Testing quark mass matrices with right-handed mixings

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In the standard model, several forms of quark mass matrices which correspond to the choice of weak bases lead to the same left-handed mixings $V_L = V_{CKM}$, while the right-handed mixings V_R are not observable quantities. Instead, in a left-right extension of the standard model, such forms are *Ansätze* and give different right-handed mixings which are now observable quantities. We partially select the reliable forms of quark mass matrices by means of constraints on right-handed mixings in some left-right models, in particular on V^R_{cb}. Hermitian matrices are easily excluded.

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In the framework of the standard model (SM), based on the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$, the right-handed mixings are not observable quantities, but they become observable in extensions of the SM such as the left-right model (LRM) $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [1], the Pati-Salam model $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$ [2], and the grand unified model $SO(10)$ [3]. Right-handed mixings are the most direct tool to test models of quark mass matrices. Let us explain how this may happen. In the LRM the quark mass and charged current terms are [4]

$$\overline{u}_L M_u u_R + \overline{d}_L M_d d_R + g_L \overline{u}_L d_L W_L + g_R \overline{u}_R d_R W_R . \tag{1}$$

Diagonalization of M_u, M_d by means of the biunitary transformations

$$U_u^\dagger M_u V_u = D_u , \quad U_d^\dagger M_d V_d = D_d$$

gives (renaming the quark fields)

$$\overline{u}_L D_u u_R + \overline{d}_L D_d d_R + g_L \overline{u}_L V_L d_L W_L + g_R \overline{u}_R V_R d_R W_R , \tag{2}$$

where

$$V_L = U_u^\dagger U_d = V_{CKM} , \quad V_R = V_u^\dagger V_d$$

are the left- and right-handed mixing matrices of quarks, and D_u, D_d have non-negative matrix elements. In the SM the last term in Eqs. (1),(2) is absent and it is possible to perform, without physical consequences, that is, without changing the observable quantities appearing in Eq. (2), the following unitary transformations on the quark fields:

$$u_L \to \mathcal{U} u_L , \quad d_L \to \mathcal{U} d_L , \tag{3}$$

$$u_R \to \mathcal{V}_u u_R , \quad d_R \to \mathcal{V}_d d_R . \tag{4}$$

In the LRM Eq. (4) must be replaced by

$$u_R \to \mathcal{V} u_R , \quad d_R \to \mathcal{V} d_R , \tag{5}$$

that is, u_R and d_R must also transform in the same way because of the last term in Eqs. (1),(2). From the point of view of quark mass matrices, the consequences of replacing Eq. (4) with Eq. (5), keeping Eq. (3), are the following. In the SM we can use the freedom in $\mathcal{U}$ and $\mathcal{V}_u$ to choose $M_u = D_u$. Further we can use the freedom in $\mathcal{V}_d$ to choose M_d to be Hermitian or to have three zeros [5–10]. In the LRM, the second freedom is not there because both the diagonalizing matrices of M_d are physical observables. This fact means that *bases in the SM become Ansätze in the LRM*, giving the same V_L but different V_R.

The aim of this paper is to begin a selection of quark mass matrices in the LRM by using informations on right-handed mixings. In fact, if without loss of generality one sets $M_u = D_u$, then $V_L^\dagger M_d V_R = D_d$, and thus

$$V_R^\dagger = D_d^{-1} V_L^\dagger M_d \tag{6}$$

permits to calculate the right-handed mixing matrix V_R (values of quark masses at the scale M_Z and of the mixing V_L are extracted from Refs. [11] and [12]). It is well known that if M_d is Hermitian or symmetric then $|V_R| = |V_L|$. These conditions correspond to manifest and pseudomanifest left-right symmetry, respectively [4]. In the general case, however, V_R is not related to V_L [4]. Notice that different quark mass and mixing matrices, which correspond to bases in the SM, are connected by a suitable unitary U_R, because

$$V_L^\dagger M_d V_R = V_L^\dagger M_d U_R U_R^\dagger V_R = V_L^\dagger M_d' V_R' , \tag{7}$$

where $M_d' = M_d U_R$ and $V_R' = U_R^\dagger V_R$. Therefore, they give different right-handed mixings in the LRM. For example, let us consider the simple case of the first two generations with real mass matrices (in the LRM mass matrices are complex in general, even for only two generations). The left-handed mixings are given by

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$$V_L \simeq \begin{pmatrix} 1 & \lambda \\ -\lambda & 1 \end{pmatrix}, \quad \lambda = 0.22.$$

In the SM, using a right-handed rotation $\mathcal{V}_d$ it is possible to put one zero in M_d in any position. In the LRM such four forms give different V_R. We can get all forms from just one, for example, from [10]

$$M_d \simeq \begin{pmatrix} 0 & \sqrt{m_d m_s} \\ \sqrt{m_d m_s} & m_s \end{pmatrix} \Rightarrow V_R \simeq \begin{pmatrix} -1 & \lambda \\ \lambda & 1 \end{pmatrix},$$

by using in Eq. (7) U_R such as

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} c & s \\ -s & c \end{pmatrix}.$$

In fact

$$M_d U_R = \begin{pmatrix} 0 & \sqrt{m_d m_s} \\ \sqrt{m_d m_s} & m_s \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \sqrt{m_d m_s} & 0 \\ m_s & \sqrt{m_d m_s} \end{pmatrix} = M_d',$$

$$U_R^\dagger V_R = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & \lambda \\ \lambda & 1 \end{pmatrix} = \begin{pmatrix} \lambda & 1 \\ -1 & \lambda \end{pmatrix} = V_R'.$$

The mixing V_{us}^R is small on the first basis and large on the second. Moreover,

$$\begin{pmatrix} 0 & \sqrt{m_d m_s} \\ \sqrt{m_d m_s} & m_s \end{pmatrix} \begin{pmatrix} c & s \\ -s & c \end{pmatrix} = \begin{pmatrix} -\sqrt{m_d m_s} s & \sqrt{m_d m_s} c \\ \sqrt{m_d m_s} c - m_s s & \sqrt{m_d m_s} s + m_s c \end{pmatrix},$$

and imposing the element 2-1 to vanish, we have

$$c = \sqrt{\frac{m_s}{m_s + m_d}} \simeq 1, \quad s = \sqrt{\frac{m_d}{m_s + m_d}} \simeq \lambda,$$

and the third basis

$$\begin{pmatrix} 0 & \sqrt{m_d m_s} \\ \sqrt{m_d m_s} & m_s \end{pmatrix} \begin{pmatrix} \sqrt{\frac{m_s}{m_s + m_d}} & \sqrt{\frac{m_d}{m_s + m_d}} \\ -\sqrt{\frac{m_d}{m_s + m_d}} & \sqrt{\frac{m_s}{m_s + m_d}} \end{pmatrix} \simeq \begin{pmatrix} -m_d & \sqrt{m_d m_s} \\ 0 & m_s \end{pmatrix},$$

$$\begin{pmatrix} 1 & -\lambda \\ \lambda & 1 \end{pmatrix} \begin{pmatrix} -1 & \lambda \\ \lambda & 1 \end{pmatrix} \simeq \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}.$$

And again we can get the fourth basis, from the third, through

$$\begin{pmatrix} -m_d & \sqrt{m_d m_s} \\ 0 & m_s \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \sqrt{m_d m_s} & -m_d \\ m_s & 0 \end{pmatrix},$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

Mixing is nearly zero on the third basis and nearly one on the fourth. In this way, also for more than two generations, one can construct different bases in the SM, which are different *Ansätze* in the LRM, from just a few of them.

Therefore, let us consider now three generations and label elements in M_d as

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}.$$

There are several SM bases with three zeros in M_d [10]. For example, zeros can be put in positions 137 [8] and 236 [7], 478 [9], 124. The last form can be obtained from 137 by just relabeling the family indices 2,3. From bases 124, 137, 478 we can calculate fifty-four bases by means of the six special rotations

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

which produce permutations of columns in M_d and of rows in V_R, and suitable unitary transformations of the type

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha}c & e^{i\beta}s \\ 0 & -e^{i\gamma}s & e^{i\delta}c \end{pmatrix}, \quad \alpha+\gamma=\beta+\delta.$$

For each of the three starting forms under examination we have calculated the matrix M_d by the relation [8]

$$M_d M_d^\dagger = V_L D_d^2 V_L^\dagger \tag{8}$$

and V_R by Eq. (6). For V_L we use the standard parametrization [12]. Moreover, to keep arbitrary representation of V_L one must put three phases (not just one SM observable) in M_d [6]. Their positions for our starting bases are 356, 256, 236, respectively. Putting three phases and their position is part of the *Ansätze* in the LRM, because for three generations of quarks there are seven observable phases; one can be inserted in V_L and six in V_R [13]. However, due to the position of the three zeros, putting the three phases in another position, or more than three phases, up to six, does not change the moduli of V_R. From the three starting bases, by means of the six rotations we get eighteen bases and with the help of the unitary transformation we get another six bases (imposing one element in the second column to vanish) which become thirty-six by using again the six rotations, making a total of fifty-four. These are all the SM bases with $M_u = D_u$ and M_d containing three zeros, out of eighty-four possibilities. We try to understand if some of the fifty-four bases satisfy constraints coming from B decay, $K_L - K_S$ mass difference and $B-\overline{B}$ mixing, within the LRM.

In fact, a recent analysis [14] of right-handed currents in B decay within the LRM suggests that $|V_{cb}^R|$ is large and perhaps near unity. In such analysis $M_{W_R} \gtrsim 720$ GeV [15] is supposed. Actually, this experimental bound is obtained by manifest left-right symmetry. From inclusive semileptonic decays of B mesons Ref. [14] gives $|V_{cb}^R| \gtrsim 0.782$. Moreover, if, as suggested in Ref. [16], right-handed currents can help to solve the B semileptonic branching fraction and charm counting problems, then Ref. [14] gives $|V_{cb}^R| \geqslant 0.908$ and $M_{W_R} \lesssim 1600$ GeV. For our purposes we assume $|V_{cb}^R| > 0.750$. We have used this constraint to select quark mass matrices, taking $\delta = 75°$ in V_L. This value is well inside the experimentally favored region [12]. A moderate variation of δ, say from 60° to 90°, does not have a relevant effect. Of course, the Hermitian form of M_d (and in general of both mass matrices) is excluded because it yields $|V_{cb}^R| = |V_{cb}^L| \simeq \lambda^2$. The successful *Ansätze* are in Table I, where the positions of phases can be changed by a diagonal phase matrix U_R. It should be noted that *Ansätze* 149, 259, 167, 268, 347, 358 give the quite particular exact result that some element in $M_d^\dagger M_d = V_R D_d^2 V_R^\dagger$ is zero. As an example of a successful *Ansätz* we report $|M_d|$ and $|V_R|$ for model 124:

TABLE I. Positions of zeros and phases in M_d and corresponding prediction for $|V_{cb}^R|$.

zeros	124	236	146	256	149	259	127	128
phases	356	145	589	479	568	467	356	346
$\lvert V_{cb}^R\rvert$	0.896	0.896	0.789	0.789	0.984	0.984	0.914	0.914
zeros	167	268	479	589	347	358	467	568
phases	589	479	235	134	256	146	235	134
$\lvert V_{cb}^R\rvert$	0.785	0.785	0.999	0.999	0.875	0.875	0.871	0.871

$$|M_d| = \begin{pmatrix} 0 & 0 & 0.023 \\ 0 & 0.106 & 0.104 \\ 0.541 & 2.687 & 1.213 \end{pmatrix},$$

$$|V_R| = \begin{pmatrix} 0.958 & 0.221 & 0.180 \\ 0.205 & 0.393 & 0.896 \\ 0.199 & 0.892 & 0.405 \end{pmatrix}.$$

The near equality of elements 5 and 6 in $|M_d|$ is due to the specific value $\delta \simeq 75°$ [10]. The matrix $|V_R|$ has an approximate symmetric expression

$$|V_R| \simeq \begin{pmatrix} 1 & \lambda & \lambda \\ \lambda & 2\lambda & 1 \\ \lambda & 1 & 2\lambda \end{pmatrix}.$$

Further constraints on the form of V_R come from $K_L - K_S$ mass difference [17] and $B-\overline{B}$ mixing. Assuming that each row and column of V_R contains only one large element and forbidding fine tuning, these constraints give [14] $|V_{us}^R| \lesssim \lambda^2$, $|V_{ub}^R| \lesssim \lambda$, $|V_{tb}^R| \lesssim \lambda$, when $|V_{ts}^R|$ is large, and $|V_{ud}^R| \lesssim \lambda$, $|V_{ub}^R| \lesssim \lambda$, $|V_{tb}^R| \lesssim \lambda^3$, when $|V_{td}^R|$ is large. Out of the sixteen models in Table I, the 128 satisfies quite well the second set of constraints:

$$|M_d| = \begin{pmatrix} 0 & 0 & 0.023 \\ 0.103 & 0.021 & 0.104 \\ 2.741 & 0 & 1.213 \end{pmatrix},$$

$$|V_R| = \begin{pmatrix} 0.199 & 0.892 & 0.405 \\ 0.088 & 0.395 & 0.914 \\ 0.976 & 0.217 & 0.0003 \end{pmatrix}.$$

The approximate expression for $|V_R|$ is

$$|V_R| \simeq \begin{pmatrix} \lambda & 1 & 2\lambda \\ 2\lambda^2 & 2\lambda & 1 \\ 1 & \lambda & \lambda^5 \end{pmatrix}.$$

Model 124 gives $|V_{us}^R| \simeq \lambda$ rather than $|V_{us}^R| \lesssim \lambda^2$. Also models 479 and 589 are reliable, with very small mixings.

To better explain the physical content of the foregoing calculation we present some comments. We have considered fifty-four forms of quark mass matrices with three zeros and three phases inside M_d and a diagonal M_u. As we said, $|V_R|$ does not change if we put more than three phases. On the other hand, the existence of three zeros in M_d is a strong restriction because in the LRM just one or two zeros settle an *Ansätz*. Nevertheless, we have found a few models that satisfy the constraints from K and B physics. Other *Ansätze* can be obtained starting from a diagonal M_d.

We stress the simple result that, if $|V_{cb}^R|$ is large, Hermitian or symmetric quark mass matrices [18] are not reliable [19]. Nonsymmetric mass matrices have important applications in the leptonic sector, mainly in connection with the large mixing of neutrinos [20].

Using Eqs. (3),(5) it is possible to change the structure of both M_u and M_d. For example, from model 128, multiplying to the right by a simple unitary transformation in the 2-3 sector, it yields M_d with one zero in position 1 and M_u with four zeros in positions 2347. Although such forms can be more interesting to discover an underlying theory of fermion masses and mixings, they lead to the same parameters of Eq. (2), and we need other observable quantities to make a selection of such models with nondiagonal mass matrices. Such new physical parameters exist in extensions of the LRM. Actually, in the SM one can get M_u diagonal and M_d with three zeros; in the LRM M_u can be diagonalized but M_d is fixed. In the Pati-Salam model, due to the relation between quarks and leptons, we cannot choose M_u diagonal in the general case. In $SO(10)$ also u_L and u_R transform in the same way and then it is never possible to choose M_u diagonal. Nonsymmetric mass matrices can be obtained by using also the antisymmetric Higgs representation **120** in the Yukawa couplings with fermions, or if one allows for effective nonrenormalizable couplings of the light generations [21].

In conclusion, for the first time, we have performed, within the LRM, a systematic study of quark mass matrices which have a general structure in the SM. Constraints on right-handed mixings, coming from various experimental and theoretical sources, permit to select three reliable forms, apart from phases.

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