Determination of the QCD color factor ratio C_A/C_F from the scale dependence of multiplicity in three jet events

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I examine the determination of the QCD color factor ratio C_A/C_F from the scale evolution of particle multiplicity in e^+e^- three jet events. I fit an analytic expression for the multiplicity in three jet events to event samples generated with QCD multihadronic event generators. I demonstrate that a one parameter fit of C_A/C_F yields the expected result $C_A/C_F=2.25$ in the limit of asymptotically large energies if energy conservation is included in the calculation. In contrast, a two parameter fit of C_A/C_F and a constant offset to the gluon jet multiplicity, proposed in a recent study, does not yield $C_A/C_F=2.25$ in this limit. I apply the one parameter fit method to recently published data of the DELPHI experiment at the e^+e^- collider LEP at CERN and determine the effective value of C_A/C_F from this technique, at the finite energy of the Z^0 boson, to be $1.74\pm0.03\pm0.10$, where the first uncertainty is statistical and the second is systematic.

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I. INTRODUCTION

At the basis of quantum chromodynamics (QCD), the gauge theory of strong interactions, are the color factors C_A and C_F, with values 3 and 4/3, respectively [1]. C_A determines the relative probability for a soft gluon to couple to another gluon, while C_F determines the corresponding probability for a soft gluon to couple to a quark. The ratio C_A/C_F is perhaps the most fundamental quantity in QCD in addition to the strong interaction coupling strength α_S. Currently, the most accurate measurements of C_A/C_F are from angular correlations between jets in four jet e^+e^- events [2] and from the ratio of soft particle multiplicities at large transverse momenta to the jet axes between unbiased gluon and quark jets [3].

Recently, a new method to measure C_A/C_F was proposed [4], based on the scale dependence of the mean particle multiplicity in e^+e^- three jet events, $N_{\text{3-jet}}$. This method utilizes a next-to-leading-order[1] (NLO) analytic expression for $N_{\text{3-jet}}$ [5], in conjunction with a constant offset term N_g^0 [4] for the gluon jet multiplicity, to perform a two parameter fit of C_A/C_F and N_g^0. The constant N_g^0 is intended to account for nonperturbative effects. The variable C_A/C_F is introduced using an analytic expression for the mean multiplicity ratio between gluon and quark jets, r. The expression used for r [6] does not incorporate recoil effects (energy conservation).

In this paper, I examine the determination of C_A/C_F from multiplicity in three jet events. My principal purpose is to test the analytic expressions for $N_{\text{3-jet}}$ and r. The theoretical expressions are tested by fitting them to event samples generated with QCD multihadronic event generators. The main conclusions are that to obtain the correct asymptotic result $C_A/C_F=9/4=2.25$ from the method it is necessary to use the pure QCD result [5] without the offset term N_g^0 and to include recoil effects in the expression for r. As a consistency check, I apply my method to Monte Carlo events with $C_A=C_F=4/3$ to verify that the fitted result for the parameter C_A/C_F is consistent with unity in this case.

Having established a fitting technique that yields the correct results in the limiting cases of (1) asymptotically large energies and (2) $C_A=C_F$, I apply the method to recently published data [4] of the DELPHI experiment at the e^+e^- collider LEP at CERN. I thereby determine the effective value of C_A/C_F from this method at the finite energy of the Z^0 boson.

[1] Also referred to as MLLA.

II. THEORETICAL FRAMEWORK

An analytic expression for the topology dependence of the mean particle multiplicity in e^+e^- three jet quark-antiquark-gluon $q\overline{q}g$ events, $N_{\text{3-jet}}$, valid in the NLO approximation of perturbation theory, is given by Eq. (6.43) of [5] (see also [7]):

$$N_{\text{3-jet}}=N_{e^+e^-}(2E^*)+r(p_\perp)\frac{N_{e^+e^-}(p_\perp)}{2}, \tag{1}$$

where $N_{e^+e^-}(Q)$ is the mean inclusive particle multiplicity of e^+e^- annihilation events at energy scale Q. The quark and gluon jet scales E^* and $p_\perp$ are [see Eqs. (6.38) and (6.41) of [5]]

$$E^*=\sqrt{\frac{p_q\cdot p_{\overline{q}}}{2}}, \tag{2}$$

$$p_\perp=\sqrt{\frac{2(p_q\cdot p_g)(p_{\overline{q}}\cdot p_g)}{p_q\cdot p_{\overline{q}}}}, \tag{3}$$

with p_q, $p_{\overline{q}}$, and p_g the four-momenta of the q, $\overline{q}$, and g. E^* is the energy of the quark or antiquark in the $q\overline{q}$ rest frame, while $p_\perp$ is the transverse momentum of the gluon with respect to the $q\overline{q}$ axis in that frame. These equations are valid for massless quarks and gluons.

The quantity $r(Q)$ is the ratio of the mean multiplicities between gluon and quark jets. It has been calculated analytically in the next-to-next-to-next-to-leading-order (3NLO) approximation of perturbation theory, including recoil effects [8]:

$$r(Q)=r_0(1-r_1\gamma_0-r_2\gamma_0^2-r_3\gamma_0^3), \tag{4}$$

where $\gamma_0(Q)=\sqrt{2C_A\alpha_S(Q)/\pi}$, $r_0=C_A/C_F$, and the correction terms r_1, r_2, and r_3 are constants in QCD, which functionally depend on the color factors through terms proportional to $1/r_0$ and $1/C_A$ [8]. r depends on the scale Q only through α_S:

$$\alpha_S(Q)=\frac{2\pi}{\beta_0 y}\left[1-\frac{\beta_1\ln(2y)}{\beta_0^2 y}\right], \tag{5}$$

with $y=\ln(Q/\Lambda)$, Λ a cutoff which defines the limit of perturbative evolution, $\beta_0=(11C_A-2n_f)/3$, $\beta_1=[17(C_A)^2-n_fC_A(5+3/r_0)]/3$, and n_f the number of active quark flavors. In this paper, I set $n_f=5$ and use the corresponding result for Λ found in a fit of the 3NLO expression for quark jet multiplicity [9] to inclusive e^+e^- data. This result, $\Lambda=0.148$ GeV [9], is similar to the value of $\Lambda_{\overline{MS}}$ [10].[2]

III. ANALYSIS TECHNIQUE

Three jet events are selected using standard jet finding algorithms (see Sec. IV). Two of the jets are identified as the quark (q or $\overline{q}$) jets, the other as the gluon jet. The four-momentum of each jet is assigned to the underlying q, $\overline{q}$, or g. Since expressions (2) and (3) are based on massless kinematics, the jet momenta are modified to obtain massless jets. First, the jets are assigned calculated energies E_{calc} based on the angles between jets, assuming the jets are massless (see e.g., [11]). Second, the jet three-momenta are scaled as follows:

$$\vec{P}=\frac{E_{\text{calc}}}{|\vec{P}_{\text{jet finder}}|}\vec{P}_{\text{jet finder}}, \tag{6}$$

with $\vec{P}_{\text{jet finder}}$ the jet three-momentum determined by the jet finder. The quark and gluon four-momenta defined by $p=(E_{\text{calc}},\vec{P})$ are used to determine the scales (2) and (3). This method of defining massless jets is referred to in the literature as the $E0$ scheme [12].

The values of $N_{e^+e^-}(2E^*)$ and $N_{e^+e^-}(p_\perp)$ in Eq. (1) are determined using parametrizations of $N_{e^+e^-}(Q)$ versus Q. These parametrizations are based on sixth-order polynomials in $\ln(Q)$. A parametrization is determined independently for each Monte Carlo event sample[3] and for the data. For the Monte Carlo samples, the parametrizations are obtained by a fit to the predicted values of $N_{e^+e^-}$ versus $Q=E_{\text{c.m.}}$ in the interval between 10 GeV and 10 TeV, where $E_{\text{c.m.}}$ is the center-of-mass (c.m.) energy. For the data, a fit is made to measurements of $N_{e^+e^-}$ for $12\,\text{GeV}\leqslant E_{\text{c.m.}}\leqslant 189\,\text{GeV}$.[4] The parametrizations provide good representations of the multiplicity in all cases.

[2] Λ and $\Lambda_{\overline{MS}}$ are strongly related to each other, but are not necessarily the same.

[3] HERWIG at the parton and hadron levels, and JETSET at the parton level with $C_A=C_F$; see Sec. IV.

[4] The data used are the same as presented in Fig. 2 of [9].

The analytic expression for r [Eq. (4)] is introduced into Eq. (1). r_1, r_2, and r_3 in expression (4) depend on $1/r_0$ and $1/C_A$, as stated above. Similarly, β_1 in Eq. (5) depends on $1/r_0$, while β_0 and β_1 depend on C_A. The leading r_0 term in Eq. (4) and the $1/r_0$ terms in r_1, r_2, r_3, and β_1 form the fitted parameter. C_A in r_1, r_2, r_3, β_0, and β_1 is set equal to its QCD value of 3. r_0 is then determined in a one parameter fit of Eq. (1) to the Monte Carlo or experimental results for $N_{\text{3-jet}}$ as a function of scale. The DELPHI Collaboration recently presented a similar study [4]. I discuss the DELPHI method and results in Sec. VII.

IV. MONTE CARLO SAMPLES AND EVENT SELECTION

For the principal Monte Carlo based results I present, I use event samples generated with the HERWIG Monte Carlo multihadronic event generator [13], version 5.9. The parameter values used for HERWIG are the same as those given in [3]. HERWIG contains the most complete computer simulation of QCD presently available, including terms up to and beyond the next-to-next-to-leading-order (NNLO) approximation. In this sense HERWIG resembles an analytic calculation. In addition, HERWIG implements exact energy-momentum conservation at each parton branching and a model for hadronization. HERWIG yields the correct asymptotic result of 2.25 for the multiplicity ratio r [14] and related quantities [3]. It provides a good description of gluon and quark jet properties up to the highest available e^+e^- energies. Thus HERWIG provides a suitable QCD reference sample. HERWIG generally predicts that QCD variables reach their asymptotic values at center-of-mass energies of several TeV or more, depending on the variable. In the following, I choose 10 TeV as the canonical c.m. energy at which to test my fit method in the asymptotic limit.

For studies with $C_A=C_F=4/3$, I employ a special version of the JETSET Monte Carlo multihadronic event generator [15], version 7.4, with parameter values given in [16]. In addition to setting $C_A=C_F$, I turn off gluon splittings, $g\to q\overline{q}$. The reason JETSET is used for these studies, and not HERWIG, is that HERWIG does not allow $C_A=C_F$. JETSET is based on leading-order (LO) QCD with a simulation of coherence effects due to higher orders. The standard version of JETSET does not yield the correct asymptotic result for r, as seen from Fig. 2 of [14], except perhaps at exceptionally high energies ($E_{\text{c.m.}}\gg 100$ TeV?). Thus the QCD predictions of JETSET should be treated with precaution. For the present purposes it is sufficient that quark and gluon jets have the same internal properties, such as multiplicity, if $C_A=C_F$. This is satisfied by the special version of JETSET at the parton level. By parton level, I mean the ensemble of quarks and gluons which are present at the termination of the perturbative stage of evolution.

Three jet events are constructed from these samples by adjusting the resolution scale(s) of a jet finding algorithm for each event so that exactly three jets are found. I choose three jet finding algorithms: the $k_\perp$ [17], JADE [18], and cone [19] jet finders. These three algorithms are very different in

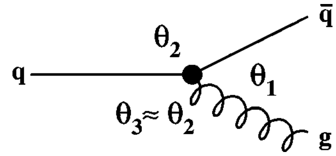

FIG. 1. Schematic representation of a three jet $q\bar{q}g$ event produced in e^+e^- annihilations with a Y event topology [11], in which the angle between the highest energy jet and each of the two lower energy jets is about the same. The angle θ_1 opposite the highest energy jet is used to specify the event topology.

their treatment of soft particles. The difference in the results found using the three algorithms therefore provides a rigorous test of the jet finder independence of the method. I retain events if the angle between the highest energy jet and each of the two other jets is the same to within 5°, the so-called ''Y events.''[5] An example of a Y event is shown in Fig. 1. Measurements of the particle multiplicity of Y events as a function of topology (i.e., scale) have recently become available [4]. I wish to utilize these data for my fits (Sec. VI). This provides my principal motivation for selecting Y events. For Y events, the three jet event multiplicity $N_{3\text{-jet}}$ and the scales (2) and (3) depend only on $E_{\text{c.m.}}$ and one angle in the event, conveniently chosen to be θ_1 (see Fig. 1). For fixed $E_{\text{c.m.}}$, θ_1 therefore determines the scale.

For the Monte Carlo events used here, the quark and gluon jets are identified using parton level Monte Carlo (MC) information. The directions of the primary quark and antiquark[6] are determined after their perturbative evolution has terminated. The jet closest to the direction of the evolved primary quark or antiquark is considered to be a quark jet. The distinct jet closest to the other evolved primary quark or antiquark is considered to be the other quark jet. The remaining jet is identified as the gluon jet. This algorithm is applied to jets at both the parton and hadron levels. By hadron level, I mean the level after hadronization, with charged and neutral particles with lifetimes greater than 3×10^{-10} s treated as stable. Hence charged particles from the decays of K_s^0 and weakly decaying hyperons are included in the definition of the hadron level multiplicity.

V. MONTE CARLO–BASED RESULTS

1 begin by studying HERWIG events at the parton level, with $E_{\text{c.m.}}=10$ TeV. This large energy is meant to ensure that the fit results are asymptotic, as mentioned above. The mean multiplicity of these events as a function of the opening angle θ_1 is shown in Fig. 2(a). The results are shown for the three jet algorithms. The results of the three algorithms are seen to be similar for angles larger than about 80°. As θ_1 becomes smaller, the two lower energy jets are not as well separated and background from two jetlike events increases. Different jet finders have different efficiencies for selecting

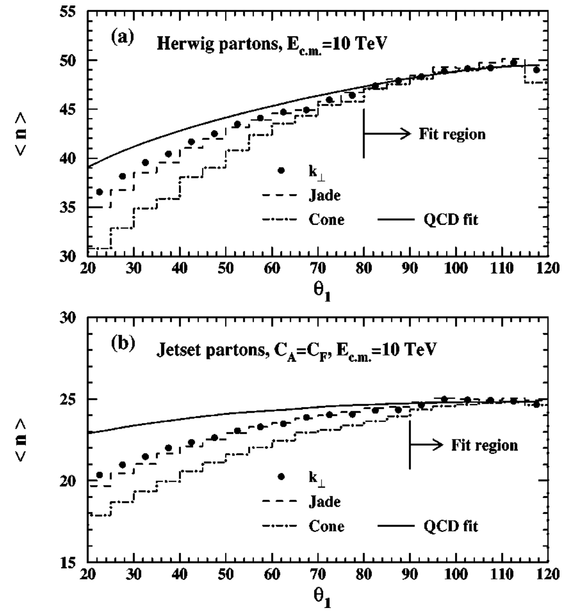

FIG. 2. (a) The mean parton level multiplicity of three jet Y events as a function of the opening angle θ_1, for events generated with the HERWIG multihadronic event generator. (b) The analogous results for events generated with the JETSET multihadronic event generator with $C_A=C_F$. The event samples in (a) and (b) are selected using the $k_\perp$, JADE, and cone jet finders. The center-of-mass energy is 10 TeV. The solid curves show the results of a one parameter QCD fit to events selected using the $k_\perp$ jet finder. The fits are performed within the regions shown.

background: thus the results of the jet finders diverge. Since the results should be independent of the choice of a jet algorithm to be sensible, I restrict the fits to the region where the predictions of the jet finders approximately agree, namely, $80°\leqslant\theta_1\leqslant120°$.

The solid curve in Fig. 2(a) shows the result of the one parameter fit of Eq. (1) to the event multiplicity determined using the $k_\perp$ jet finder. The curve provides a reasonable description of the multiplicity inside the fit region. Outside this region, i.e., for angles less than 80°, the fitted curve does not describe the event multiplicity well. The multiplicity of the events in Fig. 2(a) is not well defined for $\theta_1<80°$, however, since the results from different jet finders disagree strongly. Therefore, I do not consider the discrepancies between the fitted curve and the jet finder based results for $\theta_1<80°$ to be meaningful.

The results for r_0 are summarized in the top portion of Table I. Taking the result found using the $k_\perp$ jet finder as the central value, with half the difference between the extreme values found using the different jet finders as a systematic uncertainty, I obtain $r_0=2.248\pm0.010$ (stat) ±0.024 (syst), consistent with the QCD asymptotic expectation of 2.25.

The analogous results for JETSET at the parton level with $C_A=C_F$ are shown in Fig. 2(b). Again, the c.m. energy is 10 TeV. The predictions of the three jet finders are seen to be similar only for $\theta_1>90°$. Therefore, I limit the fit range to $90°\leqslant\theta_1\leqslant120°$ in this case. The results for r_0 are given in the bottom portion of Table I. Combining the results in the manner described in the previous paragraph yields r_0

[5] Y events were first studied in [11].

[6] I.e., the q and $\bar{q}$ produced directly in the electroweak decay of the virtual Z^0/γ in $e^+e^-\rightarrow(Z^0/\gamma)^*\rightarrow$hadrons events.

TABLE I. Results of a one parameter fit of $r_0 = C_A/C_F$ to the parton level multiplicity in three jet events, as predicted by the HERWIG QCD multihadronic event generator and by the JETSET multihadronic event generator with $C_A = C_F$. The $E_{\mathrm{c.m.}}$ value is 10 TeV for both samples. The fits are performed using the 3NLO expression for the multiplicity ratio between gluon and quark jets, r. The uncertainties are statistical. N_{DF} is the number of degrees of freedom.

HERWIG partons	$r_0 = C_A/C_F$	χ^2/N_{DF}
$k_\perp$ jet finder	2.248 ± 0.010	5.9/8
JADE jet finder	2.269 ± 0.010	12.8/8
Cone jet finder	2.221 ± 0.013	14.7/8
JETSET partons, $C_A = C_F$		
$k_\perp$ jet finder	1.012 ± 0.009	5.9/6
JADE jet finder	1.032 ± 0.007	3.9/6
Cone jet finder	0.979 ± 0.008	6.7/6

$= 1.012 \pm 0.009$ (stat) ± 0.027 (syst), consistent with unity.

Thus a one parameter fit of r_0 yields the correct results in the limiting cases of (1) QCD at asymptotically large energies and (2) $C_A = C_F$, as long as the fit range is restricted to regions where the results of the different jet finders agree or, equivalently, to regions where the fitted curves provide a good description of the multiplicity. The fits generally yield $\chi^2/N_{\mathrm{DF}} \sim 1$ (Table I), where N_{DF} is the number of degrees of freedom in the fit.

It is of interest to determine the importance of energy conservation in the expression for r. To this effect, I replace Eq. (4) by the corresponding result in the NNLO approximation both with [20] and without [6] recoil effects, and repeat the fit of HERWIG events described above. The NNLO approximation is used for this test, and not the 3NLO approximation, because a 3NLO expression for r without energy conservation is not available. For the $k_\perp$-based event sample, the NNLO calculation with energy conservation yields $r_0 = 2.254 \pm 0.010$ (stat), not very different from the 3NLO result presented above. The corresponding result without energy conservation is only 2.079 ± 0.009 (stat), however significantly smaller than 2.25. This implies that it is important to include energy conservation in the analytic expressions, even for $E_{\mathrm{c.m.}} = 10$ TeV.

In Fig. 3, I show the fitted results for r_0 as a function of $E_{\mathrm{c.m.}}$, using HERWIG events at the parton and hadron levels. The events are selected using the $k_\perp$ jet finder. The hadron level multiplicity is based on charged particles only. The fit interval is $80° \leqslant \theta_1 \leqslant 120°$, i.e., the same as in Fig. 2(a). The fitted curves provide good descriptions of the multiplicity within the fit region for all c.m. energies at both the parton and hadron levels. The fit results are observed to have only a moderate dependence on the choice of the jet algorithm, generally similar to that indicated in Table I for parton level events or in item (1) of the list presented below in Sec. VI for hadron level events. From the parton level curve (solid line) it is seen that the asymptotic result $r_0 \approx 2.25$ is reached for $E_{\mathrm{c.m.}} \sim 3$ TeV.[7] The hadron level curve (dashed line) converges to the asymptotic limit much more slowly, however. As a consequence, the hadronization correction, defined by the ratio of the parton to the hadron level results, is fairly large. The hadronization correction is predicted to be 1.30 at the mass of the Z^0 and 1.06 at 10 TeV. The principal origin of this correction is the effect of hadronization on the gluon jet scale (3): e.g., the mean value of $p_\perp$ at $E_{\mathrm{c.m.}} = 91$ GeV is 36% larger at the parton level than it is at the hadron level, as determined using HERWIG. The corresponding difference for $E_{\mathrm{c.m.}} = 10$ TeV is only 2%.

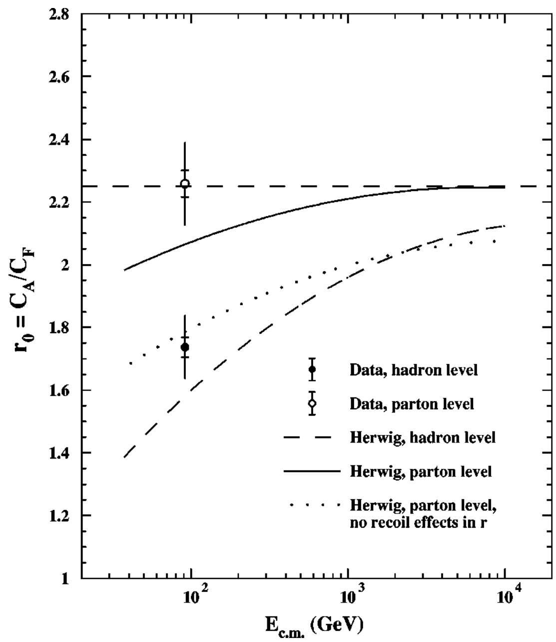

FIG. 3. Results of a one parameter fit of $r_0 = C_A/C_F$ as a function of the c.m. energy, for HERWIG Monte Carlo three jet Y events at the parton and hadron levels. The events are selected using the $k_\perp$ jet finder. The corresponding results for data [4] at $E_{\mathrm{c.m.}} = 91$ GeV are shown by the open and solid points. The hadron level results are based on charged particles only. For data points, the vertical lines show the total uncertainties, with statistical and systematic terms added in quadrature. The small horizontal lines indicate the statistical uncertainties.

The dotted curve in Fig. 3 shows the fitted results for r_0 at the parton level if the NNLO expression for r *without* recoil effects is used in place of the 3NLO expression. The QCD asymptotic limit of 2.25 is not attained in this case, again emphasizing the importance of energy conservation.

[7]The asymptotic result $r_0 = 2.25$ is not reached for values of $E_{\mathrm{c.m.}}$ below about 3 TeV because of the approximate nature of expressions (1)–(4) at finite energies. For example, the analytic result for r [expression (4)] is 1.7 at the scale of the Z^0 [8], compared to its experimental and Monte Carlo values of about 1.51 and 1.54, respectively [3]. Further, the assumption of massless kinematics employed for expressions (2) and (3) becomes strictly valid only for scales well above the Z^0.

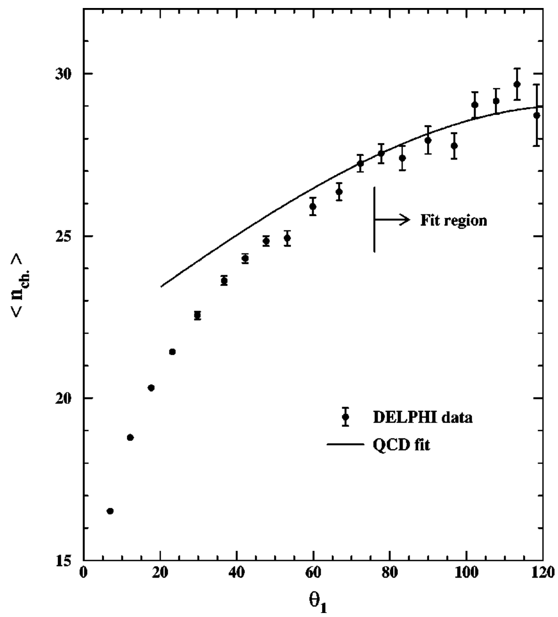

FIG. 4. Measurements [4] of the mean charged particle multiplicity of three jet Y events as a function of the opening angle θ_1, for $E_{\mathrm{c.m.}}=91$ GeV. The events are selected using the $k_\perp$ jet finder. The solid curve shows the result of a one parameter fit to the data within the fit region shown.

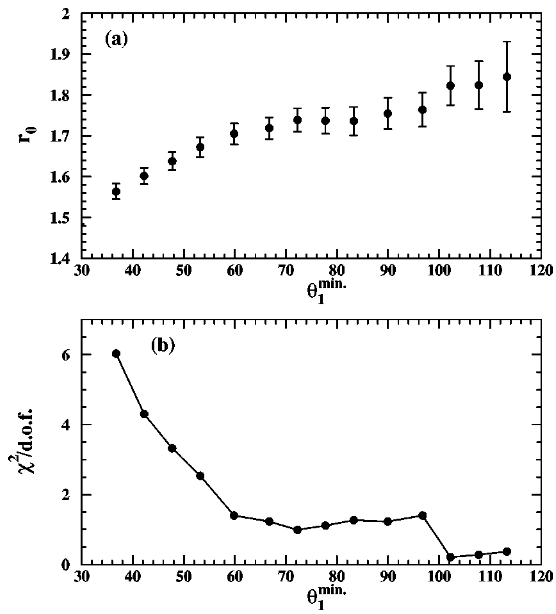

FIG. 5. (a) Results of a one parameter fit of $r_0=C_A/C_F$ to measurements [4] of the charged particle multiplicity of three jet Y events at $E_{\mathrm{c.m.}}=91$ GeV, as a function of the lower limit $\theta_1^{\min}$ of the fit range $\theta_1^{\min}\leqslant\theta_1\leqslant120°$. (b) The corresonding values of χ^2/N_{DF}. The events are selected using the $k_\perp$ jet finder.

VI. DATA-BASED RESULTS

Recently, the DELPHI experiment at LEP presented measurements of the charged particle multiplicity of Y events and the scales (2) and (3) as a function of the opening angle θ_1 [4]. The results are based on the $k_\perp$ jet finder with $E_{\mathrm{c.m.}}=91$ GeV. These data allow me the possibility to determine the effective value of r_0 at the scale of the Z^0 using my one parameter fit method. The DELPHI multiplicity measurements are shown in Fig. 4. The result of the one parameter fit is shown by the solid curve. The fit range employed is $78°\leqslant\theta_1\leqslant120°$, similar to the interval of $80°\leqslant\theta_1\leqslant120°$ I choose for HERWIG events (Figs. 2(a) and 3). The small difference in the choice of fit interval between the HERWIG and DELPHI samples is not important [see item (2) below]. The analytic curve provides a good description of the measurements within the fit region, yielding $\chi^2/N_{\mathrm{DF}}=8.9/8$.

The result for the fitted parameter is $r_0=1.737\pm0.032$ (stat). To estimate a systematic uncertainty for this result, I consider the following.

(1) *Jet finder dependence*. The DELPHI results are presented for the $k_\perp$ jet finder, but not for the JADE or cone jet finders. HERWIG at the hadron level with $E_{\mathrm{c.m.}}=91$ GeV yields $r_0=1.585$ for the $k_\perp$ jet finder, 1.601 for the JADE jet finder, and 1.516 for the cone jet finder, where the statistical uncertainty is 0.008 in all cases. Half the difference between the extreme values is taken as a systematic uncertainty.

(2) *Fit interval*. The fit interval I choose for the DELPHI data is $78°\leqslant\theta_1\leqslant120°$, as stated above. Decreasing the lower limit of this interval to 60° yields $r_0=1.705\pm0.025$ (stat), while decreasing the upper limit to 90°, with the lower limit at the standard value, yields $r_0=1.755\pm0.038$ (stat). I take half the difference between these values as a systematic uncertainty. A further test of the choice of the fit interval is presented in Fig. 5. Figure 5(a) shows the fitted results for r_0 as a function of $\theta_1^{\min}$, where $\theta_1^{\min}$ is the lower limit of the fit range $\theta_1^{\min}\leqslant\theta_1\leqslant120°$. The corresponding values of χ^2/N_{DF} are shown in Fig. 5(b). The χ^2/N_{DF} is 1.4 or less for $\theta_1^{\min}\geqslant60°$, but much larger for $\theta_1^{\min}<60°$. This provides justification for not extending the fit range below 60°; i.e., the fit is restricted to an interval where the theoretical expression (1) describes the data accurately.

(3) *Parametrization of $N_{e^+e^-}$ versus Q*. Rather than use a polynomial parametrization of $N_{e^+e^-}$ versus Q (Sec. III), I use the parametrization based on the 3NLO expression for quark jet multiplicity [9] with the parameter values in [9]. This yields $r_0=1.804\pm0.032$ (stat). The difference with respect to the standard result is taken as a systematic uncertainty. Note that the polynomial provides a better description of $N_{e^+e^-}$ versus Q than the 3NLO expression.

(4) *Value of* Λ. Setting Λ in Eq. (5) to the PDG value of $\Lambda_{\overline{\mathrm{MS}}}=0.220$ GeV [10], rather than using 0.148 GeV (Sec. II), yields $r_0=1.761\pm0.033$ (stat). I take the difference with respect to the standard result as a systematic uncertainty.

(5) *Averaging procedure for E^* and $p_\perp$*. The DELPHI results for the quark and gluon scales (2) and (3) are found by taking the *geometric means* of E^* and $p_\perp$, averaged over the event sample, as a function of θ_1. For the Monte Carlo–based results presented in Sec. V, I employ the much more common *arithmetic means*. For hadron level events at 91 GeV, HERWIG with the $k_\perp$ jet finder yields $r_0=1.629$ for geometric means and $r_0=1.585$ for arithmetic means, where

TABLE II. Summary of systematic uncertainties for the effective value of r_0 at the scale of the Z^0 as determined using data [4].

Systematic term	Δr_0
(1) Jet finder dependence	0.043
(2) Fit interval	0.025
(3) Parametrization of $N_{e^+e^-}$	0.067
(4) Value of Λ	0.024
(5) Averaging procedure for E^* and $p_\perp$	0.044
(6) Number of active flavors	0.002
Total	0.097

the statistical uncertainty is 0.008 in both cases. The difference between these values is taken as a systematic uncertainty.

(6) *Number of active flavors*, n_f. Using $n_f=3$ rather than $n_f=5$ in the analytic expressions for r and α_S [Eqs. (4) and (5)], and correspondingly evaluating α_S using $\Lambda=0.322$ GeV [9] rather than $\Lambda=0.148$ GeV, yields $r_0=1.735\pm0.029\,(\text{stat})$. The difference with respect to the standard result is taken as a systematic uncertainty.

The systematic uncertainties are summarized in Table II. The largest terms arise from the parametrization of $N_{e^+e^-}$, the averaging procedure for the scales, and the choice of jet finder, in that order. The terms are added in quadrature to define the total systematic uncertainty. The final result for the effective value of $r_0=C_A/C_F$ at the scale of the Z^0 is

$$r_0=1.737\pm0.032\ \ (\text{stat})\pm0.097\ \ (\text{syst}). \qquad (7)$$

Multiplying this value by the hadronization correction of 1.30 mentioned at the end of Sec. V yields $r_0=2.26\pm0.04\,(\text{stat})\pm0.12\,(\text{syst})$ as the corresponding result at the parton level. The data-based results I obtain at the hadron and parton levels are shown by the solid and open points in Fig. 3. The experimental results lie somewhat above the HERWIG curves, but are generally consistent with them. Because the data are not entirely consistent with HERWIG, it is possible that the numerical similarity between the parton level measurement of 2.26 and the QCD asymptotic prediction of 2.25 is somewhat coincidental.

VII. COMPARISON TO A TWO PARAMETER FIT METHOD

In their recent publication [4], the DELPHI Collaboration presented an alternative method to determine $r_0=C_A/C_F$ from three jet event particle multiplicity. I used the data of that study, shown in Fig. 4, to obtain the results of Sec. VI. The DELPHI analysis is based on a fit of the expression

$$N_{3\text{-jet}}=N_{e^+e^-}(2E^*)+r(p_\perp)\left[\frac{N_{e^+e^-}(p_\perp)}{2}-N_g^0\right] \qquad (8)$$

to the three jet event multiplicity data, where N_g^0 is a parameter meant to account for differences in the hadronization of gluons and quarks. The DELPHI analysis differs from mine principally by using expression (8) rather than expression (1), by using the NNLO result for r without energy conservation [6] rather than the 3NLO expression, and by invoking a two parameter fit of r_0 and N_g^0 rather than a one parameter fit. The DELPHI analysis also differs from mine in the choice of fit range and in the parametrization of $N_{e^+e^-}$ versus Q. In the DELPHI study, the fit range is $30°\leqslant\theta_1\leqslant120°$ rather than $78°\leqslant\theta_1\leqslant120°$ and the parametrization of $N_{e^+e^-}$ versus Q is based on the NLO expression for quark jet multiplicity in e^+e^- annihilations [21] rather than a polynomial. The DELPHI results utilize events at $E_{\text{c.m.}}=91$ GeV selected using the $k_\perp$ jet finder, as stated in Sec. VI.

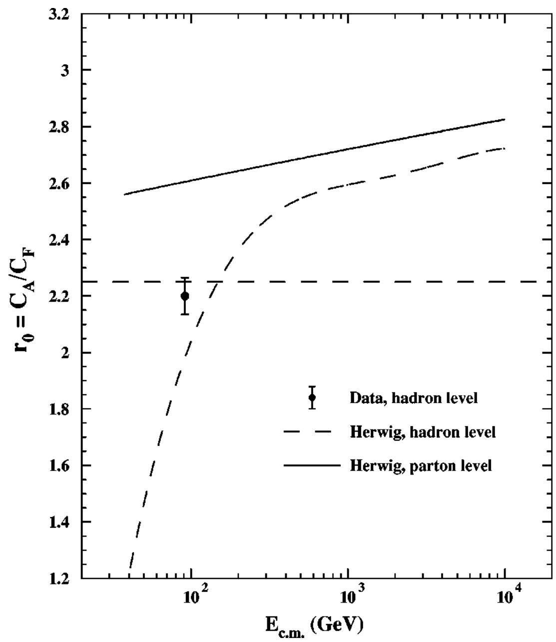

FIG. 6. Results for $r_0=C_A/C_F$ from a two parameter fit method [4] as a function of the c.m. energy, for HERWIG Monte Carlo three jet Y events at the parton and hadron levels. The events are selected using the $k_\perp$ jet finder. The corresponding hadron level result found using data [4] at $E_{\text{c.m.}}=91$ GeV is shown by the solid point. The hadron level results are based on charged particles only. The uncertainty shown for the data point is statistical.

Repeating the DELPHI analysis, viz., a two parameter fit of Eq. (8) to the data in Fig. 4, using the DELPHI values of E^* and $p_\perp$, the expression for r in [6], a fit range from 30° to 120°, and the NLO expression for quark jet multiplicity to parametrize $N_{e^+e^-}$ versus Q,[8] I obtain

$$r_0=2.200\pm0.066\ \ (\text{stat}), \qquad (9)$$

$$N_g^0=1.46\pm0.10\ \ (\text{stat}). \qquad (10)$$

The χ^2/N_{DF} of the fit is 13.2/16. The results (9) and (10) are very similar to those of DELPHI, namely, $r_0=2.251\pm0.063\,(\text{stat})$ and $1.40\pm0.10\,(\text{stat})$ [4]. The result (9) for r_0 is shown by the solid point in Fig. 6.

[8]For the NLO parametrization of quark jet multiplicity, I use the parameters in [22].

The value of r_0 derived from the DELPHI two parameter fit method is numerically very similar to the QCD asymptotic result $C_A/C_F=2.25$. On this basis, DELPHI suggests [4] that their analysis provides a measurement of that quantity. To test this hypothesis, I determine the results of the DELPHI method in the two limiting cases discussed in Sec. V: (1) asymptotically large energies and (2) $C_A=C_F$. For HERWIG events at the parton level with $E_{\mathrm{c.m.}}=10$ TeV, the DELPHI fit method yields $r_0=2.80\pm0.03$, 3.19 ± 0.03, and 4.03 ± 0.05 for events selected using the $k_\perp$, JADE, and cone jet finders, respectively, where the uncertainties are statistical. These values are much larger than 2.25 and exhibit a strong dependence on the choice of the jet finder, in contrast to the results of Sec. V (top portion of Table I). The analogous results for JETSET at the parton level with $E_{\mathrm{c.m.}}=10\,\mathrm{TeV}$ and $C_A=C_F$ are $1.76\pm0.03\,(\mathrm{stat})$, $2.05\pm0.02\,(\mathrm{stat})$, and $2.68\pm0.03\,(\mathrm{stat})$, which are inconsistent with unity and again exhibit a strong jet finder dependence. This is also in contrast to the results of Sec. V (bottom portion of Table I). For the above results, the NLO expression for quark jet multiplicity is fitted to the MC predictions of $N_{e^+e^-}$ versus $E_{\mathrm{c.m.}}$ for both the HERWIG and JETSET samples, using scale values between 20 GeV and 10 TeV. The results are similar if the polynomial parametrizations discussed in Sec. III are used instead.

The dashed and solid curves in Fig. 6 show the results I obtain for r_0 from applying the DELPHI fit method to HERWIG events at the hadron and parton levels. The results are shown as a function of $E_{\mathrm{c.m.}}$. The hadron level results are based on charged particles only. The event samples are selected using the $k_\perp$ jet finder. Thus Fig. 6 is the analogue for the DELPHI method of the results I show in Fig. 3 for my method. The hadron level curve in Fig. 6 is seen to be in general agreement with the experimental result (9) at the scale of the Z^0. Asymptotically, the hadron level prediction reaches a value of about 2.7, however much larger than 2.25. The parton level curve exceeds 2.25 by a large margin even at $E_{\mathrm{c.m.}}=91\,\mathrm{GeV}$.

On the basis of the above results, I conclude that the DELPHI fit method probably does not measure C_A/C_F and that the similarity of the hadron level result (9) to the asymptotic prediction $C_A/C_F=2.25$ is most likely a coincidence. As a last note, I remark that if energy conservation is included in the NNLO expression for r, the result (9) increases to $r_0=2.479\pm0.081\,(\mathrm{stat})$. Thus, if energy conservation is incorporated into the DELPHI fit method, the value of r_0 derived from charged hadrons at 91 GeV is no longer similar to 2.25.

VIII. SUMMARY AND CONCLUSIONS

In this paper, I have presented a test of the QCD expression for the topology (scale) dependence of particle multiplicity in e^+e^- three jet events. Using event samples generated with the HERWIG Monte Carlo multihadronic event generator as a reference, I find that the QCD expression yields the correct result $C_A/C_F=2.25$ in the asymptotic limit of large energy scales $Q\sim3\,\mathrm{TeV}$ as long as it is used in conjunction with an expression for r which incorporates energy conservation, where r is the ratio of mean particle multiplicities between gluon and quark jets. This emphasizes the importance of energy conservation in QCD analytic expressions, even at large scales. My analysis is based on a one parameter fit of C_A/C_F to three jet event mean particle multiplicity as a function of the topology of the event.

As a second test, I apply my method to a sample of Monte Carlo three jet events in which the color factors are set equal, $C_A=C_F$. I obtain $C_A/C_F\approx1$ in this case, demonstrating the self consistency of the technique.

Applying my fit method to recently published data [4] of the DELPHI experiment at LEP, I obtain $C_A/C_F=1.737\pm0.032\,(\mathrm{stat})\pm0.097\,(\mathrm{syst})$ as the effective value of the color factor ratio at $E_{\mathrm{c.m.}}=91\,\mathrm{GeV}$ from this technique. This result is based on charged particles. It is of interest to compare this result to related measurements at $E_{\mathrm{c.m.}}=91\,\mathrm{GeV}$ based on the charged particle multiplicity ratio between gluon and quark jets, r_{ch}. The experimental result for r_{ch} in full phase space is $1.514\pm0.019\,(\mathrm{stat})\pm0.034\,(\mathrm{syst})$ [3,14]. The corresponding result for r_{ch} in limited phase space, defined by soft particles with large transverse momenta to the jet axes, is $2.29\pm0.09\,(\mathrm{stat})\pm0.15\,(\mathrm{syst})$ [3]. All these measurements—the one presented here and the two based on r_{ch}—are predicted to equal 2.25 in the limit of large energies. The result presented here is seen to be intermediate to the two based on r_{ch}, both in value and in the size of the uncertainty. Whereas r_{ch} in limited phase space has already attained its asymptotic value at $E_{\mathrm{c.m.}}=91\,\mathrm{GeV}$, r_{ch} in full phase space and the result presented here are subasymptotic at this scale.

After correction for hadronization, the result I obtain from the DELPHI data is $C_A/C_F=2.26\pm0.04\,(\mathrm{stat})\pm0.12\,(\mathrm{syst})$, somewhat larger than the parton level prediction of HERWIG (for $E_{\mathrm{c.m.}}=91\,\mathrm{GeV}$) of 2.06. The numerical similarity between the parton level measurement of 2.26 and the QCD asymptotic result of 2.25 may be somewhat fortuitous, given that the data and HERWIG are not entirely consistent.

Last, I test a two parameter fit method to determine C_A/C_F from particle multiplicity in three jet events, proposed in a recent study [4]. I find that this method does not yield the correct results $C_A/C_F\approx2.25$ or $C_A/C_F\approx1$ in the two limiting cases of QCD at asymptotic energies or identical color factors $C_A=C_F$, in contrast to my method. Thus I conclude that this two parameter fit method probably does not measure the color factor ratio.

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