Hyperon semileptonic decays and quark spin content of the proton

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We investigate the hyperon semileptonic decays and the quark spin content of the proton $\Delta\Sigma$, taking into account flavor SU(3) symmetry breaking. Symmetry breaking is implemented with the help of the chiral quark-soliton model in an approach in which the dynamical parameters are fixed by the experimental data for six hyperon semileptonic decay constants. As a result we predict the unmeasured decay constants, particularly for $\Xi^0 \to \Sigma^+$, which will be soon measured and examine the effect of SU(3) symmetry breaking on the spin content $\Delta\Sigma$ of the proton. Unfortunately large experimental errors of Ξ^- decays propagate in our analysis, making $\Delta\Sigma$ and Δs practically undetermined. We conclude that statements concerning the values of these two quantities, which are based on exact SU(3) symmetry, are premature. We stress that meaningful results can be obtained only if the experimental errors for the Ξ decays are reduced.

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I. INTRODUCTION

Since the European Muon Collaboration (EMC) measured the first moment $I_p^{\rm EMC} = 0.112$ [at $Q^2 = 3\ \ ({\rm GeV}/c)^2$] of the proton spin structure function g_1^p [1], there has been a great deal of discussion about the spin content of the proton. An immediate and unexpected consequence of the EMC measurement was that the quark contribution to the spin of the proton was very small ($\Delta\Sigma \approx 0$). A series of ensuing experiments [2–4] confirmed the EMC measurement, giving, however a somewhat larger, but still small value for $\Delta\Sigma$.

This result is in contradiction with expectations based on the naive, nonrelativistic quark model, supplemented by the assumption that the contribution of strange quarks to I_p was zero ($\Delta s = 0$) [5]. The EMC measurements required $\Delta s \neq 0$ and relatively large. These two results, $\Delta\Sigma \approx 0$ and $\Delta s \neq 0$, are often referred to as the *spin crisis*. Let us shortly summarize how the crisis arises.

Theoretical analysis of recent measurements [6] indicates that the I_p is equal to

$$I_p(Q^2 = 3\ \ ({\rm GeV}/c)^2) = 0.124 \pm 0.011. \tag{1}$$

On the other hand, the I_p is related to the integrated polarized quark densities:

$$I_p = \frac{1}{18}(4\Delta u + \Delta d + \Delta s)\left(1 - \frac{\alpha_s}{\pi} + \cdots\right). \tag{2}$$

Here for simplicity we neglect higher orders and higher twist contributions. Comparing Eq. (1) with Eq. (2) and assuming $\alpha_s(Q^2 = 3\ \ ({\rm GeV}/c)^2) = 0.4$ [7], we get immediately

$$\Gamma_p \equiv 4\Delta u + \Delta d + \Delta s = 2.56 \pm 0.23. \tag{3}$$

Let us quote here for completeness the experimental value for the neutron [6]:

$$\Gamma_n \equiv 4\Delta d + \Delta u + \Delta s = -0.928 \pm 0.186. \tag{4}$$

With this definition of Γ_n the Bjorken sum rule is automatically satisfied.

Integrated polarized quark densities Δq can be in principle extracted from the hyperon semileptonic decays. It is customary to assume SU(3) symmetry to analyze these decays. Then all decay amplitudes are given in terms of two reduced matrix elements F and D. For example

$$A_1(n \to p) = F + D, \quad A_4(\Sigma^- \to n) = F - D.$$

Here by A_i we denote the ratios of axial-vector to vector coupling constants g_1/f_1 for semileptonic decays as displayed in Table I. Taking for these decays experimental values (see Table I) one gets $F = 0.46$ and $D = 0.80$. The matrix elements of diagonal operators λ_3 and λ_8 (called $g_A^{(3)}$ and $g_A^{(8)}$, respectively), which define integrated quark densities Δq, can be also expressed in terms of F and D:

$$g_A^{(3)} \equiv \Delta u - \Delta d = F + D,$$
$$g_A^{(8)} \equiv \frac{1}{\sqrt{3}}(\Delta u + \Delta d - 2\Delta s) = \frac{1}{\sqrt{3}}(3F - D). \tag{5}$$

Using the values of F and D obtained from the neutron and Σ^- decays together with Eq. (3) we get $\Delta u = 0.79$, $\Delta d = -0.47$, and $\Delta s = -0.13$. Defining the quark content of the proton's spin,

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s, \tag{6}$$

TABLE I. The parameters $r, \ldots, q'$ fixed to the experimental data of the semileptonic decays [32,33] $A_1 - A_6$. The entries for $A_1 - A_6$ for the full fit (last column) correspond to the experimental data.

		Chiral limit	With m_s
	r	-0.0892	-0.0892
	s	0.0113	0.0113
	x'	0	-0.0055
	y	0	0.0080
	z	0	-0.0038
	q'	0	-0.0140
A_1	$(g_1/f_1)^{n\to p}$	1.271 ± 0.11	1.2573 ± 0.0028
A_2	$(g_1/f_1)^{\Sigma^+\to\Lambda}$	0.769 ± 0.04	0.742 ± 0.018
A_3	$(g_1/f_1)^{\Lambda\to p}$	0.758 ± 0.08	0.718 ± 0.015
A_4	$(g_1/f_1)^{\Sigma^-\to n}$	-0.267 ± 0.04	-0.340 ± 0.017
A_5	$(g_1/f_1)^{\Xi^-\to\Lambda}$	0.246 ± 0.07	0.25 ± 0.05
A_6	$(g_1/f_1)^{\Xi^-\to\Sigma^0}$	1.271 ± 0.11	1.278 ± 0.158

we obtain $\Delta\Sigma = 0.19$. Had we used for I_p the result of the first EMC measurement, $I_p^{\rm EMC} = 0.112$, we would get an even smaller value: $\Delta\Sigma = 0.07$.

Although quite often used, the above derivation of $\Delta\Sigma$ has, however, one serious flaw. Namely, we could equally well use some other decays to extract F and D. For example using

$$A_4(\Sigma^- \to n) = F - D, \quad A_5(\Xi^- \to \Lambda) = F - \frac{D}{3}, \qquad (7)$$

together with the experimental data for these decays (see Table I) and experimental value for Γ_p, Eq. (3), we would get $F = 0.55$ and $D = 0.89$, yielding $\Delta\Sigma = 0.02$ — almost 10 times less than our previous value. It is the breaking of SU(3) symmetry which is responsible for this discrepancy. Although the symmetry breaking in hyperon decays themselves is not that large — i.e., it amounts to no more than 10% — the effect of symmetry breaking on $\Delta\Sigma$, or integrated quark density Δs, is much stronger.

There are 6 measured semileptonic hyperon decays, so that the number of combinations which one can form to extract F and D is 14 (actually 15, but two conditions are linearly dependent). Taking these 14 combinations into account and Eq. (3) we get the following values for $\Delta u = 0.75 \to 0.85$, $\Delta d = -0.39 \to -0.58$ and $\Delta s = -0.05 \to -0.25$, which in turn give $\Delta\Sigma = 0.02 \to 0.30$. These are the uncertainties of the *central values* due to the theoretical error caused by using SU(3) symmetry to describe the hyperon decays. They are further increased by the experimental errors of all individual decays and the one of Γ_p.

The authors of Ref. [8] made a similar observation trying to fit the variation of F and D for various decays with one parameter related to m_s. Assuming further $\Delta s = 0$ they were able to fit experimental data for $I_{p,n,deuter}$ with satisfactory accuracy.

Similarly in Refs. [9,10] a simple quark model has been proposed to describe the symmetry breaking in the hyperon decays. It has been observed that with the increase of the symmetry breaking parameter the value of Δs increased, while $\Delta\Sigma$ stayed almost unchanged.

Semileptonic decays and $\Delta\Sigma$ have been also investigated within the SU(3) Skyrme model [11–13], where $\Delta\Sigma = 0$ irrespective of the symmetry breaking. Symmetry breaking influences only Δs [12,13]. In this respect our analysis gives a similar result: although $\Delta\Sigma \neq 0$, it depends very weakly on m_s.

It is virtually impossible to analyze the symmetry breaking in weak decays without resorting to some specific model [7]. In this paper we will implement the symmetry breaking for the hyperon decays using the chiral quark-soliton model (χQSM; see Ref. [14] for a review). This model has proven to give satisfactory description of the axial-vector properties of hyperons [15–18]. It describes the baryons as solitons rotating adiabatically in flavor space. Thus it provides a link between the matrix elements of the octet of the axial-vector currents, responsible for hyperon decays, and the matrix elements of the singlet axial-vector current, in our normalization equal to $\Delta\Sigma$. In the present work we will study the relation between the semileptonic decays and integrated polarized quark distributions, with the help of the χQSM. However, we will use only the collective Hamiltonian of the flavor rotational degrees of freedom including the corrections linear in the strange quark mass m_s. The dynamical quantities in this Hamiltonian, certain moments of inertia calculable within the model [15], are not calculated but treated as free parameters. By adjusting them to the experimentally known semileptonic decays we allow for maximal phenomenological input and minimal model dependence. In Refs. [19,20] we have already studied the magnetic moments of the octet and decuplet in this way.

Such an approach — introduced to our knowledge for the first time by Adkins and Nappi [21] in the context of the Skyrme model — can be viewed from two perspectives. First, it can be considered as a QCD motivated tool to analyze and clasify (in terms of powers of m_s and $1/N_c$) the symmetry breaking terms for a given observable. For nontrivial operators such as magnetic moments or axial form factors a general analysis, without referring to some specific model, is often virtually impossible. Second, it also provides information for the model builders. It tells us what are the best predictions the model can ever produce. Indeed, model calculations are not as unique as one might think: they depend on adopted regularization, cutoff parameters and constituent quark mass. Moreover, in the SU(3) version of the χQSM the quantization ambiguity appears [22]. So if the ''model independent'' analysis would have failed to describe the data, that would mean that the model did not correctly include all necessary physics relevant for a given observable. On the other hand, the success of such an analysis gives a strong hint for the model builders that the model is correct and worth exploring. In fact this concerns all the hedgehog models which would give a collective structure identical to the one of the χQSM.

As far as the symmetry breaking is concerned, our results are identical to the ones obtained in Ref. [23] within large N_c QCD. Indeed, the χQSM is a specific realization of the large N_c limit. The new ingredient of our analysis is the model formula for the singlet axial-vector constant $g_A^{(0)}$, which we use to calculate quantities relevant for the polarized high energy experiments. In the χQSM one can define two interesting limits [24–26] in which the soliton size is artificially changed either to zero (so-called quark-model limit) or to ∞ (Skyrme limit). In these two limiting cases one recovers the well-known results (1) $g_A^{(0)}=1$ in the quark-model limit and (2) $g_A^{(0)}/g_A^{(3)}\to 0$ in the Skyrme limit. This concerns not only the axial couplings; it is often said that the χQSM *interpolates* between the quark model and the Skyrme model. Also these simple qualitative features make us believe that the model correctly describes the physics essential for the axial-vector properties of the nucleon.

The Skyrme limit of the χQSM can also be defined as the limit in which the constituent quark mass $M\to\infty$. The explicit *interpolating* features of the SU(2) version of the model in this limit have been discussed numerically in Ref. [27].

As we will see, in the χQSM in the chiral limit we can express the singlet axial-vector coupling through F and D: $g_A^{(0)}=9F-5D$. We see that the value of $g_A^{(0)}$ is very sensitive to small variations of F and D, since it is the difference of the two, with relatively large multiplicators. Indeed, for the 14 fits mentioned above (where as the input we use *only* semileptonic decays plus model formula for $g_A^{(0)}$) the central value for $g_A^{(0)}$ varies between -0.25 and approximately 1. So despite the fact that semileptonic decays are relatively well described by the model in the chiral limit, the singlet axial-vector coupling is basically undetermined. This is a clear signal of the importance of symmetry breaking for this quantity.

One could argue that this kind of behavior is just an artifact of the χQSM. However, the scenario of a rotating soliton (which is by the way used also in the Skyrme-type models) is very plausible and cannot be *a priori* discarded on the basis of first principles. The χQSM is a particular realization of this scenario and we use it as a tool to investigate the sensitivity of the singlet axial-vector current to the symmetry breaking effects in hyperon decays. In fact conclusions similar to ours have been obtained in chiral perturbation theory in Ref. [28].

As a result of our present analysis we will give predictions for the semileptonic decays not yet measured. More importantly, we will show that the symmetry breaking effects cannot be neglected in the analysis of the quark contribution to the spin of the proton. In other words, linking low energy data with high energy polarized experiments is meaningful only if SU(3) breaking is taken into account. We will furthermore show which semileptonic decays should be measured more accurately in order to reduce the experimental errors for $\Delta\Sigma$ and Δs.

The paper is organized as follows: in the next section we will shortly recapitulate the formalism of the χQSM needed for the calculation of semileptonic hyperon decays. In Sec. III we will discuss quantities relevant for the polarized parton distribution. Finally in Sec. IV we will draw conclusions. The formulas used to calculate hyperon decays and axial-vector constants are collected in the Appendix.

II. HYPERON DECAYS IN THE CHIRAL QUARK SOLITON MODEL

The transition matrix elements of the hadronic axial-vector current $\langle B_2|A_\mu^X|B_1\rangle$ can be expressed in terms of three independent form factors:

$$\langle B_2|A_\mu^X|B_1\rangle=\bar{u}_{B_2}(p_2)\left[\left\{g_1^{B_1\to B_2}(q^2)\gamma_\mu -\frac{ig_2^{B_1\to B_2}(q^2)}{M_1}\sigma_{\mu\nu}q^\nu +\frac{g_3^{B_1\to B_2}(q^2)}{M_1}q_\mu\right\}\gamma_5\right]u_{B_1}(p_1), \tag{8}$$

where the axial-vector current is defined as

$$A_\mu^X=\overline{\psi}(x)\gamma_\mu\gamma_5\lambda_X\psi(x), \tag{9}$$

with $X=\frac{1}{2}(1\pm i2)$ for strangeness conserving $\Delta S=0$ currents and $X=\frac{1}{2}(4\pm i5)$ for $|\Delta S|=1$. Similar expressions hold for the hadronic vector current, where the g_i are replaced by f_i ($i=1,2,3$) and γ_5 by $\mathbf{1}$.

The $q^2=-Q^2$ stands for the square of the momentum transfer $q=p_2-p_1$. The form factors g_i are real quantities depending only on the square of the momentum transfer in the case of CP-invariant processes. We can safely neglect g_3 for the reason that on account of q_μ its contribution to the decay rate is proportional to the ratio $m_l^2/M_1^2\ll 1$, where m_l represents the mass of the lepton (e or μ) in the final state and M_1 that of the baryon in the initial state.

The form factor g_2 is equal to 0 in the chiral limit. It gets the first nonvanishing contribution in the linear order in m_s. The inclusion of this effect in the discussion of the hyperon decays would require reanalyzing the experimental data, which is beyond the scope of this paper. However, the model calculations show that the m_s contribution to g_2 enters with a relatively small numerical coefficient, which means that the numerical error due to the neglect of g_2 in the full fledged analysis of the hyperon decays is small.

It is already well known how to treat hadronic matrix elements such as $\langle B_2|A_\mu^X|B_1\rangle$ within the χQSM (see, for example, [14] and references therein). Taking into account the $1/N_c$ rotational and m_s corrections, we can write the resulting axial-vector constants $g_1^{B_1\to B_2}(0)$ in the following form[1]:

[1]In the following we will assume that the baryons involved have $S_3=\frac{1}{2}$.

$$g_1^{(B_1\rightarrow B_2)}=a_1\langle B_2|D_{X3}^{(8)}|B_1\rangle+a_2 d_{pq3}\langle B_2|D_{Xp}^{(8)}\hat{S}_q|B_1\rangle+\frac{a_3}{\sqrt{3}}\langle B_2|D_{X8}^{(8)}\hat{S}_3|B_1\rangle+m_s\Bigg[\frac{a_4}{\sqrt{3}}d_{pq3}\langle B_2|D_{Xp}^{(8)}D_{8q}^{(8)}|B_1\rangle+a_5\langle B_2|(D_{X3}^{(8)}D_{88}^{(8)}+D_{X8}^{(8)}D_{83}^{(8)})|B_1\rangle+a_6\langle B_2|(D_{X3}^{(8)}D_{88}^{(8)}-D_{X8}^{(8)}D_{83}^{(8)})|B_1\rangle\Bigg]. \quad (10)$$

$\hat{S}_q$ ($\hat{S}_3$) stand for the qth (third) component of the spin operator of the baryons. The $D_{ab}^{(\mathcal{R})}$ denote the SU(3) Wigner matrices in representation $\mathcal{R}$. The a_i denote parameters depending on the specific dynamics of the chiral soliton model. Their explicit form in terms of a Goldstone mean field can be found in Ref. [15]. As mentioned already, in the present approach we will not calculate this mean field but treat a_i as free parameters to be adjusted to experimentally known semileptonic hyperon decays.

Because of the SU(3) symmetry breaking due to the strange quark mass m_s, the collective baryon Hamiltonian is no longer SU(3) symmetric. The octet states are mixed with the higher representations such as antidecuplet $\overline{\mathbf{10}}$ and eikosiheptaplet **27** [19]. In the linear order in m_s the wave function of a state $B=(Y,I,I_3)$ of spin S_3 is given as

$$\psi_{B,S_3}=(-)^{1/2-S_3}(\sqrt{8}D_{BS}^{(8)}+c_B^{(\overline{10})}\sqrt{10}D_{BS}^{(\overline{10})}+c_B^{(27)}\sqrt{27}D_{BS}^{(27)}), \quad (11)$$

where $S=(-1,\frac{1}{2},S_3)$. Mixing parameters $c_B^{(\mathcal{R})}$ can be found, for example, in Ref. [15]. They are given as products of a numerical constant $N_B^{(\mathcal{R})}$ depending on the quantum numbers of the baryonic state B and dynamical parameter $c_{\mathcal{R}}$ depending linearly on m_s (which we assume to be 180 MeV) and the model parameter I_2, which is responsible for the splitting between the octet and higher exotic multiplets [29].

Analogously to Eq. (10) one obtains in the χQSM diagonal axial-vector coupling constants. In that case X can take two values: $X=3$ and $X=8$. For $X=0$ (singlet axial-vector current) we have the following expression [15,16]:

$$\frac{1}{2}g_B^{(0)}=\frac{1}{2}a_3+\sqrt{3}m_s(a_5-a_6)\langle B|D_{83}^{(8)}|B\rangle. \quad (12)$$

This equation is remarkable, since it provides a link between an octet and singlet axial-vector current. It is perhaps the most important model input in our analysis. Pure QCD arguments based the large N_c expansion [23] do not provide such a link.

A remark concerning constants a_i is here in order. Coefficient a_1 contains terms which are leading and subleading in the large N_c expansion. The presence of the subleading terms enhances the numerical value of a_1 calculated in the χQSM for the self-consistent profile and makes model predictions, e.g., for $g_A^{(3)}$ remarkably close to the experimental data [30,31]. This feature, although very important for the model phenomenology, does not concern us here, since our procedure is based on fitting all coefficients a_i from the data. Constants a_2 and a_3 are both subleading in $1/N_c$ and come from the anomalous part of the effective chiral action in Euclidean space. In the Skyrme model they are related to the Wess-Zumino term. However, in the simplest version of the Skyrme model (which is based on the pseudoscalar mesons only) $a_3=0$ identically [11]. In the case of the χQSM $a_3\neq 0$ and it provides a link between the SU(3) octet of axial-vector currents and the singlet current of Eq. (12). It was shown in Ref. [25] that in the limit of the artificially large soliton, which corresponds to the ''Skyrme limit'' of the present model, $a_3/a_1\rightarrow 0$ is in agreement with [11]. On the contrary, for small solitons $g_A^{(0)}\rightarrow 1$, reproducing the result of the nonrelativistic quark model.

So instead of calculating seven dynamical parameters $a_i (i=1,\ldots,6)$ and I_2 (which enters into $c_{\overline{10}}$ and c_{27}) within the χQSM, we shall fit them from the hyperon semileptonic decay data. It is convenient to introduce the following set of seven new parameters:

$$r=\frac{1}{30}\left(a_1-\frac{1}{2}a_2\right), \quad s=\frac{1}{60}a_3, \quad x=\frac{1}{540}m_s a_4,$$

$$y=\frac{1}{90}m_s a_5, \quad z=\frac{1}{30}m_s a_6,$$

$$p=\frac{1}{6}m_s c_{\overline{10}}\left(a_1+a_2+\frac{1}{2}a_3\right),$$

$$q=-\frac{1}{90}m_s c_{27}\left(a_1+2a_2-\frac{3}{2}a_3\right). \quad (13)$$

Employing this new set of parameters, we can express all possible semileptonic decays of the octet baryons. Explicit formulas can be found in the Appendix [see Eq. (A1)]. Let us finally note that there is certain redundancy in Eq. (A1); namely, by redefinition of q and x we can get rid of the variable p:

$$x'=x-\frac{1}{9}p, \quad q'=q-\frac{1}{9}p. \quad (14)$$

So there are six free parameters which have to be fitted from the data.

From Eq. (A1), we can easily find that in the chiral limit the following eight sum rules for (g_1/f_1) exist:

$$
\begin{gathered}
(n\to p)=(\Xi^-\to\Sigma^0),\quad (n\to p)=(\Sigma^-\to n)+2(\Sigma^+\to\Lambda),\\
(n\to p)=\frac{4}{3}(\Sigma^+\to\Lambda)+(\Xi^-\to\Lambda),\quad (n\to p)=(\Lambda\to p)+\frac{2}{3}(\Sigma^+\to\Lambda),\\
(n\to p)=2(\Sigma^+\to\Lambda)+(\Xi^-\to\Xi^0),\quad (n\to p)=(\Sigma^-\to\Sigma^0)+(\Sigma^+\to\Lambda),\\
(\Sigma^+\to\Lambda)=(\Sigma^-\to\Lambda),\quad (\Xi^0\to\Sigma^+)=(\Xi^-\to\Sigma^0).
\end{gathered}
\tag{15}
$$

Only the first four sum rules (15) contain known decays, and the accuracy here is not worse than 10%. Apparently the symmetry breaking of SU(3) has only a small effect on the semileptonic decays.

With the linear m_s corrections turned on, we end up with only four sum rules:

$$
\begin{gathered}
(\Xi^-\to\Sigma^0)=(\Xi^0\to\Sigma^+),\quad (\Sigma^-\to\Lambda)=(\Sigma^+\to\Lambda),\\
3(\Lambda\to p)-2(n\to p)+2(\Sigma^-\to n)+4(\Sigma^+\to\Lambda)\\
-(\Xi^-\to\Sigma^0)+2(\Xi^-\to\Xi^0)-2(\Xi^-\to\Lambda)=0,\\
3(\Lambda\to p)-2(n\to p)-(\Sigma^-\to n)+2(\Sigma^+\to\Lambda)\\
-2(\Xi^-\to\Sigma^0)+2(\Sigma^-\to\Sigma^0)=0.
\end{gathered}
\tag{16}
$$

However, more experimental data are required to verify Eq. (16).

III. LINKING HYPERON DECAYS WITH DATA ON POLARIZED PARTON DISTRIBUTIONS

As we have demonstrated in the preceding section, the amplitudes of the hyperon decays are described in the χQSM by six free parameters. There are two *chiral* ones, r and s, and four proportional to m_s, x', y, z, and q'. Since there are six known hyperon decays, we can express all model parameters as linear combinations of these decay constants, and subsequently all quantities of interest can be expressed in terms of the input amplitudes. In the following we will use the experimental values of Refs. [32,33], which are presented in Table I.

Before doing this, let us, however, observe that there exist two linear combinations of the decay amplitudes which are free of the m_s corrections (within the model):

$$
\begin{gathered}
A_1+2A_6=-42r+6s,\\
3A_1-8A_2-6A_3+6A_4+6A_5=90r+90s,
\end{gathered}
\tag{17}
$$

where A_i stand for the decay constants in shorthand notation (see Table I). Solving Eq. (17) for r and s, we obtain the *chiral-limit* (i.e., with $x'=y=z=q'=0$) expressions for hyperon decays and integrated quark densities. The numerical values obtained in this way can be found in Tables I and II. Reexpressing r and s, x', y, z, and q' in terms of the A_i's allows us to write down the integrated quark densities as

$$
\begin{aligned}
\Delta u&=\frac{4A_1}{3}-\frac{16A_2}{9}-\frac{4A_3}{3}+\frac{4A_4}{3}+\frac{4A_5}{3}+\frac{4A_6}{3},\\
\Delta d&=A_1-\frac{16A_2}{9}-\frac{4A_3}{3}+\frac{4A_4}{3}+\frac{4A_5}{3}+\frac{2A_6}{3},\\
\Delta s&=\frac{2A_1}{3}-\frac{10A_2}{9}-\frac{5A_3}{6}+\frac{5A_4}{6}+\frac{5A_5}{6}+\frac{A_6}{2}.
\end{aligned}
\tag{18}
$$

The two least known amplitudes A_5 and A_6 are almost entirely responsible for the errors quoted in Tables I and II. However, since the coefficients which enter into Eq. (18) are not too large, the absolute errors are relatively small.

In Table II the yet unmeasured hyperon semileptonic decay constants are listed. The $\Xi^0\to\Sigma^+$ channel is particularly interesting, since its measurement will be soon announced by the KTeV collaboration [34].

Forming linear combinations of the quark densities we obtain the *chiral limit* expressions for $\Gamma_{p,n}$ and $\Delta\Sigma$:

$$
\begin{aligned}
\Gamma_p&=7A_1-10A_2-\frac{15A_3}{2}+\frac{15A_4}{2}+\frac{15A_5}{2}+\frac{13A_6}{2},\\
\Gamma_n&=6A_1-10A_2-\frac{15A_3}{2}+\frac{15A_4}{2}+\frac{15A_5}{2}+\frac{9A_6}{2},
\end{aligned}
$$

TABLE II. The predictions for yet unmeasured decays, integrated quark densities Δq and $\Gamma_{p,n}$ and $\Delta\Sigma$.

	Chiral limit	With m_s
$(g_1/f_1)^{\Sigma^-\to\Lambda}$	0.769 ± 0.04	0.742 ± 0.02
$(g_1/f_1)^{\Sigma^-\to\Sigma^0}$	0.502 ± 0.07	0.546 ± 0.16
$(g_1/f_1)^{\Xi^-\to\Xi^0}$	-0.267 ± 0.04	-0.12 ± 0.12
$(g_1/f_1)^{\Xi^0\to\Sigma^+}$	1.271 ± 0.11	1.278 ± 0.16
Δu	0.98 ± 0.23	0.72 ± 0.07
Δd	-0.29 ± 0.13	-0.54 ± 0.07
Δs	-0.02 ± 0.09	0.33 ± 0.51
Γ_p	3.63 ± 1.12	2.67 ± 0.33
Γ_n	-0.19 ± 0.84	-1.10 ± 0.33
$\Delta\Sigma$	0.68 ± 0.44	0.51 ± 0.41

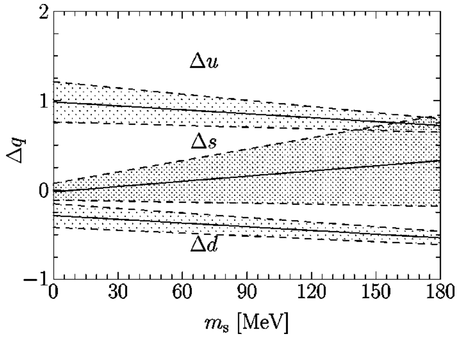

FIG. 1. Δq as a function of the strange quark mass m_s. While the Δu and Δd have less uncertainties as the m_s increases, the uncertainty of Δs becomes larger, as the m_s increases.

$$\Delta\Sigma = 3A_1 - \frac{14A_2}{3} - \frac{7A_3}{2} + \frac{7A_4}{2} + \frac{7A_5}{2} + \frac{5A_6}{2}. \quad (19)$$

The numerical values together with the error bars are listed in Table II.

The full expressions are obtained by solving the remaining four equations for m_s dependent parameters x', y, z, and q'. Also in this case we are able to link integrated quark densities Δq to the hyperon decays:

$$\Delta u = \frac{8A_2}{9} + \frac{5A_3}{3} + \frac{7A_4}{3} + \frac{A_5}{3} - \frac{A_6}{3},$$

$$\Delta d = -A_1 + \frac{8A_2}{9} + \frac{5A_3}{3} + \frac{7A_4}{3} + \frac{A_5}{3} - \frac{A_6}{3}, \quad (20)$$

$$\Delta s = \frac{15A_1}{4} - \frac{101A_2}{18} - \frac{289A_3}{48} + \frac{13A_4}{48} + \frac{43A_5}{48} + \frac{149A_6}{48}.$$

It is interesting to observe that the amplitudes A_5 and in particular A_6 come with a relatively large weight in the expression for Δs, whereas Δu and Δd are much less affected by the relatively large experimental error of these two decays. This is explicitly seen in Fig. 1, where we plot the central values and error bars of Δq's. In the same figure we draw central values and errors of Δq's in the *chiral limit* as given by Eq. (18). To guide the eye we have restored the linear dependence on the symmetry breaking m_s corrections assuming $m_s = 180$ MeV, as done in Ref. [19].

We can first see that our results in the chiral limit correspond to typical SU(3)-symmetric values: $F \approx 0.50$ and $D \approx 0.77$. However, the results for individual integrated quark densities, where the model prediction for the singlet current $g_A^{(0)}$ plays a role, are beyond the typical SU(3) symmetry values. Only when chiral symmetry breaking is taken into account are central values for Δq's shifted towards the ''standard'' values. Unfortunately the error of Δs becomes 7 times larger than the one of Δu or Δd, so that at this stage we are not able to make any firm conclusion concerning the value of Δs.

It is perhaps more interesting to look directly at the combinations relevant for the polarized scattering experiments, which take the following form:

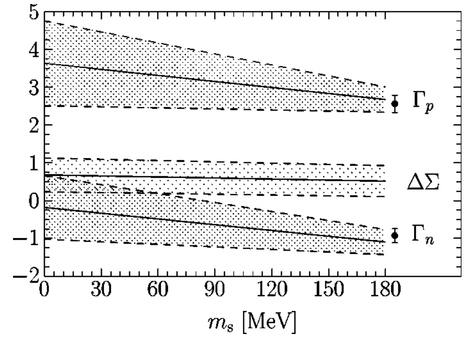

FIG. 2. $\Gamma_{p,n}$ and $\Delta\Sigma$ as functions of m_s. While the uncertainty of $\Gamma_{p,n}$ decreases, as the m_s increases, the error of the $\Delta\Sigma$ remains constant. The error bars denote the experimental data for the $\Gamma_{p,n}$.

$$\Gamma_p = \frac{11A_1}{4} - \frac{7A_2}{6} + \frac{37A_3}{16} + \frac{191A_4}{16} + \frac{41A_5}{16} + \frac{23A_6}{16},$$

$$\Gamma_n = \frac{-A_1}{4} - \frac{7A_2}{6} + \frac{37A_3}{16} + \frac{191A_4}{16} + \frac{41A_5}{16} + \frac{23A_6}{16}, \quad (21)$$

$$\Delta\Sigma = \frac{11A_1}{4} - \frac{23A_2}{6} - \frac{43A_3}{16} + \frac{79A_4}{16} + \frac{25A_5}{16} + \frac{39A_6}{16}.$$

In Fig. 2 we plot $\Gamma_{p,n}$ and $\Delta\Sigma$ both for the chiral symmetry fit and for the full fit of Eq. (21), together with experimental data for the proton and neutron. Again, to guide the eye we have restored the linear dependence of the symmetry breaking m_s corrections. We see that despite the large uncertainty of Δs, we get reasonable values for Γ_p and Γ_n. Somewhat unexpectedly we see that $\Delta\Sigma$ is almost independent of the chiral symmetry breaking[2] and stays within the range 0.1 $\rightarrow$1.1, if the errors of the hyperon decays are taken into account. Here 75% of the experimental error of $\Delta\Sigma$ comes from the two least known hyperon decays $\Xi^- \rightarrow \Lambda$, Σ^0 (corresponding to A_5 and A_6).

It is interesting to see how $\Delta\Sigma$ and Δs are correlated. To this end, instead of using two last hyperon decays A_5 and A_6 as input, we use the experimental value for Γ_p as given by Eq. (3) and $\Delta\Sigma$, which we vary in the range from 0 to 1. In Fig. 3 we plot our prediction for the two amplitudes A_5 and A_6 (solid lines), together with the experimental error bands for these two decays. It is clearly seen from Fig. 3 that the allowed region for $\Delta\Sigma$, in which the theoretical prediction falls within the experimental error bars, amounts to $\Delta\Sigma = 0.20 \rightarrow 0.45$.

In Fig. 4 we plot the variation of Δq's with respect to $\Delta\Sigma$ [with Γ_p fixed by Eq. (3)]. We see that Δu and Δd are relatively stable, whereas Δs exhibits a rather strong dependence on $\Delta\Sigma$. Within the allowed region $0.20 < \Delta\Sigma < 0.45$ the strange quark density Δs varies between -0.12 and 0.30. Interestingly, in the central region around $\Delta\Sigma \approx 0.30$ the

[2]Similar behavior has been observed in Ref. [10].

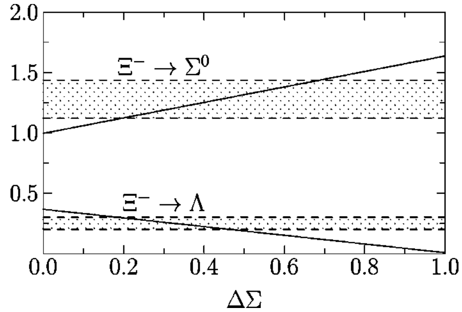

FIG. 3. A_5 (lower line) and A_6 (upper line) as functions of $\Delta\Sigma$.

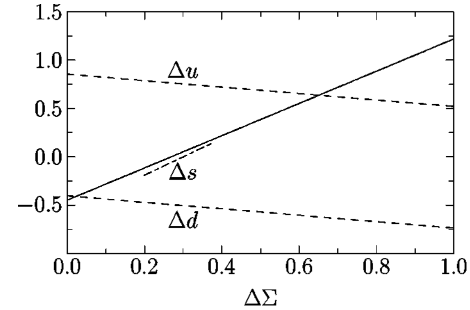

FIG. 4. Δq's as functions of $\Delta\Sigma$. The dash-dotted line below Δs corresponds to the result of Ref. [10].

strange quark density vanishes in accordance with an intuitive assumption of Ellis and Jaffe [5].

Identical behavior[3] (shown in Fig. 4 by a dash-dotted line) was obtained by Lichtenstadt and Lipkin in an analysis of the hyperon decays in which no model for $\Delta\Sigma$ has been used [10]. Indeed (assuming only the first order QCD corrections), the identity

$$\Delta\Sigma = \frac{1}{2}\Gamma_p - \frac{1}{4}(3g_A^{(3)} + \sqrt{3}g_A^{(8)}) \tag{22}$$

allows one to calculate $\Delta\Sigma$ in terms of $g_A^{(8)}$ (or equivalently Δs) by using $g_A^{(3)} = 1.257$ and Γ_p as an additional input. In the χQSM and also in large N_c QCD one can express $g_A^{(8)}$ in terms of the known hyperon semileptonic decays:

$$(3g_A^{(3)} + \sqrt{3}g_A^{(8)}) = \frac{1}{8}(-44A_1 + 104A_2 + 123A_3 + 33A_4 - 9A_5 - 55A_6). \tag{23}$$

Equation (23) gives $\Delta\Sigma = 0.46 \pm 0.31$, remarkably close to the χQSM prediction in which model formula for $\Delta\Sigma$ is used. This is, in our opinion, another strong argument in support for the model formula for $g_A^{(8)}$.

IV. SUMMARY

In this paper we studied the influence of SU(3) symmetry breaking in semileptonic hyperon decays on the determination of the integrated polarized quark densities Δq. Using the chiral quark soliton model we have obtained a satisfactory parametrization of all available experimental data on semileptonic decays. In this respect our analysis is identical to the large N_c QCD analysis of Ref. [23]. Using six known hyperon decays we have predicted g_1/f_1 for the decays not yet measured.

The new ingredient of our analysis consists in using the model formula for the singlet axial-vector current in order to make contact with the high energy polarization experiments. We have argued that our model interpolates between the quark model (the small soliton limit) and the Skyrme model (large soliton limit) [24] reproducing the value of $\Delta\Sigma$ in these two limiting cases [25,26]. This unique feature and, also, the numerical agreement with the analysis of Ref. [10] as discussed at the end of the last section make us believe that our approach contains all the necessary physics needed to analyze the symmetry breaking not only for the octet axial-vector currents, but also in the case of the singlet one.

The model contains six free parameters which can be fixed by six known hyperon decays. Unfortunately g_1/f_1 for the two known decays of Ξ^- have large experimental errors, which influence our predictions for Δq. Our strategy was very simple: using model parametrization we expressed Δq's, $\Gamma_{p,n}$, and $\Delta\Sigma$ in terms of the six known hyperon decays. Errors were added in quadrature.

The first observation which should be made is that we reproduce $\Gamma_{p,n}$ as measured in deep inelastic scattering. We obtain $\Delta u = 0.72 \pm 0.07$ and $\Delta d = -0.54 \pm 0.07$; however, Δs is practically undetermined, being equal to 0.33 ± 0.51. This large error is entirely due to the experimental errors of the Ξ^- decays, which also make $\Delta\Sigma$ to lie between 0.1 and 0.9.

There are two points which have to be stressed here. Our fit respects chiral symmetry in a sense that the leading order parameters r and s (or equivalently F and D) are fitted to linear combinations of the hyperon decays which are free from m_s corrections. Had we used this SU(3) symmetric parametrization as given by Eq. (17) we would not be able to reproduce (as far as the central values are concerned) $\Gamma_{p,n}$. With m_s corrections turned on we hit experimental values for $\Gamma_{p,n}$; however, as stated above, the value of $\Delta\Sigma$ is practically undetermined, due to the experimental error of Ξ^- decays. Therefore to confirm or invalidate our analysis it is of utmost importance to have better data for these decays. Since we predict that $(\Xi^- \to \Sigma^0) = (\Xi^0 \to \Sigma^+)$, the forthcoming experimental result for the latter decay [34] will provide a test of our approach. If the future data on this and on other decays disagrees with the predictions of our analysis (in which dynamical quantities are *fitted* to the existing data rather than *calculated* in the model), that would also mean that the model (with dynamical quantities *calculated*) fails for these particular observables. It would then be the signal for the model builders that presumably there were some physical effects which had been not included in the present version of the model.

Interestingly, if we use Γ_p and $\Delta\Sigma$ as an input instead of the Ξ^- decays, we see a very strong correlation between $\Delta\Sigma$

[3]Note that authors of Ref. [10] use a slightly different value for I_p and include higher order QCD corrections.

and Δs, whereas Δu and Δd are basically $\Delta\Sigma$ independent. This behavior has been also observed in Ref. [10].

Our analysis shows clearly that if one wants to link the low-energy hyperon semileptonic decays with high-energy polarized experiments, one cannot neglect SU(3) symmetry breaking for the former. In this respect our conclusions agree with Refs. [8,28]. Similarly to Ref. [8] we see that $\Delta s=0$ is not ruled out by present experiments. Therefore the results for Δs and $\Delta\Sigma$ which are based on exact SU(3) symmetry are in our opinion premature. Meaningful results for these two quantities can be obtained only if the experimental errors for the Ξ^- decays are reduced.

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APPENDIX

In this appendix we quote the formulas used in the fits. Semileptonic decay constants are parametrized as follows:

$$A_1=(g_1/f_1)^{(n\to p)}=-14r+2s-44x-20y-4z-4p+8q,$$

$$A_2=(g_1/f_1)^{(\Sigma^+\to\Lambda)}=-9r-3s-42x-6y-3p+15q,$$

$$A_3=(g_1/f_1)^{(\Lambda\to p)}=-8r+4s+24x-2z+2p-6q,$$

$$A_4=(g_1/f_1)^{(\Sigma^-\to n)}=4r+8s-4x-4y+2z+4q,$$

$$A_5=(g_1/f_1)^{(\Xi^-\to\Lambda)}=-2r+6s-6x+6y-2z+6q,$$

$$A_6=(g_1/f_1)^{(\Xi^-\to\Sigma^0)}=-14r+2s+22x+10y+2z+2p-4q,$$

$$(g_1/f_1)^{(\Sigma^-\to\Lambda)}=-9r-3s-42x-6y-3p+15q,$$

$$(g_1/f_1)^{(\Sigma^-\to\Sigma^0)}=-5r+5s-18x-6y+2z-2p,$$

$$(g_1/f_1)^{(\Xi^-\to\Xi^0)}=4r+8s+8x+8y-4z-8q,$$

$$(g_1/f_1)^{(\Xi^0\to\Sigma^+)}=-14r+2s+22x+10y+2z+2p-4q. \quad \text{(A1)}$$

The U(1) and SU(3) axial-vector constants $g_A^{(0,3,8)}$ can be also expressed in terms of the new set of parameters (13). For the singlet axial-vector constant, we have

$$g_A^{(0)}=60s-18y+6z, \quad \text{(A2)}$$

for the triplet one,[4]

$$g_A^{(3)}=-14r+2s-44x-20y-4z-4p+8q, \quad \text{(A3)}$$

and for the octet one, we get

$$g_A^{(8)}=\sqrt{3}(-2r+6s+12x+4p+24q). \quad \text{(A4)}$$

[4]Triplet $g_A^{(3)}$'s are proportional to I_3; the formulas in Eq. (A3) correspond to the highest isospin state.

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