Nucleosynthesis in power-law cosmologies

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(Received 5 November 1999; published 24 April 2000)

We have recently considered cosmologies in which the universal scale factor varies as a power of the age of the Universe and concluded that they cannot satisfy the observational constraints on the present age, the magnitude-redshift relation for SN Ia, and the primordial element (D, ^{3}He, ^{4}He, and ^{7}Li) abundances. This claim has been challenged in a proposal that suggested a high baryon density model ($\Omega_B h^2 \simeq 0.3$) with an expansion factor varying linearly with time could be consistent with the observed abundance of primordial helium-4, while satisfying the age and magnitude-redshift constraints. In this paper we further explore primordial nucleosynthesis in generic power-law cosmologies, including the linear case, concluding that models selected to satisfy the other observational constraints are incapable of accounting for *all* the light element abundances.

PACS number(s): 98.80.Cq

I. MOTIVATION

We have studied a class of cosmological models in which the universal scale factor grows as a power of the age of the Universe ($a \propto t^\alpha$) and concluded that such models are not viable since constraints on the present age of the Universe and from the magnitude-redshift relation favor $\alpha = 1.0 \pm 0.2$, while those from the abundances of the light elements produced during primordial nucleosynthesis require that α lie in a very narrow range around 0.55 [1]. Successful primordial nucleosynthesis provides a very stringent constraint, requiring that a viable model simultaneously account for the observationally inferred primordial abundances of deuterium, helium-3, helium-4, and lithium-7. For example, if the nucleosynthesis constraint is satisfied, the present Universe would be very young: $t_0 = 7.7$ Gyr for a Hubble parameter $H_0 = 70$ km s^{-1} Mpc^{-1} (or requiring $H_0 \leqslant 54$ km s^{-} Mpc^{-1} for $t_0 \geqslant 10$ Gyr).

Recently, Sethi *et al.* [2] noted that cosmologies where the scale factor grows linearly with time may produce the correct amount of ^{4}He *provided that* the Universal baryon fraction is sufficiently large. At first this result might seem counterintuitive since such a universe would have been very old at the time of big bang nucleosynthesis (BBN), suggesting that all neutrons have decayed and are unavailable to be incorporated in ^{4}He. In fact, as Sethi *et al.* correctly pointed out, the expansion rate is so slow that the weak reactions remain in equilibrium sufficiently long to permit a ''simmering'' synthesis of the required amount of ^{4}He. However, such an old universe also leaves more time to burn away D and ^{3}He so that no astrophysically significant amounts can survive. The observations of deuterium in high-redshift, low-metallicity quasistellar object (QSO) absorbers [3], the observations of lithium in very old, very metal-poor halo stars (the ''Spite plateau'') [4], and those of helium in low-metallicity extragalactic H II regions [5] require an internally consistent primordial origin. The claim of Sethi *et al.* that deuterium could have a nonprimordial origin is without basis as shown long ago by Epstein, Lattimer, and Schramm [6]. Nevertheless, the paper of Sethi *et al.* [2] prompted us to reinvestigate primordial nucleosynthesis in those power-law cosmologies which may produce ''interesting'' amounts of ^{4}He so as to study the predicted yields for D, ^{3}He, and ^{7}Li.

II. NUCLEOSYNTHESIS IN POWER-LAW COSMOLOGIES

Preliminaries. For a power-law cosmology it is assumed that the scale factor varies as a power of the age independent of the cosmological epoch:

$$a/a_0 = (t/t_0)^\alpha = (1+z)^{-1}, \tag{1}$$

where the subscript 0 refers throughout to the present time and z is the redshift. We may relate the present cosmic background radiation (CBR) temperature to that at any earlier epoch by $T = (1+z)\beta T_0$, where $\beta \leqslant 1$ accounts for any entropy production. For the models we consider, $\beta = 1$ after electron-positron annihilation. The Hubble parameter is then given by

$$H = \frac{\dot{a}}{a} = \frac{\alpha}{t_0}\left(\frac{T}{\beta T_0}\right)^{1/\alpha}. \tag{2}$$

The second equality should be read with the understanding that it is not valid during the epoch of electron-positron anhilation due to the nonadiabatic nature of annhilations. Power-law cosmologies with large α share the common feature that the slow universal expansion rate permits neutrinos

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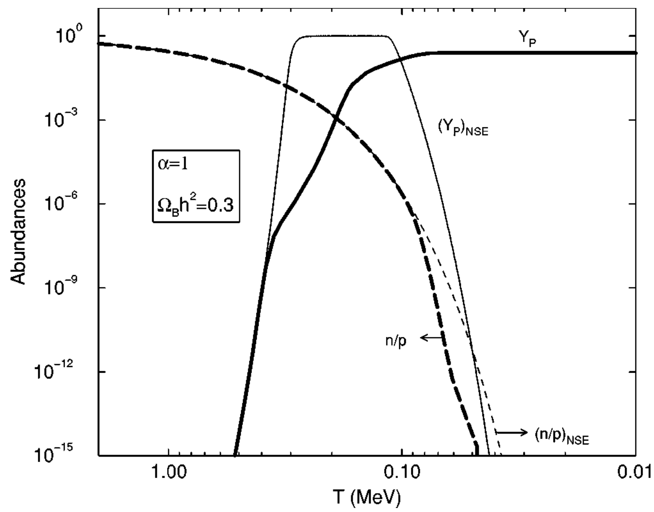

FIG. 1. Comparison of nucleosynthesis in the linear expansion model (heavy curves) for the case of $Y_P=0.24$ with the predictions of nuclear statistical equilibrium (lighter curves). The solid curves are for the ^{4}He mass fraction Y_P, while the dashed curves show the evolution of the ratio of neutrons to protons.

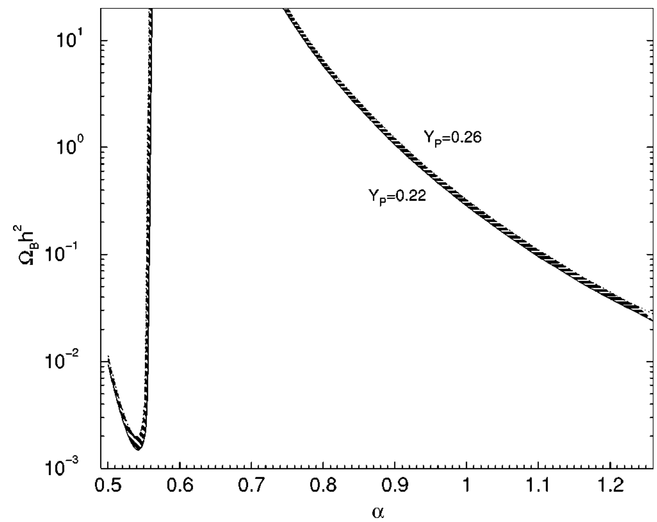

FIG. 2. Iso-abundance contours of the ^{4}He mass fraction (Y_P) in the baryon density$-\alpha$ plane. The shaded band corresponds to helium abundances in the range $0.22\leqslant Y_P\leqslant 0.26$.

to remain in equilibrium until after electron-positron annihilation has ended so that neutrino and photon temperatures remain equal. In this case the entropy factor for $T>m_e/3$ in Eq. (2) is

$$\beta=(29/43)^{1/3} \tag{3}$$

in contrast to the standard big bang nucleosynthesis (SBBN) value of $(4/11)^{1/3}$. As α increases, the expansion rate at a fixed temperature decreases due to the dominant effect of the $1/\alpha$ power. Another useful way to view this is that at a fixed temperature, a power-law universe with a larger α is older. As a consequence of the decreasing expansion rate, the reactions remain in equilibrium longer. In particular, as pointed out by Sethi *et al.* [2] for the linear expansion model ($\alpha=1$), the weak interactions remain in equilibrium to much lower temperatures than in the SBBN scenario, allowing neutrons and protons to maintain equilibrium at temperatures below 100 keV, as can be seen in Fig. 1. As is evident from Fig. 1, the ^{4}He production rate below about 0.4 MeV is too slow to maintain nuclear statistical equilibrium. However, the presence of neutrons in equilibrium and the enormous amount of time available for nucleosynthesis during neutron-proton equilibrium (compared to SBBN) make it possible to build up a significant abundance of ^{4}He [2].

The above discussion is not restricted to $\alpha=1$, but applies for all values of α which are sufficiently large (so that the expansion rate is sufficiently small) to allow neutrons to stay in equilibrium long enough to enable synthesis of ^{4}He in sufficient amounts, as we show in Fig. 2. Although we explore a larger range in α in this paper, we present detailed results for $0.75\leqslant\alpha\leqslant 1.25$, a range consistent with the age and expansion rate of the Universe, and we check these results for consistency with independent (i.e., non-BBN) constraints on the baryon density. The iso-abundance contours in Fig. 2 show clearly that as α decreases towards 0.75, a larger baryon density is required to produce the same abundance of ^{4}He. For example, although $Y_P=0.24$ can be synthesized in the $\alpha=0.75$ model, the density of baryons required is very large: $\Omega_B h^2\simeq 20$. These ''large Ω_B'' models are constrained by dynamical estimates of the mass density, an issue we discuss later.

Helium-4 abundance. In an earlier study [1], we showed that there is a very small region, centered on $\alpha=0.55$, for which the light elements can be produced in abundances similar to those predicted by SBBN. But this small window is closed by the SNIa magnitude-redshift data [7]. Here we are concerned with larger values of α and, correspondingly, larger baryon-to-photon ratios (η). First we consider the nucleosynthesis of ^{4}He in these models. Figure 2 shows the connection between the baryon density ($\Omega_B h^2=\eta_{10}/273$, where $\eta_{10}\equiv 10^{10}n_N/n_\gamma$) and α set by the requirement that the primordial helium mass fraction lie in the generous range $0.22\leqslant Y_P\leqslant 0.26$. We have included in Fig. 2 the region investigated in Ref. [1], $\alpha<0.6$, as well. To understand the features in Fig. 2, we need to isolate the important factors controlling the synthesis of helium. In SBBN, the ^{4}He abundance is essentially controlled by the number density ratio of neutrons to protons (n/p) at the start of nucleosynthesis ($T=T_{\rm BBN}\approx 80$ keV). This ratio in turn is determined by (1) the *n-p* ratio at ''freeze-out'' ($T=T_f$) of the neutron-proton interconversion rates which may be approximated by $(n/p)_f=\exp(-Q/T_f)$, where $Q=1.293$ MeV is the neutron-proton mass difference, and (2) the time available for neutrons to decay after freeze-out, $\Delta t_d=t(T_{\rm BBN})-t(T_f)$. In contrast, for power-law cosmologies another factor comes into play—the time available for nucleosynthesis, $\Delta t_{\rm BBN}$, before the nuclear reactions freeze out. For larger α, the expansion rate of the Universe (at fixed temperature) is smaller and the Universe is older. Hence, for larger α neutrons remain in equilibrium longer and the freeze-out temperature (T_f) is smaller, so that $(n/p)_f$ is smaller. However, the effect of the increase in $\Delta t_{\rm BBN}$ as α increases dominates that due to the change in T_f. For $\alpha=0.50$ the freeze-out temperature is around 4 MeV, whereas for $\alpha=0.55$, $T_f \simeq 1$ MeV which implies a decrease in $(n/p)_f$ by a factor of about 2.5. On the other hand, the age of the Universe at $T=10$ keV (about the temperature when SBBN ends) is a factor of 25 larger for $\alpha=0.55$ relative to that for $\alpha=0.50$. Thus, for the same η, increasing α from 0.50 to 0.55 has the effect of increasing the ^{4}He abundance because more time is available for nucleosynthesis. But since decreasing the baryon density decreases the nuclear reaction rates leading to a decrease in ^{4}He, we may understand the trend of the smaller baryon density requirement as α increases from 0.50 to about 0.55, even though the decrease in T_f opposes this effect. The time delay between ''freeze-out'' and BBN, Δt_d, which has, until now, been much smaller than τ_n, becomes comparable to it at $\alpha \sim 0.55$. Since a larger α results in an older Universe at a fixed temperature, Δt_d increases with α. Thus for $\alpha \gtrsim 0.55$, Y_P is increasingly suppressed (exponentially) as α is increased. The only way to compensate for this is by increasing $T_{\rm BBN}$ [since $\Delta t_d \propto (T_{\rm BBN})^{-1/\alpha}$], which may be achieved by increasing the baryon density. But since $T_{\rm BBN}$ depends only logarithmically on the baryon density [8], this accounts for the exponential rise in the required value of $\Omega_B h^2$ as α increases. This trend cannot continue indefinitely; the curve must turn over for reasons we describe below.

From Fig. 2, it is apparent that in the ''large α'' range, the required value of $\Omega_B h^2$ decreases with increasing α. In our previous analysis [1] of ^{4}He nucleosynthesis which concentrated on α in the vicinity of 0.55, we implicitly assumed that the age of the Universe at $T=T_f$ was not large enough for appreciable amounts of ^{4}He to have been built up. This assumption breaks down for large values of α *and* η. Since D, ^{3}He, and ^{3}H are not present in appreciable quantities, a large value of η is needed to boost the ^{4}He production rate. Now, the larger the value of α, the longer neutrons remain in equilibrium, thus allowing more ^{4}He to be slowly built up, with the neutrons incorporated in ^{4}He being replaced via $p \rightarrow n$ reactions. Roughly speaking, the required value of η for a given α is set by the condition

$$\left[\frac{dY_P}{dt}\right]_{T=T_f} \sim 0.24/t(T_f). \tag{4}$$

The effects of α on $t(T_f)$ and η on dY_P/dt complement each other, giving rise to the trend shown by the ^{4}He iso-abundance curves in Fig. 2 for $\alpha \gtrsim 0.75$.

Light element abundances in the linear expansion model. We now turn to the production of deuterium and ^{3}He. For large α (e.g., $\alpha=1$) (see Fig. 3), we expect the deuterium abundance to be insignificant since D can be efficiently burned to ^{3}He during the long time available for nucleosynthesis. The mean lifetime of deuterium against destructive collisions with protons at a low temperature of 10 keV is around 3 days; at this temperature the $\alpha=1$ universe is already 300 years old. The fact that the time scales are so different allows us to derive analytical expressions for the deuterium, helium-3, and lithium-7 (beryllium-7) mass fractions (to be denoted by $X_{\rm D}$, X_3, and X_7 respectively). The

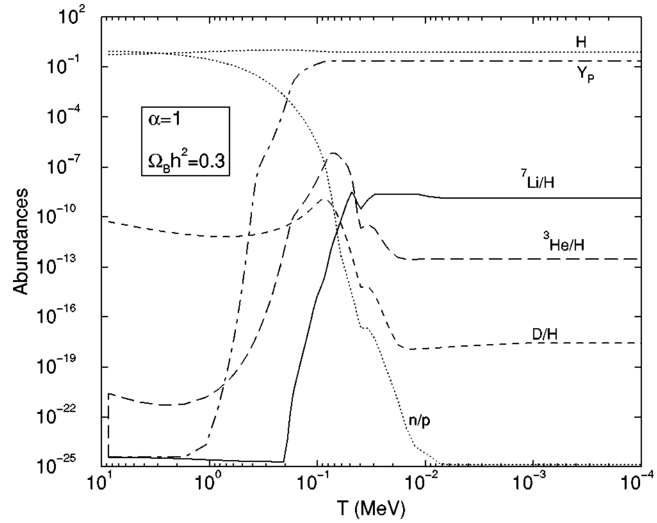

FIG. 3. Evolution of the light element abundances as a function of the photon temperature in an $\alpha=1$ universe.

generic equation for the rate of evolution of the mass fraction of nuclide ''a'' can be parametrized as

$$\frac{dX_a}{dt} = R_{\rm prod}(a) - R_{\rm dest}(a) X_a, \tag{5}$$

where ''prod'' and ''dest'' refer to the production and destruction rates of nuclide ''d.'' Given that the universe remains at the same temperature for a very long time (compared to the reaction time scales), it is not surprising that X_a achieves its steady-state value at each temperature (for a detailed discussion in the context of SBBN, see [9]):

$$X_a \approx \frac{R_{\rm prod}(a)}{R_{\rm dest}(a)}. \tag{6}$$

We can write this explicitly for the simplest case—deuterium:

$$X_{\rm D} = 2\,\frac{(\Gamma_{np} + \Gamma_{pp}/2) X_p}{\Gamma_{p\rm D} + \Gamma_{\gamma\rm D}}, \tag{7}$$

where the various Γ's represent the relevant deuterium creation ($n+p \rightarrow {\rm D} + \gamma$ and $p+p \rightarrow {\rm D} + e^+ + \nu$) rates per target proton and destruction [D$(p,\gamma)^3$He and D$(\gamma,p)n$] rates per target deuterium. All of these rates can be obtained from Ref. [10]. Once the reaction rates become smaller than the universal expansion rate (say at some temperature $T_\star$), the abundances freeze out with values close to X_a at the corresponding $T_\star$. This is illustrated in Fig. 4 which clearly shows that the steady-state solution works very well. We note here that the steady state (dotted) curves in Fig. 4 are not independent analytic derivations, but use the abundances of the various nuclei as calculated by the numerical code. The figure intends to emphasize that nucleosynthesis in this (linear expansion) model can be well represented by the steady-state solutions in Eq. (6).

In the expression for $X_{\rm D}$ [see Eq. (7)], the $n+p$ reaction term dominates until about 20 keV, after which the $p+p$

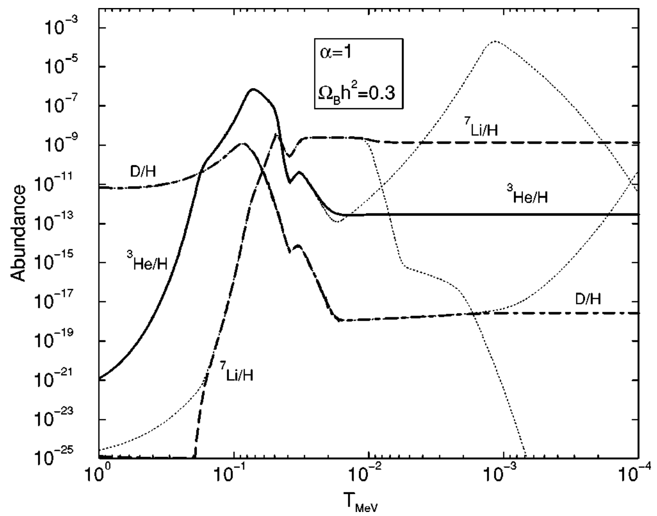

FIG. 4. Comparison of the light element abundances with their steady state values as a function of the photon temperature in an $\alpha=1$ universe. The dotted curves correspond to the equilibrium solution.

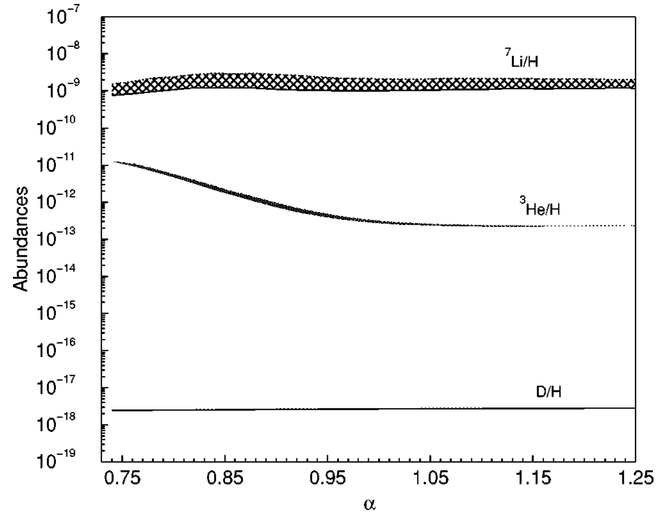

FIG. 5. Abundances of ^{3}He, D, and ^{7}Li in power-law cosmologies for different values of the expansion index (α). The shaded bands correspond to helium abundances in the range $0.22 \leqslant Y_P \leqslant 0.26$.

reaction makes the dominant contribution. The final deuterium abundance is thus determined by the weak *pp* reaction ($p+p \rightarrow \mathrm{D}+e^+ +\nu$), the effect of which can be seen in Fig. 3 as the very slow rise in X_{D} between temperatures of 10 keV and 1 keV (at which point the D abundance freezes out). Since both ^{3}He and ^{7}Li freeze out much earlier, they do not get any significant boost from the weak *pp* reaction.

From Eq. (7), X_{D} and thus X_3 (^{3}He is formed from D) are proportional to X_n, the neutron abundance. One striking feature in Fig. 3 is the boost to the neutron abundance (and hence the abundances of D and ^{3}He) at temperatures around 40 keV. The effect is subtle and may be missed in BBN codes with a limited nuclear reaction network. The slow rate of expansion of the universe during nucleosynthesis facilitates the production of a relatively large ‘‘metal’’ ($A \geqslant 8$) abundance ($X_{\mathrm{metals}} \simeq 3\times 10^{-7}$). In particular, ^{13}C is produced in these models through the chain $^{12}\mathrm{C}+p \rightarrow {}^{13}\mathrm{N}+\gamma$ and the subsequent beta decay of ^{13}N. In this environment $^{13}\mathrm{C}+{}^4\mathrm{He} \rightarrow {}^{16}\mathrm{O}+n$ leads to the production of free neutrons.

The mass-7 abundance is entirely due to the production of ^{7}Be through the reaction $^3\mathrm{He}+{}^4\mathrm{He} \rightarrow {}^7\mathrm{Be}+\gamma$. ^{7}Be decays to ^{7}Li by electron capture once the universe has cooled sufficiently to permit the formation of atoms. Once formed, it is difficult to destroy ^{7}Be at temperatures $\lesssim$ 100 keV. In contrast, ^{7}Li is very easily destroyed, specifically through its interaction with protons. Since the ^{7}Be production (and thus ^{7}Li) follows the evolution of the ^{3}He abundance, and there is very little destruction of ^{7}Be, ^{7}Li also benefits from the boost to the neutron abundance described in the last paragraph. This has the effect of boosting the ^{7}Li abundance from 10^{-11} (if this source of neutrons were not included) to 10^{-9}. This is significant in that, at the level of a few parts in 10^{10} (e.g., [11]), the primordial lithium abundance lies between these two estimates.

Light element abundances vs α. Having explored BBN in the linear model ($\alpha=1$) it is now important to ask how these results depend on α. It is clear from Fig. 5 that nothing dramatically different happens as α changes; this is simply because the key physics remains the same. In preparing Fig. 5 we adjust the value of η (baryon density) for each choice of α so that the primordial ^{4}He mass fraction lies between 22% and 26%. As α increases, the nuclei freeze out at lower temperatures since the expansion rate at the same temperature is lower for a larger α. The effect of this can be gauged by the behavior of X_{D}, X_3, and X_7 with respect to temperature, as given by Eq. (6). For deuterium this implies a small increase with α due to the *pp* (weak) reaction, which is also reflected in the behavior of ^{3}He for $\alpha \gtrsim 1$. The fall of ^{3}He with increasing α for $\alpha \lesssim 1$ is due to larger destruction of ^{3}He because of the increase in the time available for the nuclear reactions. As already mentioned, the abundance of ^{7}Be depends critically on the evolution of the ^{3}He abundance; so while the mass-7 (^{7}Be) abundance increases appreciably with the increase in baryon density, it is relatively unaffected by a change in α.

Note that in those power-law models which can simultaneously reproduce an acceptable ^{4}He abundance along with a consistent age and expansion rate, the corresponding baryon density must be very large, $0.04 \leqslant \Omega_B h^2 \leqslant 6.4$ ($11 \leqslant \eta_{10} \leqslant 1750$; see Fig. 2). Most—if not all—of this range is far too large for consistency with independent (non-BBN) estimates of the universal density of baryons ($\eta \lesssim 7.4$ [12]) or, for that matter, the total matter density [13]. Conservatively, clusters limit the total (gravitating) matter density to $\Omega_M \lesssim 0.4$, so that if there were no nonbaryonic dark matter, $\Omega_B h^2 \lesssim 0.2 (\eta \lesssim 54)$ for $h \sim 0.7$. However, if the x-ray emission from clusters is used to estimate the cluster baryon fraction (see [14]), the universal baryon density should be smaller than this very conservative estimate by a factor of 7–8 (consistent with the upper bound from the baryon inventory of Fukugita, Hogan, and Peebles [12]). Thus power-law cosmologies constrained to reproduce ^{4}He (only), an acceptable age and magnitude-redshift relation, and an acceptable baryon density must have α restricted to a very narrow range: $1 \lesssim \alpha \lesssim 1.2$. Furthermore, the baryon density in even this restricted range is large when compared with estimates [14] of the baryon density from cluster x rays. Finally, for α in the narrow range of $1 \lesssim \alpha \lesssim 1.2$ and $0.22 \leqslant Y_P \leqslant 0.26$, the other light element abundances are restricted to ^{7}Li/H $> 10^{-9}$, ^{3}He/H$< 3 \times 10^{-13}$, and D/H$< 3 \times 10^{-18}$. For deuterium and helium-3 this is in very strong disagreement (by 8–13 orders of magnitude) with observational data (for a review see [11]). Although the predicted ^{7}Li abundance is comparable to that observed in the solar system, the local ISM, and in Pop I stars, it is larger than the primordial abundance inferred from the Pop II halo stars [4,11], and marginally inconsistent with the observations of lithium in the ISM of the LMC [15].

III. CONCLUSIONS

In response to the claim [2] that a power-law universe expanding linearly with time could be consistent with the constraints on BBN, we have reexamined these models. Although it is true that observationally consistent amounts of ^{4}He can be produced in these models, this is not the case for the other light elements D, ^{3}He, ^{7}Li. Furthermore, consistency with ^{4}He at $\alpha = 1$ requires a very high baryon density ($75 \leqslant \eta_{10} \leqslant 86$ or $0.27 \leqslant \Omega_B h^2 \leqslant 0.32$), inconsistent with non-BBN estimates of the universal baryon density and even with the total mass density. We have also investigated BBN in power-law cosmologies with $\alpha > 1$ and have confirmed that although the correct ^{4}He abundance can be produced, the yields of the other light elements D, ^{3}He, and ^{7}Li are inconsistent with their inferred primordial abundances. In general, power-law cosmologies are unable to account simultaneously for the early evolution of the Universe (BBN) (which requires $\alpha \simeq 0.55$) and for its presently observed expansion (which requires $\alpha = 1 \pm 0.2$) [16–19].

ACKNOWLEDGMENTS

This work was supported at Ohio State by DOE Grant DE-AC02-76ER01545.

[1] M. Kaplinghat, G. Steigman, I. Tkachev, and T. P. Walker, Phys. Rev. D **59**, 043514 (1999).
[2] M. Sethi, A. Batra, and D. Lohiya, Phys. Rev. D **60**, 108301 (1999).
[3] S. Burles and D. Tytler, Astrophys. J. **499**, 699 (1998); **507**, 732 (1998).
[4] F. Spite and M. Spite, Astron. Astrophys. **115**, 357 (1982); L. Hobbs and J. Thorburn, Astrophys. J. Lett. **428**, L25 (1994); P. Bonifacio and P. Molaro, Mon. Not. R. Astron. Soc. **285**, 847 (1997); S. Ryan, J. Norris, and T. Beers, astro-ph/9903059.
[5] K. A. Olive and G. Steigman, Astrophys. J., Suppl. Ser. **97**, 49 (1995); K. A. Olive, E. Skillman, and G. Steigman, Astrophys. J. **483**, 788 (1997); Y. I. Izotov and T. X. Thuan, *ibid.* **500**, 188 (1998).
[6] R. I. Epstein, J. M. Lattimer, and D. N. Schramm, Nature (London) **263**, 198 (1976).
[7] P. M. Garnavich *et al.*, Astrophys. J. Lett. **493**, L53 (1998); S. Perlmutter *et al.*, Nature (London) **391**, 51 (1998).
[8] E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, Reading, MA, 1990).
[9] R. Esmailzadeh, G. D. Starkman, and S. Dimopoulos, Astrophys. J. **378**, 504 (1991).
[10] W. A. Fowler, G. R. Caughlan, and B. A. Zimmerman, Annu. Rev. Astron. Astrophys. **13**, 69 (1975).
[11] K. A. Olive, G. Steigman, and T. P. Walker, Phys. Rep. (to be published), astro-ph/9905320.
[12] M. Persic and P. Salucci, Mon. Not. R. Astron. Soc. **258**, 14P (1992); M. Fukugita, C. J. Hogan, and P. J. E. Peebles, Astrophys. J. **503**, 518 (1998).
[13] N. Bahcall, L. M. Lubin, and V. Dorman, Astrophys. J. Lett. **447**, L81 (1995).
[14] G. Steigman, N. Hata, and J. E. Felten, Astrophys. J. **510**, 564 (1999).
[15] G. Steigman, Astrophys. J. **457**, 737 (1996).
[16] F. Pont, M. Mayor, C. Turon, and D. A. VandenBerg, Astron. Astrophys. **329**, 87 (1998).
[17] R. Rood, G. Steigman, and B. M. Tinsley, Astrophys. J. Lett. **207**, L57 (1976); D. S. P. Dearborn, G. Steigman, and M. Tosi, Astrophys. J. **465**, 887 (1996).
[18] R. T. Rood, T. M. Bania, and T. L. Wilson, Nature (London) **355**, 618 (1992); D. S. Balser, T. M. Bania, R. T. Rood, and T. L. Wilson, Astrophys. J. **483**, 320 (1997).
[19] M. Bureau, J. R. Mould, and L. Staveley-Smith, Astrophys. J. **463**, 60 (1996); T. Kundić *et al.*, *ibid.* **482**, 75 (1997); G. A. Tammann and M. Federspiel, in *The Extragalactic Distance Scale*, edited by M. Livio, M. Donahue, and N. Panagia (Cambridge University Press, Cambridge, England, 1997), p. 137; J. L. Tonry, J. P. Blakeslee, E. A. Ajhar, and A. Dressler, Astrophys. J. **475**, 399 (1997).