High temperature symmetry breaking via flat directions

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We show that the natural presence of flat directions in supersymmetric theories allows for nonrestoration of global and/or gauge symmetries. This could have important cosmological consequences for supersymmetric grand unified theories and, in particular, it offers a solution of the monopole problem.

I. INTRODUCTION

In spite of everyday experience, it is well known that global symmetries may remain broken at temperatures much above the physical scale of the theory in question [1,2]. Although this has recently been confirmed by rigorous renormalization-group improved methods [3] and lattice calculations [4], it is not clear whether the same is true for the case of local symmetries [5,6]. If one is willing to consider the universe with some large external charge then even local symmetries may naturally remain broken at high T [7,8]. This has important application for the fate of topological defects and in particular could solve the cosmological problems of monopoles and domain walls [9–11]. For a recent review of these issues see [12].

However, in what follows we are interested only in the generic high-T behavior of field theories without any external charge. It is known then at the level of a no-go theorem [13,14] that in supersymmetric theories the phenomenon of nonrestoration is not operative, even when nonrenormalizable interactions are included [15–17].

We wish to make our philosophy as clear as possible. Even if there is no phase transition, one needs to justify the initial condition of a homogeneous universe. For this reason we, as everybody else, assume that inflation took place at some point. This is indispensable for our program. However, one has no guarantee that inflation takes place after the production of topological defects or that the reheating temperature is below their masses. In fact, in grand unified theories this is often not the case. For this reason we believe that it is imperative to look for other solutions to the problems of topological defects and the false vacuum. It is thus important to readdress the issue of high-T behavior of supersymmetric theories and this is precisely the aim of this paper.

The crucial point in the no-go theorem of symmetry nonrestoration in supersymmetry is based on the assumption of thermal equilibrium for all the fields of the theory. Whereas this is normally true, it is precisely the supersymmetric theories that may provide naturally a way out of this assumption. Namely, these theories are generically characterized by a large number of flat directions, generically denoted by ϕ in what follows. Such flat directions, at least for large enough values of ϕ (see below), do not have strong enough interactions to be in thermal equilibrium and, therefore, the no-go

theorem is not directly applicable to them [18]. Remarkably enough, it can be shown that these flat directions may quite naturally possess vacuum expectation values much bigger than the temperature. This is in full agreement with the recent results of Dvali and Krauss [18]. In order to set the stage, we first briefly review the conventional situation of all the fields being in thermal equilibrium.

II. SUPERSYMMETRY IN THERMAL EQUILIBRIUM

The point here is quite simple. The fields being in thermal equilibrium have high-temperature mass terms proportional to T^2. Now, in ordinary theories the mechanism of nonrestoration is based on the possibility of negative dimensionless couplings in the scalar potential, thus allowing for the negative T^2 scalar masses. In supersymmetry, however, these couplings are the squares of the relevant Yukawa couplings and so can never be negative. This is, roughly speaking, the reason behind the no-go theorem for supersymmetry which can be rigorously proven under the assumption of thermal equilibrium and renormalizable couplings. The situation is somewhat more subtle in the nonrenormalizable case [15], but still the no-go theorem goes through [16,17]. Essentially, the argument is as follows: if the main interaction is non-renormalizable (say $|\phi|^6/M^2$, where M is the large scale cutoff), its sign in the potential must be positive in order for the potential to be bounded from below. This then immediately gives a positive temperature-dependent mass term (in this case T^4/M^2).

III. FLAT DIRECTIONS AND SYMMETRY NONRESTORATION

As we already remarked, flat directions may not be in equilibrium at high temperature [18]. This implies the absence of the T^2 mass term and paves the way for the possibility of symmetry nonrestoration. Now, at zero temperature one normally lifts the flat directions (at least the dangerous ones which break charge or color) by positive soft supersymmetry breaking mass terms which push them to the origin. At high T, though, such soft terms are negligible compared to T and since the temperature breaks supersymmetry, it determines the stability point of flat directions. We will see that the run-away behavior is quite natural and can happen in a

large class of supersymmetric field theories. The situation is reminiscent of the upside-down hierarchy of Witten [19], when small supersymmetry breaking terms may induce large vacuum expectation values (vev's) (even exponentially large).

In order to illustrate this phenomenon, we describe it using a simple toy model and then address the issue of realistic gauge theories. Imagine a scalar superfield q (it will correspond to the matter superfields) with a renormalizable self-interaction and a flat (or better almost flat) field ϕ, with the following superpotential:

$$W = \lambda q^3/3 + W(\phi), \qquad (1)$$

where we take

$$W(\phi) = \frac{\phi^{n+3}}{(n+3)M^n} \quad (n \geq 1), \qquad (2)$$

so that in the limit $M \to \infty$ we have a completely flat direction. For this particular example the superpotential possesses a U(1) R symmetry

$$\phi \to e^{i3\alpha}\phi,$$

$$q \to e^{i(n+3)\alpha}q,$$

$$W \to e^{i3(n+3)\alpha}W, \qquad (3)$$

and we wish to show that this symmetry is broken at high temperature. Of course, this symmetry has an anomaly, but this is of no importance for what we are trying to do.

We choose this simplified model only to illustrate the possibility of broken symmetries at high T. It is more realistic to allow other higher dimensional terms, which would break the R symmetry. However, everything we do will hold true in this case too; the field will get a large vev, only there will be no symmetry to talk about. Our toy model is only an illustration of a more physical situation, when of course there are (gauge) symmetries in the game.

If M is the Planck scale $M_{\rm Pl}$, even the $n = 1$ case may not suffice to bring ϕ in thermal equilibrium at high T, and in any case we imagine n large enough to keep ϕ not in equilibrium (a reasonable assumption for what we call a flat direction). We quantify this more precisely below.

Now, for any reasonable $\lambda < 1$ we will have a field q in equilibrium and thus q will run in thermal loops. Next, imagine that the Kähler potential is not trivial, exemplified by

$$K(q,\phi) = q^\dagger q + \phi^\dagger \phi + a \frac{q^\dagger q \phi^\dagger \phi}{M^2}. \qquad (4)$$

Since we are interested only in the question of the vev of ϕ and since ϕ is not in equilibrium, we can take it to be a background field. Thus we can ignore its kinetic term and just concentrate on the field q. We first put its kinetic term in the canonical form by rescaling
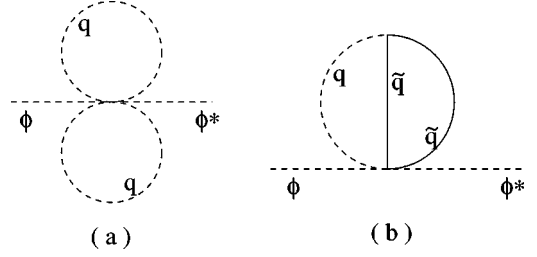
$$q \to (1 + a|\phi|^2/M^2)^{-1/2}q. \qquad (5)$$

FIG. 1. Diagrams which give the thermal mass to the flat direction ϕ. Since ϕ is not in thermal equilibrium, only scalars q and fermions $\tilde{q}$ are running in the loops.

The potential at zero temperature is given by (before rescaling)

$$V = \frac{\partial W}{\partial \phi^i}(K^{-1})^i_j \frac{\partial W^*}{\partial \phi_j^*}, \qquad (6)$$

where K^{-1} is the inverse of

$$K_i^j = \frac{\partial^2 K(\phi)}{\partial \phi^i \partial \phi_j^*}. \qquad (7)$$

The relevant potential then simply becomes (for the rescaled field q)

$$V = \lambda^2 \frac{|q|^4}{(1 + a|\phi|^2/M^2)^3} + \frac{|\phi|^{2(n+2)}}{M^{2n}}, \qquad (8)$$

which can be expanded in $|\phi|^2/M^2$ to give

$$V \approx \lambda^2|q|^4 - 3a\lambda^2|q|^4 \frac{|\phi|^2}{M^2} + \frac{|\phi|^{2(n+2)}}{M^{2n}}. \qquad (9)$$

Let us evaluate now the high-temperature effective potential for the field ϕ. Notice that there can be no term $T^2|\phi|^2$ due to the absence of $|q|^2|\phi|^2$ type terms in the potential (of course there is a $|q|^2T^2$ term which pushes the vev of q to the origin). This is precisely the statement of ϕ being a flat direction and not being in thermal equilibrium (T^2 mass terms can only arise for the fields in equilibrium). It does not mean though that the field ϕ does not feel the temperature at all. Clearly, from the $|q|^4|\phi|^2$ interaction in Eq. (9), when q is running in thermal loops, we will be left with a temperature-dependent mass term for ϕ.

Now, from the diagrams of Fig. 1 the high-T effective potential for ϕ can be easily evaluated [16]

$$V_{\rm eff}(\phi,T) = -\frac{3a\lambda^2}{32}\frac{T^4|\phi|^2}{M^2} + \frac{|\phi|^{2(n+2)}}{M^{2n}}. \qquad (10)$$

Clearly, for $a > 0$ the $-$ sign in Eq. (10) guarantees a nonvanishing vev for ϕ:

$$\langle\phi\rangle^{n+1} = \left(\frac{3a\lambda^2}{32(n+2)}\right)^{1/2}T^2M^{n-1}. \qquad (11)$$

For $n=1$, $\langle\phi\rangle\approx T$ and for large n, $\langle\phi\rangle\gg T$ (we are by definition at $T\ll M$). Thus, not only does the field ϕ (an almost flat direction at $T=0$) have a vev at large T, but in general $\langle\phi\rangle$ is expected to be much bigger than the temperature. For $a<0$, of course $\langle\phi\rangle=0$. Thus the possibility of symmetry nonrestoration depends crucially on the sign of the noncanonical piece in K.

Let us now justify the natural assumption of ϕ being out of thermal equilibrium. From interaction (2) the mass of the ϕ particles is $m_\phi\approx\langle\phi\rangle^{n+1}/M^n$, which, using the solution (11), becomes

$$m_\phi\approx T^2/M\ll T. \tag{12}$$

Thus, for $M=M_{\rm Pl}$, which is what we expect in realistic cases, the effective decay rate for ϕ particles can be at most $m_\phi^2/T\approx T^3/M^2$ and is obviously much smaller than the expansion rate of the universe $H\approx T^2/M$. As expected, the field ϕ is not in thermal equilibrium. In exactly the same manner one shows that the Kähler interaction is also too weak to bring ϕ into the equilibrium.

We can generalize the above simple example by taking an arbitrary Kähler potential [18], in which case one has

$$V(q,\phi)=\frac{\lambda^2|q|^4}{(K_q^q(\phi))^3}+\text{other terms}. \tag{13}$$

As before, at high T, $|q|^4$ becomes proportional to T^4, so that clearly the vev for ϕ depends on the behavior of $K_q^q(\phi)$: when it grows with ϕ, this implies $\langle\phi\rangle\neq 0$ (its precise value depends on whatever stabilizes the theory). Now, there is nothing unnatural about it and this is the central point of our work: not only is it possible to have symmetries broken at high T in supersymmetric theories, but the presence of flat directions provides the most natural mechanism of symmetry nonrestoration in general.

IV. REALISTIC CASE: GAUGE THEORIES

The central ingredients, as we have seen, for the fields ϕ to develop a vev at high T is supersymmetry breaking, provided by temperature, and a Kähler which grows with ϕ. Supersymmetry must be broken in order for this to take place, and temperature plays this role naturally. What about the sign of $K_q^q(\phi)$ in realistic situations, so crucial for the phenomenon of nonrestoration to take place? What about the heavy fields of the theory, those coupled to ϕ? These important issues have been addressed and discussed at length in [18]. For the sake of completeness we summarize here briefly their idea.

The answer to the second question is simple: any field which is coupled to ϕ and gets a mass from $\langle\phi\rangle$, will necessarily decouple at temperatures $T\ll\langle\phi\rangle$. As long as this is satisfied, such fields can be safely ignored, their contribution in thermal loops being suppressed by $\exp(-\langle\phi\rangle/T)$. The obvious example of such fields are the gauge bosons in the realistic case when ϕ has gauge interactions.

The main concern must be addressed to the first question, i.e., the sign in $K_q^q(\phi)$. In general the light fields q have both

gauge interactions and Yukawa couplings to heavy states, and so in general the wave-function renormalization will produce the logarithmic dependence in ϕ (through the heavy states masses)

$$K_q^q(\phi)=1+[g^2(\phi)-h^2(\phi)]\ln(\phi^2) \tag{14}$$

written symbolically (h stands for the relevant Yukawa coupling). It is argued in [18] that for sufficiently large gauge coupling, ϕ will necessarily have a large vev at high temperature.

Now, what happens in a realistic theory such as the minimal supersymmetric standard model (MSSM)? As is well known, the flat directions are characterized by the holomorphic gauge-invariant functions of the original superfields of the theory [20]. In the MSSM with R parity typical examples of such holomorphic invariants are

$$lle^c, \quad u^cd^cd^c, \ldots, \tag{15}$$

where l is the leptonic doublet superfield and the rest are positron and antiquark superfields. Of course at zero temperature these flat directions are lifted by soft supersymmetry breaking terms which drive the vevs of their scalar components to zero. These are the flat directions that we characterized above generically by ϕ and thus they are expected to have vevs at high temperature far away from the origin. If so, the electromagnetic gauge invariance will be broken at high temperature [18] leading to the fast annihilation of monopoles [21], and thus solving the monopole problem [22]. Similarly there will be flat directions in the grand unified theory (GUT) extensions of the MSSM and the GUT symmetry may never be restored. In other words monopoles may not be created in the first place [18,22].

V. SUMMARY AND OUTLOOK

It should be clear from our work (we hope) that symmetry nonrestoration at high temperature is a rather natural and generic phenomenon in supersymmetric theories. Simply, at high T the flat directions may easily be lifted far from the origin due to the supersymmetry breaking induced by temperature. These flat directions may be the ones which for $T=0$ sit at the origin due to the soft supersymmetry breaking terms, or they may have large vevs even at $T=0$ as in the Witten's exponential hierarchy scenario [19].

The large number of flat directions in the MSSM and supersymmetric GUT's would imply then a logical and natural possibility of the SM and GUT symmetries broken at high T. This offers a possible solution to the monopole problem of GUT's.

It would be interesting to see if such a phenomenon could provide a solution to another problem of supersymmetric GUT's: the problem of the wrong vacuum [23] (unbroken GUT symmetry vacuum). Namely, at $T=0$ these theories in general have discretely degenerate vacua with the SM gauge symmetry vacuum having the same ($=0$) energy as the unbroken GUT symmetry one. On the other hand, if at high T symmetry were to be restored, this would make the theory

get caught up in the unbroken vacuum forever, since the tunneling into the true SM vacuum is enormously suppressed due to the large barrier ($\approx M_X^4$) between the different vacua. If one could achieve the flat direction to be provided by the adjoint representation of the GUT symmetry group, then there may never be a troublesome phase transition and the broken vacuum would be preferred at all T.

Notice that for the solution of the monopole problem the flat direction can be anything that provides the breaking of the GUT symmetry as to avoid the creation of monopoles. Equivalently, it could just be any flat direction which allows for the breaking of the electromagnetic gauge invariance in the MSSM at high T; this in turn allows for the annihilation of monopoles through the stringlike flux tubes.

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