta_l\cos\phi+(I_5+\overline{I}_5)\sin\theta_l\cos\phi$$
$$+(I_6+\overline{I}_6)\cos\theta_l+(I_7-\overline{I}_7)\sin\theta_l\sin\phi+(I_8-\overline{I}_8)\sin 2\theta_l\sin\phi+(I_9-\overline{I}_9)\sin^2\theta_l\sin 2\phi]\beta X d\text{PS}. \tag{5.7}$$

Recalling Eq. (4.5) and defining

$$K_i^D=\frac{1}{M_B^8}\int \beta X ds_l\, ds_P\left[\int_{-1}^{0}-\int_{0}^{1}\right]I_i d\cos\theta_P, \tag{5.8}$$

we can then construct the following *CP*-violating observables:

$$\langle A_{CP}\rangle=\frac{1}{I_0}\left[(K_1-\overline{K}_1)-\frac{1}{3}(K_2-\overline{K}_2)\right], \tag{5.9a}$$

$$\langle A_3\rangle=\frac{1}{I_0}(K_3-\overline{K}_3), \tag{5.9b}$$

$$\langle A_4\rangle=\frac{1}{I_0}(K_4^D-\overline{K}_4^D), \tag{5.9c}$$

$$\langle A_5\rangle=\frac{1}{I_0}(K_5^D-\overline{K}_5^D), \tag{5.9d}$$

$$\langle A_6\rangle=\frac{1}{I_0}(K_6-\overline{K}_6), \tag{5.9e}$$

$$\langle A_7\rangle=\frac{1}{I_0}(K_7^D-\overline{K}_7^D), \tag{5.9f}$$

$$\langle A_8\rangle=\frac{1}{I_0}(K_8^D-\overline{K}_8^D), \tag{5.9g}$$

$$\langle A_9\rangle=\frac{1}{I_0}(K_9-\overline{K}_9), \tag{5.9h}$$

in which the quantity I_0 is defined as

$$I_0=(K_1+\overline{K}_1)-\frac{1}{3}(K_2+\overline{K}_2). \tag{5.10}$$

Note that the asymmetries $\langle A_{CP}\rangle$, $\langle A_{3,\ldots,6}\rangle$ involving the differences $(I_i-\overline{I}_i)$, $i=1,\ldots,6$, can be obtained from a measurement of the difference $d\Gamma_{\text{diff}}$. On the other hand, the asymmetries $\langle A_{7,8,9}\rangle$ require only a measurement of $d\Gamma_{\text{sum}}$ and, in principle, can be determined even for an untagged equal mixture of B and $\overline{B}$.

The asymmetry $\langle A_{CP}\rangle$ is simply the asymmetry in the partial decay rates of $\overline{B}\to K^-\pi^+e^-e^+$ and $B\to K^+\pi^-e^-e^+$ (or $\overline{B}\to\pi^-\pi^+e^-e^+$ and $B\to\pi^-\pi^+e^-e^+$). The other asymmetries represent *CP*-violating effects in the angular distribution of these processes. All of these effects have their origin in the *CP*-violating imaginary part of the coefficient

$$\lambda_u\equiv\frac{V_{ub}V_{uf}^*}{V_{tb}V_{tf}^*}\approx-\lambda^2(\rho-i\eta)\quad\text{for } f=s,\qquad \frac{\rho(1-\rho)-\eta^2}{(1-\rho)^2+\eta^2}-\frac{i\eta}{(1-\rho)^2+\eta^2}\quad\text{for } f=d. \tag{5.11}$$

In the case $f=d$, this imaginary part is of order η, but in the case $f=s$ it is reduced by an extra factor λ^2. In both cases, the weak phase present in the coefficient c_9^{eff} [Eq. (2.4)] is further suppressed by a factor of order $(3c_1+c_2)/c_9\simeq 0.085$. All of these factors explain the very small magnitude of the asymmetries listed in Table III. The result for the partial width asymmetry $\langle A_{CP}\rangle$ reproduces that obtained in previous literature [2].

The small value of $\langle A_6\rangle$ implies that the coefficients K_6 and $\overline{K}_6$ are almost equal, and therefore the FB asymmetry of the electron is essentially the same for B and $\overline{B}$ decay [Eq. (4.6)]. (This corrects a statement in Ref. [13] that suggested that these asymmetries are opposite in sign.)

VI. CONCLUSIONS

Our results for $d\Gamma/d\cos\theta_P$ and $d\Gamma/d\phi$ are new consequences of the short-distance Hamiltonian for $b\to s(d)e^-e^+$, which test the polarization state of the vector meson in $\overline{B}\to K^*(\rho)e^-e^+$. The asymmetries $\langle A_k\rangle$ are *CP*-

TABLE III. Estimates of the average *CP*-violating asymmetries $\langle A_k \rangle$ in units of 10^{-4} (10^{-2}) for the $B \to K^*$ $(B \to \rho)$ transition.

	$\langle A_{CP} \rangle$	$\langle A_3 \rangle$	$\langle A_4 \rangle$	$\langle A_5 \rangle$	$\langle A_6 \rangle$	$\langle A_7 \rangle$	$\langle A_8 \rangle$	$\langle A_9 \rangle$
$\bar{B} \to K^- \pi^+ e^- e^+$ vs $B \to K^+ \pi^- e^- e^+$	2.7	−0.6	−2.0	5.2	−4.6	0	0.6	−0.04
$\bar{B} \to \pi^- \pi^+ e^- e^+$ vs $B \to \pi^- \pi^+ e^- e^+$	−1.7	0.1	0.4	−1.4	1.2	0	−0.1	0.006

violating observables that can be obtained from a comparison of the angular distribution in B and $\bar{B}$ decays in the conjugate channels. Numerical estimates have been obtained within the framework of the standard model, but the formalism presented can be applied to other Hamiltonians. As seen from Table III, the asymmetries predicted for $B \to K^*$ are exceedingly small, and any significant effect would signal a non-standard source of *CP* violation. In the case of $B \to \rho$, asymmetries of 1–2 % are predicted for $\langle A_{CP} \rangle$, $\langle A_5 \rangle$, and $\langle A_6 \rangle$. It should be noted that our predictions apply to K^* (ρ) production on the mass shell. This implies that the PP' system is in a pure p-wave state, and explains why the distribution in $\cos \theta_P$ given in Eq. (4.1) is forward-backward symmetric. In the continuum region of $K\pi$ or $\pi\pi$ masses, there will be additional partial waves, as well as long-range effects associated with bremsstrahlung from $B \to K\pi$ and $B \to \pi\pi$, followed by internal conversion of the photon. While such effects are difficult to calculate, it is conceivable that they lead to larger *CP*-violating asymmetries in the continuum region of $B \to K\pi e\bar{e}$ and $B \to \pi\pi e\bar{e}$.

ACKNOWLEDGMENTS

The work of F.K. has been supported by the TMR Network of the EC under contract ERBFMRX-CT96-0090.

APPENDIX A: FORM FACTORS

In this Appendix we list the $B \to K^*$ and $B \to \rho$ form factors of Melikhov and Nikitin [9] using their ''Set 2'' of the Isgur-Scora-Grinstein-Wise parameters [14]. (See Table IV.)

APPENDIX B: THE FUNCTIONS $I_1, \ldots, I_9$

The functions $I_1, \ldots, I_9$ appearing in the expression for $I(s_l, s_P, \theta_l, \theta_P, \phi)$, Eq. (3.8), are given by

TABLE IV. The $B \to K^*$ and $B \to \rho$ form factors of Melikhov and Nikitin [9].

Form factors	$B \to K^*$	$B^- \to \rho^-$
$g(q^2)$	$0.048\ \mathrm{GeV}^{-1}\left(1-\frac{q^2}{6.67^2}\right)^{-2.61}$	$0.036\ \mathrm{GeV}^{-1}\left(1-\frac{q^2}{6.55^2}\right)^{-2.75}$
$f(q^2)$	$1.61\mathrm{GeV}\left(1-\frac{q^2}{5.86^2}+\frac{q^4}{7.66^4}\right)^{-1}$	$1.10\ \mathrm{GeV}\left(1-\frac{q^2}{5.59^2}+\frac{q^4}{7.10^4}\right)^{-1}$
$a_+(q^2)$	$-0.036\ \mathrm{GeV}^{-1}\left(1-\frac{q^2}{7.33^2}\right)^{-2.85}$	$-0.026\ \mathrm{GeV}^{-1}\left(1-\frac{q^2}{7.29^2}\right)^{-3.04}$
$h(q^2)$	$0.0037\ \mathrm{GeV}^{-2}\left(1-\frac{q^2}{6.57^2}\right)^{-3.28}$	$0.003\ \mathrm{GeV}^{-2}\left(1-\frac{q^2}{6.43^2}\right)^{-3.42}$
$g_+(q^2)$	$-0.28\left(1-\frac{q^2}{6.67^2}\right)^{-2.62}$	$-0.20\left(1-\frac{q^2}{6.57^2}\right)^{-2.76}$
$g_-(q^2)$	$0.24\left(1-\frac{q^2}{6.59^2}\right)^{-2.58}$	$0.18\left(1-\frac{q^2}{6.50^2}\right)^{-2.73}$

$$I_1=\beta^2\left\{\frac{3}{2}s_l s_P[X^2(|x_L|^2+|x_R|^2)+(|y_L|^2+|y_R|^2)]\sin^2\theta_P\right.$$
$$\left.+[(k\cdot q)^2(|y_L|^2+|y_R|^2)+X^4(|z'_L|^2+|z'_R|^2)+2X^2(k\cdot q)\ \mathrm{Re}\,(y_L^* z'_L+y_R^* z'_R)]\cos^2\theta_P\right\}, \tag{B1}$$

$$I_2=\beta^2\left\{\frac{1}{2}s_l s_P[X^2(|x_L|^2+|x_R|^2)+(|y_L|^2+|y_R|^2)]\ \sin^2\theta_P\right.$$
$$\left.-[(k\cdot q)^2(|y_L|^2+|y_R|^2)+X^4(|z'_L|^2+|z'_R|^2)+2X^2(k\cdot q)\ \mathrm{Re}\,(y_L^* z'_L+y_R^* z'_R)]\cos^2\theta_P\right\}, \tag{B2}$$

$$I_3=\beta^2 s_l s_P\{X^2(|x_L|^2+|x_R|^2)-(|y_L|^2+|y_R|^2)\}\ \sin^2\theta_P\,, \tag{B3}$$

$$I_4=\beta^2(s_l s_P)^{1/2}\{X^2\ \mathrm{Re}\,(y_L^* z'_L+y_R^* z'_R)+(k\cdot q)(|y_L|^2+|y_R|^2)\}\ \sin 2\theta_P\,, \tag{B4}$$

$$I_5=2\beta^2 X(s_l s_P)^{1/2}\{X^2\ \mathrm{Re}\,(x_L^* z'_L-x_R^* z'_R)+(k\cdot q)\ \mathrm{Re}\,(x_L^* y_L-x_R^* y_R)\}\ \sin 2\theta_P\,, \tag{B5}$$

$$I_6=4\beta^2 X s_l s_P\ \mathrm{Re}\,(x_L^* y_L-x_R^* y_R)\ \sin^2\theta_P\,, \tag{B6}$$

$$I_7=2\beta^2 X^2(s_l s_P)^{1/2}\ \mathrm{Im}\,(y_L^* z'_L-y_R^* z'_R)\ \sin 2\theta_P\,, \tag{B7}$$

$$I_8=\beta^2 X(s_l s_P)^{1/2}\{X^2\ \mathrm{Im}\,(x_L^* z'_L+x_R^* z'_R)+(k\cdot q)\ \mathrm{Im}\,(x_L^* y_L+x_R^* y_R)\}\ \sin 2\theta_P\,, \tag{B8}$$

$$I_9=-2\beta^2 X s_l s_P\ \mathrm{Im}\,(x_L^* y_L+x_R^* y_R)\ \sin^2\theta_P\,, \tag{B9}$$

where $k\cdot q=(M_B^2-s_l-s_P)/2$.

[1] W. Jaus and D. Wyler, Phys. Rev. D **41**, 3405 (1990); C. Greub, A. Ioannissian, and D. Wyler, Phys. Lett. B **346**, 149 (1995); G. Burdman, Phys. Rev. D **52**, 6400 (1995); W. Roberts, *ibid.* **54**, 863 (1996); P. Colangelo, F. De Fazio, P. Santorelli, and E. Scrimieri, *ibid.* **53**, 3672 (1996); **57**, 3186(E) (1998); D. Melikhov, N. Nikitin, and S. Simula, *ibid.* **57**, 6814 (1998), and references therein; T. M. Aliev and M. Savci, *ibid.* **60**, 014005 (1999).

[2] F. Krüger and L. M. Sehgal, Phys. Rev. D **56**, 5452 (1997); **60**, 099905(E) (1999).

[3] L. M. Sehgal and M. Wanninger, Phys. Rev. D **46**, 1035 (1992); **46**, 5209(E) (1992); P. Heiliger and L. M. Sehgal, *ibid.* **48**, 4146 (1993).

[4] KTeV Collaboration, J. Adams *et al.*, Phys. Rev. Lett. **80**, 4123 (1998); KTeV Collaboration, M. Arenton, talk given at the 1998 Workshop on Heavy Quarks at Fixed Target, Batavia, Illinois, 1998.

[5] N. Cabibbo and A. Maksymovicz, Phys. Rev. **137**, B438 (1965); **168**, 1926(E) (1968); A. Pais and S. B. Treiman, *ibid.* **168**, 1858 (1968).

[6] J. Bijnens, G. Ecker, and J. Gasser, in *The DAΦNE Physics Handbook*, edited by L. Maiani, G. Pancheri, and N. Paver (Servizio Documentazione dei Laboratori Nazionali di Frascati, Frascati, Italy, 1992), Vol. I, p. 115.

[7] A. J. Buras and M. Münz, Phys. Rev. D **52**, 186 (1995); M. Misiak, Nucl. Phys. **B393**, 23 (1993); **B439**, 461(E) (1995).

[8] For a review, see G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).

[9] D. Melikhov and N. Nikitin, hep-ph/9609503.

[10] R. Sinha, hep-ph/9608314; N. Sinha and R. Sinha, hep-ph/9707416.

[11] G. Kramer and W. F. Palmer, Phys. Rev. D **45**, 193 (1992); **46**, 2969 (1992); G. Kramer, W. F. Palmer, and H. Simma, Nucl. Phys. **B428**, 77 (1994); A. S. Dighe, I. Dunietz, H. J. Lipkin, and J. L. Rosner, Phys. Lett. B **369**, 144 (1996).

[12] F. Parodi, P. Rodeau, and A. Stocchi, Nuovo Cimento A **112**, 833 (1999); A. Ali and D. London, Eur. Phys. J. C **9**, 687 (1999).

[13] F. Krüger and L. M. Sehgal, Proceedings of the International Europhysics Conference on High-Energy Physics, Jerusalem, Israel, 1997, hep-ph/9711301, p. 745.

[14] N. Isgur, D. Scora, B. Grinstein, and M. B. Wise, Phys. Rev. D **39**, 799 (1989); D. Scora and N. Isgur, *ibid.* **52**, 2783 (1995).