Gravitino production after inflation

Renata Kallosh,[1] Lev Kofman,[2] Andrei Linde,[1] and Antoine Van Proeyen[3]
[1]*Department of Physics, Stanford University, Stanford, California 94305*
[2]*CITA, University of Toronto, 60 St. George Str., Toronto, Ontario, Canada M5S 3H8*
[3]*Instituut voor Theoretische Fysica, Katholieke Universiteit Leuven, Celestijnenlaan 200D B-3001 Leuven, Belgium*
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We investigate the production of gravitinos in a cosmological background. Gravitinos can be produced during preheating after inflation due to a combined effect of interactions with an oscillating inflation field and the absence of conformal invariance. In order to get insight into the conformal properties of a gravitino we reformulate phenomenological supergravity in an SU(2,2|1)-symmetric way. The Planck mass and F and D terms appear via the gauge-fixed value of a superfield that we call the conformon. We find that in general the probability of gravitino production is not suppressed by the small gravitational coupling. This may lead to the copious production of gravitinos after inflation. The efficiency of the new nonthermal mechanism of gravitino production is sensitive to the choice of the underlying theory. This may put strong constraints on certain classes of inflationary models.

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I. INTRODUCTION

The possibility of excessive production of gravitinos is one of the most complicated problems of cosmological models based on supergravity. Such particles decay very late and lead to disastrous cosmological consequences unless the ratio of the number density of gravitinos $n_{3/2}$ to the entropy density s is extremely small. For example, the ratio of the number density of gravitinos $n_{3/2}$ to the entropy density s should be smaller than $O(10^{-14})$ for gravitinos with a mass $O(100)$ GeV [1,2]. The standard thermal mechanism of gravitino production involves scattering of particles at high temperature in the early universe. To avoid excessive production of gravitinos one must assume that the reheating temperature of the universe after inflation was smaller than 10^8-10^9 GeV [1,2].

However, gravitinos can be produced not only in the thermal bath after reheating, but even earlier, during the oscillations of the inflaton field at the end of inflation. We already know that bosons as well as spin-$\frac{1}{2}$ fermions can be copiously produced by the coherently oscillating inflaton field. Quite often this effect occurs in a nonperturbative way during the stage of preheating [3,4]. Similar effect may occur for gravitinos. According to Ref. [5], nonthermal gravitino or moduli production may rule out certain classes of inflationary models which otherwise would be quite legitimate.

The theory of the cosmological gravitino production is very complicated. Recently production of transversal gravitino components (helicity $\frac{3}{2}$) was studied in Ref. [6] (see also Ref. [7] where an attempt has been made to study this question using perturbation theory). In this paper we will investigate production of all gravitino components, transversal (helicity $\frac{3}{2}$) and longitudinal (helicity $\frac{1}{2}$). As we will show, the production rate of the longitudinal gravitino component can be much greater than that of the transversal gravitino components.[1]

Since gravitino is a part of the gravitational multiplet, one could expect that their production must be strongly suppressed by the small gravitational coupling. Indeed, usually the production of particles occurs because their effective mass changes nonadiabatically during the oscillations of the inflaton field [3]. This is the main effect responsible for the production of the gravitinos with helicity $\frac{3}{2}$. The gravitino mass m at small values of the inflaton field ϕ is proportional to $M_P^{-2}W$, where W is a superpotential. Thus the amplitude of the oscillations of the gravitino mass is suppressed by M_P^{-2}. That is why production of the gravitino components with helicity $\frac{3}{2}$ is relatively inefficient [6,7].

There is another mechanism to be considered, which is related to breaking of conformal invariance [5]. It is well known, for example, that expansion of the universe does not lead to production of massless vector particles and massless fermions of spin $\frac{1}{2}$ because the theory of such particles is conformally invariant and the Friedmann universe is conformally flat. Meanwhile, massless scalar particles minimally coupled to gravity (as well as gravitons) are created in an expanding universe because the theory describing these particles is not conformally invariant. The rate of scalar particle production is determined by the Hubble constant $H\equiv\dot{a}/a=M_P^{-1}\sqrt{\rho/3}$, where ρ is the energy density. If similar effects are possible for gravitinos, they could be much stronger than the effects discussed in the previous paragraph. Indeed, H is

[1]In flat spacetime, transversal components, which exist even if supersymmetry is unbroken, correspond to the helicity $\frac{3}{2}$, while longitudinal components with nonvanishing ψ_0 and $\gamma^i\psi_i$ correspond to the helicity $\frac{1}{2}$. Although the helicity concept has a less precise meaning in Friedmann-Robertson-Walker (FRW) metrics, we still will use this loose definition as a shortcut to the transversal and longitudinal components.

suppressed only by the first degree of M_P^{-1}. As a result H typically is much greater than $m_{3/2}$ after inflation, so its time dependence may lead to a more efficient particle production.

The issue of conformal invariance of gravitinos is rather-nontrivial, and until now it has not been thoroughly examined. For gravitinos with helicity $\frac{3}{2}$ the effects proportional to H do not appear, and therefore the violation of conformal invariance for such particles is very small, being proportional to their mass [6]. However, as we will show, the theory of gravitinos with helicity $\frac{1}{2}$ is not conformally invariant, and therefore such particles will be produced during the expansion of the universe even if one neglects their mass.

But the most surprising effect which we have found is that the production of the gravitinos of helicity $\frac{1}{2}$ by the oscillating scalar field ϕ in general is *not* suppressed by any powers of M_P^{-1}, and therefore their production can be very efficient. The magnitude of the effect is model dependent. For example, this effect is very strong in the theory with the effective potential $\lambda\phi^4/4$. Gravitinos in this theory are produced very quickly, within about ten oscillations of the inflaton field, with occupation numbers $n_k \sim 1/2$ over a large range of momenta $k \leqslant \sqrt{\lambda}\phi$, which is very different from usual perturbation theory. Meanwhile, this effect is not so dramatic in the simplest model of a single chiral multiplet with a quadratic effective potential $m_\phi^2\phi^2/2$.

This result may have important cosmological implications, since it may allow to rule out certain classes of cosmological theories. The nature of this effect resembles the well known fact that the longitudinal components of massive vector bosons at high energy behave in the same way as the Goldstone boson that was eaten by the vector field [8]. A similar effect is known to exist in the theory of technicolor [9]. A more direct analogy is the transmutation of gravitational interactions of gravitinos with helicity $\frac{1}{2}$ to weak interactions found by Fayet [10]. In our case the effect is nonperturbative, and its adequate interpretation is achieved by finding solutions of the gravitino equations in a nontrivial self-consistent cosmological background.

In order to study conformal properties of gravitinos we reformulated the standard $N=1$ phenomenological supergravity in an SU(2,2|1)-invariant way, which makes the conformal properties of the theory manifest and explains how the conformal symmetry is broken. Our formulation describes arbitrary number of chiral and vector multiplets and is flexible enough to allow investigation of regimes where the superpotential W vanishes.

In application to the theory of gravitino production, we concentrate on the simplest models with one chiral multiplet Φ and arbitrary superpotential. We present classical equations of motion and constraints for the transverse and longitudinal gravitino in the expanding Friedmann universe interacting with the moving inflation field. We use the gauge where the goldstino is absent. Then we solve classical equations for gravitino. This solution confirms the generic prediction from the SU(2,2|1)-symmetric theory that the longitudinal gravitinos are not conformal.

We represent equations describing gravitino components with helicities $\frac{3}{2}$ and $\frac{1}{2}$ in a form analogous to the equations for the usual spin-$\frac{1}{2}$ fermions with time-dependent mass. This allows to reduce, to a certain extent, the problem of gravitino production to the problem of production of particles with spin $\frac{1}{2}$ after preheating [4].

Finally, we estimate the number density of gravitinos produced by the oscillating scalar field in several inflationary models, and show that in some models the ratio $n_{3/2}/s$ may substantially exceed the bound $n_{3/2}/s \leqslant O(10^{-14})$. A detailed account of our investigation will be given in a separate publication [11]. Here we will only outline the main points of our study and present the most interesting results.

II. SUPERGRAVITY LAGRANGIAN AND CONFORMAL PROPERTIES OF GRAVITINO

Fundamental M theory, which should encompass both supergravity and string theory, at present experiences rapid changes. One may still expect that the low-energy physics will be described by the $N=1$ $d=4$ supergravity [12] and address the issues of the early universe cosmology in the context of the most general phenomenological $N=1$ supergravity–Yang-Mills–matter theory [13].

We are interested in conformal properties of supergravity fields, which include various spin fields, in the conformally flat FRW metric describing the early universe:

$$g_{\mu\nu}(x) = a^2(\eta)\eta_{\mu\nu}. \tag{2.1}$$

In particular, we will be interested in conformal properties of gravitino. Supergravity is not a conformally invariant theory, despite the fact that it has a long history of being derived using the superconformal tensor calculus [14] as a technical tool. Therefore it is difficult even to address this issue as the supergravity fields do not have specific conformal weights. To solve this problem, the idea is to view the supergravity theory as a gauge-fixed version of the conformally invariant theory describing the most general $N=1$ gauge theory superconformally coupled to supergravity [11]. The derivation SU(2,2|1) invariant Lagrangian of $N=1$ supergravity coupled to $n+1$ chiral multiplets (with complex scalars X_I and fermions Ω_I) and Yang-Mills vector multiplets (with gauginos λ^α and vectors W_μ^α) superconformally will be presented in Ref. [11]. It has no dimensional parameters. It consists of three parts, depending, respectively, on a real function $\mathcal{N}$, a holomorphic function $\mathcal{W}$ and a gauge group two-tensor $f_{\alpha\beta}$. Each of them is conformally invariant by itself:

$$\mathcal{L}_{\text{superconf}} = [\mathcal{N}(X,\bar{X})]_D + [\mathcal{W}(X)]_F + [f_{\alpha\beta}(X)\bar{\lambda}_L^\alpha\lambda_L^\beta]_F. \tag{2.2}$$

The statement of the SU(2,2|1) symmetry of the action (2.2) includes, among others, the symmetries under the following set of local dilatations, with parameters $\sigma(x)$, for the metric and gravitino, for the scalars and spinors of the chiral multiplets, and for the vectors and spinors of the gauge multiplet, respectively:

$$g'_{\mu\nu} = e^{-2\sigma(x)}g_{\mu\nu}, \quad \psi'_\mu = e^{-(1/2)[\sigma(x)]}\psi_\mu,$$
$$X'_I = e^{\sigma(x)}X_I, \qquad \Omega'_I = e^{3/2\sigma(x)}\Omega_I, \tag{2.3}$$

$$W_\mu^{\alpha\prime}=W_\mu^\alpha, \qquad \lambda^{\alpha\prime}=e^{3/2\sigma(x)}\lambda^\alpha.$$

The function of scalars $\mathcal{N}(X,\bar{X})$ codifies the information on Kähler manifold. The holomorphic function of scalars $\mathcal{W}(X)$ codifies the superpotential. They transform as follows under local dilatations:

$$\mathcal{N}(X',\bar{X}')=e^{2\sigma(x)}\mathcal{N}(X,\bar{X}),$$
$$\mathcal{W}(X')=e^{3\sigma(x)}\mathcal{W}(X), \quad f_{\alpha\beta}(X')=f_{\alpha\beta}(X). \tag{2.4}$$

The important term in the conformal action which allows us to distinguish between conformal properties of helicity $\pm\frac{3}{2}$ and $\pm\frac{1}{2}$ gravitino is the following:

$$[\mathcal{N}]_D e^{-1}=\frac{1}{6}\mathcal{N}(X,\bar{X})[R+\bar{\psi}_\mu R^\mu+e^{-1}\partial_\mu(e\bar{\psi}\cdot\gamma\psi^\mu)]+\cdots. \tag{2.5}$$

The gauge fixing of the local dilatation of the conformally invariant action presented in Ref. [11] leads to the standard[2] Poincaré supergravity theory.

The dimensionful constants, Planck mass, and F and D terms appear via the gauge fixed value of the conformal compensator superfield, which we call *conformon*. The original $n+1$ complex variables X_I are split into one complex scalar conformon field Y, and n physical complex scalars z_i, which are Hermitian coordinates for parametrizing the Kähler manifold in the Poincaré theory. One defines

$$X_I=Yx_I(z_i), \tag{2.6}$$

and the local dilatation takes the form in which only the conformon Y transforms and the physical scalars do not transform under the local dilatation

$$Y'=e^{\sigma(x)}Y, \quad z_i'=z_i. \tag{2.7}$$

In these variables the gauge-fixing of the dilatational invariance (2.3), (2.7) is given by

$$\mathcal{N}(Y,z,\bar{z})=-3|Y|^2\exp\left[-\frac{1}{3}K(z,\bar{z})\right]=-3M_P^2, \tag{2.8}$$

where the first equation defines $K(z,\bar{z})$, which is the Kähler potential, and the second is the gauge fixing. Here $M_P \equiv M_{\rm Planck}/\sqrt{8\pi}\sim 2\times 10^{18}$ GeV. Thus the conformon field Y is frozen to $|Y|=M_P\exp[\frac{1}{6}K(z,\bar{z})]$, i.e., it becomes a functional of the Kähler potential and not an independent field. The theory becomes that of the Poincaré supergravity theory with

$$[\mathcal{N}]_D e^{-1}=-\frac{1}{2}M_P^2(R+\bar{\psi}_\mu R^\mu)+\cdots. \tag{2.9}$$

Here $R^\mu=\gamma^{\mu\rho\sigma}\mathcal{D}_\rho\psi_\sigma$, where $\mathcal{D}_\rho$ is a covariant derivative. The local dilatation of the metric and gravitino in Eq. (2.3) is not compensated anymore by the local dilatation of scalars,

$$Y=Y'\neq e^{\sigma(x)}Y, \quad \mathcal{N}(X,\bar{X})=\mathcal{N}(X',\bar{X}')\neq e^{2\sigma(x)}\mathcal{N}(X,\bar{X}). \tag{2.10}$$

The same happens with the $\mathcal{W}$ part of the theory:

$$\mathcal{W}(X)=Y^3M_P^{-3}W(z)\Rightarrow\mathcal{W}(X)=\mathcal{W}(X')\neq e^{3\sigma(x)}\mathcal{W}(X). \tag{2.11}$$

We have chosen here to give W mass dimension 3.

Thus, after the freezing of the conformon field some part of the transformation cannot be performed and therefore some parts of the phenomenological supergravity Lagrangian are not invariant under dilatations. One can try to change the dilatational weight for these fields to compensate the appearance of powers of M_P. However, this does not help, since the terms with derivatives on the conformon field are absent after the gauge fixing. The gravitino field equation which follows from the superconformal action is

$$R^\mu-\gamma^\mu\psi^\nu\partial_\nu\ln\sqrt{-\mathcal{N}}+\gamma\cdot\psi\partial_\mu\ln\sqrt{-\mathcal{N}}+\cdots=0. \tag{2.12}$$

In the FRW cosmological problems only time derivatives of the scalar fields are important, therefore in $\psi^\nu\partial_\nu\ln\sqrt{-\mathcal{N}}$ only the term $\bar{\psi}^0\partial_0\ln\sqrt{-\mathcal{N}}$ is relevant. After gauge fixing the conformal symmetry will be broken for configurations for which either

$$\gamma\cdot\psi\neq 0 \quad\text{or}\quad \psi_0\neq 0. \tag{2.13}$$

Only such terms will be sensitive to the absence of the terms $\partial_0\ln\sqrt{-\mathcal{N}}$ due to gauge fixing[3] when $-\mathcal{N}=3M_P^2$. The gravitino in the general theory with spontaneously broken supersymmetry will be massive. The states of a free massive spin-$\frac{3}{2}$ particle were studied by Auvil and Brehm in Ref. [18] (see also Ref. [2] for the nice review). A free massive gravitino has $\gamma\cdot\psi=0$. Helicity $\pm\frac{3}{2}$ states are given by transverse space components of gravitino ψ_i^T. Helicity $\pm\frac{1}{2}$ states are given by

[2] In fact two different gauges can be used to fix the R part of the superconformal symmetry. With the first choice $\mathcal{W}=\mathcal{W}^*$ we get the $N=1$ phenomenological supergravity [13] depending on $\mathcal{G}$, while the second one $Y=Y^*$ gives the version which is nonsingular in the limit of the vanishing superpotential $\mathcal{W}=0$. This version is closer to the one in Ref. [15] which was obtained by the superspace methods. The limit $\mathcal{W}=0$ from the first version has been discussed at the end of Ref. [16]. However, we find the second version more suitable for cosmology. It will be presented below.

[3] In Ref. [17], where an attempt to study conformal properties of gravitino has been made, it was assumed that $\gamma\cdot\psi=\psi^0=0$ and conformal symmetry of gravitino was deduced in the context of pure supergravity. Without scalars, however, pure supergravity does not support a cosmological background. In the presence of matter the assumption that $\gamma\cdot\psi=\psi^0=0$ is not valid and conformal symmetry is broken.

the time component of the gravitino field ψ_0. In cases when gravitino interacts with gravity and other fields, we will find that $\gamma\cdot\psi\neq 0$. It will be a function of ψ_0. Thus the consideration of superconformal symmetry lead us to a conclusion that helicity $\pm\frac{1}{2}$ states of gravitino are not conformally coupled to the metric. When these states are absent, the $\pm\frac{3}{2}$ helicity states are conformally coupled (up to the mass terms, as usual). Thus the conformal properties of gravitino are simple, as it is known for scalars: if the action has an additional term $\frac{1}{12}\phi^2 R$, the massless scalars are conformal. If this term is absent, the scalars are not conformal. Note that both these statements are derivable from the superconformal action. We will see the confirmation of this prediction in the solutions of the gravitino equations below.

As we already explained, our formulation starting with superconformal action [11] provides flexibility in the choice of the form of $N=1$ phenomenological supergravity. If we take $\mathcal{W}=\mathcal{W}^*$ gauge for R-symmetry, we get the action [13] depending on the combination of the Kähler potential and the superpotential, called $\mathcal{G}$. Here we use the $Y=Y^*$ gauge for R symmetry and present the form of the phenomenological Lagrangian in which the Kähler potential and the superpotential are not combined in one function. This allows to avoid problems which sometimes appear when the superpotential W vanishes. The action can be written as

$$\begin{aligned}
e^{-1}\mathcal{L}=&-\frac{1}{2}M_P^2[R+\overline{\psi}_\mu R^\mu+\mathcal{L}_{\text{SG,torsion}}]-M_P^2 g_i^j[(\hat\partial_\mu z^i)(\hat\partial^\mu z_j)+\overline{\chi}_j \not{\mathcal{D}}\chi^i+\overline{\chi}^i\not{\mathcal{D}}\chi_j]+(\operatorname{Re} f_{\alpha\beta})\left[-\frac{1}{4}F_{\mu\nu}^{\alpha}F^{\mu\nu\beta}-\frac{1}{2}\overline{\lambda}^\alpha\hat{\not{\mathcal{D}}}\lambda^\beta\right]\\
&+\frac{1}{4}i(\operatorname{Im} f_{\alpha\beta})[F_{\mu\nu}^{\alpha}\tilde{F}^{\mu\nu\beta}-\hat\partial_\mu(\overline{\lambda}^\alpha\gamma_5\gamma^\mu\lambda^\beta)]-M_P^{-2}e^K[-3WW^*+(\mathcal{D}^iW)g_i^{-1j}(\mathcal{D}_jW)]-\frac{1}{2}(\operatorname{Re} f)^{-1\alpha\beta}\mathcal{P}_\alpha\mathcal{P}_\beta\\
&+\frac{1}{8}(\operatorname{Re} f_{\alpha\beta})\overline{\psi}_\mu\gamma^{\nu\rho}(F_{\nu\rho}^{\alpha}+\hat{F}_{\nu\rho}^{\alpha})\gamma^\mu\lambda^\beta+\Bigg\{M_P^2 g_j^i\overline{\psi}_{\mu L}(\hat{\not\partial}z^j)\gamma^\mu\chi_i-\frac{1}{4}f_{\alpha\beta}^i\overline{\chi_i}\gamma^{\mu\nu}\hat{F}_{\mu\nu}^{-\alpha}\lambda_L^\beta+\frac{1}{2}e^{K/2}W\overline{\psi}_{\mu R}\gamma^{\mu\nu}\psi_{\nu R}\\
&+\overline{\psi}_R\cdot\gamma\left[\frac{1}{2}i\lambda_L^\alpha\mathcal{P}_\alpha+\chi_i e^{K/2}\mathcal{D}^iW\right]-e^{K/2}(\mathcal{D}^i\mathcal{D}^jW)\overline{\chi_i}\chi_j+\frac{1}{2}i(\operatorname{Re} f)^{-1\alpha\beta}\mathcal{P}_\alpha f_{\beta\gamma}^i\overline{\chi_i}\lambda^\gamma-2M_P^2\xi_\alpha^i g_i^j\overline{\lambda}^\alpha\chi_j\\
&+\frac{1}{4}M_P^{-2}e^{K/2}(\mathcal{D}^jW)g_j^{-1i}f_{\alpha\beta i}\overline{\lambda}_R^\alpha\lambda_R^\beta-\frac{1}{4}f_{\alpha\beta}^i\overline{\psi}_R\cdot\gamma\chi_i\overline{\lambda}_L^\alpha\lambda_L^\beta+\frac{1}{4}(\mathcal{D}^i\partial^j f_{\alpha\beta})\overline{\chi_i}\chi_j\overline{\lambda}_L^\alpha\lambda_L^\beta+\text{H.c.}\Bigg\}\\
&+M_P^2 g_j^i\left(\frac{1}{8}e^{-1}\varepsilon^{\mu\nu\rho\sigma}\overline{\psi}_\mu\gamma_\nu\psi_\rho\overline{\chi}^j\gamma_\sigma\chi_i-\overline{\psi}_\mu\chi^j\overline{\psi}^\mu\chi_i\right)+M_P^2\left(R_{ij}^{kl}-\frac{1}{2}g_i^k g_j^l\right)\overline{\chi}^i\chi^j\overline{\chi}_k\chi_l+\frac{3}{64}M_P^{-2}[(\operatorname{Re} f_{\alpha\beta})\overline{\lambda}^\alpha\gamma_\mu\gamma_5\lambda^\beta]^2\\
&-\frac{1}{16}M_P^{-2}f_{\alpha\beta}^i\overline{\lambda}_L^\alpha\lambda_L^\beta g_i^{-1j}f_{\gamma\delta j}\overline{\lambda}_R^\gamma\lambda_R^\delta+\frac{1}{8}(\operatorname{Re} f)^{-1\alpha\beta}(f_{\alpha\gamma}^i\overline{\chi_i}\lambda^\gamma-f_{\alpha\gamma i}\overline{\chi}^i\lambda^\gamma)(f_{\beta\delta}^j\overline{\chi_j}\lambda^\delta-f_{\beta\delta j}\overline{\chi}^j\lambda^\delta).
\end{aligned}\tag{2.14}$$

The L and R denote left and right chirality, e.g., $\lambda_L=\frac{1}{2}(1+\gamma_5)\lambda$, while for the χ, the chirality is indicated by the position of the index: χ_i is left chiral, while χ^i is right chiral. For the scalars, z^i is the complex conjugate of z_i. The Kähler metric is g_i^j, which is used also for covariant derivatives and Kähler curvature

$$\Gamma_i^{jk}=g_i^{-1l}\partial^j g_l^k,\quad R_{ij}^{kl}\equiv g_i^m\partial_j\Gamma_m^{kl}. \tag{2.15}$$

Extra i indices on quantities, e.g., $f_{\alpha\beta}^i$ denote derivatives, here the derivative of $f_{\alpha\beta}$ with respect to z_i. Other covariant derivatives and notations are (antisymmetrization $[\mu\nu]$ with weight 1, metric signature $(-,+,+,+)$, notation $\partial^i=\partial/\partial z_i$ and $\gamma_5=i\gamma_0\gamma_1\gamma_2\gamma_3$)

$$\hat{F}_{\mu\nu}^{\alpha}=F_{\mu\nu}^{\alpha}+\overline{\psi}_{[\mu}\gamma_{\nu]}\lambda^\alpha,\quad F_{\mu\nu}^{\alpha}=2\partial_{[\mu}W_{\nu]}^{\alpha}+W_\mu^\beta W_\nu^\gamma f_{\beta\gamma}^{\alpha},$$

$$\tilde{F}^{\mu\nu\alpha}=\frac{1}{2}e^{-1}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}^{\alpha},\quad \varepsilon^{0123}=i,\quad \hat\partial_\mu=\partial_\mu-W_\mu^\alpha\delta_\alpha,$$

$$R^\mu=\gamma^{\mu\rho\sigma}\mathcal{D}_\rho\psi_\sigma,$$

$$\mathcal{D}_{[\mu}\psi_{\nu]}=\left(\partial_{[\mu}+\frac{1}{4}\omega_{[\mu}^{ab}(e)\gamma_{ab}+\frac{i}{2}A_{[\mu}^B\gamma_5\right)\psi_{\nu]},$$

$$\begin{aligned}\hat{\mathcal{D}}_\mu\lambda^\alpha=&\left[\partial_\mu+\frac{1}{4}\left(\omega_\mu^{ab}(e)+\frac{1}{4}(2\tilde{\psi}_\mu\gamma^{[a}\psi^{b]}+\overline{\psi}^a\gamma_\mu\psi^b)\right)\gamma_{ab}\right.\\ &\left.+\frac{i}{2}A_\mu^B\gamma_5\right]\lambda^\alpha-W_\mu^\gamma\lambda^\beta f_{\beta\gamma}^{\alpha},\end{aligned}$$

$$\begin{aligned}\mathcal{D}_\mu\chi_i=&\left(\partial_\mu+\frac{1}{4}\omega_\mu^{ab}(e)\gamma_{ab}-\frac{i}{2}A_\mu^B\right)\chi_i\\ &+\Gamma_i^{jk}\chi_j\hat\partial_\mu z_k-W_\mu^\alpha\chi_j\partial^j\delta_\alpha z_i,\end{aligned}$$

$$A_\mu^B=\frac{i}{2}[(\partial_i K)\hat\partial z^i-(\partial^i K)\hat\partial z_i+3W_\mu^\alpha(r_\alpha^*-r_\alpha)],$$

$$\mathcal{D}^i W = \partial^i W + W \partial^i K,$$
$$\mathcal{D}^i \mathcal{D}^j W \equiv \partial^i \mathcal{D}^j W + (\partial^i K) \mathcal{D}^j W - \Gamma^{ij}_k \mathcal{D}^k W, \tag{2.16}$$

where δ_α is a symbol for the transformation under the gauge group for all fields. For the conformon field Y and for the rest of the scalars z_i we have

$$\delta_\alpha Y = Y r_\alpha(z), \quad \delta_\alpha z_i = \xi_{\alpha i}(z), \tag{2.17}$$

where r and ξ are $n+1$ holomorphic functions for every symmetry, such that the quantities in Eq. (2.2) are invariant. That determines also

$$\mathcal{P}_\alpha(z,z^*) = i M_P^2 [\xi_{\alpha i}(z) \partial^i K(z,z^*) - 3 r_\alpha(z)]$$
$$= i M_P^2 [-\xi^i_\alpha \partial_i K(z,z^*) + 3 r^*_\alpha(z^*)]. \tag{2.18}$$

See Ref. [11] for details.

The appearance of $M_P = |Y| e^{-K/B}$ in various places in this Lagrangian shows that the conformal symmetry is broken. One can rescale the fields with M_P so that they have standard kinetic terms. For our purpose it will be convenient to replace the scalar field z^i by Φ^i / M_P, chiral fermions χ_i by χ_i / M_P, and similar for the gravitino, $\psi_\mu \to \psi_\mu / M_P$.

III. GRAVITINO EQUATIONS

In general background metrics in the presence of complex scalar fields with nonvanishing vacuum expectation values (VEV's), the starting equation for the gravitino has in the left hand side the kinetic part R^μ and a rather lengthy right hand side which will be given in [11]. Apart of varying gravitino mass $m = M_P^{-2} e^{K/2} W$, the right hand side contains a chiral connection A^B_μ [see Eq. (2.16)] and various mixing terms such those in the fourth, fifth, and sixth lines of the phenomenological Lagrangian (2.14). For a self-consistent setting of the problem, the gravitino equation should be supplemented by the equations for the fields mixing with gravitino, as well as by the equations determining the gravitational background and the evolution of the scalar fields.

Let us make some simplifications. We consider the supergravity multiplet and a single chiral multiplet containing a complex scalar field $z = \Phi / M_P$ with a superpotential W and a single chiral fermion χ. This is a simple nontrivial extension which allows us to study gravitino in the nontrivial FRW cosmological metric supported by the scalar field. A nice feature of this model is that the chiral fermion χ can be gauged to zero so that the mixing between ψ_μ and χ in Eq. (2.14) is absent. We also can choose the nonvanishing VEV of the scalar field in the real direction, $\mathrm{Re}\,\Phi = \phi/\sqrt{2}$, $\mathrm{Im}\,\Phi = 0$, so that $A^B_\mu = 0$. The field $\phi = \sqrt{2}\,\mathrm{Re}\,\Phi$ plays the role of the inflation field.[4] Then from Eq. (2.14) we can obtain the master equation for the gravitino field

$$\not{\mathcal{D}} \psi_\mu + m \psi_\mu = \left(\mathcal{D}_\mu - \frac{m}{2} \gamma_\mu \right) \gamma^\nu \psi_\nu, \tag{3.1}$$

where gravitino mass $m = m[\phi(\eta)]$ is given by

$$m = e^{K/2} \frac{W}{M_P^2}. \tag{3.2}$$

Gravitino equation (3.1) is a curved spacetime generalization of the familiar gravitino equation $(\not{\partial} + m_\alpha) \psi_\mu = 0$ in a flat metric, where m_0 is a constant gravitino mass.

The generalization of the constraint equations $\partial^\mu \psi_\mu = 0$ and $\gamma^\mu \psi_\mu = 0$ reads

$$\mathcal{D}^\mu \psi_\mu - \not{\mathcal{D}} \gamma^\mu \psi_\mu + \frac{3}{2} m \gamma^\mu \psi_\mu = 0, \tag{3.3}$$

$$\frac{3}{2} m^2 \gamma^\mu \psi_\mu + m' a^{-2} \gamma^0 \gamma^i \psi_i = -\frac{1}{2} G_{\mu\nu} \gamma^\mu \psi^\nu,$$
$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R, \tag{3.4}$$

where primes stand for conformal time derivative ∂_η. It is important that the covariant derivative in these equations must include both the spin connection and the Christoffel symbols, otherwise equation $\mathcal{D}_\mu \gamma_\nu = 0$ used for the derivation of these equations is not valid.

The last equation will be especially important for us. Naively, one could expect that in the limit $M_P \to \infty$, gravitinos should completely decouple from the background. However, this equation implies that this is not the case for the gravitinos with helicity $\frac{1}{2}$. Indeed, from Eq. (3.4) one can find an algebraic relation between $\gamma^0 \psi_0$ and $\gamma^i \psi_i$:

$$\gamma^0 \psi_0 = \hat{A} \gamma^i \psi_i. \tag{3.5}$$

Here $\hat{A}$ is a matrix which will play a crucial role in our description of the interaction of gravitino with the varying background fields. If ρ and p are the background energy-density and pressure, we have $G^0_0 = M_P^{-2} \rho$, $G^i_k = -M_P^{-2} p \delta^i_k$, and one can represent the matrix $\hat{A}$ as follows:

$$\hat{A} = \frac{p - 3m^2 M_P^2}{\rho + 3m^2 M_P^2} + \gamma_0 \frac{2m' a^{-1} M_P^2}{\rho + 3m^2 M_P^2} = A_1 + \gamma_0 A_2. \tag{3.6}$$

[4]Typical time evolution of the homogeneous inflation field starts with the regime of inflation when ϕ slowly rolls down. One can construct a superpotential W which provides chaotic inflation for $\phi > M_P$. When $\phi(t)$ drops below $\simeq M_P$, it begins to oscillate coherent oscillations around the minimum of its the effective potential $V(\phi)$.

Note that in the limit $M_P\to\infty$ with fixed Φ in $z=\Phi M_P^{-1}$, one has $m=M_P^{-2}W$. If W does not blow up in this limit, this matrix $\hat{A}$ is given by

$$\hat{A}=\frac{p}{\rho}+\gamma_0\frac{2\dot{W}}{\rho}, \tag{3.7}$$

where the overdot stands for derivative ∂_t, and the relation between physical and conformal times is given by $dt=a(\eta)d\eta$. In the limit of flat case without moving scalars, $A=-1$.

For definiteness, we will consider the minimal Kähler potential $K=zz^*=\Phi\Phi^*/M_P^2$. In the models where the energy-momentum tensor is determined by the energy of a classical scalar field and Φ depends only on time we have

$$\rho=|\dot{\Phi}|^2+V, \quad p=|\dot{\Phi}|^2-V,$$
$$V(\Phi)=e^K\left(\left|\partial_\Phi W+\frac{\Phi^*}{M_P^2}W\right|^2-\frac{3}{M_P^2}|W|^2\right). \tag{3.8}$$

We will use the representation of gamma matrices where $\gamma_0=\mathrm{diag}(i,i,-i,-i)$. Then in Eq. (3.6) the combination $A=A_1+iA_2$ emerges. For a single chiral multiplet we obtain $|A|=1$. (One can show that $|A|=1$ for the theories with one chiral multiplet even if the Kähler is not minimal.) Therefore A can be represented as[5]

$$A=-\exp\left(2i\int_{-\infty}^{t}dt\,\mu(\eta)\right). \tag{3.9}$$

Using the Einstein equations, one obtains for μ (for minimal Kähler potential, and real scalar field):

$$\mu=M_P^{-2}e^{K/2}(\mathcal{D}\mathcal{D}W+W)+3(HM_P^{-1}\dot{\Phi}-mm_1)\times\frac{m_1}{M_P^{-2}\dot{\Phi}^2+m_1^2},$$
$$m_1\equiv m'=M_P^{-2}e^{K/2}\mathcal{D}W, \quad H\equiv\dot{a}a^{-1}=a'a^{-2}. \tag{3.10}$$

The expression for μ becomes much simpler and its interpretation is more transparent if the amplitude of oscillations of the field Φ is much smaller than M_P. In the limit $\Phi/M_P\to 0$ one has

$$\mu=\partial_\Phi\partial_\Phi W. \tag{3.11}$$

This coincides with the mass of both fields of the chiral multiplet (the scalar field and spin-$\frac{1}{2}$ fermion) in rigid supersymmetry. When supersymmetry is spontaneously broken, the chiral fermion, goldstino, is ''eaten'' by gravitino which becomes massive and acquires helicity $\pm\frac{1}{2}$ states in addition to helicity $\pm\frac{3}{2}$ states of the massless gravitino.

The matrix $\hat{A}$ does not become constant in the limit $M_P\to\infty$. The phase (3.9) rotates when the background scalar field oscillates. The amplitude and sign of A change two times within each oscillation. Consequently, the relation between $\gamma^0\psi_0$ and $\gamma^i\psi_i$ also oscillates during the field oscillations. This means that the gravitino with helicity $\frac{1}{2}$ (which is related to ψ_0) remains coupled to the changing background even in the limit $M_P\to\infty$. In a sense, the gravitino with helicity $\frac{1}{2}$ remembers its goldstino nature. This is the main reason why the gravitino production in this background in general is not suppressed by the gravitational coupling. The main dynamical quantity which is responsible for the gravitino production in this scenario will not be the small changing gravitino mass $m(t)$, but the mass of the chiral multiplet μ, which is much larger than m. As we will see, this leads to efficient production of gravitinos in the models where the mass of the ''Goldstino'' nonadiabatically changes with time.

We shall solve the master equation (3.1) using the constraint equations in the form (3.4) and (3.3). We use plane-wave ansatz $\psi_\mu\sim e^{\mathbf{ik\cdot x}}$ for the space-dependent part. Then ψ_i can be decomposed[6] [19] into its transverse part ψ_i^T, the trace $\gamma^i\psi_i$ and the trace $\mathbf{k}\cdot\psi$:

$$\psi_i=\psi_i^T+\left(\frac{1}{2}\gamma_i-\frac{1}{2}\hat{k}_i(\hat{\mathbf{k}}\cdot\gamma)\right)\gamma^j\psi_j+\left(\frac{3}{2}\hat{k}_i-\frac{1}{2}\gamma_i(\hat{\mathbf{k}}\cdot\gamma)\right)\hat{\mathbf{k}}\cdot\psi, \tag{3.12}$$

where $\hat{k}_i=k_i/|\mathbf{k}|$, so that $\gamma^i\psi_i^T=\hat{k}^i\psi_i^T=0$. We will relate $\gamma^i\psi_i$ with ψ_0 and with $\hat{\mathbf{k}}\cdot\psi$, so that, after use of the field equations there are two degrees of freedom associated with the transverse part ψ_i^T, which correspond to helicity $\pm\frac{3}{2}$; and two degrees of freedom associated with $\gamma^i\psi_i$ (or ψ_0) which correspond to helicity $\pm\frac{1}{2}$.

For the helicity $\pm\frac{3}{2}$ states we have to derive the equation for ψ_i^T. We apply decomposition (3.12) to the master equation (3.1) for $\mu=i$ and obtain[7]

$$\left(\gamma^a\delta_a^\mu\partial_\mu+\frac{a'}{2a}\gamma^0+ma\right)\psi_i^T=0. \tag{3.13}$$

In the limit of vanishing gravitino mass, the transverse part ψ_i^T is conformal with a weight $+\frac{1}{2}$. The transformation $\psi_i^T=a^{-1/2}\Psi_i^T$ reduces the equation for the transverse part to the free Dirac equation with a time-varying mass term ma. It is well known how to treat this type of equation (e.g., see Ref.

[5]Initial conditions at inflation at $\eta\to-\infty$ correspond to $p=-\rho$, $m'=0$, and $A=-1$, which gives $\mu(-\infty)=0$. Alternatively, we can start with inflation oscillations at $\eta=0$, which defines the phase up to some constant. The final results depend only on μ.

[6]We use now ψ_i with $i=1,2,3$ for the space components of ψ_μ, while for gamma matrices γ^i are space components of flat γ^a, and similarly for the 0 index.

[7]A similar equation obtained in Ref. [6] has a different coefficient in the term $(a'/2d)\gamma^0$ since they have omitted the Christoffel symbols in the covariant derivative. One can still use their equation if one replaces the curved space gravitino vector ψ_μ for which the equation was derived by the tangent space vector $\psi_a=e_a^\mu\psi_\mu$.

[4]). The essential part of Ψ_i^T is given by the time-dependent part of the eigenmode of the transversal component $y_T(\eta)$, which obeys second-order equation

$$y_T''+(k^2+\Omega_T^2-i\Omega_T')y_T=0, \tag{3.14}$$

where the effective mass is $\Omega_T=m(\eta)a(\eta)$.

The corresponding equation for gravitino with helicity $\frac{1}{2}$ is more complicated. We have to find $k^i\psi_i$ and $\gamma^i\psi_i$. The equation for the components $k^i\psi_i$ can be obtained from the constraint equation (3.3)

$$i\mathbf{k}\cdot\psi=\left(-\frac{a'}{a}\gamma_0+i\gamma\cdot\mathbf{k}-ma\right)\gamma^i\psi_i. \tag{3.15}$$

Combining all terms together, we obtain the on-shell decomposition for the longitudinal part

$$\psi_i=\psi_i^T+\left(\hat{k}_i\gamma\cdot\hat{\mathbf{k}}+\frac{i}{2\mathbf{k}^2}(3k_i-\gamma_i\mathbf{k}\cdot\gamma)\left(\frac{a'}{a}\gamma_0+ma\right)\right)\gamma^j\psi_j. \tag{3.16}$$

Now we can derive an equation for $\gamma^j\psi_j$. From the zero component of Eq. (3.1) we have

$$\frac{3a'}{2a}\gamma^0\psi_0+\left(\frac{3}{2}ma+i\mathbf{k}\cdot\gamma\right)\psi_0=(\gamma^i\psi_i)'-\frac{ma}{2}\gamma_0\gamma^i\psi_i. \tag{3.17}$$

This equation does not contain the time derivative of ψ_0. Substituting ψ_0 from Eq. (3.5) into Eq. (3.17), we get an equation for $\gamma^i\psi_i$

$$(\partial_\eta+\hat{B}-i\mathbf{k}\cdot\gamma\gamma_0\hat{A})\gamma^i\psi_i=0, \tag{3.18}$$

where

$$\hat{A}=-\exp\left(2\gamma_0\int_{-\infty}^{t}dt\mu(\eta)\right) \tag{3.19}$$

and

$$\hat{B}=-\frac{3a'}{2a}\hat{A}-\frac{ma}{2}\gamma_0(1+3\hat{A}). \tag{3.20}$$

We can split the spinors $\gamma^i\psi$ in eigenvectors of γ_0, $\gamma^i\psi_i=\theta_++\theta_-$, and $\theta_\pm=\frac{1}{2}(1\mp i\gamma_0)\gamma^i\psi_i$. From the Majorana condition it follows that $\theta_\pm(k)^*=\mp\mathcal{C}\theta_\mp(-k)$, where $\mathcal{C}$ is the charge conjugation matrix. In a representation with diagonal γ_0 the components $\theta_\pm$ correspond to the γ_0 eigenvalues $\pm i$. Acting on Eq. (3.18) with the Hermitian conjugate operation gives us a second-order differential equation on the θ_+. We choose for each k a spinor basis $u_{1,2}(k)$ for the two components of θ_+, and two independent solutions of the second-order differential equations $f_{1,2}(k,\eta)$. The general solution is given by

$$\theta_+=\sum_{\alpha,\beta=1}^{2}a^{\alpha\beta}(k)f_\alpha(k,\eta)u_\beta(k),$$

$$\theta_-=\frac{A^*}{|A|^2}\frac{k\cdot\gamma}{k^2}(\partial_\eta+B)\theta_+ =-\mathcal{C}^{-1}\sum_{\alpha,\beta=1}^{2}a^{*\alpha\beta}(-k)f_\alpha^*(-k,\eta)u_\beta^*(-k). \tag{3.21}$$

The last equality determines reality properties of the coefficients. Here we represented $\hat{B}$ as $B_1+\gamma_0B_2$ and defined $B=B_1+iB_2$, by analogy with the definitions for the matrix $\hat{A}$. By the substitution $f_\alpha(k,\eta)=E(\eta)y_L(\eta)$, with $E=(-A^*)^{1/2}\exp(-\int^\eta d\eta\,\mathrm{Re}\,B)$, equation for the functions $f_\alpha(k,\eta)$ is reduced to the final oscillatorlike equation for the time-dependent mode function $y_L(\eta)$:

$$y_L''+(k^2+\Omega_L^2-i\Omega_L')y_L=0. \tag{3.22}$$

Here

$$a^{-1}\Omega_L=\mu-\frac{3}{2}H\sin 2\int\mu dt+\frac{1}{2}m\left(1-3\cos 2\int\mu dt\right). \tag{3.23}$$

In the derivation of Eq. (3.22) it was essential that A has the form (3.9).

Finally we give the expression for the energy density of the longitudinal mode

$$a^3T_0^0=-\frac{3}{4\mathbf{k}^2}\overline{\psi^i}(-\mathbf{k})\gamma_i\left[i\gamma\cdot\mathbf{k}\left(ma-\frac{a'}{3a}\gamma_0\right)-\left(ma-\frac{a'}{a}\gamma_0\right)^2\right]\left(ma+\frac{a'}{a}\gamma_0\right)\gamma^j\psi_j(\mathbf{k}). \tag{3.24}$$

In the flat spacetime limit $A_1=-1$, $\gamma^i\psi_j\sim\mathbf{k}/m_0$. From T_0^0 we can define the occupation number of gravitinos n_k of energy ω_k at given mode $T_0^0=\int d^3k\omega_kn_k$, where n_k is expressed through a bilinear combination of mode functions $y_L(\eta)$.

IV. GRAVITINO PRODUCTION

One could expect that the gravitino production may begin already at the stage of inflation, due to the breaking of conformal invariance. However, there is no massive particle production in de Sitter space (i.e., as long as one can neglect the motion of the scalar field during inflation). Indeed, expansion in de Sitter space is in a sense fictitious; one can always use coordinates in which it is collapsing or even static. An internal observer living in de Sitter space would not see any time-dependence of his surroundings caused by particle production; he will only notice that he is surrounded by particle excitations at the Hawking temperature $H/2\pi$.

Gravitino production may occur at the stage of inflation due to the (slow) motion of the scalar field, but the most interesting effects occur at the end of inflation, when the scalar field ϕ rapidly rolls down toward the minimum of its effective potential $V(\phi)$ and oscillates there. During this stage the vacuum fluctuations of the gravitino field are amplified, which corresponds to the gravitino production (in a squeezed state).

Production of gravitinos with helicity $\frac{3}{2}$ is described in terms of the mode function $y_T(\eta)$. This function obeys Eq. (3.14) with $\Omega_T = ma$, which is suppressed by M_P^{-2}. Nonadiabaticity of the effective mass $\Omega_T(\eta)$ results in the departure of $y_T(\eta)$ from its positive frequency initial condition $e^{ik\eta}$, which can be interpreted as particle production. The theory of this effect is completely analogous to the theory of production of usual fermions of spin $\frac{1}{2}$ and mass m [4]. Indeed, Eq. (3.14) coincides with the basic equation which was used in [4] for the investigation of production of Dirac fermions during preheating.

The description of production of gravitinos with helicity $\frac{1}{2}$ is similar but somewhat more involved. The wave function of the helicity $\frac{1}{2}$ gravitino is a product of the factor $E(\eta)$ and the function $y_L(\eta)$. The factor $E(\eta)$ does not depend on momenta and controls only the overall scaling of the solution. It is the function $y_L(\eta)$ that controls particle production which occurs because of the nonadiabatic variations of the effective mass parameter $\Omega_L(\eta)$. The function $y_L(\eta)$ obeys Eq. (3.22) with the effective mass $\Omega_L(\eta)$, which is given by the superposition (3.23) of all three mass scales in the problem μ, H, and m.

In different models of the inflation, different terms of Ω_L will have different impact on the helicity $\frac{1}{2}$ gravitino production. The strongest effect usually comes from the largest mass scale μ, if it is varying with time. This makes the production of gravitinos of helicity $\frac{1}{2}$ especially important.

To fully appreciate this fact, one should note that if instead of considering supergravity one would consider supersymmetry (SUSY) with the same superpotential, then the goldstino χ (which is eaten by the gravitino in supergravity) would have the mass $\partial_\Phi \partial_\Phi W$, which coincides with μ in the limit of large M_P, see Eq. (3.11). As a result, Eq. (3.22) describing creation of gravitinos with helicity $\frac{1}{2}$ at $\phi \ll M_P$ looks exactly as the equation describing creation of Goldstinos in SUSY. That is why production of gravitinos with helicity $\frac{1}{2}$ may be very efficient: in a certain sense it is not a gravitational effect. (On the other hand, the decay rate of gravitinos $\Gamma \sim m^3/M_P^2$ is very small because it is suppressed by the gravitational coupling M_P^{-2}.)

This does not mean that one can always neglect terms proportional to H and m as compared to μ, and that production of gravitinos with helicity $\frac{3}{2}$ can always be neglected. In order to understand the general picture, we will consider several toy models where the effective potential at the end of inflation has simple shape such as $V \sim \phi^n$. We will not discuss here the problem of finding superpotentials which lead to such potentials (and inflation) at $\phi > M_P$ [20], because we are only interested in what happens after the end of inflation, which occurs at $\phi \sim M_P$.

First consider the superpotential $W = \frac{1}{2} m_\phi \Phi^2$. At $\phi \ll M_P$ it leads to the simple quadratic potential $V = (m_\phi^2/2)\phi^2$. The parameter μ in this case coincides with the inflaton mass m_ϕ. In a realistic inflationary model one should take $m_\phi \sim 10^{13}$ GeV, which is equal to $5 \times 10^{-6} M_P$ [20]. Hubble constant during the field oscillations is given by $m_\phi \phi_0/\sqrt{6} M_P$, where $\phi_0(t)$ here is the amplitude of the field oscillations, which decreases during the expansion of the universe. The gravitino mass is given by $m = m_\phi \phi^2/4M_P^2$.

Thus, at the end of inflation in this model, which occurs at $\phi \sim M_P$, all parameters determining the behavior of the gravitino wave function are of the same order, $\mu \sim m_\phi \sim H \sim m$. However, later the amplitude of ϕ decreases as $\phi_0 \sim 1.5 M_P / m_\phi t \sim M_P/4N$, where N is the number of oscillations of the field ϕ after the end of inflation [3]. Thus already after a single oscillation there emerges a hierarchy of scales, $\mu \sim m_\phi \gg H \gg m$.

Since $m_\phi = \text{const}$, after the first oscillation the parameter μ becomes nearly constant, the parameters H and m become very small, and their contribution to the gravitino production becomes strongly suppressed. As a result, the dominant contribution to the gravitino production in this model occurs within the first oscillation of the scalar field after the end of inflation. Each of the parameters μ, H, and m at the end of inflation changes by $O(m_\phi)$ within the time $O(m_\phi^{-1})$. This means that (because of uncertainty relation) gravitinos of both helicities will be produced, they will have physical momenta $k = O(m_\phi)$, and their occupation numbers n_k will be not much smaller than $O(1)$. This leads to the following conservative estimate of the number density of produced gravitinos $n_{3/2} \sim 10^{-2} m_\phi^3$.

Now let us assume for a moment that all energy of the oscillating field ϕ transfers to thermal energy $\sim T^4$ within one oscillation of the field ϕ. This produces gas with entropy density $s \sim T^3 \sim (m_\phi^2 M_P^2/2)^{3/4}$. As a result, the ratio $n_{3/2}$ to the entropy density becomes

$$\frac{n_{3/2}}{s} \sim 10^{-2} \left(\frac{m_\phi}{M_P} \right)^{3/2} \sim 10^{-10}. \tag{4.1}$$

This violates the bound $n_{3/2}/s < 10^{-14}$ for the gravitino with $m \sim 10^2$ GeV by about 4 orders of magnitude. Thus one may encounter the gravitino problem even if one neglects their thermal production.

In this particular model one can overcome the gravitino problem if reheating and thermalization occurs sufficiently late. Indeed, during the post-inflationary expansion the number density of gravitinos decreases as a^{-3}. The energy density of the oscillating massive scalar field $\rho = m_\phi^2 \phi_0^2(t)/2$ also decreases as a^{-3}. But the entropy produced at the moment of reheating is proportional to $\rho^{3/4}$, so it depends on the scale factor at the moment of reheating as $a^{-9/4}$. If reheating occurs late enough (which is necessary anyway to avoid thermal production of gravitinos), the ratio $n_{3/2} s \sim 10^{-10} a^{-3/4}$ becomes small, and the gravitino problem does not appear.

But this simple resolution is not possible in some other models. As an example, consider the model with the superpotential $W = \sqrt{\lambda} \Phi^3/3$, which at $\phi \ll M_P$ leads to the effective potential $\lambda \phi^4/4$. The oscillations of the scalar field near the minimum of this potential are described by elliptic cosine

$\phi(\eta)=(\phi_0/a)\mathrm{cn}(\sqrt{\lambda}\phi_0\eta,1/\sqrt{2})$. The frequency of oscillations is $0.8472\sqrt{\lambda}\phi_0$ and initial amplitude $\phi_0\simeq M_P$ [3].

The parameter μ for this model is given by $\mu=\sqrt{2\lambda}\phi$. It rapidly changes in the interval between 0 and $\sqrt{2\lambda}\phi_0$. Initially it is of the same order as H and m, but then H and m rapidly decrease as compared to μ, and therefore the oscillations of μ remain the main source of the gravitino production. In this case production of gravitinos with helicity $\frac{1}{2}$ is much more efficient than that of helicity $\frac{3}{2}$.

The theory of production of gravitinos with helicity $\frac{1}{2}$ in this model is similar to the theory of production of spin-$\frac{1}{2}$ fermions with mass $\sqrt{2\lambda}\phi$ by the coherently oscillating scalar field in the theory $\lambda\phi^4/4$. This theory has been investigated in Ref. [4]. The result can be formulated as follows. Even though the expression for Ω contains a small factor $\sqrt{2\lambda}$, one cannot use the perturbation expansion in λ. This is because the frequency of the background field oscillations is also proportional to $\sqrt{\lambda}$. Growth of fermionic modes (3.22) occurs in the nonperturbative regime of parametric excitation. The modes get fully excited with occupation numbers $n_p\simeq\frac{1}{2}$ within about ten oscillations of the field ϕ, and the width of the parametric excitation of fermions in momentum space is about $\sqrt{\lambda}\phi_0$. This leads to the following estimate for the energy density of created gravitinos:

$$\rho_{3/2}\sim(\sqrt{\lambda}\phi_0)^4\sim\lambda V(\phi_0), \tag{4.2}$$

and the number density of gravitinos

$$n_{3/2}\sim\lambda^{3/4}V^{3/4}(\phi_0). \tag{4.3}$$

Now let us suppose that at some later moment reheating occurs and the energy density $V(\phi_0)$ becomes transferred to the energy density of a hot gas of relativistic particles with temperature $T\sim V^{1/4}$. Then the total entropy of such particles will be $s\sim T^3\sim V^{3/4}$, so that

$$\frac{n_{3/2}}{s}\sim\lambda^{3/4}\sim10^{-10}. \tag{4.4}$$

This result violates the cosmological constraints on the abundance of gravitinos with mass $\sim10^2$ GeV by 4 orders of magnitude. In this model the ratio $n_{3/2}/s$ does not depend on the time of thermalization, because both $n_{3/2}$ and $V(\phi_0)^3/4$ decrease as a^{-3}. To avoid this problem one may, for example, change the shape of $V(\phi)$ at small ϕ, making it quadratic.

The situation in models with $V(\phi)\sim\phi^n$ for $n>4$ is even more dangerous because the energy density of the oscillating field ϕ in such models decreases faster than a^{-4}, so the entropy of the particles produced during reheating is suppressed stronger than by the factor of a^{-3}. The later reheating occurs in such models, the greater will be the resulting ratio $n_{3/2}/s$.

The most dangerous situation occurs in the class of inflationary models where the effective potential $V(\phi)$ does not have a minimum, but instead monotonically decreases and becomes flat at $\phi\to\infty$. Such models have been studied recently by many authors [21,5], which gave them many different names: deflation, kination, and quintessential inflation. Following Ref. [5], we will call them nonoscillatory (NO) models, which reflects an unusual nonoscillatory behavior of the scalar field after inflation. This behavior implies that the standard mechanism of reheating does not work in NO models. Therefore until very recently it was assumed that in such models all particles are produced gravitationally, due to the breaking of conformal invariance [21]. A typical model of such type has the effective potential which is given by $V(\phi)\approx\lambda\phi^4/4$ at $\phi<0$, and then it rapidly vanishes as ϕ becomes positive, so that $V(\phi)\to0$ at $\phi\to\infty$.

We will leave apart the question whether it is easy to obtain realistic versions of NO models in supergravity. For us it is only important that in such models the parameters μ, H, and m change by $O(\sqrt{\lambda}\phi_0)=O(H)$ during the time when the field ϕ rolls from $\phi_0\sim M_P$ to 0. Just as in one of the examples considered above, this should lead to production of gravitinos with number density which can be estimated as $\sim10^{-2}H^3$. This number is of the same order as the number density of all other conformally noninvariant particles produced by gravitational effects in the scenario of Ref. [21]. Thus, barring the subsequent dilution of gravitinos by some late-time entropy release, one has $n_{3/2}/s=O(1)$, which contradicts observational data by 14 orders of magnitude.

This problem of NO models can be resolved if one assumes that the scalar field ϕ interacts with some other particles σ with a sufficiently large coupling constant g. This leads to production of particles in the context of the instant preheating scenario [22]. This mechanism is much more efficient than the gravitational particle production studied in [21], and the entropy s of produced particles becomes much greater than $O(H^3)$. This leads to a strong suppression of $n_{3/2}/s$ [5].

But do we really have the gravitino problem in all of these models? In our investigation we studied only the models with one chiral multiplet. This is good enough to show that nonthermal gravitino production may indeed cause a serious problem, but much more work should be done in order to check whether the problem actually exists in realistic models with several different chiral and vector multiplets.

First of all, one should write and solve a set of equations involving several multiplets. Even in the case of one multiplet it is extremely difficult, and the results which we obtained are very unexpected. The situation with many multiplets is even more involved. One possibility is to consider the limit $M_P\to\infty$, since the most interesting effects should still exist in this limit.

But this is not the only problem to be considered. In the toy models studied in this section with $W=\frac{1}{2}m_\phi\Phi^2$ and $W=\frac{1}{3}\sqrt{\lambda}\Phi^3$, the superpotential W and the gravitino mass vanish in the minimum of the potential at $\phi=0$. Then after the end of oscillations supersymmetry is restored, superhiggs effect does not occur and instead of massive gravitinos we have ordinary chiral fermions. In order to study production of gravitino with helicity $\frac{1}{2}$ with nonvanishing mass $m\sim10^2$ GeV one must introduce additional terms in the superpotential, and make sure that these terms do not lead to a large vacuum energy density.

Models with one chiral superfield which satisfy all of these requirements do exist. The simplest one is the Polonyi model with $W=\alpha[(2-\sqrt{3})M_p+\Phi]$. One can introduce various generalizations of this model. However, potentials in all models of this type that we were able to construct are much more complicated than the potentials of the toy models studied in this section. In particular, if one simply adds small terms $\alpha+\beta\Phi+\ldots$ to the superpotentials $\sim\Phi^2$ or Φ^3, one typically finds that V becomes negative in the minimum of the potential, which sometimes becomes shifted to the direction ϕ_2, where $\Phi=(\phi_1+i\phi_2)/\sqrt{2}$. This problem can be easily cured in realistic theories with many multiplets, which is another reason to study such models.

It would be most important to verify, in the context of these models, validity of our observation that the probability of production of gravitinos of helicity $\frac{1}{2}$ is not suppressed by the gravitational coupling. We have found, for example, that the ratio $n_{3/2}/s$ for the gravitinos with helicity $\frac{1}{2}$ in the model $\lambda\phi^4/4$ is suppressed by $\lambda^{-3/4}$. This suppression is still rather strong because the coupling constant λ is extremely small in this model, $\lambda\sim10^{-13}$. However, in such models as the hybrid inflation scenario all coupling constants typically are $O(10^{-1})$ [23]. If production of gravitinos in such models is suppressed only by powers of the coupling constants, one may need to take special precautions in order to avoid producing excessively large number of gravitinos during preheating. We will return to this question and present a more detailed description of the effects discussed above in a separate publication [11].

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