Naked singularity explosion

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It is known that the gravitational collapse of a dust ball results in naked singularity formation from an initial density profile which is physically reasonable. In this paper, we show that explosive radiation is emitted during the formation process of the naked singularity.

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It is known that the gravitational collapse of an inhomogeneous dust ball results in shell-focusing naked singularity formation [1–4]. It also has been shown that naked singularity formation is possible from the spherical collapse of a perfect fluid with a very soft equation of state [5–8]. Moreover, a kind of runaway collapse in Newtonian gravity is similar to naked singularity formation in general relativity in many respects. These strongly suggest that the gravitational collapse will often involve the drastic growth of spacetime curvature outside the event horizon.

From this point of view, a number of researchers have examined emission during naked singularity formation. In classical theory, Nakamura, Shibata, and Nakao [9] suggested that the forming naked singularity in the collapse of a prolate spheroid may be a strong source of gravitational waves. Recently, Iguchi, Nakao, and Harada [10] and Iguchi, Harada, and Nakao [11,12] examined the behavior of nonspherical linear perturbations of spherical dust collapse. They reported rather milder instability in [12].

Hawking [13] showed that thermal radiation is emitted from the gravitational collapse to a black hole by quantum effects. In the formation of a globally naked singularity, the spacetime curvature grows unboundedly and the strongly curved region can be seen by a distant observer, unlike in the formation of a black hole. This fact suggests that the forming naked singularity may be a strong source of radiation owing to quantum effects. In this context, Ford and Parker [14] calculated the radiation during the formation of a shell-crossing naked singularity and found that the luminosity is finite. This will be because the shell-crossing singularity is very weak. It is known that the shell-focusing singularity is stronger than the shell-crossing singularity. Hiscock, Williams, and Eardley [15] showed the diverging luminosity during the formation of the shell-focusing singularity in the spherically symmetric, self-similar collapse of a null dust. Barve, Singh, Vaz, and Witten [16] also showed the diverging luminosity during the shell-focusing singularity in the self-similar collapse of a dust ball. See also [17,18].

The last two examples in which the diverging luminosity is emitted are self-similar collapse. However, the self-similar collapse is a particular solution among gravitational collapse solutions in general relativity. Moreover, it is uncertain whether or not the central part of the realistic spherical collapse with nonzero pressure tends to be self-similar in a strong-gravity regime such as shell-focusing singularity formation (cf. [8]). For the self-similar collapse of a dust ball, it has been shown that the redshift at the center diverges to infinity and that the curvature strength of the naked singularity is very strong. In fact, it is known that this solution does not allow an initial density profile which is a C^∞ function with respect to the local Cartesian coordinates. For the C^∞ case, the features are much different. The redshift is finite and the curvature strength is not very strong [19–21]. Though the Einstein equation does not require such strong differentiability of initial data, we usually set such initial data in most astrophysical numerical simulations. We should also comment that this model of the collapsing dust ball will be valid until perturbations sufficiently grow due to the reported mild instability. In this paper, we examine radiation during the naked singularity formation in the collapse of an inhomogeneous dust ball from an initial density profile which is a C^∞ function. We use the units in which $G = c = \hbar = 1$.

We consider both minimally and conformally coupled massless scalar fields in four-dimensional spacetime which is spherically symmetric and asymptotically flat. Let u and v be null coordinates such that they are written as $u \approx T - R$ and $v \approx T + R$ in the asymptotic region with the quasi-Minkowskian spherical coordinates (T, R, θ, ϕ). An outgoing null ray $u = $ const arriving on $\mathscr{I}^+$ can be traced back through the geometry becoming an incoming null ray $v = $ const originating from $\mathscr{I}^-$ with v. This gives the relation between u and v and we define the function $G(u)$ by $v \equiv G(u)$. Here, we assume that geometrical optics approximation is valid, which implies that the trajectory of the null ray gives a surface of a constant phase of the scalar field. Then, the luminosity L_{lm} for the minimally coupled scalar field and the luminosity $\hat{L}_{lm}$ for the conformally coupled scalar field for fixed l and m are given through the point-splitting regularization as [14]

$$L_{lm} = \frac{1}{48\pi}\left(\frac{G''}{G'}\right)^2 - \frac{1}{24\pi}\left(\frac{G''}{G'}\right)', \qquad \hat{L}_{lm} = \frac{1}{48\pi}\left(\frac{G''}{G'}\right)^2.$$

(1)

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This implies that the luminosity depends on how the scalar field couples with gravity. However, if $G''/G'|_{u=a} = G''/G'|_{u=b}$ holds, the amounts of the radiated energy during $a \leq u \leq b$ for both fields are the same. We should note that the geometrical optics approximation is only valid for smaller l. Hereafter we omit the suffix l and m. It is noted that these results are free of ambiguity coming from local curvature because the regularization is done in flat space-time.

The spherically symmetric collapse of a dust fluid is exactly solved [22,23]. The solution is called the Lemaître-Tolman-Bondi (LTB) solution. For simplicity, we assume the marginally bound collapse. The metric is given by

$$ds^2 = -dt^2 + R_{,r}^2(t,r)dr^2 + R^2(t,r)d\Omega^2 \qquad (2)$$

in the synchronous comoving coordinates with $d\Omega^2 \equiv d\theta^2 + \sin^2\theta d\phi^2$. The energy density is given by

$$\epsilon = \frac{F'(r)}{8\pi R^2 R_{,r}}, \qquad (3)$$

where $F(r)$ is equal to twice the Misner-Sharp mass. We rescale the radial coordinate r as $R(0,r)=r$. Then, R is given as

$$R = r\left(1 - \frac{3}{2}\sqrt{\frac{F}{r^3}}t\right)^{2/3}. \qquad (4)$$

The singularity occurs at the time $t_s(r) \equiv (2/3)\sqrt{r^3/F}$. We denote the time of occurrence of singularity at the center as $t_0 \equiv t_s(0)$. From Eq. (3), if we require that an initial density profile at $t=0$ is a C^∞ function with respect to the local Cartesian coordinates, $F(r)$ is expanded around $r=0$ as

$$F(r) = F_3 r^3 + F_5 r^5 + F_7 r^7 + \cdots, \qquad (5)$$

where we assume that F_3 is positive. If we assume the marginally bound collapse and the mass function as Eq. (5), then the central shell-focusing singularity is naked if and only if F_5 is negative [1–4,24–26]. At an arbitrary radius $r=r_{sf}$, the LTB spacetime can be matched with the Schwarzschild spacetime

$$ds^2 = -\left(1 - \frac{2M}{R}\right)dT^2 + \left(1 - \frac{2M}{R}\right)^{-1}dR^2 + R^2 d\Omega^2, \qquad (6)$$

with $M = F(r_{sf})/2$. The relation between T and t is given at $r=r_{sf}$ as

$$T = t - \frac{2r^{3/2}}{3\sqrt{2M}} - 2\sqrt{2MR} + 2M\ln\frac{\sqrt{R}+\sqrt{2M}}{\sqrt{R}-\sqrt{2M}}. \qquad (7)$$

The null coordinates u and v in the Schwarzschild spacetime are defined as

$$u \equiv T - R_*, \quad v \equiv T + R_* \qquad (8)$$

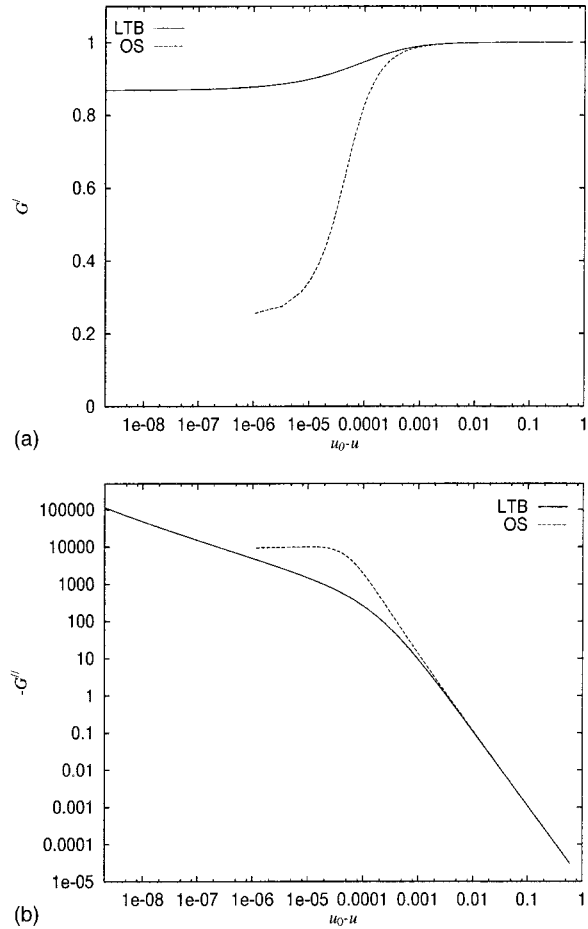with $R_* \equiv R + 2M\ln[(R/2M)-1]$.

FIG. 1. (a) $G'(u)$ and (b) $G''(u)$ for the LTB spacetime and the OS spacetime.

We can determine the function G by solving the trajectories of outgoing and ingoing null rays in the dust and calculating u and v through Eq. (8) at the time when the outgoing and ingoing null rays reach the surface boundary. The trajectories of null rays in the dust are given by the following ordinary differential equation:

$$\frac{dt}{dr} = \pm R_{,r}, \qquad (9)$$

where the upper and lower signs denote outgoing and ingoing null rays, respectively. We have numerically solved Eq. (9) by the Runge-Kutta method of the fourth order. We have carried out the quadruple precision calculation for retaining accuracy. We have chosen the mass function as $F(r) = F_3 r^3 + F_5 r^5$. We find that the central singularity is globally naked for very small r_{sf} if we fix the value of F_3 and F_5. Although we have calculated several models, we only display the numerical results for the model with $F_3=1$, $F_5 = -2$ and $r_{sf}=0.02$ in an arbitrary unit because the features are the same if the singularity is globally naked. The total gravitational mass M is given by $M = 3.9968 \times 10^{-6}$ for this model.

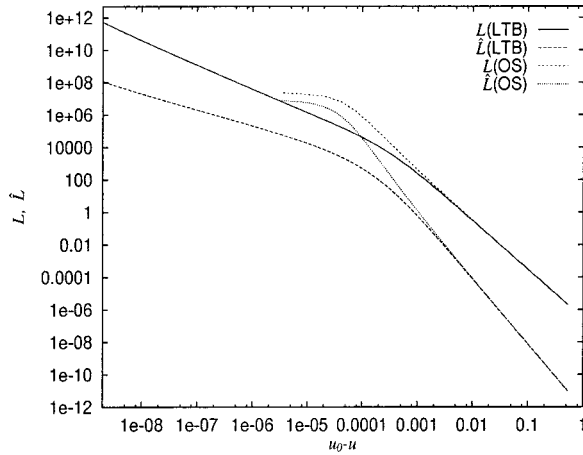See Fig. 1. We define u_0 as the retarded time of the ear-

FIG. 2. Luminosity for minimally and conformally coupled scalar fields in the LTB spacetime and in the OS spacetime.

liest light ray which originates from the singularity. In Fig. 1(a), the first derivative $G'(u)$ is plotted. This shows that $G'(u)$ does not diverge but converges to some positive value A with $0<A<1$. In Fig. 1(b), it is found that the second derivative $G''(u)$ does diverge as $u \to u_0$. This figure shows that the behaviors of growth of $G''(u)$ are different from each other for $10^{-4} \lesssim u_0 - u$ and for $0 < u_0 - u \lesssim 10^{-4}$. For $10^{-4} \lesssim u_0 - u$, the dependence on u is written as $G'' \propto -(-u)^{-2}$, while, for $0 < u_0 - u \lesssim 10^{-4}$, the dependence is written as $G'' \propto -(u_0 - u)^{-1/2}$. Here we determine a dimensionful constant of proportion by physical consideration. We assume that the early time behavior is due to the collapse of the dust as a whole while the late time behavior is due to the growth of the central curvature. This assumption will be justified later. First we should note that the coefficient must be written using the initial data because it must not depend on time. Next we should note that we can regard any $t = \text{const} < t_0$ hypersurface as an initial hypersurface. Therefore, the coefficient must be independent of the choice of an initial slice. For the early time behavior, the only possible quantity is the gravitational mass M of the dust cloud. For the late time behavior, the only possible quantity is $\omega_s \equiv l_0^6/t_0^7 = (3/2)^7 F_3^{13/2} (-F_5)^{-3}$, where t_0 is given as $t_0 = (2/3) F_3^{-1/2}$ and l_0 denotes the scale of inhomogeneity defined as $l_0 \equiv (-F_5/F_3)^{-1/2}$. We call ω_s a singularity frequency. The ω_s is independent of the choice of an initial slice because the mass function $F(r)$ is written in terms of R as

$$F(r) = F_3 \left(\frac{t_0 - t}{t_0} \right)^{-2} R^3 + F_5 \left(\frac{t_0 - t}{t_0} \right)^{-13/3} R^5 + \cdots, \quad (10)$$

around the center. Thus, we can write $G'' \approx -f_e M(-u)^{-2}$ for $-u \gg M$, and $G'' \approx -f_l A \omega_s^{1/2} (u_0 - u)^{-1/2}$ for $0 < u_0 - u \ll \omega_s^{-1}$, where f_e and f_l are dimensionless positive constants of order unity. The turning point from the early time behavior to the late time behavior is roughly estimated as $u_0 - u \approx (M \omega_s)^{2/3} \omega_s^{-1}$. These estimates show a good agreement with the numerical results.

Let us consider the luminosity which is calculated by Eq. (1). The numerical results are displayed in Fig. 2. We can

also write an analytic expression for the luminosity using Eq. (1). In Fig. 1(a), we can find $G' \approx A$ for the late time behavior. Then, the luminosity for the late time behavior is obtained as

$$L \approx \frac{1}{48\pi} f_l \omega_s^{1/2} (u_0 - u)^{-3/2}, \quad \hat{L} \approx \frac{1}{48\pi} f_l^2 \omega_s (u_0 - u)^{-1}. \quad (11)$$

Therefore the luminosity diverges to positive infinity for both fields as $u \to u_0$. The radiated energy is obtained by integrating the luminosity with u. The radiated energy for the late time behavior is estimated as

$$E \approx \frac{1}{24\pi} f_l \omega_s^{1/2} (u_0 - u)^{-1/2}, \quad \hat{E} \approx \frac{1}{48\pi} f_l^2 \omega_s \ln \frac{(M\omega_s)^{2/3}}{\omega_s (u_0 - u)}. \quad (12)$$

Therefore, the amounts of the total radiated energy diverge to positive infinity for the both fields as $u \to u_0$. In realistic situations, we may assume that the naked singularity formation is prevented by some mechanism and that the quantum particle creation is ceased at the time $u_0 - u \approx \Delta t$, which implies that G''/G' vanishes for $u_0 - u \lesssim \Delta t$. Then, the second term in the expression of the luminosity L gives no contribution to the total radiated energy. Therefore, the amounts of the total energy for both fields are the same, i.e.,

$$E = \hat{E} \approx \frac{1}{48\pi} f_l^2 \omega_s \ln \frac{(M\omega_s)^{2/3}}{\omega_s \Delta t}. \quad (13)$$

From the numerical results and physical discussion above, we obtain the following formula for the late time behavior:

$$G(u) \approx A(u - u_0) - \frac{4}{3} A f_l \omega_s^{1/2} (u_0 - u)^{3/2} + \text{const.} \quad (14)$$

For comparison and for a test of our numerical code, we have also calculated the function G for the Oppenheimer-Snyder (OS) collapse to a black hole which is given by $F(r) = F_3 r^3$. We only display the numerical results for the model with $F_3 = 1$ and $r_{sf} = 0.02$ in an arbitrary unit. M is given by $M = 4 \times 10^{-6}$ for this model. The results are plotted also in Figs. 1 and 2. Because we cannot define u_0 for the OS spacetime, we plot the numerical results by setting $u_0 = 0$. For $u > 0$, the numerical results show the well-known behavior

$$G(u) \approx -\text{const} \times \exp\left(\frac{-u}{4M} \right) + v_h, \quad (15)$$

$$L \approx \hat{L} \approx \frac{1}{768\pi M^2}, \quad (16)$$

where the ingoing null ray with v_h is reflected to the outgoing null ray which is on the event horizon. The numerical results of L and $\hat{L}$ for the OS collapse show a good agree-

ment with Eq. (16). In Figs. 1 and 2, we can find that the assumption concerning the early time and late time behaviors is justified.

If a back reaction of quantum effects would not become important until a considerable fraction of the total energy of the system is radiated away, the emitted energy could amount to $E \sim 10^{54}(M/M_{\odot})$erg. Thus, the naked singularity explosion would be a new candidate for a source of ultra-high-energy cosmic rays. It may also be a candidate for the central engine of a gamma ray burst. In order to study such possibilities, we need to transform the obtained intrinsic quantities to observed quantities by taking possible reactions of created energetic particles into consideration. It is noted that, since we have obtained the formula for the late time behavior of the function of $G(u)$, we can determine the spectrum of the radiation and thereby estimate the validity of the geometrical optics approximation. We are now obtaining positive evidences for the consistency [27].

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