Quark-loop amplitudes for $W^{\pm}H^{\mp}$ associated hadroproduction

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In this addendum to our paper, Phys. Rev. D **59**, 015009 (1999), we list analytic results for the helicity amplitudes of the partonic subprocess $gg\to W^-H^+$ induced by virtual quarks.

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In a recent paper [1], we studied the hadroproduction of a charged Higgs boson in association with a W boson at the CERN Large Hadron Collider (LHC) in the context of the two-Higgs-doublet model of type II, which serves as the Higgs sector for the minimal supersymmetric extension of the standard model (SM). This reaction dominantly proceeds via the partonic subprocesses $b\overline{b}\to W^{\pm}H^{\mp}$ at the tree level (see Fig. 1 in Ref. [1]) and $gg\to W^{\pm}H^{\mp}$, which is mediated by triangle- and box-type diagrams involving virtual top and bottom quarks (see Fig. 2 in Ref. [1]). In Ref. [1], we presented analytic expressions for the cross section of $b\overline{b}\to W^{\pm}H^{\mp}$ and the transition-matrix element of $gg\to W^{\pm}H^{\mp}$ arising from the quark triangles. However, we refrained from listing our formulas for the quark box contributions because we found that they were somewhat lengthy. In the meantime, a signal-versus-background analysis of $W^{\pm}H^{\mp}$ associated production at the LHC was carried out by Moretti and Odagiri [2], who generated the signal cross section by using the formulas published in Ref. [1], thus omitting the quark box contributions. This motivated us to further compactify our expressions for the latter by introducing helicity amplitudes. The purpose of this Brief Report is to provide these results, which may be useful for other authors as well.

Calling the four-momenta of the two gluons and the W boson p_a, p_b, and p_W, respectively, we define the partonic Mandelstam variables as $s=(p_a+p_b)^2$, $t=(p_a-p_W)^2$, and $u=(p_b-p_W)^2$. Furthermore, we introduce the following shorthand notation: $w=m_W^2$, $h=m_H^2$, $d=t-u$, $t_1=t-w$, $t_2=t-h$, $u_1=u-w$, $u_2=u-h$, $N=tu-wh$, $\lambda=s^2+w^2+h^2-2(sw+wh+hs)$, and $q=m_t^2-m_b^2$. We label the helicity states of the two gluons and the W boson in the partonic center-of-mass frame by $\lambda_a=-1/2,1/2$, $\lambda_b=-1/2,1/2$, and $\lambda_W=-1,0,1$. For reference, we first list the helicity amplitudes for the quark triangle contributions, $\mathcal{M}^{\triangle}_{\lambda_a\lambda_b\lambda_W}$. They may be extracted from Eq. (5) of Ref. [1] and read

$$\mathcal{M}^{\triangle}_{\lambda_a\lambda_b 0}=\frac{s\sqrt{\lambda}}{m_W}[(1+\lambda_a\lambda_b)\Sigma(s)-(\lambda_a+\lambda_b)\Pi(s)],\quad (1)$$

where Σ and Π are the vector and axial-vector form factors given in Eq. (6) of Ref. [1]. In this case, the W boson can only be longitudinally polarized because it couples to two Higgs bosons, so that $\mathcal{M}^{\triangle}_{\lambda_a\lambda_b\lambda_W}=0$ for $\lambda_W=\pm1$. As for the quark box contributions, all 12 helicity amplitudes $\mathcal{M}^{\square}_{\lambda_a\lambda_b\lambda_W}$ contribute. Because of Bose[1] and weak-isospin symmetry, they are related by

$$\begin{aligned}
&\mathcal{M}^{\square}_{\lambda_a\lambda_b 0}(t,u,m_b^2,m_t^2,\tan\beta)=\mathcal{M}^{\square}_{\lambda_b\lambda_a 0}(u,t,m_b^2,m_t^2,\tan\beta),\\
&\mathcal{M}^{\square}_{\lambda_a\lambda_b\lambda_W}(t,u,m_b^2,m_t^2,\tan\beta)\\
&\qquad=-\mathcal{M}^{\square}_{\lambda_b\lambda_a\lambda_W}(u,t,m_b^2,m_t^2,\tan\beta),\\
&\mathcal{M}^{\square}_{\lambda_a\lambda_b 0}(t,u,m_b^2,m_t^2,\tan\beta)\\
&\qquad=-\mathcal{M}^{\square}_{-\lambda_a-\lambda_b 0}(t,u,m_t^2,m_b^2,\cot\beta),\\
&\mathcal{M}^{\square}_{\lambda_a\lambda_b\lambda_W}(t,u,m_b^2,m_t^2,\tan\beta)\\
&\qquad=\mathcal{M}^{\square}_{-\lambda_b-\lambda_a-\lambda_W}(u,t,m_t^2,m_b^2,\cot\beta).
\end{aligned}\quad (2)$$

Keeping $\lambda_W=\pm1$ generic, we thus only need to specify four expressions. These read

[1]Notice that the interchange of t and u also affects the representation of the W-boson polarization four-vector through its dependence on the angle between the three-momenta of gluon a and the W boson. This explains the minus sign in the second line of Eq. (2), which is not expected from pure Bose symmetry.

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$$\mathcal{M}^{\square}_{++0}=\frac{2}{m_W s\sqrt{\lambda}}[(m_b^2\tan\beta+m_t^2\cot\beta)F^0_{++}+m_t^2\cot\beta G^0_{++}+(t\leftrightarrow u)],$$

$$\mathcal{M}^{\square}_{+-0}=\frac{1}{m_W N\sqrt{\lambda}}[(m_b^2\tan\beta+m_t^2\cot\beta)F^0_{+-}+m_t^2\cot\beta G^0_{+-}-(t\leftrightarrow u,m_b^2\leftrightarrow m_t^2,\tan\beta\leftrightarrow\cot\beta)],$$

$$\mathcal{M}^{\square}_{++\lambda_W}=\sqrt{\frac{2}{sN}}\left[\frac{m_b^2\tan\beta+m_t^2\cot\beta}{s}\left(\frac{F^1_{++}}{\sqrt{\lambda}}+\lambda_W F^2_{++}\right)+m_t^2\cot\beta\left(\frac{G^1_{++}}{\sqrt{\lambda}}+\lambda_W G^2_{++}\right)-(t\leftrightarrow u)\right], \tag{3}$$

$$\mathcal{M}^{\square}_{+-\lambda_W}=\frac{1}{\sqrt{2sN}}\left[\frac{m_b^2\tan\beta+m_t^2\cot\beta}{N}\left(\frac{F^1_{+-}}{\sqrt{\lambda}}+\lambda_W F^2_{+-}\right)+m_t^2\cot\beta\left(\frac{G^1_{+-}}{\sqrt{\lambda}}+\lambda_W G^2_{+-}\right)\right.$$
$$\left.+(t\leftrightarrow u,m_b^2\leftrightarrow m_t^2,\tan\beta\leftrightarrow\cot\beta,\lambda_W\rightarrow-\lambda_W)\right],$$

where $F^i_{+\pm}$ and $G^i_{+\pm}$, with $i=0,1,2$, are complex functions of t, u, m_b^2, and m_t^2. The normalization of $\mathcal{M}^{\triangle}_{\lambda_a\lambda_b\lambda_W}$ and $\mathcal{M}^{\square}_{\lambda_a\lambda_b\lambda_W}$ is such that the differential cross section of $gg\rightarrow W^-H^+$ is given by

$$\frac{d\sigma}{dt}(gg\rightarrow W^-H^+)=\frac{\alpha_s^2(\mu)G_F^2m_W^2}{256(4\pi)^3s^2}\sum_{\lambda_a,\lambda_b,\lambda_W}|\mathcal{M}^{\triangle}_{\lambda_a\lambda_b\lambda_W}+\mathcal{M}^{\square}_{\lambda_a\lambda_b\lambda_W}|^2. \tag{4}$$

We now express the form factors $F^i_{+\pm}$ and $G^i_{+\pm}$ in terms of the standard scalar two-, three-, and four-point functions:

$$B_0(p_1^2,m_0^2,m_1^2)=\int\frac{d^Dq}{i\pi^2}\,\frac{1}{(q^2-m_0^2+i\epsilon)[(q+p_1)^2-m_1^2+i\epsilon]},$$

$$C_0(p_1^2,(p_2-p_1)^2,p_2^2,m_0^2,m_1^2,m_2^2)=\int\frac{d^Dq}{i\pi^2}\,\frac{1}{(q^2-m_0^2+i\epsilon)[(q+p_1)^2-m_1^2+i\epsilon][(q+p_2)^2-m_2^2+i\epsilon]}, \tag{5}$$

$$D_0(p_1^2,(p_2-p_1)^2,(p_3-p_2)^2,p_3^2,p_2^2,(p_3-p_1)^2,m_0^2,m_1^2,m_2^2,m_3^2)$$
$$=\int\frac{d^Dq}{i\pi^2}\,\frac{1}{(q^2-m_0^2+i\epsilon)[(q+p_1)^2-m_1^2+i\epsilon][(q+p_2)^2-m_2^2+i\epsilon][(q+p_3)^2-m_3^2+i\epsilon]},$$

where D is the space-time dimensionality. The B_0 function is ultraviolet (UV) divergent in the physical limit $D\rightarrow 4$, while the C_0 and D_0 functions are UV finite in this limit. We evaluate the B_0, C_0, and D_0 functions numerically with the aid of the program package *FF* [3]. To simplify the notation, we introduce the abbreviations $C^{ab}_{ijk}(c)=C_0(a,b,c,m_i^2,m_j^2,m_k^2)$ and $D^{abcd}_{ijkl}(e,f)=D_0(a,b,c,d,e,f,m_i^2,m_j^2,m_k^2,m_l^2)$. We find

$$F^0_{++}=-2s(t_1+u_1)[m_b^2C^{00}_{bbb}(s)-m_t^2C^{00}_{ttt}(s)]+f_1(t,u,q)[t_2C^{h0}_{btt}(t)+t_1C^{w0}_{tbb}(t)]$$
$$+f_1(u,t,q)[t_2C^{h0}_{tbb}(t)+t_1C^{w0}_{btt}(t)]-f_1(t,u,q)[N+s(m_b^2+m_t^2)]D^{h0w0}_{bttb}(t,u)$$
$$-2sm_b^2[2wt_2+f_1(t,u,q)]D^{hw00}_{btbb}(s,t)+2sm_t^2[2wu_2-f_1(t,u,q)]D^{hw00}_{tbtt}(s,t),$$

$$G^0_{++}=-2s^2(t+u)C^{00}_{ttt}(s)-t_2f_1(-t_2,u_2,h)[C^{h0}_{btt}(t)+C^{h0}_{tbb}(t)]$$
$$-t_1f_1(-t_2,u_2,h)[C^{w0}_{btt}(t)+C^{w0}_{tbb}(t)]+[2s\lambda m_b^2+(N+sq)f_1(-t_2,u_2,h)]D^{h0w0}_{bttb}(t,u)$$
$$+2s\lambda m_b^2D^{hw00}_{btbb}(s,t)+2s[2swh-sm_b^2(t+u)+m_t^2f_1(-t_2,u_2,h)]D^{hw00}_{tbtt}(s,t),$$

$$F^0_{+-}=2s\{2wN+(t+u)[N-f_2(0,t,\lambda)]+qf_2(t,u,2\lambda)+2q^2(t_1+u_1)\}C^{00}_{bbb}(s)$$
$$-2t_2[wh(t_1+u_1)+qf_1(2u,2h,t)]C^{h0}_{btt}(t)+2t_2[w(hd-2tt_2-2N)$$
$$-qf_1(2u,2h,t)]C^{h0}_{tbb}(t)-2t_1[w(hd-2ut_2)+qf_1(2t,2h,t)]C^{w0}_{btt}(t)$$
$$+2t_1[w(hd-2tt_2)-qf_1(2t,2h,t)]C^{w0}_{tbb}(t)-2\{\lambda(ud-2wt_2)-(t_1+u_1)[N(t+u)$$

$$
\begin{aligned}
&+q(d^2+2N)]\}C^{hw}_{btb}(s)-[ud-2N-q(t_1+u_1)][N(m_b^2+m_t^2)+sq^2]D^{h0w0}_{bttb}(t,u)\\
&-\{2N[wN-m_b^2f_3(t,u,2w)]-Nq[ud+2u_1^2+2m_b^2(t_1+u_1)]-q^2[du_1(t_1+u_1)\\
&-t_2f_3(t,u,w)]-sq^3(t_1+u_1)\}D^{h0w0}_{tbbt}(t,u)-2\{stw(hd-2tt_2)-2Nm_b^2f_2(0,t,\lambda)\\
&-q[sN(t+u)-stf_2(t,0,2\lambda)+2Nm_b^2(t_1+u_1)]-sq^2f_2(t,0,\lambda)-sq^3(t_1+u_1)\}D^{hw00}_{btbb}(s,t)\\
&+2(t_1+u_1)\{stwh+2uNm_t^2+q[st(t+2u)-N(s-2m_t^2)]+sq^2(2t+u)+sq^3\}D^{hw00}_{tbtt}(s,t),
\end{aligned}
$$

$$
\begin{aligned}
G^0_{+-}=&\,2s[(t+u)(d^2+2N)-2\lambda q]C^{00}_{bbb}(s)+2[t^2(t+u)-wh(3t-u)]\{t_2[C^{h0}_{btt}(t)+C^{h0}_{tbb}(t)]\\
&+t_1[C^{w0}_{btt}(t)+C^{w0}_{tbb}(t)]\}-2\lambda(d^2+2N)C^{hw}_{btb}(s)-\lambda[N(m_b^2+m_t^2)+sq^2]D^{h0w0}_{bttb}(t,u)\\
&+[2N(2wN-\lambda m_b^2)-\lambda q(N+sq)]D^{h0w0}_{tbbt}(t,u)-2f_4(m_b^2,m_t^2)D^{hw00}_{btbb}(s,t)\\
&-2f_4(m_t^2,m_b^2)D^{hw00}_{tbtt}(s,t),
\end{aligned}
$$

$$
\begin{aligned}
F^1_{++}=&\,2s^2d[m_b^2C^{00}_{bbb}(s)-m_t^2C^{00}_{ttt}(s)]-f_5(w,m_b^2,m_t^2)[t_2C^{h0}_{btt}(t)+t_1C^{w0}_{tbb}(t)]\\
&+f_5(w,m_t^2,m_b^2)[t_2C^{h0}_{tbb}(t)+t_1C^{w0}_{btt}(t)]+[N+s(m_b^2+m_t^2)]f_5(w,m_b^2,m_t^2)D^{h0w0}_{bttb}(t,u)\\
&+2sf_6(m_b^2,m_t^2)D^{hw00}_{btbb}(s,t)-2sf_6(m_t^2,m_b^2)D^{hw00}_{tbtt}(s,t),
\end{aligned}
$$

$$
\begin{aligned}
F^2_{++}=&-(N-sq)[t_2C^{h0}_{btt}(t)-t_1C^{w0}_{btt}(t)]+(N+sq)[t_2C^{h0}_{tbb}(t)-t_1C^{w0}_{tbb}(t)]\\
&+\{N[N+2s(m_b^2+m_t^2)]+s^2q^2\}D^{h0w0}_{bttb}(t,u),
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
G^1_{++}=&-sd(t_2+u_2)C^{00}_{ttt}(s)+t_2f_3(t,u,h)[C^{h0}_{btt}(t)+C^{h0}_{tbb}(t)]-t_1f_3(u,t,h)[C^{w0}_{btt}(t)+C^{w0}_{tbb}(t)]\\
&+(N+sq)f_3(u,t,h)D^{h0w0}_{bttb}(t,u)+s(t_2+u_2)[2N+d(t+q)]D^{hw00}_{tbtt}(s,t),
\end{aligned}
$$

$$
\begin{aligned}
G^2_{++}=&-sdC^{00}_{ttt}(s)+t_2u_2[C^{h0}_{btt}(t)+C^{h0}_{tbb}(t)]+(st+N)C^{w0}_{btt}(t)-t_1t_2C^{w0}_{tbb}(t)+t_2(N+sq)D^{h0w0}_{bttb}(t,u)\\
&+s[2N+d(t+q)]D^{hw00}_{tbtt}(s,t),
\end{aligned}
$$

$$
\begin{aligned}
F^1_{+-}=&-4sdNB_0(s,m_b^2,m_b^2)-2s\{sd(t^2+u^2)-N[4su+d(w-h)+\lambda]\\
&-2sq[d(t+u)-2N]+2sdq^2\}C^{00}_{bbb}(s)+2t_2[2sN(t+u)+f_7(m_b^2,m_t^2)]C^{h0}_{btt}(t)\\
&-2t_2f_7(m_t^2,m_b^2)C^{h0}_{tbb}(t)-2t_1[2t_2N(t_1+u_1)+f_8(m_b^2,m_t^2)]C^{w0}_{btt}(t)+2t_1f_8(m_t^2,m_b^2)C^{w0}_{tbb}(t)\\
&-2s\lambda[N-d(t+u-2q)]C^{hw}_{btb}(s)+f_5(h,m_t^2,m_b^2)[N(m_b^2+m_t^2)+sq^2]D^{h0w0}_{bttb}(t,u)\\
&-\{N^2[f_3(t,u,w)+2m_t^2(3s+w-h)]-Nq[s\lambda-N(t_1+u_1)-sd(t_1+2m_t^2)]+2sNq^2(2s+u_2)\\
&+s^2dq^3\}D^{h0w0}_{tbbt}(t,u)-2s(t-q)\{Nf_3(u,t,h)-d[st(t-2m_t^2)-2t_1t_2m_b^2+sq^2]\}D^{hw00}_{btbb}(s,t)\\
&+2s\{stdN-(ud+\lambda)(st^2+2Nm_t^2)-q[sd(t(t+2u)-N)+2st\lambda+2dNm_t^2]\\
&-sq^2[d(2t+u)+\lambda]-sdq^3\}D^{hw00}_{tbtt}(s,t),
\end{aligned}
$$

$$
\begin{aligned}
F^2_{+-}=&-4sNB_0(s,m_b^2,m_b^2)-2s[s(t^2+u^2)+N(t_2-u_1)+4Nm_b^2-2sq(t+u)+2sq^2]C^{00}_{bbb}(s)\\
&-2t_2[u_1N-f_9(t_2,m_b^2,m_t^2)]C^{h0}_{btt}(t)-2t_2[u_1N+f_9(-t_2,m_t^2,m_b^2)]C^{h0}_{tbb}(t)\\
&+2t_1f_9(t_2,m_b^2,m_t^2)C^{w0}_{btt}(t)-2t_1f_9(-t_2,m_t^2,m_b^2)C^{w0}_{tbb}(t)+2s[d^2(t+u)+N(t+3u)\\
&-2q(d^2+2N)]C^{hw}_{btb}(s)-q[N^2+2sN(m_b^2+m_t^2)+s^2q^2]D^{h0w0}_{bttb}(t,u)\\
&-[u_1N^2+Nq(tu_1+ut_2+u_1^2+4sm_t^2)-2sNq^2+s^2q^3]D^{h0w0}_{tbbt}(t,u)\\
&-2f_{10}(t_2,m_b^2,m_t^2)D^{hw00}_{btbb}(s,t)+2f_{10}(s,m_t^2,m_b^2)D^{hw00}_{tbtt}(s,t),
\end{aligned}
$$

$$G^1_{+-} = -2f_3(u,t,h)\{2sC^{00}_{bbb}(s) + t_1[C^{w0}_{btt}(t) + C^{w0}_{tbb}(t)]\}$$
$$+2t_2 f_3(t,u,h)[C^{h0}_{btt}(t) + C^{h0}_{tbb}(t)] - [dN(t_1+u_1) + s\lambda q]D^{h0w0}_{tbbt}(t,u)$$
$$+2s(t-q)f_3(u,t,h)D^{hw00}_{btbb}(s,t) - 2s(t+q)f_3(t,u,h)D^{hw00}_{tbtt}(s,t),$$
$$G^2_{+-} = 2t_2\{2sC^{00}_{bbb}(s) + u_2[C^{h0}_{btt}(t) + C^{h0}_{tbb}(t)] + t_1[C^{w0}_{btt}(t) + C^{w0}_{tbb}(t)]\}$$
$$+f_5(w,m_b^2,m_t^2)D^{h0w0}_{tbbt}(t,u) - 2st_2(t-q)D^{hw00}_{btbb}(s,t) - 2su_2(t+q)D^{hw00}_{tbtt}(s,t),$$

where we have used the auxiliary functions

$$\begin{aligned}
f_1(t,u,q) &= -w(t-u) + q(t_1+u_1),\\
f_2(t,u,\lambda) &= \lambda - (3t+u)(t_1+u_1),\\
f_3(t,u,h) &= 2N - (t-u)(u-h),\\
f_4(m_b^2,m_t^2) &= -st[N(t+u) - t\lambda] - s(m_b^2 - m_t^2)[N(t+u) - 2t\lambda]\\
&\quad + \lambda[2Nm_b^2 + s(m_b^2 - m_t^2)^2],\\
f_5(w,m_b^2,m_t^2) &= N(t+u-2w) - sd(m_b^2 - m_t^2),\\
f_6(m_b^2,m_t^2) &= -sm_b^2[2N + d(t + m_b^2 - m_t^2)],\\
f_7(m_b^2,m_t^2) &= sd(t^2+N) - (m_b^2 - m_t^2)[2std + N(3s+w-h)],\\
f_8(m_b^2,m_t^2) &= 2N^2 - d(st^2 - t_2N) + (m_b^2 - m_t^2)[d(2st+N) - 2t_2N],\\
f_9(t_2,m_b^2,m_t^2) &= st^2 - t_2N - (m_b^2 - m_t^2)(2st+N),\\
f_{10}(t_2,m_b^2,m_t^2) &= -s(t+m_b^2-m_t^2)[st(t+2m_b^2-2m_t^2) + N(t_2+4m_b^2) + sq^2].
\end{aligned} \tag{7}$$

Here it is understood that all variables appearing on the right-hand sides are to be taken as independent. E.g., N should be treated as independent of t, u, w, and h. Notice that the UV divergences of F^1_{+-} and F^2_{+-} cancel in the expression for $\mathcal{M}^{\square}_{+-\lambda_W}$ in Eq. (3). Finally, we remark that we recover the SM result for $d\sigma(gg\to ZH)/dt$ due to one quark flavor [4] from Eq. (4) by substituting $m_W = m_Z$, $m_b = m_t$, and $\tan\beta = 1$ and adjusting the strengths of the axial-vector and Yukawa couplings. In particular, the contribution proportional to the weak vector coupling then vanishes as required by charge-conjugation invariance. This serves as a useful check for our analytical and numerical analyses.

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