Applicability of perturbative QCD to $\Lambda_b \to \Lambda_c$ decays

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We develop a perturbative QCD factorization theorem for the semileptonic heavy baryon decay $\Lambda_b \to \Lambda_c l \bar{\nu}$, whose form factors are expressed as the convolutions of hard b quark decay amplitudes with universal Λ_b and Λ_c baryon wave functions. Large logarithmic corrections are organized to all orders by Sudakov resummation, which renders perturbative expansions more reliable. It is observed that perturbative QCD is applicable to $\Lambda_b \to \Lambda_c$ decays for velocity transfer greater than 1.2. Under the requirement of heavy quark symmetry, we predict the branching ratio $B(\Lambda_b \to \Lambda_c l \bar{\nu}) \sim 2\%$, and determine the Λ_b and Λ_c baryon wave functions.

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I. INTRODUCTION

Analyses of exclusive heavy hadron decays are a challenging subject because of their complicated QCD dynamics. Recently, we have proposed a rigorous theory for these processes based on perturbative QCD (PQCD) factorization theorems [1,2]. In this approach heavy hadron decay rates are expressed as convolutions of hard heavy quark decay amplitudes with heavy hadron wave functions. The former are calculable in perturbation theory, if processes involve large momentum transfer. The latter, absorbing nonperturbative dynamics of processes, must be obtained by means outside the PQCD regime. Since wave functions are universal, they can be determined once and for all, and then employed to make predictions for other processes containing the same hadrons. With this prescription for nonperturbative wave functions, PQCD factorization theorems possess a predictive power.

For semileptonic decays, the PQCD approach complements heavy quark symmetry in studies of heavy hadron transition form factors [3]. Heavy quark symmetry determines the normalization of transition form factors at zero recoil of final-state heavy hadrons, up to power corrections in $1/M$, M being the heavy quark mass, and up to perturbative corrections in the coupling constant α_s, while PQCD is appropriate for fast recoil, the region with large energy release, and gives a dependence of transition form factors on velocity transfer. For nonleptonic decays, PQCD is a more systematic approach compared with the phenomenological Bauer-Stech-Wirbel (BSW) model [4]. In PQCD factorization theorems contributions to nonleptonic decay rates characterized by different scales are carefully absorbed into different subprocesses, among which renormalization-group (RG) evolutions are constructed [2], leading to a scale and scheme independent, gauge invariant, and infrared finite theory [5]. Not only factorizable but nonfactorizable contributions can be evaluated [6]. The BSW model considers only factorizable contributions: two fitting parameters a_1 and a_2 are associated with external and internal W-emission form factors, respectively. Nonfactorizable contributions must be included as additional parameters [7].

The above PQCD formalism has been applied to heavy meson decays successfully. It is then natural to extend the formalism to more complicated heavy baryon decays. In [8] we have developed a factorization theorem for the semileptonic decay $\Lambda_b \to p l \bar{\nu}$, in which Sudakov resummation of double logarithmic corrections to the Λ_b baryon wave function was included, and a full set of diagrams for the hard b quark decay amplitudes was calculated. This is an analysis more complete than the work in the literature [9]. On the other hand, b baryons have been observed in experiments at LEP and at the Tevatron. Masses and decay widths of the lightest b baryons, as compared with theoretical predictions, have stimulated many interesting discussions and investigations [10–14]. When run II of the Tevatron comes up with a vertex trigger employed, it will be expected to collect more than 10^6 b baryon events. Therefore, an intensive study of exclusive heavy baryon decays is urgent.

Exclusive heavy baryon decays are dominated by $b \to c$ modes. In this paper we shall develop a factorization theorem for the semileptonic decay $\Lambda_b \to \Lambda_c l \bar{\nu}$, and locate the kinematic region where PQCD is applicable. It will be shown that PQCD predictions for the involved transition form factors are reliable at fast recoil of the Λ_c baryon with velocity transfer greater than 1.2. Under the requirement of heavy quark symmetry, we predict the branching ratio $B(\Lambda_b \to \Lambda_c l \bar{\nu}) \sim 2\%$. We shall also determine the unknown parameters in the Λ_b and Λ_c baryon wave functions, which can be employed to study nonleptonic Λ_b baryon decays because of the universality.

In Sec. II we develop a factorization theorem for the semileptonic decay $\Lambda_b \to \Lambda_c l \bar{\nu}$. Sudakov resummation of double logarithmic corrections to the process is performed. The factorization formulas for the involved heavy baryon transition form factors and their numerical results are pre-

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sented in Sec. III and in Sec. IV, respectively. Section V is the conclusion.

II. FACTORIZATION THEOREM

The amplitude for the semileptonic decay $\Lambda_b \to \Lambda_c l \bar{\nu}$ is written as

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{cb} \bar{l} \gamma^{\mu}(1-\gamma_5)\nu_l \langle \Lambda_c(p') | \bar{c} \gamma_{\mu}(1-\gamma_5) b | \Lambda_b(p) \rangle, \tag{1}$$

where G_F is the Fermi coupling constant, V_{cb} is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element, and p and p' are the Λ_b and Λ_c baryon momenta, respectively. All QCD dynamics is contained in the hadronic matrix element

$$\begin{aligned}\mathcal{M}_{\mu} &\equiv \langle \Lambda_c(p') | \bar{c} \gamma_{\mu}(1-\gamma_5) b | \Lambda_b(p) \rangle, \\ &= \bar{\Lambda}_c(p')[f_1(q^2)\gamma_{\mu} - i f_2(q^2)\sigma_{\mu\nu}q^{\nu} + f_3(q^2) q_{\mu}]\Lambda_b(p) \\ &\quad + \bar{\Lambda}_c(p')[g_1(q^2)\gamma_{\mu}\gamma_5 - i g_2(q^2)\sigma_{\mu\nu}\gamma_5 q^{\nu} \\ &\quad + g_3(q^2)\gamma_5 q_{\mu}]\Lambda_b(p). \end{aligned} \tag{2}$$

In the second expression $\mathcal{M}_{\mu}$ has been expressed in terms of six form factors f_i and g_i, where $\Lambda_b(p)$ and $\Lambda_c(p')$ are the Λ_b and Λ_c baryon spinors, respectively, and the variable q denotes $q = p - p'$. In the case of massless leptons with

$$q_{\mu} \bar{l} \gamma^{\mu}(1-\gamma_5)\nu_l = 0, \tag{3}$$

the form factors f_3 and g_3 do not contribute. Since the contributions from f_2 and g_2 are small, we shall concentrate on f_1 and g_1 in the present work.

The idea of PQCD factorization theorems is to sort out nonperturbative dynamics involved in QCD processes and factorize it into hadron wave functions. Nonperturbative dynamics is reflected by infrared divergences in radiative corrections to quark-level amplitudes in perturbation theory. The construction of a factorization theorem for the decay $\Lambda_b \to \Lambda_c l \bar{\nu}$ is basically similar to that for the decay $\Lambda_b \to p l \bar{\nu}$ in [8]. The lowest-order diagrams for $b \to c$ decays are shown in Fig. 1, where two hard gluons attach the three incoming and outgoing quarks in all possible ways. We then investigate infrared divergences from radiative corrections to these diagrams. Small transverse momenta k_T are associated with the valence quarks, such that they are off mass shell a bit. The transverse momenta k_T serve as a factorization scale, below which dynamics is regarded as being nonperturbative, and absorbed into Λ_b and Λ_c baryon wave functions, and above which perturbation theory is reliable, and radiative corrections are absorbed into hard $b \to c$ decay amplitudes.

Infrared divergences from radiative corrections are collinear, when loop momenta are parallel to an energetic light quark, and soft, when loop momenta are much smaller than the Λ_b baryon mass M_{Λ_b}. Collinear and soft enhancements may overlap to give double logarithms. Three-particle reducible corrections on the Λ_b baryon side are absorbed into the Λ_b baryon wave function. If the light valence quarks move slowly, collinear divergences associated with these quarks will not be pinched [1], and soft divergences are important. However, there is a probability, though small, of finding light quarks in the Λ_b baryon with longitudinal momenta of order M_{Λ_b}. Therefore, reducible corrections on the Λ_b baryon side are dominated by soft dynamics, but contain weak double logarithms with collinear ones suppressed. Similarly, three-particle reducible corrections on the Λ_c baryon side are absorbed into the Λ_c baryon wave function. In the fast recoil region collinear divergences become stronger, and double logarithms associated with the Λ_c baryon wave function are more important. The remaining part of radiative corrections, with all collinear and soft divergences subtracted, is characterized by a scale of order M_{Λ_b}, and absorbed into the hard b quark decay amplitudes. Irreducible corrections, with a gluon attaching a quark in the Λ_b baryon and a quark in the Λ_c baryon, are infrared finite in the large recoil region [15] and also absorbed into the hard decay amplitudes.

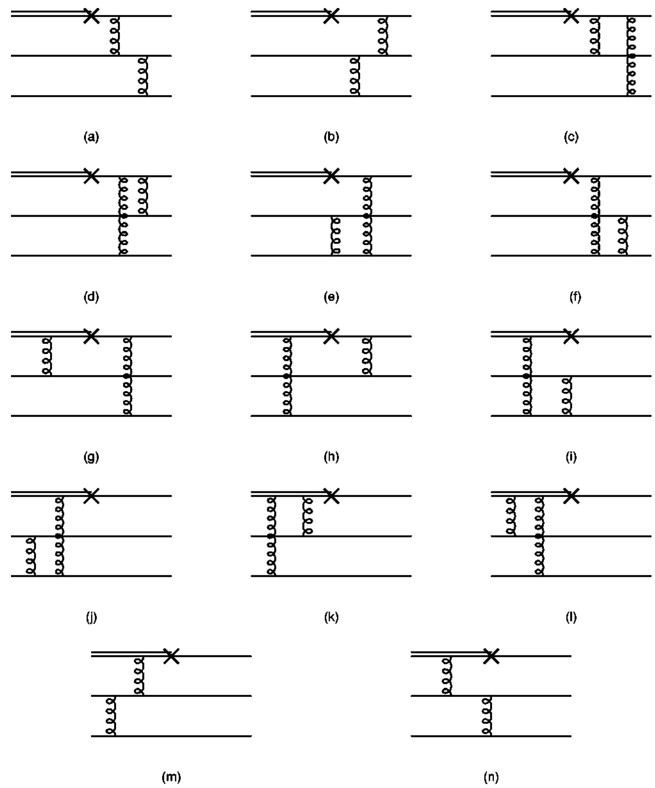

FIG. 1. Lowest-order diagrams for $\Lambda_b \to \Lambda_c l \bar{\nu}$ decay.

The kinematic variables are defined as follows. The Λ_b baryon is assumed to be at rest with momentum

$$p \equiv (p^+, p^-, \mathbf{p}_T) = \frac{M_{\Lambda_b}}{\sqrt{2}}(1,1,\mathbf{0}). \tag{4}$$

The valence quark momenta in the Λ_b baryon are parametrized as

$$\begin{aligned} &k_1 = (p^+, x_1 p^-, k_{1T}), \quad k_2 = (0, x_2 p^-, k_{2T}), \\ &k_3 = (0, x_3 p^-, k_{3T}), \end{aligned} \tag{5}$$

where k_1 is associated with the b quark. The momentum fractions and the transverse momenta obey the conservation laws

$$x_1+x_2+x_3=1, \quad k_{1T}+k_{2T}+k_{3T}=0. \tag{6}$$

The Λ_c baryon momentum is chosen as $p' \equiv (p'^+, p'^-, \mathbf{0})$ with $p'^+ \gg p'^-$ at fast recoil. We define the velocity transfer ρ,

$$\rho=\frac{p\cdot p'}{M_{\Lambda_b}M_{\Lambda_c}}, \quad 1<\rho<\frac{M_{\Lambda_b}^2+M_{\Lambda_c}^2}{2M_{\Lambda_b}M_{\Lambda_c}}, \tag{7}$$

M_{Λ_c} being the Λ_c baryon mass. Using the on-shell condition $p'^2=M_{\Lambda_c}^2$, the plus and minus components of p' are written as

$$p'^+=\rho_+p^+, \quad p'^-=\rho_-p^-, \tag{8}$$

with

$$\rho_+=(\rho+\sqrt{\rho^2-1})r, \quad \rho_-=(\rho-\sqrt{\rho^2-1})r, \tag{9}$$

and $r=M_{\Lambda_c}/M_{\Lambda_b}$. The valence quark momenta in the Λ_c baryon are parametrized as

$$\begin{aligned} &k_1'=(x_1'p'^+,p'^-,k_{1T}'), \quad k_2'=(x_2'p'^+,0,k_{2T}'),\\ &k_3'=(x_3'p'^+,0,k_{3T}'), \end{aligned} \tag{10}$$

where k_1' is associated with the c quark. The primed variables obey similar relations to Eq. (6).

According to the factorization theorem, the hadronic matrix element is expressed as

$$\begin{aligned}\mathcal{M}_\mu=&\int_0^1[dx][dx']\int[d^2k_T][d^2k_T']\overline{\Psi}_{\Lambda_c\alpha'\beta'\gamma'}(k_i',\mu)\\ &\times H_\mu^{\alpha'\beta'\gamma'\alpha\beta\gamma}(k_i',k_i,\rho,M_{\Lambda_b},\mu)\Psi_{\Lambda_b\alpha\beta\gamma}(k_i,\mu),\end{aligned} \tag{11}$$

with the notation

$$[dx]=dx_1dx_2dx_3\delta\left(1-\sum_{i=1}^3 x_i\right),$$

$$[d^2k_T]=d^2k_{1T}d^2k_{2T}d^2k_{3T}\delta^2\left(\sum_{i=1}^3 k_{iT}\right). \tag{12}$$

$[dx']$ and $[d^2k_T']$ associated with the Λ_c baryon are defined in a similar way. The hard amplitude H_μ will be computed in Sec. III. The dependence on the factorization (renormalization) scale μ will disappear after performing a RG analysis.

The structure of the Λ_b baryon distribution amplitude $\Psi_{\Lambda_b\alpha\beta\gamma}$ is simplified under the assumptions that the spin and orbital degrees of freedom of the light quark system are decoupled and that the Λ_b baryon is in the ground state (s wave). The distribution amplitude is then expressed as [9]

$$\begin{aligned}\Psi_{\Lambda_b\alpha\beta\gamma}(k_i,\mu)&=\frac{1}{2\sqrt{2}N_c}\int\prod_{l=1}^2\frac{dy_l^-d\mathbf{y}_l}{(2\pi)^3}e^{ik_l\cdot y_l}\epsilon^{abc}\langle 0|T[b_\alpha^a(y_1)u_\beta^b(y_2)d_\gamma^c(0)]|\Lambda_b(p)\rangle\\ &=\frac{f_{\Lambda_b}}{8\sqrt{2}N_c}[(\not p+M_{\Lambda_b})\gamma_5C]_{\beta\gamma}[\Lambda_b(p)]_\alpha\Phi(k_i,\mu),\end{aligned} \tag{13}$$

where $N_c=3$ is the number of colors, b, u, and d are quark fields, a, b, and c are color indices, α, β, and γ are spinor indices, f_{Λ_b} is a normalization constant, C is the charge conjugation matrix, and Φ is the Λ_b baryon wave function. Under similar assumptions, the Λ_c baryon distribution amplitude $\Psi_{\Lambda_c\alpha\beta\gamma}$ is written as

$$\begin{aligned}\Psi_{\Lambda_c\alpha\beta\gamma}(k_i',\mu)&=\frac{1}{2\sqrt{2}N_c}\int\prod_{l=1}^2\frac{dy'^-_l d\mathbf{y}'_l}{(2\pi)^3}e^{ik_l'\cdot y_l'}\epsilon^{abc}\langle 0|T[c_\alpha^a(y_1')u_\beta^b(y_2')d_\gamma^c(0)]|\Lambda_c(p')\rangle\\ &=\frac{f_{\Lambda_c}}{8\sqrt{2}N_c}[(\not p'+M_{\Lambda_c})\gamma_5C]_{\beta\gamma}[\Lambda_c(p')]_\alpha\Pi(k_i',\mu),\end{aligned} \tag{14}$$

where the normalization constant f_{Λ_c} and the wave function Π are associated with the Λ_c baryon.

Because of the inclusion of parton transverse momenta, Sudakov resummation for a hadron wave function should be performed in impact parameter b space with b conjugate to k_T [1,16]. The result is [8]

$$\begin{aligned}\Phi(k_i^-,b_i,\mu)=\exp\Bigg[&-\sum_{l=2}^3 s(w,k_l^-)\\ &-3\int_w^\mu\frac{d\bar\mu}{\bar\mu}\gamma_q(\alpha_s(\bar\mu))\Bigg]\phi(x_i),\end{aligned} \tag{15}$$

where $\gamma_q=-\alpha_s/\pi$ is the quark anomalous dimension, and the factorization scale w is chosen as

$$w=\min\left(\frac{1}{b_1},\frac{1}{b_2},\frac{1}{b_3}\right), \tag{16}$$

with $b_3=|\mathbf{b}_1-\mathbf{b}_2|$. The explicit expression of the Sudakov exponent s is given by [17]

$$s(w,Q)=\int_w^Q\frac{dp}{p}\left[\ln\left(\frac{Q}{p}\right)A(\alpha_s(p))+B(\alpha_s(p))\right], \tag{17}$$

where the anomalous dimensions A to two loops and B to one loop are

$$A=C_F\frac{\alpha_s}{\pi}+\left[\frac{67}{9}-\frac{\pi^2}{3}-\frac{10}{27}n_f+\frac{8}{3}\beta_0\ln\left(\frac{e^{\gamma_E}}{2}\right)\right]\left(\frac{\alpha_s}{\pi}\right)^2,$$

$$B=\frac{2}{3}\frac{\alpha_s}{\pi}\ln\left(\frac{e^{2\gamma_E-1}}{2}\right), \tag{18}$$

$C_F=4/3$ being a color factor, $n_f=4$ the flavor number, and γ_E the Euler constant. The one-loop running coupling constant

$$\frac{\alpha_s(\mu)}{\pi}=\frac{1}{\beta_0\ln(\mu^2/\Lambda_{\rm QCD}^2)}, \tag{19}$$

with the coefficient $\beta_0=(33-2n_f)/12$ and QCD scale $\Lambda_{\rm QCD}$, will be substituted into Eq. (17). The initial condition ϕ of the Sudakov evolution absorbs nonperturbative dynamics below the factorization scale w.

Following the derivation in [3,18], we obtain the Sudakov resummation for the Λ_c baryon distribution amplitude:

$$\Pi(k_i'^+,b_i,\mu)=\exp\left[-\sum_{l=1}^{3}s(w,k_l'^+)-3\int_w^\mu\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu}))\right]\pi(x_i'). \tag{20}$$

We have included the Sudakov exponent s associated with the c quark, which carries large longitudinal momentum in the fast recoil region. Notice the same transverse extents b_i as those for the Λ_b baryon. This is the consequence of neglecting the transverse momenta which flow through the virtual quark lines in H_μ [18].

The RG analysis of H_μ leads to

$$\begin{aligned}H_\mu(k_i'^+,k_i^-,b_i,\rho,M_{\Lambda_b},\mu)&=\exp\left[-3\sum_{l=1}^{2}\int_\mu^{t_l}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu}))\right]\\&\quad\times H_\mu(x_i',x_i,b_i,\rho,M_{\Lambda_b},t_1,t_2),\end{aligned} \tag{21}$$

where the superscripts α', β', ..., have been suppressed. Since large logarithms have been collected by the exponential, the initial condition H_μ of the RG evolution on the right-hand side of the above expression can be computed reliably in perturbation theory. To simplify the formalism, we shall make the approximations $M_b\approx M_{\Lambda_b}$ and $M_c\approx M_{\Lambda_c}$, and neglect the transverse momentum dependence of the virtual quark propagators as mentioned before. The two arguments t_1 and t_2 of H_μ, which will be specified in the next section, imply that each running coupling constant α_s is evaluated at the mass scale of the corresponding hard gluon. Substituting Eqs. (13)–(21) into Eq. (11), we derive the factorization formula for the semileptonic decay $\Lambda_b\to\Lambda_c l\bar{\nu}$, where the μ dependence has disappeared as stated before.

For the Λ_b baryon wave function $\phi(x_1,x_2,x_3)$, we adopt the model proposed in [19],

$$\phi(\zeta,\eta)=N\eta^2\zeta(1-\eta)(1-\zeta)\exp\left[-\frac{M_b^2}{2\beta^2(1-\eta)}-\frac{m_l^2}{2\beta^2\eta\zeta(1-\zeta)}\right], \tag{22}$$

with N being a normalization constant, β a shape parameter, and m_l the mass of light degrees of freedom in the Λ_b baryon. The new variables ζ and η are defined by

$$\zeta=\frac{x_2}{x_2+x_3},\qquad \eta=x_2+x_3. \tag{23}$$

In terms of ζ and η, the normalization of $\phi(\zeta,\eta)$ is given by

$$\int d\zeta\eta d\eta\phi(\zeta,\eta)=1, \tag{24}$$

which determines the constant N, once the parameters β and m_l are fixed. The above wave function with the factor $\eta^2\zeta(1-\eta)(1-\zeta)=x_1x_2x_3$ suppresses contributions from the end points of momentum fractions. The exponents proportional to $M_b^2/(1-\eta)=M_b^2/x_1$ and to $m_l^2/[\eta\zeta(1-\zeta)]=m_l^2/x_2+m_l^2/x_3$ with $M_b\gg m_l$ indicate that ϕ has a maximum at large x_1 and at small x_2 and x_3, and that the b quark momentum k_1^2 is roughly equal to M_b^2. For $\phi(x_3,x_1,x_2)$ which will appear in the factorization formulas presented in Sec. III, the above expression is transformed into

$$\phi(\zeta,\eta)=N\eta^2\zeta(1-\eta)(1-\zeta)\exp\left[-\frac{M_b^2}{2\beta^2\eta(1-\zeta)}-\frac{m_l^2(1-\eta+\eta\zeta)}{2\beta^2\eta\zeta(1-\eta)}\right]. \tag{25}$$

For convenience, we assume that the Λ_c wave function $\pi(\zeta',\eta')$ possesses the same functional form and the same parameters β and m_l as of $\phi(\zeta,\eta)$, but with the b quark mass M_b replaced by the c quark mass M_c. The wave function $\pi(x_1',x_2',x_3')$ also has a maximum at large x_1', such that the c quark momentum $k_1'^2$ is roughly equal to M_c^2.

III. TRANSITION FORM FACTORS

In this section we present the factorization formulas for the form factors f_1 and g_1, which are associated with the spin structures $\overline{\Lambda}_c \gamma_\mu \Lambda_b$ and $\overline{\Lambda}_c \gamma_\mu \gamma_5 \Lambda_b$ in M_μ, respectively. Working out the contraction of $\overline{\Psi}_{\Lambda_c \alpha' \beta' \gamma'} H_\mu^{\alpha' \beta' \gamma' \alpha\beta\gamma} \Psi_{\Lambda_b \alpha\beta\gamma}$ in momentum space, we extract the hard part H. Employing a series of permutations of the valence quark kinematic variables as in [8], the summation over the leading diagrams in Fig. 1 reduces to two terms for each form factor. The factorization formulas for the form factors $f_1(\rho)$ and $g_1(\rho)$ are written as

$$f_1(\rho)=\frac{4\pi}{27}\int_0^1 [dx'][dx]\int_0^\infty b_1 db_1 b_2 db_2 \int_0^{2\pi} d\theta f_{\Lambda_c} f_{\Lambda_b} \sum_{j=1}^{2} H_j(x_i',x_i,b_i,\rho,M_{\Lambda_b},t_{jl})\mathcal{F}_j(x_i',x_i,\rho) \times\exp[-S(x_i',x_i,w,\rho,M_{\Lambda_b},t_{jl})], \tag{26}$$

$$g_1(\rho)=\frac{4\pi}{27}\int_0^1 [dx'][dx]\int_0^\infty b_1 db_1 b_2 db_2 \int_0^{2\pi} d\theta f_{\Lambda_c} f_{\Lambda_b} \sum_{j=1}^{2} H_j(x_i',x_i,b_i,\rho,M_{\Lambda_b},t_{jl})\mathcal{G}_j(x_i',x_i,\rho) \times\exp[-S(x_i',x_i,w,\rho,M_{\Lambda_b},t_{jl})], \tag{27}$$

where θ is the angle between $\mathbf{b}_1$ and $\mathbf{b}_2$.

The functions $\mathcal{F}_j$ and $\mathcal{G}_j$, which group together the products of the initial and final baryon wave functions, are, in terms of the notation,

$$\phi_{123}\equiv \phi(x_1,x_2,x_3), \qquad \pi_{123}\equiv \pi(x_1',x_2',x_3'), \tag{28}$$

given by

$$\begin{aligned}\frac{\mathcal{F}_1}{\phi_{123}\pi_{123}}=&\frac{r^2}{[(1-x_1'-\rho_-)\rho_+ + r^2](1-x_1')x_2\rho_+}\{2(2\sqrt{\rho^2-1}-1)(1-x_1')+[2(1+r)\rho-4r-1]x_2+[2(2\rho-1)\\&+(2\rho-3)\rho_1]x_2x_1'\}+\frac{r^2}{[(1-x_1'-\rho_-)\rho_+ + r^2](1-x_1)x_2'\rho_+}\{(\rho_1+2r\sqrt{\rho^2-1}+3+4r-r\rho)(1-x_1)\\&-(\rho_1+3)(1-x_1)x_1'+2[2(\rho-1)(\sqrt{\rho+1}+\rho)-1]x_2'\}+\frac{r}{(1-x_1)^2x_2'\rho_+^2}\{2r(2\sqrt{\rho^2-1}-1+2\rho)(1-x_1)\\&+2(\sqrt{\rho^2-1}-2+\rho)x_2'-r[(2-\rho)\rho_1+1-2\rho](1-x_1)x_2'\}+\frac{r}{(1-x_1)(1-x_1')x_2\rho_+^2}\{2(\sqrt{\rho^2-1}+2-\rho)+r(\rho_1+3)\\&\times(1-x_1)(1-x_1')+2r[2(\rho-1)(\sqrt{\rho^2-1}+\rho)-1]x_2\},\end{aligned} \tag{29}$$

$$\begin{aligned}\frac{\mathcal{F}_2}{\phi_{312}\pi_{312}}=&\frac{r}{[(1-x_3'-\rho_-)\rho_+ + r^2](1-x_3)x_1'\rho_+}\{2r\rho_1(1-x_3')+4r^2(1+\rho)(1-x_1)+2r^2(3-\sqrt{\rho^2-1})x_1\\&-2r[(\rho-1)\sqrt{\rho^2-1}-\rho^2]x_2'-(1+\rho_1)[r(\rho-1)x_1+x_2'](1-x_3')\}\\&+\frac{2r}{[(x_2'-\rho_-)(1-x_1)\rho_+ + r^2][1-(1-x_1\rho_+)(1-x_2')]}\{r(\rho+\sqrt{\rho^2-1})[x_1x_2'-\rho_1(x_1+x_2')]\\&+\rho_1(2r^2x_1+x_2'+2r\sqrt{\rho^2-1})\}+\frac{r}{[1-(1-x_2\rho_+)(1-x_1')](1-x_3)\rho_+}\{2r\rho_1(1-x_3)+4(1+\rho)(1-x_1')\\&-2(\sqrt{\rho^2-1}-3)x_1'-2r[(\rho-1)(\rho+\sqrt{\rho^2-1})+1]x_2-r(1+\rho_2)(rx_2+\sqrt{\rho^2-1}x_1')(1-x_3)\},\end{aligned} \tag{30}$$

$$\frac{\mathcal{G}_1}{\phi_{123}\pi_{123}}=\frac{r^2}{[(1-x_1'-\rho_-)\rho_++r^2](1-x_1')x_2\rho_+}[(2\rho-3+(2\rho-1)\rho_2)x_2(1-x_1')+2(2-\rho-\sqrt{\rho^2-1})x_2$$
$$-2(2\rho-1+2\sqrt{\rho^2-1})(1-x_1')]+\frac{r^2}{[(1-x_1'-\rho_-)\rho_++r^2](1-x_1)x_2'\rho_+}\{2[2(\rho-1)(\sqrt{\rho^2-1}+\rho)-1]x_2'$$
$$+2r(\sqrt{\rho^2-1}+1)(1-x_1)-(3\rho_2+1)(1-x_1)(1-x_1')\}+\frac{r^2}{(1-x_1)^2x_2'\rho_+^2}[(2\rho-3+(2\rho-1)\rho_2)x_2'(1-x_1)$$
$$+2(2-\rho-\sqrt{\rho^2-1})x_2'-2(2\rho-1+2\sqrt{\rho^2-1})(1-x_1)]+\frac{r}{(1-x_1)(1-x_1')x_2\rho_+^2}\{-2r[2(\rho-1)$$
$$\times(\sqrt{\rho^2-1}+\rho)-1]x_2-2(\sqrt{\rho^2-1}+2-\rho)(1-x_1')+r(3\rho_2+1)(1-x_1)(1-x_1')\}, \quad (31)$$

$$\frac{\mathcal{G}_2}{\phi_{312}\pi_{312}}=\frac{r}{[(1-x_3'-\rho_-)\rho_++r^2](1-x_3)x_1'\rho_+}\{-4r^2(1+\rho)+r\rho_2(4-\rho-\rho^2)x_1+r(\rho-1)(\rho_2-1)x_3'x_1+2r(1-x_3')$$
$$+2r[(\rho-1)(\sqrt{\rho^2-1}+\rho)+1]x_2'-(\rho_2+1)x_2'(1-x_3')\}$$
$$+\frac{2r}{[(x_2'-\rho_-)(1-x_1)\rho_++r^2][1-(1-x_1\rho_+)(1-x_2')]}[r(\sqrt{\rho^2-1}-2-\rho)+r^2x_1+x_2'-2r(\rho+\sqrt{\rho^2-1})(1-x_1)$$
$$\times(1-x_2')]+\frac{r}{[1-(1-x_2\rho_+)(1-x_1')](1-x_3)\rho_+}\{-4(1+\rho)+2r(1-x_3)-2(\sqrt{\rho^2-1}+2\rho-1)x_1'+2r[(\rho-1)$$
$$\times(\sqrt{\rho^2-1}+\rho)+1]x_2-r(\rho_2+1)(rx_2+(\rho-1)x_1')(1-x_3)\}, \quad (32)$$

with $\rho_1=\sqrt{(\rho+1)/(\rho-1)}$ and $\rho_2=1/\rho_1$.

The hard parts are given by

$$H_1=\alpha_s(t_{11})\alpha_s(t_{12})K_0(\sqrt{(1-x_1)(1-x_1')\rho_+}M_{\Lambda_b}b_1)$$
$$\times K_0(\sqrt{x_2x_2'\rho_+}M_{\Lambda_b}b_2), \quad (33)$$

$$H_2=\alpha_s(t_{21})\alpha_s(t_{22})K_0(\sqrt{x_1x_1'\rho_+}M_{\Lambda_b}b_1)$$
$$\times K_0(\sqrt{x_2x_2'\rho_+}M_{\Lambda_b}b_2), \quad (34)$$

with K_0 being the modified Bessel function of order zero. The complete Sudakov exponent S is written as

$$S(x_i',x_i,w,\rho,M_{\Lambda_b},t_{jl})=S_d(x_i',x_i,w,\rho,M_{\Lambda_b})+S_s(w,t_{jl}), \quad (35)$$

with

$$S_d=\sum_{l=2}^{3}s(w,x_lp^-)+\sum_{l=1}^{3}s(w,x_l'p'^+), \quad (36)$$

$$S_s=3\int_w^{t_{j1}}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu}))+3\int_w^{t_{j2}}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu})). \quad (37)$$

The hard scales t_{jl} are chosen as

$$t_{11}=\max[\sqrt{(1-x_1)(1-x_1')\rho_+}M_{\Lambda_b},1/b_1],$$
$$t_{21}=\max[\sqrt{x_1x_1'\rho_+}M_{\Lambda_b},1/b_1],$$
$$t_{21}=t_{22}=\max[\sqrt{x_2x_2'\rho_+}M_{\Lambda_b},1/b_2], \quad (38)$$

which are always greater than w. It is possible that the hard scales t_{jl} are small and the running coupling constants become large as b_i are close to $1/\Lambda_{\rm QCD}$. These nonperturbative enhancements are, however, suppressed by the Sudakov exponential $\exp(-S_d)$, which decreases quickly in the large b_i region and vanishes as $b_i \geqslant 1/\Lambda_{\rm QCD}$. The exponential $\exp(-S_d)$ approaches unity; that is, there is no Sudakov suppression from the all-order summation of infrared logarithmic corrections at small b_i. In these short-distance regions higher-order corrections are regarded as being hard and should be absorbed into H [20]. Another exponential $\exp(-S_s)$, as a consequence of single-logarithm summation, describes the RG evolution from the factorization scale w to the hard scales t_{jl}.

For the case with massless leptons, it is easy to show that the differential decay rate in the rest frame of the Λ_b baryon is given by

$$\frac{d\Gamma}{d\rho} = \frac{M_{\Lambda_b}^5 r^3}{24\pi} G_F^2 |V_{cb}|^2 \sqrt{\rho^2-1}\{|f_1|^2(\rho-1) \times[3+3r^2-2(2\rho-1)r]+|g_1|^2(\rho+1) \times[3+3r^2-2(2\rho+1)r]\}, \tag{39}$$

where only the contributions from the form factors f_1 and g_1 are considered. It is straightforward to obtain the total decay rate

$$\Gamma \equiv \int d\rho \frac{d\Gamma}{d\rho} \tag{40}$$

from Eq. (39) and thus the branching ratio $B(\Lambda_b \to \Lambda_c l \bar{\nu})$, if the form factors $f_1(\rho)$ and $g_1(\rho)$ in the whole range of ρ are known.

IV. RESULTS

In order to reduce the number of unknown parameters, we make an approximation. Consider the baryonic decay constant $\tilde{f}_\Lambda$ defined, in heavy quark effective theory, by

$$\langle 0|\tilde{j}^v|\Lambda_Q\rangle = \tilde{f}_\Lambda \Lambda_Q, \tag{41}$$

in terms of the Λ baryonic current [21,22]

$$\tilde{j}^v = \epsilon^{abc}(u^a C\gamma_5 d^b)h_v^c, \tag{42}$$

where Λ_Q is the heavy baryon spinor, h_v the heavy quark field, and a, b, c denote the color indices. We contract a Dirac tensor $(C\gamma_5)_{\beta\gamma}$ with a heavy Λ baryon distribution amplitude such as $\Psi_{\Lambda_b \alpha\beta\gamma}$ in Eq. (13) and integrate out the valence quark momenta k_i. Compared with Eq. (41), we extract the baryonic decay constant

$$\tilde{f}_\Lambda = f_{\Lambda_Q} M_{\Lambda_Q}. \tag{43}$$

It implies that in the heavy quark limit the normalization constants f_{Λ_b} and f_{Λ_c} are related by

$$f_{\Lambda_b} M_{\Lambda_b} = f_{\Lambda_c} M_{\Lambda_c}. \tag{44}$$

Therefore, f_{Λ_c} associated with the Λ_c baryon will not be treated as a free parameter in the numerical analysis below.

We are now ready to compute the form factors $f_1(\rho)$ and $g_1(\rho)$ from Eqs. (26) and (27), adopting the CKM matrix element $V_{cb}=0.04$, the masses $M_{\Lambda_b}=5.624$ GeV and $M_{\Lambda_c}=2.285$ GeV, and the QCD scale $\Lambda_{\rm QCD}=0.2$ GeV. We examine the self-consistency of our calculation by considering the percentage of the full contribution to the form factor f_1 that arises from the short-distance region with all $\alpha_s(t_{jl})/\pi < 0.5$. The percentages for different β with m_l fixed at 0.3 GeV are listed in Table I. It is observed that the perturbative contributions become dominant gradually as ρ and β increase: a larger ρ corresponds to larger momentum transfer involved in decay processes, and a larger β corresponds to heavy baryon wave functions which are less sharp at the high ends of the momentum fractions x_1 and x_1'. We conclude that the PQCD analysis of the transition form factors is self-consistent for $\beta>1.0$ GeV and $\rho>1.2$, viewing the perturbative percentage of about 80%. Compared to the corresponding meson decay $B\to Dl\bar{\nu}$ [3], a perturbative expansion is less reliable in the baryon case, because partons in a baryon are softer, such that Sudakov suppression is weaker.

TABLE I. Percentages of perturbative contributions for various β and ρ.

Percentage	$\rho=1.2$	$\rho=1.3$	$\rho=1.4$
$\beta=1.0$ GeV	77.7%	83.6%	85.2%
$\beta=2.0$ GeV	79.3%	83.0%	85.7%
$\beta=4.0$ GeV	82.3%	84.7%	86.3%

To obtain the total decay rate, we need the information on f_1 and g_1 in the whole range of ρ. Since the perturbative analysis is reliable only in the fast recoil region, we extrapolate the PQCD predictions at large ρ to small ρ. Hinted at by [23], we propose the following parametrization for the form factors:

$$f_1(\rho) = \frac{c_f}{\rho^{\alpha_f}}, \quad g_1(\rho) = \frac{c_g}{\rho^{\alpha_g}}, \tag{45}$$

where the constants c_f and c_g and the powers α_f and α_g are determined by the PQCD results at large ρ. The constants c_f and c_g, equal to the values of the form factors at zero recoil $(\rho=1)$, should be close to unity according to heavy quark symmetry. We fit Eq. (45) to the PQCD results in the range with $\rho>1.3$ for $\beta=1.0$, where the perturbative contribution has exceeded 80%. The powers $\alpha_f=5.18$ and $\alpha_g=5.14$, close to $\alpha_f \sim 4.6$ at large ρ from the method of wave function overlap integrals [24], are obtained. These values are larger than 1.8 extracted from the transition form factors associated with the corresponding meson decay $B\to Dl\bar{\nu}$ [3]. This is expected, because perturbative baryon decays involve more hard gluon exchanges.

On the experimental side, there exist only the data of the semileptonic branching ratio $B(\Lambda_b \to Xl\bar{\nu}) \sim 10\%$ [25], where the final-state particles X are dominated by the charm baryons. The data of the B meson semileptonic decays show $B(B\to D^* l\bar{\nu}) \sim 3B(B\to Dl\bar{\nu})$, indicating that each of the three polarization states of the D^* meson contributes the same amount of branching ratio as the D meson does. It is possible that this observation applies to dominant modes in the $\Lambda_b \to Xl\bar{\nu}$ decays with the excited charm baryons $\Lambda_c(2593)$ of spin $J=1/2$ and $\Lambda_c(2625)$ of $J=3/2$. That is, the branching ratio $B(\Lambda_b\to\Lambda_c l\bar{\nu})$ is about 1/4 of $B(\Lambda_b \to Xl\bar{\nu})$, i.e., about 2–3 %. This estimation is consistent with the experimental upper bound of the branching ratio from the data $B(\Lambda_b\to\Lambda_c l\bar{\nu}+X)=(8.27\pm3.38)\%$ [25].

We substitute Eq. (45) for the form factors f_1 and g_1 into the decay rate Γ in Eq. (40), and adjust the normalization

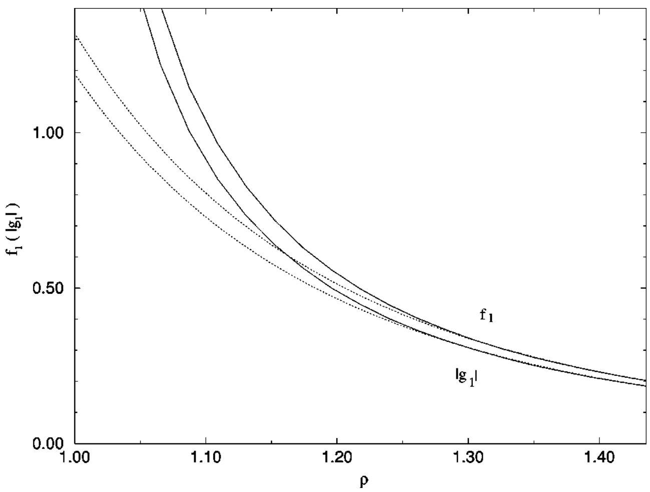

FIG. 2. Dependence of f_1 and $|g_1|$ on ρ for $\beta=1.0$ and $m_l=0.3$ obtained from PQCD (solid lines) and from the extrapolation in Eq. (46) (dashed lines). The upper (lower) set of curves represents the form factor f_1 ($|g_1|$).

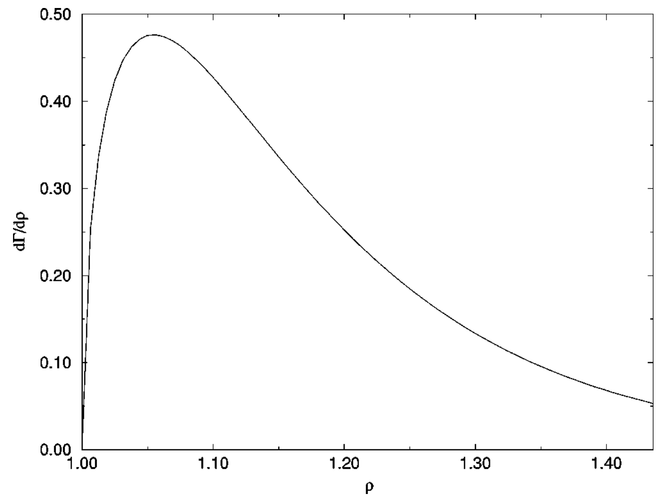

FIG. 3. Dependence of $d\Gamma/d\rho$ on ρ obtained from Eq. (46) in units of 10^{-13} GeV.

constant f_{Λ_b} such that our predictions for the branching ratio are located in the range of 2–3 %. The Λ_c baryon normalization constant f_{Λ_c} changes according to Eq. (44). We adopt the Λ_b baryon lifetime $\tau=(1.24\pm0.08)\times10^{-12}$ s [25]. The value of f_{Λ_b} determines the parameters c_f and c_g. It is then found that $f_{\Lambda_b}=2.71\times10^{-3}$ GeV^2, corresponding to

$$f_1(\rho)=\frac{1.32}{\rho^{5.18}}, \quad g_1(\rho)=\frac{-1.19}{\rho^{5.14}}, \tag{46}$$

gives a branching ratio 2%, and $f_{\Lambda_b}=3.0\times10^{-3}$ GeV^2, corresponding to

$$f_1(\rho)=\frac{1.62}{\rho^{5.18}}, \quad g_1(\rho)=\frac{-1.46}{\rho^{5.14}}, \tag{47}$$

gives the branching ratio 3%. Since the values of the form factors at zero recoil should be close to unity as stated above, we prefer Eq. (46) with $f_1(1)=1.32$ and $g_1(1)=-1.19$, which are also consistent with the conclusion in [24]. The corresponding normalization constant $f_{\Lambda_b}=2.71\times10^{-3}$ GeV^2, of the same order as $f_P=(5.2\pm0.3)\times10^{-3}$ GeV^2 for the proton [26], is reasonable. The PQCD predictions and the corresponding extrapolations are displayed in Fig. 2, which deviate from each other at small ρ. Applying the PQCD formalism to the zero recoil region, we shall obtain divergent form factors as shown in Fig. 2, which imply the failure of PQCD. Note that our results of the form factors exhibit slopes larger than the dipole behavior assumed in [23].

We then examine the sensitivity of our predictions for the branching ratio $B(\Lambda_b\to\Lambda_c l\bar{\nu})$ to the variation of the parameter β. Choosing $\beta=2.0$ GeV and $\beta=4.0$ GeV, and normalizing the corresponding form factors in the way that they have similar values to those for $\beta=1.0$ GeV in Eq. (46), we obtain the form factors

$$f_1(\rho)=\frac{1.34}{\rho^{5.04}}, \quad g_1(\rho)=\frac{-1.17}{\rho^{4.92}} \tag{48}$$

and

$$f_1(\rho)=\frac{1.34}{\rho^{4.94}}, \quad g_1(\rho)=\frac{-1.18}{\rho^{4.79}}, \tag{49}$$

respectively. Equations (48) and (49) lead to increases of the branching ratio by 4% and 8%, respectively. That is, our predictions for the branching ratio are not sensitive to the choice of baryon wave functions. This observation is attributed to the fact that the PQCD results of the transition form factors at large recoil are insensitive to the variation of baryon wave functions.

We present in Fig. 3 the differential decay rate $d\Gamma/d\rho$ derived from the form factors in Eq. (46), which can be compared with experimental data in the future. The Λ_b and Λ_c baryon wave functions determined in this work are given by

$$\begin{aligned}\phi(\zeta,\eta)=&6.67\times10^{12}\eta^2\zeta(1-\eta)(1-\zeta)\\&\times\exp\left[-\frac{M_b^2}{2(1.0\ \mathrm{GeV})^2(1-\eta)}\right.\\&\left.-\frac{m_l^2}{2(1.0\ \mathrm{GeV})^2\eta\zeta(1-\zeta)}\right],\end{aligned} \tag{50}$$

$$\pi(\zeta,\eta)=6.94\times 10^4\,\eta^2\zeta(1-\eta)(1-\zeta)\times\exp\left[-\frac{M_c^2}{2(1.0\ \text{GeV})^2(1-\eta)}-\frac{m_l^2}{2(1.0\ \text{GeV})^2\eta\zeta(1-\zeta)}\right]. \qquad (51)$$

At last, we compare our predictions with those derived from other approaches in the literature. The $\Lambda_b \to \Lambda_c$ transition form factors have been evaluated by means of overlap integrals of infinite-momentum-frame (IMF) wave functions, nonrelativistic and relativistic quark models, and QCD sum rules. For a review, refer to [27]. Basically, they are nonperturbative methods without involving hard gluons. QCD dynamics is completely parametrized into IMF wave functions in the overlap-integral approach [24,28] and into baryon–three-quark vertex form factors in the relativistic quark model [29]. Information on the above bound-state quantities can be obtained by solving Bethe-Salpeter equations [30]. Most of the analyses, including QCD sum rules [22,31,32], led to branching ratios about or below 6%. The prediction $B(\Lambda_b \to \Lambda_c l \bar{\nu}) \sim 9\%$ in [28] is a bit higher compared to the data of $B(\Lambda_b \to \Lambda_c l \bar{\nu} + X)$. Our result is close to $(3.4 \pm 0.6)\%$ derived in [31].

V. CONCLUSION

In this paper we have developed a PQCD factorization theorem for the semileptonic heavy baryon decay $\Lambda_b \to \Lambda_c l \bar{\nu}$, whose form factors are expressed as the convolutions of hard b quark decay amplitudes with universal Λ_b and Λ_c baryon wave functions. It is observed that the PQCD formalism with Sudakov suppression in the long-distance region is applicable to $\Lambda_b \to \Lambda_c$ decays for the velocity transfer greater than 1.2. This observation indicates that PQCD is an appropriate approach to analyses of two-body exclusive nonleptonic Λ_b baryon decays. Requiring that the normalizations of the form factors at zero recoil be consistent with heavy quark symmetry, we have predicted the branching ratio $B(\Lambda_b \to \Lambda_c l \bar{\nu}) \sim 2\%$. We have also determined the Λ_b and Λ_c baryon wave functions shown in Eqs. (50) and (51), respectively. These wave functions, because of their universality, will be employed to study nonleptonic Λ_b baryon decays in the future.

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