Fermion dipole moments in supersymmetric models with explicitly broken *R* parity

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We present a simple analysis that allows us to extract the leading mass dependence of the dipole moment of matter fermions that might be induced by new physics. We present explicit results for the supersymmetric model with broken R parity as an illustration. We show that the extra contributions to the electric dipole moment of fermions from $R\!\!\!/_p$ interactions can occur only at the two-loop level, contrary to claims in the literature. We further find that unlike the generic leptoquark models, the extra contributions to the dipole moments of the leptons can only be enhanced by m_b/m_l and not by m_t/m_l relative to the expectations in the standard model. An interesting feature about this enhancement of these dipole moments is that it does not involve unknown mixing angles. We then use experimental constraints on the electric dipole moments of e^- and n to obtain bounds on (the imaginary part of) products of $R\!\!\!/_p$ couplings, and show that bounds claimed in the literature are too stringent by many orders of magnitude.

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I. INTRODUCTION

The dipole moments of fermions have played a very important role in testing our understanding about particles and their interactions, $(g-2)_\mu$ being the first and foremost example. Indeed magnetic and electric dipole moments, (EDMs), whether diagonal or transition, have proved to be excellent tools to constrain extra physics beyond the standard model [1,2]. The stringent experimental upper limits on lepton number violating processes such as $\mu \rightarrow e\gamma$ as well as the small value of a possible (Majorana) mass for neutrinos both lead to strong constraints on transition moments of leptons that might be present in extensions of the standard model (SM) that allow for lepton flavor violation. In the SM the CP odd neutron electric dipole moment (EDM) vanishes at two loops [3]. At three loops it has been estimated [4,2] to be $d_n \sim 10^{-32\pm 1}$ e cm. Since there are no purely leptonic sources of CP violation in the SM, an electron dipole moment can only be induced from d_n at second order in G_F and thus may be estimated to be $d_e \sim (G_F m_n^2)^2 d_n \sim 10^{-42}$ e cm, to be compared to the estimate 8×10^{-41} e cm quoted in the literature [5,1]. These expectations are much lower than the current experimental limits [6,7]. An observation of the EDM of either the electron or the neutron in current experiments would clearly be very exciting since it would almost certainly signal new physics. Many extensions of the SM, including the minimal supersymmetric standard model (MSSM) where R parity is assumed to be conserved include additional sources of CP violation so that fermion EDM's are induced at even the one-loop level. Such models are, of course, severely constrained by the experimental bounds on neutron and electron EDM's [8].

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Dipole moment operators flip chirality, and hence have either to be proportional to *some* fermion mass (this may not be the mass of the external fermion), or to a chirality flipping Yukawa-type coupling [9]. The theoretical predictions for the moments of the heavier fermions like the t,b or the τ, are larger than for first generation particles due to the linear dependence on m_f [10]. In models with leptoquarks, particularly large enhancements of the predicted values of the τ moments by a factor of m_t/m_τ, are possible [11]. Hence, measurements of the dipole moments of the τ and the t, at current [12] or future e^+e^- colliders [13] or $\gamma\gamma$ colliders [14] form a potentially interesting probe of nonstandard physics. In this note, we set up a method of analysis, which would allow us to extract the leading fermion mass dependence of the coefficient of the induced dipole moment in any theory. We illustrate our method using the example of $R\!\!\!/_p$ supersymmetric (SUSY) interactions [15].

Even assuming the MSSM field content, the most general renormalizable, gauge-invariant superpotential allows terms which do not conserve R parity. These terms also violate lepton number (L) and/or baryon number (B) conservation. Phenomenologically, many of these $R\!\!\!/_p$ couplings have been constrained very tightly using a large number of low-energy and collider measurements [16]. While R-parity violation does not appear to be required by any theoretical or phenomenological considerations, it is not excluded either. One of the theoretical challenges then is to understand why some of the $R\!\!\!/_p$ couplings in the superpotential are as small as they are (of course, we have no understanding of why the electron Yukawa coupling is as small as it is either), and especially, why products of B and L violating couplings are so tiny. Many authors have examined phenomenological implications of R-parity violating models which differ in many ways from MSSM expectations.

The $R\!\!\!/_p$ part of the supersymmetric Lagrangian (assuming the MSSM field content) is given by

$$\mathcal{L}_{R\!\!\!/_p} = [\lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \lambda''_{ijk} U_i^c D_j^c D_k^c + \kappa_i L_i H_2]_F + \text{H.c.}, \tag{1}$$

where L_i, Q_i, are the left-handed lepton and quark SU(2) doublet superfields corresponding to the three generations and E_i^c, D_i^c, U_i^c are the SU(2) singlet lepton and quark superfields, with $i, j, \ldots$ being the generation index, while H_2 is the Higgs superfield with $Y=1$. The first, second, and the fourth terms violate lepton number conservation, while the third one violates conservation of *B*. The λ_{ijk} and λ''_{ijk} are antisymmetric in the indices i,j and j,k, respectively. The stability of the proton requires that both *B* and *L* violating operators not be simultaneously present.

Since these $R\!\!\!/_p$ interactions violate *L* and/or *B*, their contributions to masses and magnetic moments of the neutrinos, as well as the flavor changing off diagonal moment $\mu \to e\gamma$ have been considered in literature. The dipole moment (direct or transition) of fermions can be obtained by considering the matrix element of the electromagnetic or the neutral weak current between on-shell fermions, as the momentum transfer q between these vanishes. The (electromagnetic and weak) magnetic and electric dipole moments are then given by the values of the tensor form factors $F_T^{\mathcal{V}}$ and $F_T'^{\mathcal{V}}$ (at zero momentum transfer) that can be read off as coefficients of the terms

$$\overline{u}_{f_1}(p-q)\sigma_{\mu\nu}q^\nu(F_T^{\mathcal{V}}+\gamma_5 F_T'^{\mathcal{V}})u_{f_2}(p), \ (\mathcal{V}=\gamma, Z)$$

in this matrix element. Of course, in any renormalizable theory such as *R*-parity violating SUSY, these coefficients which are only induced at the loop level must be finite. It has been claimed [17] that *current* bounds on the EDM of the electron can be translated to very stringent bounds, e.g., $|\lambda'_{133}|^2 < 4\times 10^{-10}$ if the additional *CP*-violating phases that are intrinsically present in these models are large. This is traced to enhancement factors $\sim m_t/m_e \sim 10^5$ that are claimed to be present.

In this note, we develop a method that enables us to extract the dependence of the coefficients $F_T^{\mathcal{V}}$ and $F_T'^{\mathcal{V}}$ on various fermion masses in any extension of the SM. We then apply this to the MSSM and to $R\!\!\!/_p$ SUSY. In the first part of the analysis we consider only the trilinear $R\!\!\!/_p$ terms and come back to the bilinears at the end. While our analysis of the mass dependence of the induced quark and lepton dipole moments yields answers consistent with many previous explicit calculations, we find that the claimed enhancement of the electron EDM by the factor m_t/m_e does not occur, and conclude that the bounds on $R\!\!\!/_p$ couplings coming from the EDM and frequently quoted [17–19] in the literature are too stringent.

II. SELECTION RULES FOR NONZERO DIPOLE MOMENTS

We begin by defining five different global charges Q_{l_L}, Q_{e_R}, Q_{q_L}, Q_{d_R}, and Q_{u_R} corresponding to five different U(1) transformations. Only the members of the superfield indicated by the subscript have nonzero value of the particular charge, e.g.,

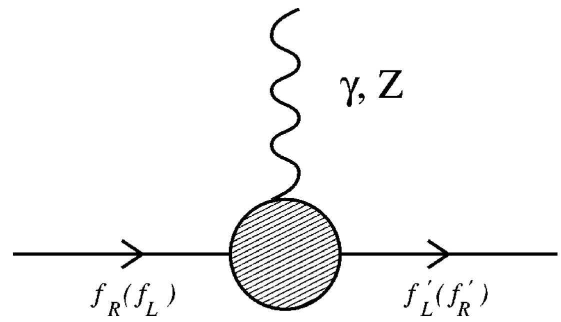

FIG. 1. Generic diagram which will contribute to the dipole moment.

$$\begin{aligned} Q_{l_L} &= 1 \text{ for } e_{iL}, \nu_i, \tilde{e}_{iL}, \tilde{\nu}_i \quad (i=1-3) \\ &= 0 \text{ for all the other particles/sparticles.} \end{aligned} \tag{2}$$

Notice that *all* left-handed lepton fields and their superpartners have the same charge, regardless of generation. The other charges are similarly defined. The value of all the charges for all gauge and Higgs bosons and their SUSY partners is zero.

We see that gauge (and gaugino) interactions conserve these charges, while superpotential Yukawa interactions and the soft SUSY breaking *A* terms as well as the $R\!\!\!/_p$ terms in the Lagrangian, do not. These charges are a kind of ''superchirality'' in that they are nonzero even for spin zero sfermions, as they must be in order to be compatible with supersymmetry. They differentiate fermions of different chirality, and also right-handed quarks of different electrical charge. They do not, however, differentiate between flavors of leptons or quarks with the same chirality, and so are conserved by intergenerational quark mixing.

The induced dipole moment operator will have to flip the chirality of the fermion involved as symbolically shown in Fig. 1. In the SM, Yukawa interactions (we regard fermion masses as Yukawa interactions) are the only source of chirality flip. Within the MSSM, this is still the case if we include the interactions of higgsinos as well as *A* terms (proportional to the same Yukawa coupling) in our definition of Yukawa interactions.[1] $R\!\!\!/_p$ interactions are yet another source of matter fermion chirality flip.

The charge assignments that we introduced in Eq. (2) above provide a systematic way of analyzing when a dipole moment can be induced. For instance, a leptonic moment requires Q_{l_L} and Q_{e_R} to change by one unit in equal and opposite directions, with no change in the other charges. Likewise, a moment for the $u(d)$ quark requires a corresponding change in Q_{q_L} and Q_{u_R} (Q_{d_R} and Q_{q_L}). Since the change induced in each of these charges by any (chirality-flipping) interaction is known (see Table I), it is straightforward to derive relations between the number of vertices of

[1]Gaugino masses can flip the chirality of gauginos, but in order for the chirality flipped gaugino component to couple, one also needs $\tilde{f}_L - \tilde{f}_R$ mixing which has its origin in Yukawa interactions.

TABLE I. The change in the charges Q_{l_L}, Q_{e_R}, Q_{q_L}, Q_{u_R}, and Q_{d_R} defined in the text for different interactions that might be present in SUSY models with MSSM field content. Gauge and gaugino interactions or Higgs boson and higgsino self-interactions do not change any of these charges. $\mathcal{H}^0$ indicates any of the neutral Higgs bosons in the MSSM.

Interaction	ΔQ_{l_L}	ΔQ_{e_R}	ΔQ_{q_L}	ΔQ_{u_R}	ΔQ_{d_R}
Lepton Yukawa interactions	-1	$+1$	0	0	0
Up quark Yukawa Interactions	0	0	-1	$+1$	0
Down quark Yukawa Interactions	0	0	-1	0	$+1$
$\mathcal{H}^0 H^- \tilde{d}_R^* \tilde{u}_R$, $H^- \tilde{d}_R^* \tilde{u}_R$	0	0	0	-1	$+1$
$\lambda_{ijk} L_i L_j E_k^c$ interactions	-2	$+1$	0	0	0
$\lambda'_{ijk} L_i Q_j D_k^c$ interactions	-1	0	-1	0	1
$\lambda''_{ijk} U_i^c D_j^c D_k^c$ interactions	0	0	0	1	2

various types of chirality flipping interactions in order that these collectively induce a dipole moment for any particular matter fermion.[2]

The changes in these charges for each of the vertices in the $R\!\!\!/_p$ SUSY model with MSSM field content are shown in Table I. Of course, the Hermitian conjugate of any interaction would lead to exactly the opposite change in the charges. It should be clarified that by Yukawa interactions, we mean all interactions with the corresponding Yukawa coupling (with one exception listed in the fourth row of Table I whose origin is clarified below): for instance, the lepton Yukawa interaction would include the (charged and neutral) Higgs couplings to leptons, the lepton mass term, Higgs slepton/sneutrino couplings from the superpotential, as well as the corresponding A terms (assumed to be proportional to the lepton Yukawa coupling) and left-right slepton mixing terms, and likewise for the up (down) type Yukawa interactions. In addition to the Higgs sfermion couplings discussed above, there are trilinear Higgs-sfermion interactions from both D terms as well as F terms of the type

$$1 \; : \; \mathcal{H}^0(\tilde{u}_L^* \tilde{u}_L + \tilde{u}_R^* \tilde{u}_R),$$

$$2 \; : \; \mathcal{H}^0(\tilde{d}_L^* \tilde{d}_L + \tilde{d}_R^* \tilde{d}_R),$$

$$3 \; : \; H^- \tilde{d}_L^* \tilde{u}_L,$$

$$4 \; : \; H^- \tilde{d}_R^* \tilde{u}_R, \tag{3}$$

where $\mathcal{H}^0$ indicates any of the neutral Higgs bosons in the MSSM. Out of these, the first three are just like gauge interactions, as far as our charges defined in Eq. (2) are concerned and hence will not affect the selection rules we will derive below. However, the fourth one, which arises from the superpotential and is $\propto (m_u m_d / m_W) \tilde{d}_R^* \tilde{u}_R$, violates the charges we have defined in Eq. (2) but changes them in a way different than the Yukawa interactions. This happens, even though the term arises from the superpotential, because it comes from a cross term between a down-type Yukawa interaction and the (Hermitian conjugate) of the up-type Yukawa interaction. Note that this kind of vertex can exist only for the squarks and not for the sleptons as there is no $\tilde{\nu}_R$ in the MSSM. There are also quartic scalar couplings between a pair of Higgs bosons and $\tilde{f}_R^* \tilde{f}_R / \tilde{f}_L^* \tilde{f}_L$ pairs. Of these, only the term $\mathcal{H}^0 H^- \tilde{d}_R^* \tilde{u}_R$, causes a nonzero charge change which is different from those caused by the Yukawa interactions. The changes in the charges for these interactions are given in the fourth row of Table I, and are as expected, the difference of the changes for down- and up-type Yukawa interactions.

We are now ready to compute the change in each of these charges for any graph. Let us denote by P, S, and R the number of down-quark, up-quark, and lepton Yukawa interactions (in the generalized sense explained above), and by P^*, S^*, R^* the number of insertions corresponding to the Hermitian conjugate (H.c.) of these interactions. Similarly, let N, M, L denote the number of vertices corresponding to interactions proportional to $\lambda, \lambda', \lambda''$ of Eq. (1), respectively, again with N^*, M^*, and L^* indicating the number of vertices corresponding to the H.c. of these interactions. Finally, let $T(T^*)$ denote the number of trilinear or quartic scalar vertices corresponding to the interactions in the fourth row of Table I.

It is easy to see from Table I that the net change in various charges, is given by

$$\Delta Q_{l_L} = -2\Delta N - \Delta M - \Delta R,$$

$$\Delta Q_{e_R} = \Delta N + \Delta R,$$

$$\Delta Q_{q_L} = -\Delta M - \Delta P - \Delta S,$$

$$\Delta Q_{d_R} = 2\Delta L + \Delta P + \Delta M + \Delta T,$$

$$\Delta Q_{u_R} = \Delta L + \Delta S - \Delta T, \tag{4}$$

[2]These would only be necessary conditions since, without further study, it cannot be guaranteed that the answer would not vanish.

where ΔM is given by $\Delta M = M - M^*$, etc. Now we can solve this general system of equations for the special cases of the moments of the leptons as well as the up/down quarks.

Leptonic moments. Let us now consider the case where f, f' in Fig. 1 are leptons. In this case we must have

$$\Delta Q_{l_L} = -1, \quad \Delta Q_{e_R} = 1, \tag{5}$$

or vice versa and all the other remaining charges should remain unchanged, i.e.,

$$\Delta Q_{q_L} = 0, \quad \Delta Q_{u_R} = 0, \quad \Delta Q_{d_R} = 0. \tag{6}$$

Note that our analysis does not distinguish between direct and transition moments. In this case using Eqs. (4) we get

$$\begin{aligned} \Delta N &= 1 - \Delta R, \\ \Delta M &= \Delta R - 1, \\ \Delta P &= 1 - \Delta R - \Delta T, \\ \Delta L &= 0, \quad \Delta S = \Delta T. \end{aligned} \tag{7}$$

It is clear that any dipole moment $\mathcal{D}_l$ that this diagram can give rise to will be

$$\mathcal{D}_l \propto m_{l_i}^{R+R^*} m_{d_j}^{P+P^*} m_{u_k}^{S+S^*} (m_{u_l} m_{d_l})^{T+T^*}$$

with an appropriate numbers of the large masses (at least M_W or M_{SUSY} depending on the graph) coming from the loops in the denominator to give the right dimension. Here, m_{l_i}, m_{u_k}, and m_{d_j} denote *some* lepton, up-type quark, and down-type quark mass. We first see that if there are no R-parity violating interactions (so that $\Delta L = \Delta M = \Delta N = 0$), $\Delta R = 1$, so that at least R or R^* must be nonzero. In the MSSM (or the SM) the leptonic dipole moment must, therefore, be proportional to some lepton mass.[3] Since there are no sources of lepton flavor violation, this must be the external lepton mass. In $\not{R}_p$ models, the third of Eqs. (7) implies that no moment is possible without at least one Yukawa interaction insertion corresponding to a lepton or a down-type quark. Because of the lepton number violation inherent in the $\not{R}_p$ interactions, the fermion can be a b quark, and hence in principle, an enhancement of the loop contribution to the moments by a factor of m_b/m_l (relative to the case with the SM/MSSM) is possible. The last of these equations tells us further that up-type quark masses enter only as even powers so that these can never be the sole source of the required chirality flip for a lepton dipole moment. Indeed these masses have to be *in addition to* the lepton or down-type mass as mentioned above, and so will necessarily be accompanied by the same power of some high mass in the denominator, and so will actually suppress the moment. Clearly the claims [17–19] that the EDM of the electron may be enhanced by factors of m_t/m_e in $\not{R}_p$ models do not seem tenable.[4]

We mention in passing that the conditions of Eqs. (7) that we have derived have also to be satisfied by the diagrams that lead to the Majorana mass [20–22] or dipole moments of the neutrinos [23], as well as $\mu \to e\gamma$ in $\not{R}_p$ theories [24].

Down-type quark moments. For the case of the down-quark moments we have to solve the system of equations

$$\Delta Q_{q_L} = -1, \quad \Delta Q_{d_R} = 1, \tag{8}$$

with all the other charges remaining unchanged, i.e.,

$$\Delta Q_{l_L} = 0, \quad \Delta Q_{u_R} = 0, \quad \Delta Q_{e_R} = 0. \tag{9}$$

From Eq. (4) we find

$$\begin{aligned} \Delta M &= 1 - \Delta P - \Delta T, \\ \Delta N &= \Delta P - 1 + \Delta T, \\ \Delta R &= 1 - \Delta P - \Delta T, \\ \Delta L &= 0, \quad \Delta S = \Delta T. \end{aligned} \tag{10}$$

Once again we first see that if there are no $\not{R}_p$ interactions, the dipole moment would vanish in the absence of all down-type Yukawa couplings. Again the $\not{R}_p$ contributions to the dipole moments of down-type quarks are nonzero only if either ΔR or ΔP are nonzero, and thus are proportional either to a lepton mass or a down-type quark mass. Thus for a d quark, for example, enhancements of m_b/m_d are possible. Any dependence on m_t will come only in even powers and suppressed by heavier masses in the denominator compared to this leading-order mass dependence, and possibly also by small Kobayashi-Maskawa (KM) matrix elements. Thus no big enhancements of the $\not{R}_p$ contributions to the dipole moment due to the large top quark mass are possible.

Up-type quark moments. For this case, we need

$$\Delta Q_{q_L} = -1, \quad \Delta Q_{u_R} = 1, \tag{11}$$

with the other charges remaining unchanged, i.e.,

$$\Delta Q_{l_L} = 0, \quad \Delta Q_{d_R} = 0, \quad \Delta Q_{e_R} = 0. \tag{12}$$

Using Eqs. (4) we get

$$\begin{aligned} \Delta R &= -\Delta N, \\ \Delta P &= \Delta N - \Delta T, \\ \Delta M &= -\Delta N, \\ \Delta L &= 0, \quad \Delta S = 1 + \Delta T. \end{aligned} \tag{13}$$

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[3]This assumes that the leptonic A terms are proportional to the lepton Yukawa coupling, which need not be the case.

[4]We may also add here that the analysis in Ref. [17] appears to be based on the computation of one-loop diagrams that do not exist.

In this case a solution without an up-type Yukawa interaction is not allowed as opposed to the earlier two cases where a solution was allowed where a single power of quark (lepton) mass could appear for the lepton (quark) moment. The leading mass dependence of an up-quark moment generated by $\not{R}_p$ interactions is necessarily an up-type mass. This happens because neither the λ or the λ' interactions involve a $\tilde{u}_R$ or u_R.

We also see that for contributions that will involve only the λ'' part of the $\not{R}_p$ interactions, the dipole moment for the down quark will thus be proportional to $m_{d_i}m_{u_i}^{2n}$ as opposed to the up-quark moment which will be proportional to $m_{u_i}m_{d_i}^{2n}$ $(n=0,1,2\ldots)$. This is in agreement with the result for the EDM due to the λ'' couplings that was derived long ago [25].

A few more general comments are in order here. A natural question to ask is why is it possible to get an enhancement of the dipole moments by a factor of m_t/m_f in the case of theories with general leptoquarks [11]. This can be traced back to the fact that in SUSY with $\not{R}_p$ even though the squarks/sleptons do play the role of leptoquarks which have $\not{L}$ or $\not{B}$ interactions, their couplings are chiral as a result of the supersymmetry, which allowed us the charge assignment made in Eqs. (2) in the first place. The chiral nature of the couplings, therefore, forbids the enhancement of dipole moments of the leptons and down-type quarks as compared to the expectations in SM/MSSM, by a factor of m_t/m_l or m_t/m_d.

III. NUMERICAL ESTIMATES OF ELECTRIC DIPOLE MOMENTS OF THE ELECTRON AND NEUTRON

We have discussed necessary conditions for any diagram in the MSSM or $\not{R}_p$ framework to contribute to a lepton or quark dipole moment. Of course, in order to conclude that the induced moment is an electric dipole moment (weak or electromagnetic), one has to further check that it has a CP-violating piece. With the usual convention for phases of the fields, if the amplitude in Fig. 1 is complex we would ensure that the induced EDM is nonzero. We will see below that even for arbitrary phases in the $\not{R}_p$ couplings, the dominant contributions to the EDM from $\not{R}_p$ couplings come only at the two-loop level.

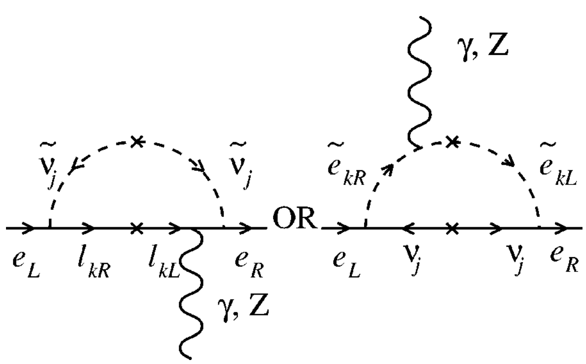

FIG. 2. One-loop diagrams contributing to the EDM of electron in the presence of Majorana masses for the neutrinos (sneutrinos). Note that here and in all the other diagrams, the γ/Z is to be attached to all possible lines to which it couples.

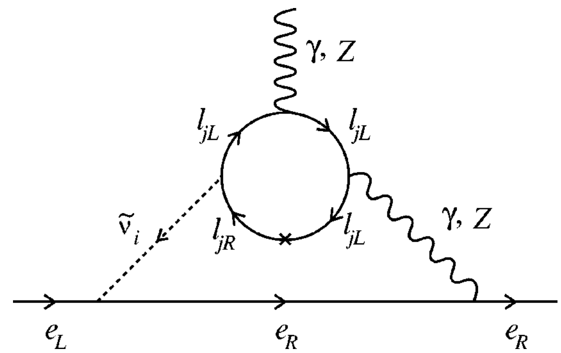

FIG. 3. An example of the leading two-loop contribution to the EDM of electron due to λ couplings.

Let us start with the case of a lepton. The lowest-order diagrams involving $\not{R}_p$ couplings will need $N=N^*=1$ or $M=M^*=1$. It is then easy to check that within our framework, each of these diagrams is proportional to some $|\lambda|^2$ $(|\lambda'|^2)$ and so cannot contain a complex piece from the $\not{R}_p$ coupling.

We note in passing that if the model is extended to allow for Majorana masses for neutrinos (and lepton number violating sneutrino masses to preserve supersymmetry), an electron EDM would be possible from diagrams shown in Fig. 2. The "cross" on the neutrino line (or the corresponding sneutrino line) essentially corresponds to this "Majorana" mass insertion for neutrinos (lepton number breaking sneutrino mass insertion), which is not present in the MSSM.[5] The "cross" on the charged lepton line (the charged scalar lepton) corresponds to a mass insertion (the insertion of the $L-R$ slepton mixing). We see that each of these diagrams is proportional to products of λ's (not λ times λ^*) and so can lead to an EDM for the electron. We expect though that the contribution will be extremely small due to the smallness of the neutrino masses. The corresponding contribution from a (SUSY violating) sneutrino mass insertion may be worthy of examination. The same mechanism is clearly not possible with λ'-type couplings as there is no neutral particle in the loop.

Once we go to two loops, it is simple to see that there are many types of diagrams involving $\not{R}_p$ couplings, where the product of the relevant couplings is complex. An example is shown in Fig. 3. This corresponds to the case $N=N^*=1$. According to our general analysis therefore, the contribution to the dipole moment should be proportional to some m_{l_j} which need not be the mass of the external lepton (electron in this case). This amplitude involves two *different* λ couplings and hence is complex in general. Here the source of the complex nature of the amplitude and hence for the EDM, is the irremovable phases of the $\not{R}_p$ couplings. We estimate the order of magnitude of the real part of the EDM as the

[5]The selection rules above would, of course, then have to be modified to allow for these additional couplings.

product of explicit factors of couplings, mass insertions and color factors, a factor of $1/(4\pi^2)$ for each loop, and finally appropriate powers of the ''large mass'' ($m_{\tilde{\nu}}$ in this case) in the denominator to get the appropriate dimension. We then take the EDM to be the imaginary part (Im) of this product.[6] We will, of course, overestimate the answer in the event there are significant cancellations between several diagrams. For the diagram in Fig. 3 we obtain

$$d_e \sim \frac{(e^2,g_Z^2)}{4\pi^2}\frac{1}{4\pi^2}\mathrm{Im}\left[\sum_{ij,i\neq 1,j} m_{l_j}\lambda^*_{ijj}\lambda_{i11}\frac{1}{m^2_{\tilde{\nu}_i}}\right]. \quad (14)$$

As long as $|\lambda_{233}|$ is not unduly small, the dominant contribution will be the one corresponding to $j=3$. Due to the antisymmetry of the λ couplings in the first two indices this piece is then given by

$$d_e \sim \frac{(e^2,g_Z^2)}{4\pi^2}\frac{1}{4\pi^2}\frac{m_\tau}{m^2_{\tilde{\nu}_2}}\mathrm{Im}(\lambda_{211}\lambda^*_{233}), \quad (15)$$

and hence we see that this diagram gives an enhancement of the dipole moment by a factor m_τ/m_e. The interesting feature of this enahancement is that the large mass has appeared without paying any price for the mixing angles.

At the two-loop level one can also get a diagram corresponding to $\Delta M=-1,\Delta N=1,\Delta P=1$ but with $\Delta R=0$, which gives an enhancement of the EDM of the electron by a factor of m_b/m_e as we have already discussed. An example is shown in Fig. 4. Again the corresponding contribution is similar to one in Eq. (14) and is dominated by the $j=3$ term. The dominant piece is estimated by

$$d_e \sim \frac{(e^2,g_Z^2)}{4\pi^2}\frac{1}{4\pi^2}m_b\,\mathrm{Im}\left[\sum_{i\neq 1} 3\lambda'^*_{i33}\lambda_{i11}\frac{1}{m^2_{\tilde{\nu}_i}}\right], \quad (16)$$

where a color factor of 3 has been inserted. It should be noted here that in both the cases there are additional diagrams where the $\tilde{\nu}$ and the neutral gauge boson lines are crossed, as well as where the $\tilde{\nu}_L$ is replaced by $\tilde{e}_L$ and γ/Z replaced by W. They would give contributions similar to the one given above, except that the $m^2_{\tilde{\nu}_i}$ will be replaced by $m^2_{\tilde{e}_{iL}}$. There exist a large number of two-loop contributions involving the Higgs exchanges, but in all the cases the resulting contributions are proportional to m_e or even higher powers in agreement with the expectations from our general rules.

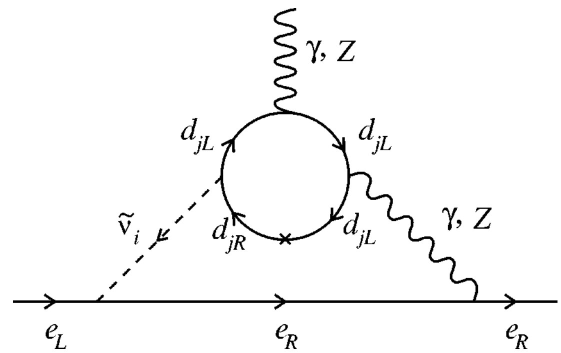

FIG. 4. An example of the leading two-loop contribution to the electron EDM due to λ' and λ couplings.

We can use these estimates to constrain products of λ couplings or those of λ and λ' couplings using the current experimental limits on the EDM of the electron. The potentially largest contributions, and hence the best limit will come from the contributions of the diagrams shown in Figs. 3 and 4. Using Eqs. (15) and (16) and the current bound [6] $d_e < 10^{-27}$ e cm on the EDM of the e, we get

$$\mathrm{Im}(\lambda_{211}\lambda^*_{233}) < 5\times 10^{-4}\left(\frac{\tilde{m}}{1\ \mathrm{TeV}}\right)^2,$$

$$\mathrm{Im}\sum_{i\neq 1}\lambda_{i11}\lambda'^*_{i33} < 0.6\times 10^{-4}\left(\frac{\tilde{m}}{1\ \mathrm{TeV}}\right)^2. \quad (17)$$

Here $\tilde{m}$ stands for the mass of the appropriate SUSY scalar. The improvement in the second case is simply the factor of m_b/m_τ and the color factor. We stress that our estimates are crude: we have clearly not made a complete computation of any amplitude, and also not included contributions from other diagrams. We add though that for some cases where explicit computations [26] are available in the literature, we did check that our crude estimate gives reasonable agreement with the complete calculation.

In the case of the d-quark dipole moment, again there is a counterpart of the diagram in Fig. 3 where λ couplings will be replaced by λ'. The diagram is similar to the ones shown in Figs. 3 and 4. It is obtained by replacing in Fig. 3 e_L, e_R by d_L, d_R and the leptons in the central loop by quarks. In this the dominant contribution is obtained from the b quark in the loop. The λ' couplings have no antisymmetry and the dominant contribution in this case, is estimated by

$$d_d \sim \frac{(e^2,g_Z^2)}{4\pi^2}\frac{1}{4\pi^2}m_b\,\mathrm{Im}\left[\sum_{i} 3\lambda'^*_{i33}\lambda'_{i11}\frac{1}{m^2_{\tilde{\nu}_i}}\right]. \quad (18)$$

---

[6]We tacitly assume that the $R\!\!\!/_p$ couplings that we have written are in the mass eigenstate basis for matter fermions and sfermions. The mass insertions in the figures are merely to show explicit fermion mass factors that would arise from the computation. In other bases, there could be complex contributions from off-diagonal terms in the propogators, and it would be difficult to isolate the imaginary part of the contribution. Since supersymmetry is broken by sfermion mass terms, SUSY would appear to be explicitly broken by some interactions such as trilinear scalar couplings. But this is not relevant to our analysis as we include soft SUSY breaking couplings anyway. The important thing is that our selection rules of the previous section are not affected by this. We stress that it is only for the extraction of the imaginary part, and *not for the derivation of the selection rules* are we forced to go into this basis.

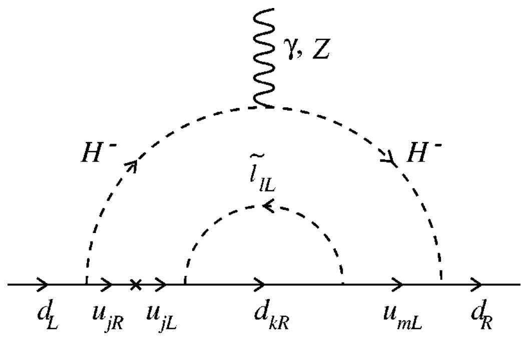

FIG. 5. An example of the Higgs mediated two-loop contribution to the EDM of the down quark due to λ' couplings.

There exist possible two-loop diagrams with charged Higgs exchange and λ' couplings. An example is shown in Fig. 5. Here the dominant contribution is again given by $j=3$ and can be estimated to be

$$d_d \sim \frac{(e^2,g_Z^2)}{4\pi^2}\frac{1}{4\pi^2}\frac{m_t^2 m_d}{m_W^2}\frac{1}{M^2}\mathrm{Im}\left[\sum_{l,k}\lambda'^*_{l3k}\lambda'_{l1k}V^*_{td}V_{ud}\right], \tag{19}$$

where $M=\max(m_{\tilde{e}_L},m_{H^-})$, and V_{ud}, etc. are the elements of the KM matrix. We see that this is proportional to $m_t^2 m_d/m_W^2$. In addition there is suppression due to the small KM mixing angles as well. Hence, the contribution of the diagram shown in Fig. 4 and the related diagrams, proportional to m_b, is still the dominant one in spite of the m_t^2 factor here. This is an illustration of our general statement that the dipole moment for the down-type quark (and also of the lepton) does not receive enhancements due to the large top mass.

The EDM of the d-quark due to the λ'' couplings had been estimated previously [25] and the corresponding diagram is shown in Fig. 6. Here the dominant contribution is again given by $j=m=3$. The antisymmetry of the λ'' couplings ensures that the dominant contribution is then given by

$$d_d \sim \frac{(e^2,g_Z^2)}{4\pi^2}\frac{2}{4\pi^2}\frac{m_t^2 m_b}{m_W^2}\frac{1}{m^2_{\widetilde{bL}}}\mathrm{Im}[V^*_{td}V_{tb}\lambda''_{323}\lambda''^*_{321}]. \tag{20}$$

Due to the relative values of the KM matrix elements V_{td} and V_{cd}, the contribution corresponding to $j=2,m=3$ is down

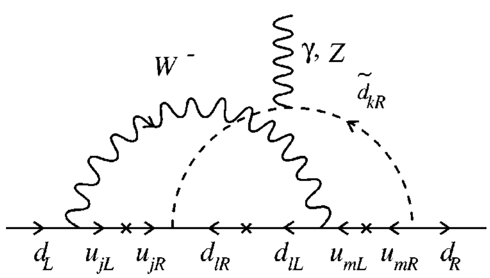

FIG. 6. An example of the leading two-loop contribution to the down-quark EDM due to λ'' couplings.

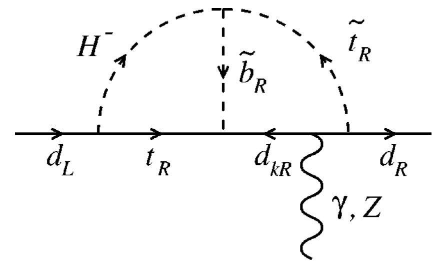

FIG. 7. An example of the two-loop contribution to the EDM of the down quark due to λ'' couplings and supersymmetric interactions of the Higgs.

compared to the one above by about a factor 5, assuming comparable R_p couplings in the two cases. Even this contribution corresponding to $j=3,m=3$, is still small compared to that of Eq. (18) for similar values of the λ' and λ'' couplings, due to the small KM matrix elements. The diagram obtained from Fig. 6, by replacing W^- by H^- also makes a similar contribution. In this case, the crosses corresponding to the mass insertions on the u_j and u_m lines in the diagram are not present as the chirality flip is achieved at the two H^- vertices. This will contain an additional factor of $\cot^2\beta$, where $\tan\beta$ is the ratio of the vacuum expectation values (VEV's) of the $Y=1$ and $Y=-1$, neutral Higgs fields.

At this point it is in order to look at potential contributions to the EDM of d from diagrams involving the scalar couplings in the fourth row of Table I. There are no analogous contributions to lepton moments. An example is shown in Fig. 7. The contribution of this diagram is given by

$$d_d \sim \frac{g^2}{4\pi^2}\frac{2}{4\pi^2}\frac{m_t^2 m_b}{m_W^2}\frac{1}{M^2}\cot\beta\ \mathrm{Im}[V^*_{td}\tilde{U}^R_{tb}\lambda''_{323}\lambda''^*_{321}], \tag{21}$$

with $M=\max(m_{\tilde{q}},m_{H^-})$, and g being the SU(2) gauge coupling. Since the term originates from the superpotential it is not surprising that the the contribution is given by an expression similar to Eq. (20), with V_{tb} replaced by $\tilde{U}^R_{tb}$, the mixing matrix element in the right-squark sector. Note however, the EDM is nonzero even with no intergenerational mixing in the squark sector. It is amusing to note that there is also a contribution to the EDM of the d quark if there is no intergenerational mixing even in the quark sector—simply replace the t quark by a u quark. This contribution is smaller than that in Fig. 7 by a factor $m_u/|V_{td}|m_t$.

The up-quark moment will receive contributions from the R_p interactions from diagrams similar to those shown in Figs. 5 and 6, by simply interchanging u and d and changing $\tan\beta$ by $\cot\beta$. There does not exist a counterpart of the diagrams shown in Figs. 3 and 4, for the u-type quark. The dominant contribution will be given by an expression similar to that of Eq. (20) involving different elements of the KM matrix and again the roles of m_{u_i} and m_{d_i} reversed. Hence the dominant piece is now sum of two terms which are proportional to $m_t m_b^2 V_{ub}$ and $m_t m_b m_s V_{us}$, in accordance with the general results obtained from Eq. (13). Both these terms are comparable to each other in size because of the relative size of different elements of the KM matrix. The contributions due to diagrams involving charged Higgs will involve a further factor of $\tan^2\beta$. Since we have just one factor of m_t, this contribution is suppressed relative to that of Eq. (20) by a factor 40 or so, which in turn is smaller than that of Eq. (18) by about a factor 5. Hence, while estimating the *n* EDM, we will neglect the *u* contribution completely.

Using then the current experimental result $d_n < 6.0 \times 10^{-26}$ e cm, and Eq. (18), we get

$$\mathrm{Im}\left[\sum_k \lambda'_{k11}\lambda'^*_{k33}\right] < 10^{-2}\left(\frac{\tilde{m}}{1\ \mathrm{TeV}}\right)^2. \tag{22}$$

Equation (20) yields a weaker constraint:

$$\mathrm{Im}[\lambda''^*_{312}\lambda''_{332}] < 0.03 \times \frac{0.01}{V_{td}}\left(\frac{\tilde{m}}{1\ \mathrm{TeV}}\right)^2. \tag{23}$$

The contribution in Eq. (21), as well as from other diagrams involving a charged Higgs boson in the loop, leads to a similar bound. In obtaining these we have assumed that $d_n \sim d_d$. In view of the fact that we have not really computed any of the diagrams, but merely estimated the various contributions, it did not seem reasonable to attempt to include the long distance contributions which could only strengthen these bounds.

We briefly mention the possibility of using the EDM of heavier fermions, in particular the tau and the top to constrain $R\!\!\!/_p$ couplings. As far as d_τ is concerned, the counterpart of the diagram of Fig. 3 (as well as the corresponding diagrams with *W*) will not contribute as the dominant piece proportional to m_τ will have no imaginary part. The diagram analogous to Fig. 4, however, does contribute to d_τ. The real and imaginary parts of the *weak* dipole moment of the τ have recently been constrained by the OPAL Collaboration [12] to be smaller than 6×10^{-18} and 1.5×10^{-17} e cm, respectively. Clearly, these limits do not give any significant constraints on the corresponding $R\!\!\!/_p$ couplings. There are no data on d_t at this time.

Our analysis up to now has ignored the bilinear terms in the superpotential of Eq. (1), and also corresponding soft SUSY breaking scalar bilinears in the potential. In the case of exact SUSY, the former can be rotated away [15] and, of course, the latter are absent, i.e., *R*-parity violation occurs only through the trilinear interactions that we have analyzed. This is not, however, true in the realistic case where supersymmtery is broken, because the bilinear soft terms in the scalar potential cannot simultaneously be rotated away. Even if we assume that these are absent at some very high scale, these terms, which are an additional source to the changes of the superchiral charges in Table I, are generated [27,21], by radiative corrections. A more important difference, however, is that in the presence of the scalar bilinears the sneutrino fields generically acquire a VEV, so that the charge Q_{l_L} is now no longer conserved. In principle, it would be possible to include modifications to our analysis by allowing diagrams where sneutrino fields disappear or are created from the vacuum: but the result then depends on the number of fields that disappear into, or are created from, the vacuum and the simple predictions that we have obtained are lost. In models where bilinears are only radiatively generated, sneutrino vaccum expectation values (VEV's) are smaller than a few GeV, and these contributions are small, and our analysis yields a reasonable approximation. There are also models [28] where due to additional discrete symmetries, sneutrino VEV's are absent. We have, however, not analyzed the generic case where the bilinear mass terms and sneutrino VEV's are all of the order of the SUSY breaking scale. It would be interesting to investigate whether this situation yields a new possibility of generating large electric dipole moments for matter fermions.

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