

# Strong and electromagnetic decays of $p$ -wave heavy baryons $\Lambda_{c1}, \Lambda_{c1}^*$

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We first calculate the binding energy and the pionic and electromagnetic coupling constants of the lowest-lying  $p$ -wave heavy baryon doublet  $\Lambda_{c1}, \Lambda_{c1}^*$  in the leading order of heavy quark expansion. Then we calculate the two-body decay widths with these couplings and compare our results with other approaches. Our results are  $\Gamma(\Lambda_{c1} \rightarrow \Sigma_c \pi, \Sigma_c \gamma, \Sigma_c^* \gamma) = 2.7, 0.011, 0.001$  MeV and  $\Gamma(\Lambda_{c1}^* \rightarrow \Sigma_c \pi, \Sigma_c \gamma, \Sigma_c^* \gamma, \Lambda_{c1} \gamma) = 33, 5.6, 0.014$  keV, respectively. We find that  $\Lambda_{c1}, \Lambda_{c1}^* \rightarrow \Lambda_c \gamma$  is strictly forbidden in the leading order of heavy quark expansion. At the order of  $\mathcal{O}(1/m_c)$  their widths are 36 and 48 keV, respectively.

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## I. INTRODUCTION

Most of the ground state charm baryons have now been found experimentally [1]. Important progress has been made in the search for orbitally excited heavy baryons. The ARGUS [2], E687 [3], and CLEO [4] Collaborations have observed a pair of states in the channel  $\Lambda_c^+ \pi^+ \pi^-$ , which were interpreted as the lowest-lying orbitally excited states:  $\Lambda_{c1}(2593)$  with  $J^P = \frac{1}{2}^-$  and  $\Lambda_{c1}^*(2625)$  with  $J^P = \frac{3}{2}^-$ . The total decay width of the  $\Lambda_{c1}(2593)$  is  $3.6_{-1.3}^{+2.0}$  MeV while only an upper limit of  $< 1.9$  MeV has been set for  $\Lambda_{c1}^*(2625)$  up to now [1]. Recently there emerges evidence for the  $\Xi_{c1}^{*+}$  with  $J^P = \frac{3}{2}^-$ , the strange partner of the  $\Lambda_{c1}^*(2625)$ . Its width is less than 2.4 MeV. In the near future much more data will be expected. We will focus on the strong and electromagnetic decays of the  $\Lambda_{c1}$  doublet since they are the only well-established states [1].

There exist many theoretical discussions on this topic. In [5] the single-pion and two-pion strong decays and radiative decays of the  $\Lambda_{c1}$  doublet were discussed within the framework of heavy baryon chiral perturbation theory. Because of unknown coupling constants in the chiral Lagrangian, no actual decay widths were given. Within the same framework the pionic decay widths were calculated assuming that the heavy quark effective theory is still valid for the strange quark [6]. The coupling constants in the chiral Lagrangian were fixed using the  $p$ -wave strange baryon decay widths, which were later used to predict the strong decays of the  $p$ -wave charm baryons [6]. The two-pion width of  $\Lambda_{c1}$  was estimated to be around 2.5 MeV, which was comparable to the total one-pion width of 3.0 MeV. And the decay of  $\Lambda_{c1}^*$  was suppressed by more than an order [6]. In [7] the  $p$ -wave doublet was treated as the bound state of the nucleon and heavy meson. It was found that the decays  $\Lambda_{c1}, \Lambda_{c1}^* \rightarrow \Lambda_c \gamma$  were suppressed due to the kinematic suppression of the electric dipole moment [7]. In [8] the constituent quark model was employed to study the orbitally excited heavy baryons. Sum rules were derived to constrain the coupling constants. The light front quark model, together with underlying  $SU(2N_f) \times O(3)$  symmetry for the light diquark system, was used to relate and analyze the pionic coupling [9–12]. However, the results have strong dependence on the constituent quark mass  $m_q$ . Varying  $m_q$  from 220 MeV to 340 MeV, the decay widths increase by more than a factor of

2 [12]. Within the same framework the electromagnetic decays of the  $p$ -wave baryons were calculated in [13]. In [14] both strong and radiative decays were calculated using a relativistic three-quark model. After this paper was submitted there appeared an interesting paper discussing the radiative decays of the ground state heavy baryon multiplets in the framework of heavy baryon chiral perturbation theory. In some cases the loop corrections yield sizable enhancement of the decay widths [15].

It will be helpful to extract these pionic and photonic coupling constants at the quark gluon level using a QCD Lagrangian. We will treat this problem using QCD sum rules (QSRs) [16], which have been successful in extracting the low-lying hadron masses and couplings. In this approach the nonperturbative effects are introduced via various condensates in the vacuum. The light cone QCD sum rule (LCQSR) differs from conventional short-distance QSRs in that it is based on an expansion over the twists of the operators. The main contribution comes from the lowest twist operators. Matrix elements of nonlocal operators sandwiched between a hadronic state and the vacuum define the hadron wave functions. In the present case our sum rules involve the pion and photon wave functions. When the LCQSR is used to calculate the coupling constant, the double Borel transformation is always invoked so that the excited states and the continuum contribution can be subtracted quite cleanly. We have calculated the pionic and electromagnetic coupling constants and decay widths of the ground state heavy hadrons [17–19] and possible hybrid heavy mesons [20]. In this work we extend the same framework to study the strong and radiative decays of the lowest  $p$ -wave heavy baryons, i.e.,  $\Lambda_{c1}$  doublet.

Our paper is organized as follows: Section I is an introduction. In the next section we derive the mass sum rule. The light cone sum rules for the pionic coupling constants are derived in Sec. III. Numerical analysis is presented. In Sec. IV we extend the same framework to analyze the electromagnetic processes  $\Lambda_{c1} \rightarrow \Sigma_c \gamma$ , etc. In Sec. V we discuss the processes  $\Lambda_{c1}, \Lambda_{c1}^* \rightarrow \Lambda_c \gamma$  and compare our results with other theoretical approaches. The last section is a summary.

## II. MASS SUM RULES FOR THE HEAVY HYBRID MESONS IN HEAVY QUARK EFFECTIVE THEORY

### A. Heavy quark effective theory

The effective Lagrangian of heavy quark effective theory (HQET), up to order  $1/m_Q$ , is

$$\mathcal{L}_{\text{eff}} = \bar{h}_v i v \cdot D h_v + \frac{1}{2m_Q} \mathcal{K} + \frac{1}{2m_Q} \mathcal{S} + \mathcal{O}(1/m_Q^2), \quad (1)$$

where  $h_v(x)$  is the velocity-dependent field related to the original heavy quark field  $Q(x)$  by

$$h_v(x) = e^{im_Q v \cdot x} \frac{1 + \not{v}}{2} Q(x), \quad (2)$$

and  $v_\mu$  is the heavy hadron velocity.  $\mathcal{K}$  is the kinetic operator defined as

$$\mathcal{K} = \bar{h}_v (iD_t)^2 h_v, \quad (3)$$

where  $D_t^\mu = D^\mu - (v \cdot D) v^\mu$ , with  $D^\mu = \partial^\mu - ig A^\mu$  the gauge-covariant derivative, and  $\mathcal{S}$  is the chromomagnetic operator:

$$\mathcal{S} = \frac{g}{2} C_{\text{mag}}(m_Q/\mu) \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v, \quad (4)$$

where  $C_{\text{mag}} = [\alpha_s(m_Q)/\alpha_s(\mu)]^{3/\beta_0}$ , and  $\beta_0 = 11 - 2n_f/3$ . Note that the heavy quark propagator has a simple form in coordinate space:

$$\langle 0 | T \{ h_v(x), \bar{h}_v(0) \} | 0 \rangle = \int_0^\infty dt \delta(x - vt) \frac{1 + \not{v}}{2}. \quad (5)$$

### B. Interpolating currents

We introduce the interpolating currents for the relevant heavy baryons:

$$\eta_{\Lambda_c}(x) = \epsilon_{abc} [u^{aT}(x) C \gamma_5 d^b(x)] h_v^c(x), \quad (6)$$

$$\eta_{\Sigma_c^+}(x) = \epsilon_{abc} [u^{aT}(x) C \gamma_\mu d^b(x)] \gamma_t^\mu \gamma_5 h_v^c(x), \quad (7)$$

$$\eta_{\Sigma_c^{++*}}^\mu(x) = \epsilon_{abc} [u^{aT}(x) C \gamma_\nu u^b(x)] \Gamma_t^{\mu\nu} h_v^c(x), \quad (8)$$

$$\eta_{\Lambda_{c1}}(x) = \epsilon_{abc} [u^{aT}(x) C \gamma_5 d^b(x)] \gamma_t^\mu \gamma_5 D_\mu^t h_v^c(x), \quad (9)$$

$$\eta_{\Lambda_{c1}}^\mu(x) = \epsilon_{abc} [u^{aT}(x) C \gamma_5 d^b(x)] \Gamma_t^{\mu\nu} D_\nu^t h_v^c(x), \quad (10)$$

where  $a, b, c$  are color indices,  $u(x)$ ,  $d(x)$ , and  $h_v(x)$  are the up, down, and heavy quark fields,  $T$  denotes the transpose,  $C$  is the charge conjugate matrix,  $\Gamma_t^{\mu\nu} = -g_t^{\mu\nu} + \frac{1}{3} \gamma_t^\mu \gamma_t^\nu$ ,  $g_t^{\mu\nu} = g^{\mu\nu} - v^\mu v^\nu$ ,  $\gamma_t^\mu = \gamma^\mu - \hat{v} v^\mu$ , and  $v^\mu$  is the velocity of the heavy hadron.

The overlap amplitudes of the interpolating currents with the heavy baryons are defined as

$$\langle 0 | \eta_{\Lambda_c} | \Lambda_c \rangle = f_{\Lambda_c} u_{\Lambda_c}, \quad (11)$$

$$\langle 0 | \eta_{\Lambda_{c1}} | \Lambda_{c1} \rangle = f_{\Lambda_{c1}} u_{\Lambda_{c1}}, \quad (12)$$

$$\langle 0 | \eta_{\Lambda_{c1}}^\mu | \Lambda_{c1}^* \rangle = \frac{f_{\Lambda_{c1}}^*}{\sqrt{3}} u_{\Lambda_{c1}}^\mu, \quad (13)$$

$$\langle 0 | \eta_{\Sigma_c} | \Sigma_c \rangle = f_{\Sigma_c} u_{\Sigma_c}, \quad (14)$$

$$\langle 0 | \eta_{\Sigma_c}^\mu | \Sigma_c^* \rangle = \frac{f_{\Sigma_c}^*}{\sqrt{3}} u_{\Sigma_c}^\mu, \quad (15)$$

where  $u_{\Lambda_{c1}}^\mu$ ,  $u_{\Sigma_c}^\mu$  are the Rarita-Schwinger spinors in HQET. In the leading order of HQET,  $f_{\Sigma_c} = f_{\Sigma_c}^*$  and  $f_{\Lambda_{c1}} = f_{\Lambda_{c1}}^*$  due to heavy quark symmetry.

### C. $\Lambda_{Q1}$ mass sum rules

In order to extract the binding energy of the  $p$ -wave heavy baryons in the leading order of HQET, we consider the correlators

$$i \int d^4x e^{ikx} \langle 0 | T \{ \eta_{\Lambda_{c1}}(x), \bar{\eta}_{\Lambda_{c1}}(0) \} | 0 \rangle = \Pi(\omega) \frac{1 + \not{v}}{2}, \quad (16)$$

with  $\omega = k \cdot v$ .

The dispersion relation for  $\Pi(\omega)$  reads

$$\Pi(\omega) = \int \frac{\rho(s)}{s - \omega - i\epsilon} ds, \quad (17)$$

where  $\rho(s)$  is the spectral density in the limit  $m_Q \rightarrow \infty$ .

On the phenomenological side,

$$\Pi(\omega) = \frac{f_{\Lambda_{c1}}^2}{\Lambda_{c1} - \omega} + \text{continuum}. \quad (18)$$

In order to suppress the continuum and higher excited state contribution we make a Borel transformation with the variable  $\omega$  to Eq. (17). We have

$$f_{\Lambda_{c1}}^2 e^{-(\bar{\Lambda}_{\Lambda_{c1}}/T)} = \int_0^{s_0} \rho(s) e^{-s/T} ds, \quad (19)$$

where  $\bar{\Lambda}_{\Lambda_{c1}}$  is the  $\Lambda_{c1}$  binding energy in leading order and  $s_0$  is the continuum threshold. Starting from  $s_0$  we have modeled the phenomenological spectral density with the parton like one including both the perturbative term and various condensates.

The spectral density  $\rho(s)$  at the quark level reads

$$\rho(s) = \frac{3}{140\pi^4} s^7 - \frac{1}{384\pi^4} \langle g_s^2 G^2 \rangle s^3 + \frac{m_0^2 a^2}{128\pi^4} \delta(s), \quad (20)$$

where  $a = -4\pi^2 \langle \bar{q}q \rangle = 0.55 \text{ GeV}^3$ ,  $\langle g_s^2 G^2 \rangle = 0.48 \text{ GeV}^4$ ,  $\langle \bar{q}g_s \sigma \cdot Gq \rangle = m_0^2 \langle \bar{q}q \rangle$ , and  $m_0^2 = 0.8 \text{ GeV}^2$ . An interesting feature of Eq. (20) is that the gluon condensate is of the opposite sign as the leading perturbative term, in contrast with the ground state baryon mass sum rules. This may be interpreted as some kind of gluon excitation since we are

considering  $p$ -wave baryons. In the present case the gluon in the covariant derivative also contributes to various condensates.

Two common approaches exist to extract the masses. One is the derivative method:

$$\bar{\Lambda}_{\Lambda_{c1}} = \frac{\int_0^{s_0} s \rho(s) e^{-s/T} ds}{\int_0^{s_0} \rho(s) e^{-s/T} ds}. \quad (21)$$

The other is the fitting method, which involves fitting the left hand side and right hand side of Eq. (19) with the most suitable parameters  $\bar{\Lambda}_{\Lambda_{c1}}, f_{\Lambda_{c1}}, s^0$  in the working region of the Borel parameter. With both methods we get consistent results

$$\begin{aligned} \bar{\Lambda}_{\Lambda_{c1}} &= (1.1 \pm 0.2) \text{ GeV}, \\ f_{\Lambda_{c1}} &= (0.025 \pm 0.005) \text{ GeV}^4, \\ s_{\Lambda_{c1}}^0 &= (1.45 \pm 0.2) \text{ GeV} \end{aligned} \quad (22)$$

in the working region 0.5–1.3 GeV for the Borel parameter  $T$ . For later use we also need the mass and overlapping amplitude of the  $\Sigma$ ,  $\Lambda$  heavy baryon doublet,  $\bar{\Lambda}_{\Sigma_c}$ ,  $\bar{\Lambda}_{\Lambda_c}$ ,  $f_{\Sigma_c}$ ,  $f_{\Lambda_c}$  in leading order of  $\alpha_s$  [21,22]:

$$\begin{aligned} \bar{\Lambda}_{\Sigma_c} &= (1.0 \pm 0.1) \text{ GeV}, \\ f_{\Sigma_c} &= (0.04 \pm 0.004) \text{ GeV}^3, \\ s_{\Sigma_c}^0 &= (1.25 \pm 0.15) \text{ GeV}, \\ \bar{\Lambda}_{\Lambda_c} &= (0.8 \pm 0.1) \text{ GeV}, \\ f_{\Lambda_c} &= (0.018 \pm 0.004) \text{ GeV}^3, \\ s_{\Lambda_c}^0 &= (1.2 \pm 0.15) \text{ GeV}. \end{aligned} \quad (23)$$

### III. LCQSR FOR THE PIONIC COUPLINGS

#### A. Correlator for pionic couplings

We introduce the following amplitudes:

$$M(\Lambda_{c1} \rightarrow \Sigma_c \pi) = g_s \bar{u}_{\Sigma_c} u_{\Lambda_{c1}}, \quad (25)$$

$$M(\Lambda_{c1}^* \rightarrow \Sigma_c^* \pi) = \sqrt{3} g_d \bar{u}_{\Sigma_c} \gamma_5 q_\mu^t \hat{q} u_{\Lambda_{c1}}^{\mu*}, \quad (26)$$

$$M(\Lambda_{c1} \rightarrow \Sigma_c^* \pi) = \sqrt{3} g_d^1 \bar{u}_{\Sigma_c}^{\mu*} \gamma_5 q_\mu^t \hat{q} u_{\Lambda_{c1}}, \quad (27)$$

$$M(\Lambda_{c1}^* \rightarrow \Sigma_c^* \pi) = \bar{u}_{\Sigma_c}^{\mu*} \left[ g'_s g'_{\mu\nu} + 3 g_d^2 \left( q_\mu^t q_\nu^t - \frac{1}{3} g'_{\mu\nu} q_t^2 \right) \right] u_{\Lambda_{c1}}^{\nu*}, \quad (28)$$

where  $\hat{q} = q_\mu \gamma^\mu$ ,  $q_\mu$  is the pion momentum. Only the first two decay processes are kinematically allowed. Because of heavy quark symmetry,  $g'_s = g_s$ ,  $g_d^1 = g_d^2 = g_d$  in the limit of  $m_Q \rightarrow \infty$ . In other words there are two independent coupling constants corresponding to  $s$ -wave and  $d$ -wave decays. Note we are unable to determine the sign of  $g_s$  and  $g_d$ . And we are mainly interested in the decay widths of the  $p$ -wave heavy baryons. In the following our convention is to let both couplings be positive.

We consider the following correlators:

$$\begin{aligned} &\int d^4x e^{ik \cdot x} \langle 0 | T [ \eta_{\Lambda_{c1}}(x) \bar{\eta}_{\Sigma_c}(0) ] | \pi(q) \rangle \\ &= \frac{1 + \hat{v}}{2} G_s(\omega, \omega'), \end{aligned} \quad (29)$$

$$\begin{aligned} &\int d^4x e^{ik \cdot x} \langle 0 | T [ \eta_{\Lambda_{c1}}^{\mu*}(x) \bar{\eta}_{\Sigma_c}(0) ] | \pi(q) \rangle \\ &= \frac{1 + \hat{v}}{2} q_\alpha q_\nu \Gamma_t^{\mu\alpha} \gamma_t^\nu \gamma_5 G_d(\omega, \omega'), \end{aligned} \quad (30)$$

where  $k' = k - q$ ,  $\omega = v \cdot k$ ,  $\omega' = v \cdot k'$ , and  $q^2 = m_\pi^2 = 0$ .

The function  $G_{s,d}(\omega, \omega')$  has the following pole terms from double-dispersion relation. For  $G_s$  we have

$$\frac{f_{\Lambda_{c1}} f_{\Sigma_c} g_s}{(\bar{\Lambda}_{\Lambda_{c1}} - \omega')(\bar{\Lambda}_{\Sigma_c} - \omega)} + \frac{c}{\bar{\Lambda}_{\Lambda_{c1}} - \omega'} + \frac{c'}{\bar{\Lambda}_{\Sigma_c} - \omega}. \quad (31)$$

#### B. Pion light cone wave functions

To go further we need the two- and three-particle pion light cone wave functions [23]

$$\begin{aligned} &\langle \pi(q) | \bar{d}(x) \gamma_\mu \gamma_5 u(0) | 0 \rangle \\ &= -i f_\pi q_\mu \int_0^1 du e^{iuqx} [ \varphi_\pi(u) + x^2 g_1(u) + \mathcal{O}(x^4) ] \\ &\quad + f_\pi \left( x_\mu - \frac{x^2 q_\mu}{qx} \right) \int_0^1 du e^{iuqx} g_2(u), \end{aligned} \quad (32)$$

$$\begin{aligned} &\langle \pi(q) | \bar{d}(x) i \gamma_5 u(0) | 0 \rangle \\ &= \frac{f_\pi m_\pi^2}{m_u + m_d} \int_0^1 du e^{iuqx} \varphi_P(u), \end{aligned} \quad (33)$$

$$\begin{aligned} & \langle \pi(q) | \bar{d}(x) \sigma_{\mu\nu} \gamma_5 u(0) | 0 \rangle \\ &= i(q_\mu x_\nu - q_\nu x_\mu) \frac{f_\pi m_\pi^2}{6(m_u + m_d)} \int_0^1 du e^{iuqx} \varphi_\sigma(u), \end{aligned} \quad (34)$$

$$\begin{aligned} & \langle \pi(q) | \bar{d}(x) \sigma_{\alpha\beta} \gamma_5 g_s G_{\mu\nu}(ux) u(0) | 0 \rangle \\ &= if_{3\pi} [(q_\mu q_\alpha g_\nu \beta - q_\nu q_\alpha g_\mu \beta) - (q_\mu q_\beta g_\nu \alpha \\ & \quad - q_\nu q_\beta g_\mu \alpha)] \int \mathcal{D}\alpha_i \varphi_{3\pi}(\alpha_i) e^{iqx(\alpha_1 + \nu\alpha_3)}, \end{aligned} \quad (35)$$

$$\begin{aligned} & \langle \pi(q) | \bar{d}(x) \gamma_\mu \gamma_5 g_s G_{\alpha\beta}(vx) u(0) | 0 \rangle \\ &= f_\pi \left[ q_\beta \left( g_{\alpha\mu} - \frac{x_\alpha q_\mu}{q \cdot x} \right) - q_\alpha \left( g_{\beta\mu} - \frac{x_\beta q_\mu}{q \cdot x} \right) \right] \\ & \quad \times \int \mathcal{D}\alpha_i \varphi_\perp(\alpha_i) e^{iqx(\alpha_1 + \nu\alpha_3)} + f_\pi \frac{q_\mu}{q \cdot x} (q_\alpha x_\beta - q_\beta x_\alpha) \\ & \quad \times \int \mathcal{D}\alpha_i \varphi_\parallel(\alpha_i) e^{iqx(\alpha_1 + \nu\alpha_3)}, \end{aligned} \quad (36)$$

and

### C. Pionic sum rules

Now the expressions of  $G_s$ ,  $G_d$  at the quark level read

$$\begin{aligned} G_s(\omega, \omega') &= if_\pi \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega t} e^{iu\omega' t} \left\{ \frac{6\mu_\pi}{\pi^2 t^4} \varphi_P(u) + \frac{\mu_\pi}{3\pi^2 t^4} \{3\varphi'_\sigma(u) + [u\varphi_\sigma(u)]''\} + \left( \langle \bar{q}q \rangle + \frac{t^2}{16} \langle \bar{q}g_s \sigma \cdot Gq \rangle \right) g_2(u) \right. \\ & \quad \left. + \left( \langle \bar{q}q \rangle + \frac{t^2}{16} \langle \bar{q}g_s \sigma \cdot Gq \rangle \right) \frac{[u\varphi_\pi(u)]'' + t^2[uG_2(u) + ug_1(u)]''}{3t^2} \right\} + \frac{i}{\pi^2} f_{3\pi} \int \frac{dt}{t^2} \int_0^1 du (1-u) \\ & \quad \times \int \mathcal{D}\alpha_i e^{i\omega t[1-(\alpha_1+u\alpha_3)]} e^{i\omega' t(\alpha_1+u\alpha_3)} [(q \cdot v)^2 - it(q \cdot v)^3(\alpha_1 + u\alpha_3)] \varphi_{3\pi}(\alpha_i) \\ & \quad - \frac{2i}{\pi^2} f_{3\pi} \int \frac{dt}{t^2} \int_0^1 duu \int \mathcal{D}\alpha_i e^{i\omega t[1-(\alpha_1+u\alpha_3)]} e^{i\omega' t(\alpha_1+u\alpha_3)} (q \cdot v)^2 \varphi_{3\pi}(\alpha_i), \end{aligned} \quad (38)$$

$$\begin{aligned} G_d(\omega, \omega') &= if_\pi \int_0^\infty dt \int_0^1 duu e^{i(1-u)\omega t} e^{iu\omega' t} \left\{ \frac{\mu_\pi}{3\pi^2 t^2} \varphi_\sigma(u) + \frac{1}{3} \left( \langle \bar{q}q \rangle + \frac{t^2}{16} \langle \bar{q}g_s \sigma \cdot Gq \rangle \right) \{ \varphi_\pi(u) + t^2[G_2(u) + g_1(u)] \} \right\} \\ & \quad - \frac{i}{\pi^2} f_{3\pi} \int \frac{dt}{t^2} \int_0^1 du (1-u) \int \mathcal{D}\alpha_i e^{i\omega t[1-(\alpha_1+u\alpha_3)]} e^{i\omega' t(\alpha_1+u\alpha_3)} [1 - it(q \cdot v)(\alpha_1 + u\alpha_3)] \varphi_{3\pi}(\alpha_i) \\ & \quad - \frac{i}{\pi^2} f_{3\pi} \int \frac{dt}{t^2} \int_0^1 duu \int \mathcal{D}\alpha_i e^{i\omega t[1-(\alpha_1+u\alpha_3)]} e^{i\omega' t(\alpha_1+u\alpha_3)} \varphi_{3\pi}(\alpha_i), \end{aligned} \quad (39)$$

where  $\mu_\pi = 1.65$  GeV,  $f_\pi = 132$  MeV,  $F'(u) = dF(u)/du$ , and  $F''(u) = d^2F(u)/du^2$ . There are two three-particle terms in the

$$\begin{aligned} & \langle \pi(q) | \bar{d}(x) \gamma_\mu g_s \tilde{G}_{\alpha\beta}(vx) u(0) | 0 \rangle \\ &= if_\pi \left[ q_\beta \left( g_{\alpha\mu} - \frac{x_\alpha q_\mu}{q \cdot x} \right) - q_\alpha \left( g_{\beta\mu} - \frac{x_\beta q_\mu}{q \cdot x} \right) \right] \\ & \quad \times \int \mathcal{D}\alpha_i \tilde{\varphi}_\perp(\alpha_i) e^{iqx(\alpha_1 + \nu\alpha_3)} \\ & \quad + if_\pi \frac{q_\mu}{q \cdot x} (q_\alpha x_\beta - q_\beta x_\alpha) \int \mathcal{D}\alpha_i \tilde{\varphi}_\parallel(\alpha_i) e^{iqx(\alpha_1 + \nu\alpha_3)}. \end{aligned} \quad (37)$$

The operator  $\tilde{G}_{\alpha\beta}$  is the dual of  $G_{\alpha\beta}$ .  $\tilde{G}_{\alpha\beta} = \frac{1}{2} \epsilon_{\alpha\beta\delta\rho} G^{\delta\rho}$ . Here  $\mathcal{D}\alpha_i$  is defined as  $\mathcal{D}\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3)$ . Because of the choice of the gauge  $x^\mu A_\mu(x) = 0$ , the path-ordered gauge factor  $P \exp[ig_s \int_0^1 du x^\mu A_\mu(ux)]$  has been omitted.

The wave function  $\varphi_\pi(u)$  is associated with the leading twist-2 operator,  $g_1(u)$  and  $g_2(u)$  correspond to twist-4 operators, and  $\varphi_P(u)$  and  $\varphi_\sigma(u)$  to twist-3 ones. The function  $\varphi_{3\pi}$  is of twist 3, while all the wave functions appearing in Eqs. (36), (37) are of twist 4. The wave functions  $\varphi(x_i, \mu)$  ( $\mu$  is the renormalization point) describe the distribution in longitudinal momenta inside the pion, the parameters  $x_i$  ( $\sum_i x_i = 1$ ) representing the fractions of the longitudinal momentum carried by the quark, antiquark, and gluon.

The wave function normalizations immediately follow from the definitions (32)–(37):  $\int_0^1 du \varphi_\pi(u) = \int_0^1 du \varphi_\sigma(u) = 1$ ,  $\int_0^1 du g_1(u) = \delta^2/12$ ,  $\int \mathcal{D}\alpha_i \varphi_\perp(\alpha_i) = \int \mathcal{D}\alpha_i \varphi_\parallel(\alpha_i) = 0$ ,  $\int \mathcal{D}\alpha_i \tilde{\varphi}_\perp(\alpha_i) = -\int \mathcal{D}\alpha_i \tilde{\varphi}_\parallel(\alpha_i) = \delta^2/3$ , with the parameter  $\delta$  defined by the matrix element:  $\langle \pi(q) | \bar{d} g_s \tilde{G}_{\alpha\mu} \gamma^\alpha u | 0 \rangle = i\delta^2 f_\pi q_\mu$ .

form of  $\varphi_{3\pi}$  in Eqs. (38), (39). The gluon arises from the light quark propagator in the first term and from the covariant derivative in the second term. For large euclidean values of  $\omega$  and  $\omega'$  this integral is dominated by the region of small  $t$ ; therefore, it can be approximated by the first few terms with lowest twists.

After Wick rotations and making a double Borel transformation with the variables  $\omega$  and  $\omega'$  the single-pole terms in Eq. (31) are eliminated. Subtracting the continuum contribution which is modeled by the dispersion integral in the region  $s, s' \geq s_0$ , we arrive at

$$g_s f_{\Lambda_{c1}} f_{\Sigma_c} = \frac{f_\pi}{\pi^2} e^{(\bar{\Lambda}_{\Lambda_{c1}} + \bar{\Lambda}_{\Sigma_c})/2T} \left\{ 6\mu_\pi \varphi_P(u_0) T^5 f_4\left(\frac{s_0}{T}\right) + \frac{f_{3\pi}}{f_\pi} (2I_3 - I_4 - I_6) T^5 f_4\left(\frac{s_0}{T}\right) + \frac{\mu_\pi}{3} \{3\varphi'_\sigma(u_0) + [u\varphi_\sigma(u)]''_{u_0}\} T^5 f_4\left(\frac{s_0}{T}\right) \right. \\ \left. + \frac{a}{12} [u\varphi_\pi(u)]''_{u_0} \left(1 - \frac{m_0^2}{16T^2}\right) T^3 f_2\left(\frac{s_0}{T}\right) - \frac{a}{4} \left\{ g_2(u_0) + \frac{1}{3} [uG_2(u) + u g_1(u)]''_{u_0} \right\} \left(1 - \frac{m_0^2}{16T^2}\right) T f_0\left(\frac{s_0}{T}\right) \right\}, \quad (40)$$

where  $f_n(x) = 1 - e^{-x \sum_{k=0}^n x^k/k!}$  is the factor used to subtract the continuum, and  $s_0$  is the continuum threshold.  $u_0 = T_1/(T_1 + T_2)$ ,  $T \equiv T_1 T_2/(T_1 + T_2)$ , and  $T_1, T_2$  are the Borel parameters. The functions  $I_i$  are defined below. In obtaining Eq. (40) we have used the Borel transformation formula  $\hat{\mathcal{B}}_\omega^T e^{\alpha\omega} = \delta(\alpha - 1/T)$  and integration by parts to absorb the factors  $(q \cdot v)$  and  $1/(q \cdot v)$ . In this way we arrive at the simple form after the double Borel transformation.

Similarly we have

$$g_d f_{\Lambda_{c1}^*} f_{\Sigma_c} = \frac{f_\pi}{\pi^2} e^{(\bar{\Lambda}_{\Lambda_{c1}^*} + \bar{\Lambda}_{\Sigma_c})/2T} \left\{ \frac{\mu_\pi}{3} u_0 \varphi_\sigma(u_0) T^3 f_2\left(\frac{s_0}{T}\right) - \frac{f_{3\pi}}{f_\pi} (I_1 + I_2 + I_5) T^3 f_2\left(\frac{s_0}{T}\right) + \frac{a}{12} u_0 \varphi_\pi(u_0) \left(1 - \frac{m_0^2}{16T^2}\right) T f_0\left(\frac{s_0}{T}\right) \right. \\ \left. - \frac{a}{12T} u_0 [G_2(u_0) + g_1(u_0)] \left(1 - \frac{m_0^2}{16T^2}\right) \right\}. \quad (41)$$

The functions  $G_2(u_0)$ ,  $I_i$  are defined as

$$G_2(u_0) = - \int_0^{u_0} g_2(u) du, \quad (42)$$

$$I_1 = \int_0^{u_0} d\alpha_1 \int_0^{1-u_0} d\alpha_2 \frac{u_0 - \alpha_1}{\alpha_2^2} \varphi_{3\pi}(\alpha_i), \quad (43)$$

$$I_2 = \int_0^{u_0} d\alpha_1 \int_0^{1-u_0} d\alpha_2 \frac{1-u_0-\alpha_2}{\alpha_2^2} \varphi_{3\pi}(\alpha_i), \quad (44)$$

$$I_3 = \int_0^{u_0} d\alpha_1 \frac{d}{d\alpha_3} \left[ \frac{\varphi_{3\pi}(\alpha_1, 1-\alpha_1-\alpha_3, \alpha_3)}{\alpha_3} \right] \Big|_{\alpha_3=u_0-\alpha_1} - \int_0^{u_0} d\alpha_1 \frac{\varphi_{3\pi}(\alpha_1, 1-u_0, u_0-\alpha_1)}{(u_0-\alpha_1)^2} \\ + \int_0^{1-u_0} d\alpha_2 \frac{\varphi_{3\pi}(u_0, \alpha_2, 1-u_0-\alpha_2)}{(1-u_0-\alpha_2)^2}, \quad (45)$$

$$I_4 = \int_0^{1-u_0} \frac{d\alpha_3}{\alpha_3} \left[ \frac{d\varphi_{3\pi}(\alpha_1, 1-\alpha_1-\alpha_3, \alpha_3)}{d\alpha_1} \right] \Big|_{\alpha_1=u_0} + \int_0^{u_0} d\alpha_1 \frac{\varphi_{3\pi}(\alpha_1, 1-u_0, u_0-\alpha_1)}{(u_0-\alpha_1)^2} - \int_0^{1-u_0} d\alpha_2 \frac{\varphi_{3\pi}(u_0, \alpha_2, 1-u_0-\alpha_2)}{(1-u_0-\alpha_2)^2}, \quad (46)$$

$$I_5 = - \int_0^{1-u_0} u_0 \frac{d\alpha_3}{\alpha_3} \varphi_{3\pi}(u_0, 1-u_0-\alpha_3, \alpha_3) + \int_0^{u_0} d\alpha_1 \int_0^{1-u_0} d\alpha_2 \frac{2u_0-1+\alpha_2}{\alpha_2^2} \varphi_{3\pi}(\alpha_i), \quad (47)$$

$$\begin{aligned}
I_6 = & \frac{d[\alpha_1 \varphi_{3\pi}(\alpha_1, 1 - \alpha_1 - \alpha_3, \alpha_3)]}{d\alpha_1} \Big|_{\alpha_3=1-u_0}^{\alpha_1=u_0} - \int_0^{1-u_0} d\alpha_3 \frac{d^2}{d\alpha_1^2} \left[ \varphi_{3\pi}(\alpha_1, 1 - \alpha_1 - \alpha_3, \alpha_3) \frac{\alpha_1}{\alpha_3} \right]_{\alpha_1=u_0} \\
& + \left[ \varphi_{3\pi}(\alpha_1, 1 - \alpha_1 - \alpha_3, \alpha_3) \frac{\alpha_3 - \alpha_1}{\alpha_3^2} \right]_{\alpha_3=u_0}^{\alpha_1=0} + \int_0^{u_0} d\alpha_3 \int_0^{u_0-\alpha_3} d\alpha_1 \frac{d}{d\alpha_1} \left[ \varphi_{3\pi}(\alpha_1, 1 - \alpha_1 - \alpha_3, \alpha_3) \frac{\alpha_3 - \alpha_1}{\alpha_3^2} \right] \\
& + \left[ \varphi_{3\pi}(\alpha_1, 1 - \alpha_1 - \alpha_3, \alpha_3) \frac{\alpha_3 - \alpha_1}{\alpha_3^2} \right]_{\alpha_3=1-u_0}^{\alpha_1=u_0} - \int_0^{1-u_0} d\alpha_3 \frac{d}{d\alpha_1} \left[ \varphi_{3\pi}(\alpha_1, 1 - \alpha_1 - \alpha_3, \alpha_3) \frac{\alpha_3 - \alpha_1}{\alpha_3^2} \right]_{\alpha_1=u_0} \\
& - 2 \left[ \frac{\varphi_{3\pi}(\alpha_i)}{\alpha_3} \right]_{\alpha_3=u_0}^{\alpha_1=0} - 2 \int_0^{u_0} d\alpha_3 \int_0^{u_0-\alpha_3} d\alpha_1 \frac{d}{d\alpha_1} \left[ \frac{\varphi_{3\pi}(\alpha_1, 1 - \alpha_1 - \alpha_3, \alpha_3)}{\alpha_3} \right] \\
& - 2 \int_0^{u_0} d\alpha_3 \int_0^{u_0-\alpha_3} d\alpha_1 \frac{\varphi_{3\pi}(\alpha_1, 1 - \alpha_1 - \alpha_3, \alpha_3)}{\alpha_3^2} + 2 \int_0^{1-u_0} d\alpha_3 \int_0^{1-u_0-\alpha_3} d\alpha_2 \frac{\varphi_{3\pi}(1 - \alpha_2 - \alpha_3, \alpha_2, \alpha_3)}{\alpha_3^2}, \quad (48)
\end{aligned}$$

where  $\alpha_3$  and  $\alpha_1$  are the longitudinal momentum fraction of gluon and down quark inside the pion, respectively.

#### D. Determination of the parameters for pionic sum rules

The mass difference between  $\Lambda_{c1}$  and  $\Sigma_c$  is only about 0.1 GeV in the leading order of HQET. And the values of the Borel parameter  $T_1, T_2$  are around 2 GeV in the working region. So we choose to work at the symmetric point  $T_1 = T_2 = 2T$ , i.e.,  $u_0 = \frac{1}{2}$ , which diminishes the uncertainty arising from the pion wave functions and enables a rather clean subtraction of the continuum contribution.

The pion wave functions and their values at the middle point are discussed in [23–25]. At the scale  $\mu = 1.0$  GeV the values of the various functions appearing in Eqs. (40),(41) at  $u_0 = \frac{1}{2}$  are  $\varphi_\pi(u_0) = (1.5 \pm 0.2)$  [25],  $\varphi_P(u_0) = 1.142$ ,  $\varphi_\sigma(u_0) = 1.463$ ,  $g_1(u_0) = 0.034$  GeV<sup>2</sup>,  $G_2(u_0) = 0.02$  GeV<sup>2</sup> [24],  $\varphi'_\sigma(u_0) = 0$ ,  $g_2(u_0) = 0$ ,  $[u\varphi_\pi(u)]''_{u=u_0} = [u\varphi_\sigma(u)]''_{u=u_0} = -6$ ,  $[ug_1(u) + uG_2(u)]''_{u=u_0} = -0.29$ ,  $I_1 = 1.17$ ,  $I_2 = 1.17$ ,  $I_3 = 31.9$ ,  $I_4 = -31.9$ ,  $I_5 = -1.64$ ,  $I_6 = 247.5$ ,  $f_{3\pi} = 0.0035$  GeV<sup>2</sup>. We have used the forms in [24] for  $\varphi_{3\pi}(\alpha_i)$  to calculate integrals  $I_i$ . The three-particle wave functions have been known to next order in the conformal spin expansion up to now. The second derivatives need knowledge of the detailed shape of the pion wave functions at the middle point. Various sources indicate that  $\varphi_\pi(u)$  is very close to the asymptotic form [25], which is exactly known. Based on these considerations we have employed the asymptotic forms to extract the second derivatives for  $\varphi_\sigma(u)$  and  $\varphi_\pi(u)$ .

#### E. Numerical analysis of pionic sum rules

Note that the spectral density of the sum rule (40),(41) is either proportional to  $s^2$  or  $s^4$ ; the continuum has to be subtracted carefully. We use  $s_0 = (1.3 \pm 0.15)$  GeV, which is the average of the thresholds of the  $\Lambda_{c1}$  and  $\Sigma_c$  mass sum rules. The variation of  $g_{s,d}$  with the Borel parameter  $T$  and  $s_0$  is presented in Fig. 1 and Fig. 2. The curves correspond to  $s_0$

= 1.2, 1.3, 1.4 GeV from bottom to top respectively. Stability develops for these sum rules in the region  $0.5$  GeV  $< T < 1.5$  GeV; we get

$$g_s f_{\Lambda_{c1}} f_{\Sigma_c} = (0.5 \pm 0.3) \times 10^{-3} \text{ GeV}^7, \quad (49)$$

$$g_d f_{\Lambda_{c1}}^* f_{\Sigma_c} = (2.8 \pm 0.6) \times 10^{-3} \text{ GeV}^5, \quad (50)$$

where the errors refer to the variations with  $T$  and  $s_0$  in this region. And the central value corresponds to  $T = 1$  GeV and  $s_0 = 1.3$  GeV.

Combining Eqs. (22), (23) we get

$$g_s = (0.5 \pm 0.3), \quad (51)$$

$$g_d = (2.8 \pm 0.6) \text{ GeV}^{-2}. \quad (52)$$

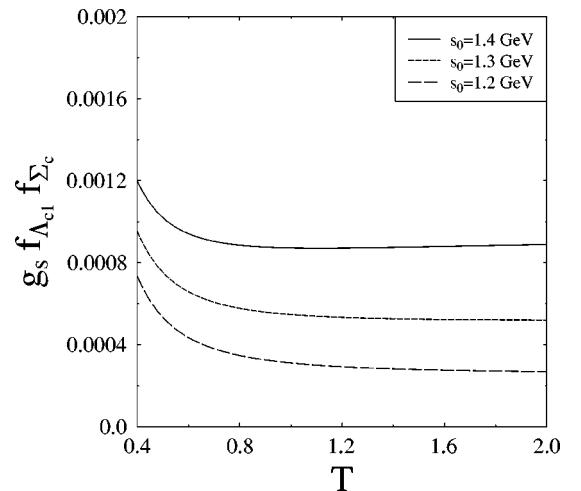
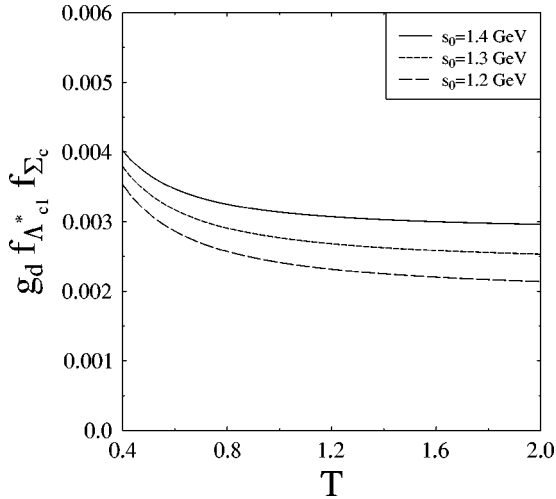


FIG. 1. Dependence of  $g_s f_{\Lambda_{c1}} f_{\Sigma_c}$  on the Borel parameter  $T$  for different values of the continuum threshold  $s_0$ . From top to bottom the curves correspond to  $s_0 = 1.4, 1.3, 1.2$  GeV.


 FIG. 2. Dependence of  $g_d f_{\Lambda_{c1}^*} f_{\Sigma_c}$  on  $T, s_0$ .

We collect the values of the pionic couplings from various approaches in Table I. Note that in our notation  $3g_d$  corresponds to that in [14].

We use the following formulas to calculate the pionic decay widths of  $p$ -wave heavy baryons:

$$\Gamma(\Lambda_{c1} \rightarrow \Sigma_c \pi) = \frac{g_s^2}{2\pi} \frac{m_{\Sigma_c}}{m_{\Lambda_{c1}}} |q|, \quad (53)$$

$$\Gamma(\Lambda_{c1}^* \rightarrow \Sigma_c \pi) = \frac{g_d^2}{2\pi} \frac{m_{\Sigma_c}}{m_{\Lambda_{c1}^*}} |q|^5, \quad (54)$$

where  $|q|$  is the pion decay momentum. We use the values  $m_{\Lambda_{c1}} = 2.593$  GeV,  $m_{\Lambda_{c1}^*} = 2.625$  GeV,  $m_{\Sigma_c} = 2.452$  GeV [1]. In the  $\Lambda_{c1}$  decays, as a result of isospin symmetry violations of the pion and  $\Sigma_c$  multiplet masses, the pion decay momentum is 17,23,32 MeV for the final states  $\Sigma_c^{++} \pi^-, \Sigma_c^0 \pi^+, \Sigma_c^+ \pi^0$ , respectively. This effect causes a significant difference in the decay widths, which are collected in Table II. Summing all the three isospin channels we get  $\Gamma(\Lambda_{c1} \rightarrow \Sigma_c \pi) = 2.7$  MeV and  $\Gamma(\Lambda_{c1}^* \rightarrow \Sigma_c \pi) = 33$  keV. The latter is nearly suppressed by two orders of magnitude due to  $d$ -wave decays.

TABLE II. Single-pion decay widths.

	Our results	Ref. [6]	Ref. [14]	Ref. [10]	Experiment
<i>s</i> -wave transitions					
$\Lambda_{c1;S}(2593) \rightarrow \Sigma_c^0 \pi^+$	0.86 MeV	$0.89 \pm 0.86$ MeV	$0.83 \pm 0.09$ MeV	$1.775 \pm 0.695$ MeV	$0.86_{-0.56}^{+0.73}$ MeV
$\Lambda_{c1;S}(2593) \rightarrow \Sigma_c^+ \pi^0$	1.2 MeV	$1.7 \pm 0.49$ MeV	$0.98 \pm 0.12$ MeV	$1.18 \pm 0.46$ MeV	$\Gamma(\Lambda_{c1;S}) = 3.6_{-1.3}^{+2.0}$ MeV
$\Lambda_{c1;S}(2593) \rightarrow \Sigma_c^{++} \pi^-$	0.64 MeV	$0.55_{-0.55}^{+1.3}$ MeV	$0.79 \pm 0.09$ MeV	$1.47 \pm 0.57$ MeV	$0.86_{-0.56}^{+0.73}$ MeV
<i>d</i> -wave transitions					
$\Lambda_{c1;S}^*(2625) \rightarrow \Sigma_c^0 \pi^+$	0.011 MeV	0.013 MeV	$0.080 \pm 0.009$ MeV	$0.465 \pm 0.245$ MeV	$< 0.13$ MeV
$\Lambda_{c1;S}^*(2625) \rightarrow \Sigma_c^+ \pi^0$	0.011 MeV	0.013 MeV	$0.095 \pm 0.012$ MeV	$0.42 \pm 0.22$ MeV	$\Gamma(\Lambda_{c1}^*) < 1.9$ MeV
$\Lambda_{c1;S}^*(2625) \rightarrow \Sigma_c^{++} \pi^-$	0.011 MeV	0.013 MeV	$0.076 \pm 0.009$ MeV	$0.44 \pm 0.23$ MeV	$< 0.15$ MeV

TABLE I. Pionic coupling constants.

Coupling	Our results	Ref. [14]	Ref. [10]
$g_s$	$0.5 \pm 0.3$	0.52	$0.665 \pm 0.135$
$3g_d$	$(8.4 \pm 1.8) \text{ GeV}^{-2}$	$21.5 \text{ GeV}^{-2}$	$50.85 \pm 14.25 \text{ GeV}^{-2}$

From Table II we see that our results are numerically close to those from fixing the unknown coupling constants from the  $p$ -wave strange baryon strong decay widths assuming heavy quark effective theory could be extended to the strange quark case [6]. The values of  $d$ -wave decay widths from the above approach and ours are much smaller than those from the quark models [14,10,12]. As for the  $s$ -wave decays various approaches yield consistent results.

#### IV. RADIATIVE DECAYS OF $p$ -WAVE HEAVY BARYONS

##### A. Correlator

The light cone photon wave functions have been used to discuss radiative decay processes in [26–30,19] in the framework of QCD sum rules. We extend the same formalism to extract the electromagnetic coupling constants for the  $\Lambda_{c1}$  doublet decays.

The radiative coupling constants are defined through the following amplitudes:

$$M(\Lambda_{c1} \rightarrow \Sigma_c \gamma) = e \epsilon_{\beta\nu\rho\mu} q^\beta e^{\nu*} \bar{u}_{\Sigma_c} [f_s g_t^{\rho\alpha} + f_d q^\alpha v^\rho] \gamma'_\alpha \gamma'_t u_{\Lambda_{c1}}, \quad (55)$$

$$M(\Lambda_{c1} \rightarrow \Sigma_c^* \gamma) = \sqrt{3} e \epsilon_{\beta\nu\rho\mu} q^\beta e^{\nu*} \bar{u}_{\Sigma_c^*} [f_s^1 g_t^{\rho\alpha} + f_d^1 q^\alpha v^\rho] \gamma_5 \gamma'_t u_{\Lambda_{c1}}, \quad (56)$$

$$M(\Lambda_{c1}^* \rightarrow \Sigma_c \gamma) = \sqrt{3} e \epsilon_{\beta\nu\rho\mu} q^\beta e^{\nu*} \bar{u}_{\Sigma_c} [f_s^2 g_t^{\rho\alpha} + f_d^2 q^\alpha v^\rho] \gamma_5 \gamma'_t u_{\Lambda_{c1}^*}, \quad (57)$$

$$M(\Lambda_{c1}^* \rightarrow \Sigma_c^* \gamma) = 3e \epsilon_{\beta\nu\rho\mu} q^\beta e^{\nu*} \bar{u}_{\Sigma_c^*} [f_s^3 g_t^{\rho\alpha} + f_d^3 q^\alpha v^\rho] u_{\Lambda_{c1}^*}^{\mu}, \quad (58)$$

where  $e_\mu(\lambda)$  and  $q_\mu$  are the photon polarization vector and momentum, respectively, and  $e$  is the charge unit. Because of heavy quark symmetry, we have  $f_s^1=f_s^2=f_s^3=f_s$ ,  $f_d^1=f_d^2=f_d^3=f_d$ . As in the case of pionic couplings there are only two independent coupling constants associated with  $E1$  and  $M2$  decays.

We consider the correlator

$$\begin{aligned} & \int d^4x e^{-ik \cdot x} \langle \gamma(q) | T[\eta_{\Lambda_{c1}}(0) \bar{\eta}_{\Sigma_c}(x)] | 0 \rangle \\ &= e^{-\frac{1+\hat{v}}{2}} \gamma_\alpha^t \gamma_t^\mu \epsilon_{\beta\nu\rho\mu} v^\beta e^{\nu*} \{ F_s(\omega, \omega') g_t^{\rho\alpha} \\ & \quad + F_d(\omega, \omega') q^\alpha v^\rho \}, \end{aligned} \quad (59)$$

where  $F_{s,d}(\omega, \omega')$  has the same pole structures as  $G_{s,d}(\omega, \omega')$ .

The light-cone two-particle photon wave functions are [26]

$$\begin{aligned} & \langle \gamma(q) | \bar{q}(x) \sigma_{\mu\nu} q(0) | 0 \rangle \\ &= ie_q e \langle \bar{q}q \rangle \int_0^1 du e^{iuqx} \{ (e_\mu q_\nu - e_\nu q_\mu) \\ & \quad \times [\chi\phi(u) + x^2 h_1(u)] \\ & \quad + [(qx)(e_\mu x_\nu - e_\nu x_\mu) + (ex)(x_\mu q_\nu - x_\nu q_\mu) \\ & \quad - x^2(e_\mu q_\nu - e_\nu q_\mu)] h_2(u) \}, \end{aligned} \quad (60)$$

$$\begin{aligned} & \langle \gamma(q) | \bar{q}(x) \gamma_\mu \gamma_5 q(0) | 0 \rangle \\ &= \frac{f}{4} e_q e \epsilon_{\mu\nu\rho\sigma} e^\nu q^\rho x^\sigma \int_0^1 du e^{iuqx} \psi(u). \end{aligned} \quad (61)$$

The  $\phi(u), \psi(u)$  are associated with the leading twist-2 photon wave function (PWF), while  $g_1(u)$  and  $g_2(u)$  are twist-4 PWFs. All these PWFs are normalized to unity,  $\int_0^1 du f(u) = 1$ .

We want to emphasize that the photon light cone wave functions include the complete perturbative and nonperturbative electromagnetic interactions for the light quarks in principle. Yet the interaction of the photon with the heavy quark is not parametrized and constrained by the photon light cone wave functions. It seems possible that the photon couples directly to the heavy quark line. This is different from the QCD sum rules for the pionic couplings since pions cannot couple directly to the heavy quark. However, the real photon coupling to heavy quarks involves a spin-flip transition, which is suppressed by a factor of  $1/m_Q$  [9]. So it vanishes in the leading order of  $1/m_Q$  expansion. Since we are interested in the leading order couplings  $f_{s,d}$ , it is enough to keep the photon light cone wave functions for the light quarks only.

Expressing Eq. (59) with the photon wave functions, we arrive at

$$\begin{aligned} F_s(\omega, \omega') &= \frac{1}{\pi^2} (e_u - e_d) \langle \bar{q}q \rangle \\ & \times \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega t} e^{iu\omega' t} \left\{ \left[ \frac{1}{t^4} \chi\phi(u) \right. \right. \\ & \quad \left. \left. + \frac{1}{t^2} [h_1(u) - h_2(u)] \right] + \frac{\pi^2}{24} f\psi(u)t \right\} + \dots, \end{aligned} \quad (62)$$

$$\begin{aligned} F_d(\omega, \omega') &= \frac{i}{\pi^2} (e_u - e_d) \langle \bar{q}q \rangle \\ & \times \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega t} e^{iu\omega' t} u \left\{ \left[ \frac{1}{t^3} \chi\phi(u) \right. \right. \\ & \quad \left. \left. + \frac{1}{t} [h_1(u) - h_2(u)] \right] + \frac{\pi^2}{24} f\psi(u)t \right\} + \dots \end{aligned} \quad (63)$$

The final sum rules are

$$\begin{aligned} f_s f_{\Lambda_{c1}} f_{\Sigma_c} &= -\frac{a}{4\pi^4} (e_u - e_d) e^{(\bar{\Lambda}_{\Lambda_{c1}} + \bar{\Lambda}_{\Sigma_c})/2T} \left\{ \chi\phi(u_0) T^5 f_4 \left( \frac{s_0}{T} \right) \right. \\ & \quad \left. - [h_1(u_0) - h_2(u_0)] T^3 f_2 \left( \frac{s_0}{T} \right) \right. \\ & \quad \left. + \frac{\pi^2}{24} f\psi(u_0) T^1 f_0 \left( \frac{s_0}{T} \right) \right\}, \end{aligned} \quad (64)$$

$$\begin{aligned} f_d f_{\Lambda_{c1}} f_{\Sigma_c} &= -\frac{a}{4\pi^4} (e_u - e_d) e^{(\bar{\Lambda}_{\Lambda_{c1}} + \bar{\Lambda}_{\Sigma_c})/2T} u_0 \\ & \times \left\{ \chi\phi(u_0) T^4 f_3 \left( \frac{s_0}{T} \right) - [h_1(u_0) \right. \\ & \quad \left. - h_2(u_0)] T^2 f_1 \left( \frac{s_0}{T} \right) + \frac{\pi^2}{24} f\psi(u_0) \right\}. \end{aligned} \quad (65)$$

## B. Numerical analysis of the photonic sum rules

The leading photon wave functions receive only small corrections from the higher conformal spins [23], so they do not deviate much from the asymptotic form. We shall use [27]

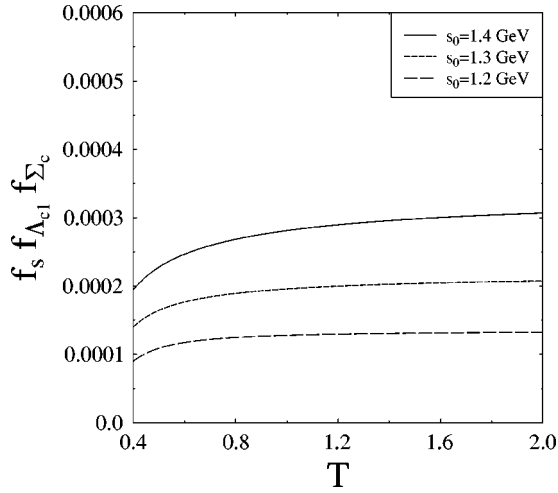
$$\phi(u) = 6u(1-u), \quad (66)$$

$$\psi(u) = 1, \quad (67)$$

$$h_1(u) = -\frac{1}{8}(1-u)(3-u), \quad (68)$$

$$h_2(u) = -\frac{1}{4}(1-u)^2, \quad (69)$$




 FIG. 3. Dependence of  $f_s f_{\Lambda_{c1}} f_{\Sigma_c}$  on  $T, s_0$ .

with  $f=0.028 \text{ GeV}^2$  and  $\chi=-4.4 \text{ GeV}^2$  [31–34] at the scale  $\mu=1 \text{ GeV}$ .

The variation of  $f_{s,d}$  with the Borel parameter  $T$  and  $s_0$  is presented in Fig. 3 and Fig. 4. Stability develops for the sum rules (64), (65) in the region  $0.5 \text{ GeV} < T < 1.5 \text{ GeV}$ ; we get:

$$f_s f_{\Lambda_{c1}} f_{\Sigma} = (2.0 \pm 0.8) \times 10^{-4} \text{ GeV}^6, \quad (70)$$

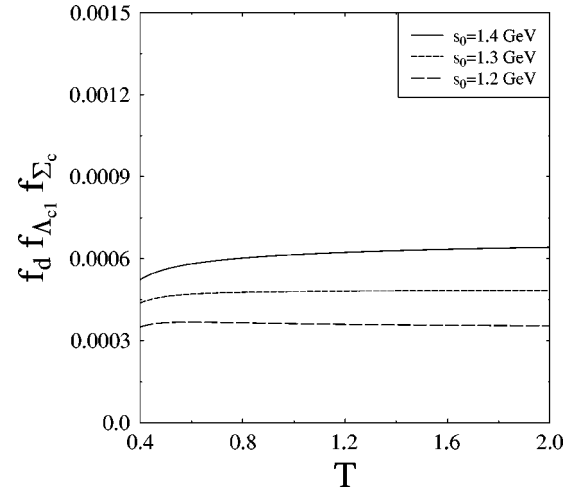
$$f_d f_{\Lambda_{c1}} f_{\Sigma} = (4.8 \pm 1.2) \times 10^{-4} \text{ GeV}^5, \quad (71)$$

where the errors refer to the variations with  $T$  and  $s_0$  in this region. And the central value corresponds to  $T=1.0 \text{ GeV}$  and  $s_0=1.3 \text{ GeV}$ . Our final result is

$$f_s = (0.20 \pm 0.08) \text{ GeV}^{-1}, \quad (72)$$

$$f_d = (0.48 \pm 0.12) \text{ GeV}^{-2}. \quad (73)$$

The decay width formulas in the leading order of HQET are


 FIG. 4. Dependence of  $f_d f_{\Lambda_{c1}} f_{\Sigma_c}$  on  $T, s_0$ .

$$\Gamma(\Lambda_{c1} \rightarrow \Sigma_c \gamma) = 16\alpha |\vec{q}|^3 \frac{m_{\Sigma_c}}{m_{\Lambda_{c1}}} \left[ f_s^2 + \frac{1}{2} f_d^2 |\vec{q}|^2 \right],$$

$$\Gamma(\Lambda_{c1} \rightarrow \Sigma_c^* \gamma) = 8\alpha |\vec{q}|^3 \frac{m_{\Sigma_c^*}}{m_{\Lambda_{c1}}} \left[ f_s^2 + \frac{1}{2} f_d^2 |\vec{q}|^2 \right],$$

$$\Gamma(\Lambda_{c1}^* \rightarrow \Sigma_c \gamma) = 4\alpha |\vec{q}|^3 \frac{m_{\Sigma_c}}{m_{\Lambda_{c1}^*}} \left[ f_s^2 + \frac{1}{2} f_d^2 |\vec{q}|^2 \right],$$

$$\Gamma(\Lambda_{c1}^* \rightarrow \Sigma_c^* \gamma) = 20\alpha |\vec{q}|^3 \frac{m_{\Sigma_c^*}}{m_{\Lambda_{c1}^*}} \left[ f_s^2 + \frac{1}{2} f_d^2 |\vec{q}|^2 \right], \quad (74)$$

where  $|\vec{q}| = 134, 72, 164, 103 \text{ MeV}$  is the photon decay momentum for the above four processes. The d-wave decay width is negligible. The decay width values are collected in Table III. The uncertainty is typically about 50%.

TABLE III. Radiative decay widths.

	Our results	Ref. [13]	Ref. [14]	Others	Experiment [1]
$\Lambda_{c1;s}(2593) \rightarrow \Lambda_c^+ \gamma$	0.036 MeV	0	$0.115 \pm 0.001 \text{ MeV}$ $0.016 \text{ MeV}$ [7]	$0.191 c_{RT}^2 \text{ MeV}$ [5]	$< 2.36_{-0.85}^{+1.31} \text{ MeV}$
$\Lambda_{c1;s}(2593) \rightarrow \Sigma_c^+ \gamma$	0.011 MeV	0.071 MeV	$0.077 \pm 0.001 \text{ MeV}$	$0.127 c_{RS}^2$ [5]	
$\Lambda_{c1;s}(2593) \rightarrow \Sigma_c^{*+} \gamma$	0.001 MeV	0.011 MeV	$0.006 \pm 0.0001 \text{ MeV}$	$0.006 c_{RS}^2$ [5]	
$\Lambda_{c1;s}^*(2625) \rightarrow \Lambda_c^+ \gamma$	0.048 MeV	0	$0.151 \pm 0.002 \text{ MeV}$ $0.021 \text{ MeV}$ [7]	$0.253 c_{RT}^2 \text{ MeV}$ [5]	$< 1 \text{ MeV}$
$\Lambda_{c1;s}^*(2625) \rightarrow \Sigma_c^+ \gamma$	0.005 MeV	0.13 MeV	$0.035 \pm 0.0005 \text{ MeV}$	$0.058 c_{RS}^2$ [5]	
$\Lambda_{c1;s}^*(2625) \rightarrow \Sigma_c^{*+} \gamma$	0.006 MeV	0.032 MeV	$0.046 \pm 0.0006 \text{ MeV}$	$0.054 c_{RS}^2$ [5]	

The decays  $\Lambda_{Q1} \rightarrow \Sigma_Q \gamma$  do not occur in the leading order in the bound state picture [7]. Because of the unknown coupling constant  $c_{RS}$  in the chiral Lagrangian for the heavy quark electromagnetic interactions, no numerical values are available [5]. However, the decay width ratios of the four final states are exactly the same as ours if we ignore the isospin violations of the heavy multiplet masses in the heavy quark limit. Our results are much smaller than those from various versions of quark models [13,14], which may indicate that the  $1/m_c$  correction is important.

### V. PROCESS $\Lambda_{c1} \rightarrow \Lambda_c \gamma$ , etc.

As can be seen later the radiative decay processes of the  $p$ -wave  $\Lambda_{c1}$  doublet to  $\Lambda_c$  is quite different from those in the previous section. We present more details here. The possible  $E1$  decay amplitudes are

$$M(\Lambda_{c1} \rightarrow \Lambda_c \gamma) = e h_p e_\mu^* \bar{u}_{\Lambda_c} [g_t^{\mu\nu} v \cdot q - v^\mu q^\nu] \gamma_\mu \gamma_5 u_{\Lambda_{c1}}, \quad (75)$$

$$M(\Lambda_{c1}^* \rightarrow \Lambda_c \gamma) = \sqrt{3} e h_p' e_\mu^* \bar{u}_{\Lambda_c} [g_t^{\mu\nu} v \cdot q - v^\mu q^\nu] u_{\Lambda_{c1}}^v. \quad (76)$$

As a result of heavy quark symmetry,  $h_p = h_p'$ .

We consider the correlator

$$\begin{aligned} \Pi &= i \int d^4x e^{ik \cdot x} \langle \gamma(q) | T[\eta_{\Lambda_{c1}}(x) \bar{\eta}_{\Lambda_c}(0)] | 0 \rangle \\ &= \frac{1+\hat{v}}{2} e_\mu^* [g_t^{\mu\nu} v \cdot q - v^\mu q^\nu] \gamma_\mu \gamma_5 H_p(\omega, \omega'). \end{aligned} \quad (77)$$

We first calculate the part solely involved with the light quark, which can be expressed with the photon wave functions. We get

$$\begin{aligned} \Pi &= 2i \int_0^\infty dt \int d^4x e^{ik \cdot x} \hat{D}_t \delta(x-vt) \gamma_5 \frac{1+\hat{v}}{2} \\ &\quad \times \{ \text{Tr}[\gamma_5 C i S_u^T(x) C \gamma_5 \langle \gamma(q) | d(x) \bar{d}(0) | 0 \rangle] + (u \leftrightarrow d) \}, \end{aligned} \quad (78)$$

where a summation over color has been performed. There are two types of terms with even  $\gamma$  matrices in the trace. The first one is connected with  $\psi(u)$  and the trace looks like  $\text{Tr}[\gamma_5 C \hat{x}^T C \gamma_5 \gamma_\mu \gamma_5]$ . The second is involved with  $\phi(u), h_1(u), h_2(u)$  and the trace looks like  $\text{Tr}[\gamma_5 C 1 C \gamma_5 \sigma_{\mu\nu}]$ . In both  $\Lambda_{Q1}, \Lambda_Q$  states the up and down quarks are in the  $0^+$  state, which leads to the presence of  $\gamma_5 C$  and  $C \gamma_5$  in both traces. Clearly both traces vanish. This property results from the underlying flavor and spin structure of the light quark sector. In other words the light quark contribution is zero to all orders of the heavy quark expansion in the framework of the LCQSR with the commonly used interpolating currents (6) and (9) for  $\Lambda_Q$  and  $\Lambda_{Q1}$ , respectively. The decays  $\Lambda_{Q1} \rightarrow \Lambda_Q \gamma$  and  $\Lambda_{Q1}^* \rightarrow \Lambda_Q \gamma$  happen only when the photon couples directly to the heavy quark line.

Now let us move to the part involved with the heavy quark. At first sight there are two types of terms in the leading order of heavy quark expansion. The first one comes from insertion of the operator  $i \int \bar{h}_v(y) i v \cdot D h_v(y) d^4y$  in Eq. (77), which contributes a factor  $v \cdot e^*(\lambda)$  to the decay amplitude. For the real photon  $v \cdot e^*(\lambda) = 0$ , so it drops out. The other possible term arises from the covariant derivative in  $\eta_{\Lambda_{c1}}$ , which leads to a nonzero correlator. For the tensor structure  $i \hat{e}^* \gamma_5 (1 + \hat{v})/2$  we have

$$\Pi(\omega, \omega') = -\frac{e}{\pi^4} \int_0^\infty dt e^{i\omega' t} \left\{ \frac{6}{t^6} + \frac{\langle g_s^2 G^2 \rangle}{64t^2} - \frac{a^2}{96} \right\}, \quad (79)$$

where the photon field has contributed a factor  $e^{-iq \cdot x}$ . It is important to note that only the variable  $\omega'$  appears in Eq. (79). It is a single-pole term which must vanish after we make a double Borel transformation to the variables  $\omega, \omega'$  simultaneously. We have shown that there is no leading order  $E1$  transition in Eq. (75) arising from the photon couplings to the heavy quark line in the leading order of heavy quark expansion. Based on the same spin and flavor consideration we know that radiative decay processes like  $\Sigma_{Q1} \rightarrow \Lambda_{Q1} \gamma$ ,  $\Lambda_{Q1} \rightarrow \Lambda_{Q1} \gamma$ ,  $\Sigma_{Q1} \rightarrow \Sigma_{Q1} \gamma$  are also forbidden in the leading order of  $1/m_Q$  expansion, where we have used notations in [9].

We may rewrite the decay amplitudes as

$$M(\Lambda_{c1} \rightarrow \Lambda_c \gamma) = e f_p F_{\mu\nu} \bar{u}_{\Lambda_c} \sigma^{\mu\nu} \gamma_5 u_{\Lambda_{c1}}, \quad (80)$$

$$M(\Lambda_{c1}^* \rightarrow \Lambda_c \gamma) = 2\sqrt{3} e f_p^1 F_{\mu\nu} v^\mu \bar{u}_{\Lambda_c} u_{\Lambda_{c1}}^v, \quad (81)$$

$$M(\Lambda_{c1}^* \rightarrow \Lambda_{c1} \gamma) = 2\sqrt{3} e f_p^2 F_{\mu\nu} \bar{u}_{\Lambda_{c1}} \gamma_t^\mu \gamma_5 u_{\Lambda_{c1}}^v. \quad (82)$$

As a result of heavy quark symmetry, we have

$$f_p = f_p^1 = f_p^2. \quad (83)$$

Note that  $f_p = \frac{1}{4} h_p$ .

In these decays we know that the light quarks do not contribute. However, the  $J^P$  of the light diquark changes from  $1^-$  to  $0^+$  which ensures that the decay  $\Lambda_{c1} \rightarrow \Lambda_c \gamma$  is an  $E1$  transition. The angular momentum and parity  $J^P = \frac{1}{2}^+$  of the heavy quark do not change, so the coupling constant  $f_p$  is the same as that for the heavy quark  $M1$  transition, which is induced by the magnetic moment operator

$$f_p = \frac{\mu_c}{2} = \frac{e_c}{4m_c}. \quad (84)$$

Another approach is to consider the three-point correlation function for the tensor structure  $\hat{e}_t \gamma_5 (1 + \hat{v})/2$ ,

$$\begin{aligned}
& i \int d^4x d^4z e^{ikx - ik'z} \langle 0 | T \\
& \times \left\{ \eta_{\Lambda_{c1}}(x), \frac{\mathcal{K}(0) + \mathcal{S}(0)}{2m_c}, \bar{\eta}_{\Lambda_{c1}}(z) \right\} | 0 \rangle \\
& = \Pi_3(\omega, \omega') \frac{1 + \hat{v}}{2}, \tag{85}
\end{aligned}$$

with  $\omega = k \cdot v, \omega' = k' \cdot v$ :

$$\begin{aligned}
\Pi_3(\omega, \omega') &= \frac{2e_c}{m_c} \frac{1}{\pi^4} \int_0^\infty dt_1 dt_2 e^{i\omega t_1 + i\omega' t_2} \left\{ \frac{18}{(t_1 + t_2)^8} \right. \\
& \left. + \frac{\langle g_s^2 G^2 \rangle}{64(t_1 + t_2)^4} \right\}, \tag{86}
\end{aligned}$$

After the double Borel transformation and continuum subtraction we get the sum rule for  $h_p$ :

$$\begin{aligned}
& h_p (\bar{\Lambda}_{\Lambda_{c1}} - \bar{\Lambda}_{\Lambda_c}) f_{\Lambda_{c1}} f_{\Lambda_c} e^{-(\bar{\Lambda}_{\Lambda_{c1}} + \bar{\Lambda}_{\Lambda_c})/2T} \\
& = \frac{1}{\pi^4} \frac{e_c}{m_c} \left\{ 36T^8 f_7 \left( \frac{s_0}{T} \right) + \frac{\langle g_s^2 G^2 \rangle}{32} T^4 f_3 \left( \frac{s_0}{T} \right) \right\}. \tag{87}
\end{aligned}$$

Dividing Eq. (87) by Eq. (19) we get

$$\begin{aligned}
h_p &= \frac{e_c}{m_c} e^{-(\bar{\Lambda}_{\Lambda_{c1}} - \bar{\Lambda}_{\Lambda_c})/2T} \frac{f_{\Lambda_{c1}}}{3(\bar{\Lambda}_{\Lambda_{c1}} - \bar{\Lambda}_{\Lambda_c}) f_{\Lambda_c}} \\
& \times \frac{T^8 f_7 \left( \frac{s_0}{T} \right) + \frac{\langle g_s^2 G^2 \rangle}{1152} T^4 f_3 \left( \frac{s_0}{T} \right)}{T^8 f_7 \left( \frac{s_0}{T} \right) - \frac{\langle g_s^2 G^2 \rangle}{6912} T^4 f_3 \left( \frac{s_0}{T} \right) + \frac{m_0^2 a^2}{8192}}. \tag{88}
\end{aligned}$$

Numerically we have  $h_p \approx e_c/m_c$ , which is consistent with Eq. (84). The decay width formulas are

$$\begin{aligned}
\Gamma(\Lambda_{c1} \rightarrow \Lambda_c \gamma) &= e_c^2 \alpha |\vec{q}|^3 \frac{m_{\Lambda_c}}{m_{\Lambda_{c1}} m_c^2}, \\
\Gamma(\Lambda_{c1}^* \rightarrow \Lambda_c \gamma) &= e_c^2 \alpha |\vec{q}|^3 \frac{m_{\Lambda_c}}{m_{\Lambda_{c1}^*} m_c^2}, \\
\Gamma(\Lambda_{c1}^* \rightarrow \Lambda_{c1} \gamma) &= e_c^2 \alpha |\vec{q}|^3 \frac{m_{\Lambda_{c1}}}{m_{\Lambda_{c1}^*} m_c^2}. \tag{89}
\end{aligned}$$

The decay momentum is 290,320,32 MeV, respectively. We take  $m_c = 1.4$  GeV. The numerical values are collected in Table III. These widths come solely from the  $\mathcal{O}(1/m_Q)$  correction. But their numerical values are greater than those leading order widths for the channels  $\Sigma_c \gamma, \Sigma_c^* \gamma$ . The reason is purely kinematical. The decay momentum for the final

state  $\Lambda_c \gamma$  is 3 times larger. For the  $E1$  decay there appears an enhancement factor of 27.

These widths in Eqs. (89) are proportional to  $e_c^2/m_c^2$ . Therefore the corresponding radiative decays  $\Lambda_{b1} \rightarrow \Lambda_b \gamma, \Lambda_{b1}^* \rightarrow \Lambda_b \gamma, \Lambda_{b1}^* \rightarrow \Lambda_{b1} \gamma$  are further suppressed by a factor  $[(e_b/e_c)/(m_c/m_b)]^2 \sim 40$ . The widths of the first two decays are around 1 keV.

If we use naive dimensional analysis to let  $c_{RT}$  [5] in Table III be of the order of unity or simply assume that the  $E1$  transition coupling constant  $h_p$  in Eq. (75) is of the same order as the  $M1$  transition one [9], we would get a width  $\mathcal{O}(100)$  keV. Our result is in strong contrast with those from the bound state picture [7], where  $\Gamma(\Lambda_{c1}, \Lambda_{c1}^* \rightarrow \Lambda_c \gamma) = 16,21$  keV and  $\Gamma(\Lambda_{b1}, \Lambda_{b1}^* \rightarrow \Lambda_b \gamma) = 90,119$  keV. Future experiments should be able to judge which mechanism is correct.

It was noted that the radiative decay  $\Lambda_{Q1} \rightarrow \Lambda_Q$  was forbidden in the leading order of heavy quark symmetry assuming one-body transition operators, which arises from a complete cancellation due to the specific spins of light constituent quarks in the antisymmetric initial and final states [13]. The point is consistent with our observation of the vanishing contribution of the light quark sector to this radiative process.

From our calculation we know that the  $d$ -wave single-pion width of  $\Lambda_{c1}^*$  is 33 keV and the estimate in [6] yielded 35 keV for the two-pion decay width. It is interesting to note that the radiative decay widths are 48,5,6,0.014 keV for the final states  $\Lambda_c \gamma, \Sigma_c \gamma, \Sigma_c^* \gamma, \Lambda_{c1} \gamma$ , respectively. The width of the decay channel  $\Lambda_c \gamma$  is bigger than either of that of the strong decay modes. The  $\Lambda_{c1}^*$  should be a narrow state with a total width about 130 keV.

The two-pion width of  $\Lambda_{c1}$  is about 2.5 MeV [6]. From Tables II and III the one-pion and electromagnetic widths are  $\Gamma(\Sigma_c \pi, \Lambda_c \gamma, \Sigma_c \gamma, \Sigma_c^* \gamma) = 2.7, 0.048, 0.011, 0.001$  MeV. Its total width is about 5.4 MeV.

It is believed that  $\Lambda_{b1}$  lies below the  $\Sigma_b \pi, \Lambda_b \pi \pi$  threshold. If so, its dominant decays are electromagnetic. From our calculation we see that  $\Gamma(\Lambda_{b1} \rightarrow \Lambda_b \gamma, \Sigma_b \gamma, \Sigma_b^* \gamma) = 1, 11, 1$  keV if we assume the same decay momentum as in the  $\Lambda_{c1}$  decays. Its total width is about 13 keV. It will be a very narrow state. Clearly the radiative channels  $\Sigma_b \gamma$  will be very useful to find them experimentally.

The major decay modes of  $\Lambda_{b1}^*$  might be  $d$ -wave one-pion decay and electromagnetic decays to a  $\Sigma_b$  doublet if the two-pion mode is not allowed. Their widths are  $\Gamma(\Lambda_{b1}^* \rightarrow \Sigma_b \pi, \Lambda_b \gamma, \Sigma_b \gamma, \Sigma_b^* \gamma) = 33, 1.5, 6$  keV if we assume the same decay momentum as in the  $\Lambda_{c1}^*$  case. It is also a very narrow state with a width of 45 keV.

Before ending this section we want to improve our previous calculation of radiative decays of excited heavy mesons [19]. (1) First the  $s$ -wave terms involved with  $g_s$  should not appear in  $(1^+, 2^+) \rightarrow (0^-, 1^-) \gamma$  processes. All decays are  $M2$  transitions. The  $g_s^2$  in the decay width formulas should be replaced by  $\frac{1}{9} g_s^2 |\vec{q}|^4$ . The last eight widths in Eq. (94) should read 2,8,3,11,6,23,7,27 keV, respectively, which is much smaller than original wrong ones. (2) The  $E1$  transition

$(0^+, 1^+) \rightarrow (0^-, 1^-) \gamma$  decay was identified as  $s$ -wave decay. This was misleading. The factor  $(q \cdot v)$  should be in the tensor structure to ensure the  $E1$  transition structure in Eq. (47) in [19]. We present the correct sum rules for  $g_1$  below:

$$g_1 f_{-,1/2} f_{+,1/2} = -\frac{a}{4\pi^2} e^{(\Lambda_{-,1/2} + \Lambda_{+,1/2})/2T} \left\{ \chi \phi(u_0) T f_0 \left( \frac{s_0}{T} \right) - g_1(u_0) \frac{1}{T} \right\}, \quad (90)$$

where  $s_0 = \omega_c/2 = (1.5 \pm 0.2)$  GeV. Numerically we have  $g_1 = (1.6 \pm 0.2)$  GeV $^{-1}$ .

## VI. DISCUSSIONS

In our calculation only the errors due to the variations of  $T$  and  $s_0$  are included in the final results for  $g_{s,d}$ ,  $f_{s,d}$ . The various input parameters such as quark condensate, gluon condensate,  $\chi$ ,  $f$ , etc., also have some uncertainty. Among these the values of the pion and photon wave functions introduce largest uncertainty. Although their values are constrained by either experimental data or other QCD sum rule analyses, they may still lead to a  $\sim 25\%$  uncertainty. Keeping the light cone wave functions up to twist 4 also leads to some errors. However, the light cone sum rules are dominated by the lowest twist wave functions. Take the sum rule (65) for  $f_d$  for an example. At  $T=1$  GeV, the twist-4 term involved with  $h_1, h_2$  is only 9% of the leading twist term after the continuum subtraction. In other words the light cone expansion converges quickly. So we expect the contribution of higher twist terms to be small. There are other sources of uncertainty which are difficult to estimate. One is the QCD

radiative correction, which is not small in both the mass sum rule and LCQSRs for the pionic coupling constants of the ground state heavy hadrons in HQET. But their ratio depends only weakly on these corrections because of a large cancellation [35]. Numerically the radiative corrections are around 10% of the tree level result.

Another possible source is the  $1/m_Q$  correction for the charmed  $p$ -wave baryons. The leading order coupling constants  $g_{s,d}$ , etc., will be corrected by terms such as  $g'_{s,d}/m_Q$ , which will affect the decay widths. For the charmed hadrons  $1/m_Q$  corrections are sizable and may reach 30% while such corrections are generally less than 10% of the leading order term for the bottom system [17]. Especially for the  $E1$  transition coupling constant  $f_s$ , the correction is of the order  $e_c/4m_c$ , which may be comparable with the leading order one for the charm system. One is justified in using these coupling constants to calculate the decay widths of the  $p$ -wave bottom baryons. Unfortunately data are still not available for the  $p$ -wave bottom baryons. So we have calculated the  $p$ -wave  $\Lambda_{c1}$  doublet decay widths with some reservation.

In summary we have calculated the pionic and electromagnetic coupling constants and decay widths of the lowest  $p$ -wave heavy baryon doublet. We compared our calculation with different approaches in the literature. We hope these results will be useful in future experimental searches for  $\Lambda_{b1}, \Lambda_{b1}^*$  baryons.

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