Splitting between up-type and down-type quark masses via mixing with exotic fermions in E_6

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(Received 26 July 1999; published 6 April 2000)

The average masses of (d,s,b) quarks are smaller than those of (u,c,t) quarks. In contrast with mechanisms relying on different Higgs boson vacuum expectation values or different Yukawa couplings, this mass difference is explained as a consequence of mixing of (d,s,b) with exotic quarks implied by the electroweak-strong unification group E_6.

PACS number(s): 12.15.Ff, 12.10.Dm, 12.60.Cn, 14.80.−j

The currently known fermions consist of quarks (u,c,t) with charge 2/3, quarks (d,s,b) with charge $-1/3$, leptons (e,μ,τ) with charge -1, and neutrinos (ν_e,ν_μ,ν_τ). Some proposals address certain broad features of their masses. Specifically:

(1) The evidence [1,2] that neutrino masses are non-zero but tiny with respect to those of other fermions may be evidence for large Majorana masses of right-handed neutrinos, which overwhelm Dirac mass terms and lead to extremely small Majorana masses for left-handed neutrinos [3].

(2) Many unified theories of the electroweak and strong interactions [4] imply a relation between the masses of charged leptons and quarks of charge $-1/3$ at the unification scale. Such a relation does seem to be approximately satisfied for the members τ,b of the heaviest family.

(3) The larger (average) masses of the (u,c,t) quarks with respect to the (d,s,b) quarks could be a consequence of different vacuum expectation values in a two-Higgs-doublet model [5], where two different doublets are responsible for the masses of quarks of different charges. [In such a picture we would view the masses of the lightest quarks, which have the inverted order $m(d)>m(u)$, as due, for example, to a radiative effect, and not characteristic of the gross pattern.]

In the present Brief Report we propose another potential source of difference between masses of quarks of different charges, which arises in unified electroweak theories based on the gauge group E_6 [6–8]. The fundamental (27-dimensional) representation of this group contains additional quarks of charge $-1/3$ and additional charged and neutral leptons, but no additional quarks of charge 2/3. We have identified a simple mixing mechanism which can depress the average mass of (d,s,b) quarks (and charged leptons) with respect to that of (u,c,t) quarks without the need for different Higgs vacuum expectation values. This mixing can occur in such a way as to have minimal effect on the weak charged-current and neutral-current couplings of quarks and leptons, but offers the possibility of observable deviations from standard couplings if the new states participating in the mixing are not too heavy.

The proposed mechanism was first observed in Ref. [9]. Similar mixing with isosinglet quarks was discussed in Refs. [10] and [11], but with different emphases (including mechanisms for understanding the peculiar behavior of m_d). Comprehensive studies of mixing with isosinglet quarks [12,13] contain formalism equivalent to ours, but without raising the possibility that down-type quark masses can be depressed by such mixing. A related (''seesaw'') effect was used to describe the top quark mass in a particular theory of electroweak symmetry breaking [14]. We stress, however, that the present mechanism is distinct from the ''universal seesaw mechanism'' proposed some time ago [15] in which *all* fermion masses are depressed by mixing with isosinglet quarks.

We first recall some basic features of E_6 and mass matrices, and then describe a scenario in which (d,s,b) masses (and those of charged leptons) can be depressed by mixing with their exotic E_6 counterparts. Some consequences of the mixing hypothesis are then noted.

The fundamental 27-dimensional representation of E_6 contains representations of dimension 16, 10, and 1 of SO(10). We assume there exist three 27-plets, corresponding to the three quark-lepton families. We may regard ordinary matter (including right-handed neutrinos) of a single quark-lepton family as residing in an SO(10) 16-plet, with SU(5) content 5^*+10+1. The additional (''exotic'') states in the 10-plet and singlet of SO(10) are summarized in Table I for one family. Here I_L and I_{3L} refer to left-handed isospin and its third component.

All the new states are vector-like. They consist of an isosinglet quark h^c of charge 1/3, a lepton isodoublet (E^-,ν_E), the corresponding antiparticles, and a Majorana neutrino n_e.

For simplicity we consider only mixings within a single family, which we shall denote (u,d,e,ν_e). We shall discuss only mass matrices of charged fermions. The neutral lepton sector is of potential interest since it contains possibilities for

TABLE I. Exotic fermions in a 27-plet of E_6.

SO(10)	SU(5)	State	Q	I_L	I_{3L}
10	5	h^c	1/3	0	0
		E^-	-1	1/2	$-1/2$
		ν_E	0	1/2	1/2
	5*	h	$-1/3$	0	0
		E^+	1	1/2	1/2
		$\overline{\nu_E}$	0	1/2	$-1/2$
1	1	n_e	0	0	0

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TABLE II. Simplest transformation properties of terms in $\mathcal{M}^d$.

Term	SO(10)	SU(5)
m_2	10	5
m_3	16*	5
M_1	16*	1
M_2	1	1

''sterile'' neutrinos not excluded by the usual cosmological and accelerator-based experimental considerations [16].

The simplest mass is that of the u quark, which cannot mix with any other. The simplest Higgs representation giving rise to m_u belongs to the [27*,10,5*] of [E_6, SO(10), SU(5)].

The corresponding mass matrix for quarks of charge $-1/3$ takes account of the possible mixing between non-exotic d and exotic h quarks. Its most general form can be written

$$[\overline{d_L} \quad \overline{h_L}]\mathcal{M}^d\begin{bmatrix} d_R \\ h_R \end{bmatrix} = [\overline{d_L} \quad \overline{h_L}]\begin{bmatrix} m_2 & m_3 \\ M_1 & M_2 \end{bmatrix}\begin{bmatrix} d_R \\ h_R \end{bmatrix}. \tag{1}$$

Here small letters refer to $\Delta I_L = 1/2$ masses, which are expected to be of electroweak scale or less, while large letters refer to $\Delta I_L = 0$ masses, which can be of any magnitude (including the unification scale). We shall assume $m_i \ll M_i$. If the masses in Eq. (1) arise through vacuum expectation values of a Higgs 27*-plet (the simplest possibility), their transformation properties are summarized in Table II.

Two distinct unitary transformations diagonalize $\mathcal{M}^d\mathcal{M}^{d\dagger}$ and $\mathcal{M}^{d\dagger}\mathcal{M}^d$. For each of these, the eigenvalues λ_1 and λ_2 satisfy

$$\lambda_1 + \lambda_2 = m_2^2 + m_3^2 + M_1^2 + M_2^2,$$
$$\lambda_1\lambda_2 = (M_1 m_3 - M_2 m_2)^2. \tag{2}$$

Suppose, to begin with, that $\overline{h_L}$ and h_R pair up to form a Dirac particle with large mass $M_2 \gg (M_1, m_2, m_3)$. Then the two eigenvalues are $\lambda_1 \simeq m_2^2$ and $\lambda_2 \simeq M_2^2$, corresponding to light and heavy Dirac particles d and h, respectively. If we label basis states with zeros as subscripts, and physical states without subscripts, this solution corresponds to $d = d_0$, $h = h_0$, for both left-handed and right-handed states.

For the more general case where M_1 is not negligible in comparison with M_2, we can write

$$M_1 = M\cos\theta, \quad M_2 = M\sin\theta,$$
$$m_3 = m\cos\phi, \quad m_2 = m\sin\phi. \tag{3}$$

Then for $m \ll M$, we have

$$\lambda_1 \simeq m^2\cos^2(\theta+\phi), \quad \lambda_2 \simeq m^2\sin^2(\theta+\phi) + M^2. \tag{4}$$

This is our central result. It is possible to choose $\theta+\phi$ in such a way that the down-type quark mass is arbitrarily small in comparison with m, whose value is a typical electroweak scale (as in the case of m_t). The opposite situation, in which up-type quarks are lighter than down-type quarks, is unnatural in the present scheme. In order to reproduce the hierarchy $m_b/m_t \simeq 0.03$, one must choose $\cos(\theta+\phi)$ to be correspondingly small. The need for this ''fine-tuning'' should then be regarded as a guide to possible additional symmetries of the problem.

The physical right-handed states (d_R, h_R) are eigenstates of the matrix

$$\mathcal{M}^{d\dagger}\mathcal{M}^d = \begin{bmatrix} m^2\sin^2\phi + M^2\cos^2\theta & m^2\cos\phi\sin\phi + M^2\cos\theta\sin\theta \\ m^2\cos\phi\sin\phi + M^2\cos\theta\sin\theta & m^2\cos^2\phi + M^2\sin^2\theta \end{bmatrix}. \tag{5}$$

For $M \gg m$ the approximate eigenstates are

$$d_R \simeq \sin\theta d_{0R} - \cos\theta h_{0R}, \quad h_R \simeq \cos\theta d_{0R} + \sin\theta h_{0R}. \tag{6}$$

In the limit $\theta = \pi/2$ in which $M_2 \gg M_1$, leading to a large Dirac mass for the exotic quark h, one thus has $d_R = d_{0R}$, $h_R = h_{0R}$.

The physical left-handed states (d_L, h_L) are eigenstates of the matrix

$$\mathcal{M}^d\mathcal{M}^{d\dagger} = \begin{bmatrix} m^2 & mM\sin(\theta+\phi) \\ mM\sin(\theta+\phi) & M^2 \end{bmatrix}, \tag{7}$$

specifically

$$d_L \simeq d_{0L} - (m/M)\sin(\theta+\phi)h_{0L},$$
$$h_L \simeq (m/M)\sin(\theta+\phi)d_{0L} + h_{0L}. \tag{8}$$

Thus, for $m \ll M$, there is little mixing between the isosinglet and isodoublet quarks, and hence little violation of unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Some consequences of this mixing have been explored, for example, in Refs. [7–13,17]. The mixing parameter $\zeta \equiv (m/M)\sin(\theta+\phi)$ and the suppression of d-type masses are both maximal for $\theta+\phi = \pm\pi/2$.

The production of $h\bar{h}$ pairs in hadronic collisions should be governed by standard perturbative QCD, which gives a reasonable account of top quark pair production [18]. For the data sample of approximately 100 pb^{-1} obtained in $p\bar{p}$ collisions at a center-of-mass energy of 1.8 TeV in run I at the

Fermilab Tevatron, it should be possible to observe or exclude values of $m(h)$ well in excess of $m(t)$ [19]. It may also be possible to produce or exclude h quarks singly through the neutral flavor-changing interaction at the CERN e^+e^- collider LEP II via the reaction $e^+e^- \to Z^* \to h + (\overline{d}, \overline{s}, \overline{b})$. Both charged-current decays $h \to W + (u,c,t)$ and neutral-current decays $h \to Z + (d,s,b)$ should be characterized by multiple leptons and missing energy in an appreciable fraction of events.

The mixing proposed here applies in an almost identical manner to the charged leptons under suitable replacements. The charged leptons' masses, just like those of the d-type quarks, thus may be depressed relative to their unmixed values. One could expect small modifications of *right-handed* lepton couplings since one is then mixing an isosinglet e with an isodoublet E.

To conclude, we have presented a mechanism which accounts for the depression in the average masses of down-type quarks and charged leptons relative to that of up-type quarks, without the need for differences in Higgs vacuum expectation values or in values of the largest Yukawa coupling for each type of fermion. This mechanism relies on mixings between ordinary fermions and their exotic counterparts in E_6 multiplets. It may be of use in building more realistic models of quark and lepton masses. Although the exotic E_6 fermions need not be accessible to present experimental searches in order for this mechanism to be effective, they could well be observable in forthcoming searches at the Fermilab Tevatron, the LEP II e^+e^- collider, or the Large Hadron Collider under construction at CERN.

I am indebted to T. André, F. del Aguila, B. Kayser, R. N. Mohapatra, S. T. Petcov, T. Rizzo, and L. Wolfenstein for useful discussions. I wish to thank the Institute for Nuclear Theory at the University of Washington for hospitality during this work, which was supported in part by the United States Department of Energy under Grant No. DE FG02 90ER40560.

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