

## Neutrino masses and mixings in a seesaw framework

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Assuming the seesaw mechanism for hierarchical neutrino masses, we calculate the heavy neutrino masses under the hypotheses that the mixing in the Dirac leptonic sector is similar to the quark mixing ( $V_D \sim V_{CKM}$ ) and that  $M_\nu \sim M_u$  or  $M_e$ , where  $M_\nu$  is the Dirac mass matrix of neutrinos. As a result we find that for  $M_\nu \sim M_u$  the vacuum oscillation solution of the solar neutrino problem leads to a scale for the heavy neutrino mass well above the unification scale, while for the MSW solutions there is agreement with this scale. For  $M_\nu \sim M_e$  the vacuum solution is consistent with the unification scale and the MSW solutions with an intermediate scale. The mass of the lightest heavy neutrino can be as small as  $10^5$  GeV.

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In the minimal standard model (MSM) [1] the neutrino is massless. This is because with only the left-handed neutrino  $\nu_L$  we cannot build a Dirac mass term, and with only the Higgs doublet  $\phi$  we cannot build a Majorana mass term for  $\nu_L$  after spontaneous symmetry breaking. However, strong indications for a nonzero neutrino mass come from solar and atmospheric neutrino experiments and on the theoretical side there are several extensions of the MSM that lead to a nonzero neutrino mass [2].

The simplest one is to add the right-handed neutrino  $\nu_R$  in order to have the analogue of the quark  $u_R$  in the leptonic sector. When this is done, it becomes possible to give a Dirac mass to the neutrino by means of the same mechanism used for the other fermions. Thus we expect this mass to be of the same order of magnitude as the other fermion masses. Moreover, it is now also possible to have a bare Majorana mass term for  $\nu_R$ , and the corresponding value of the mass is not constrained if the gauge group is the same as the MSM. Therefore we have a new mass scale in the extended theory [3], and it is a key problem to understand if this new scale is associated with new physics—that is, a larger gauge group—and at what energy it eventually happens.

If the Dirac mass of the neutrino is of the same order as the other quark or lepton masses, the seesaw mechanism [4] relates the smallness of the neutrino mass to a very large scale in the Majorana term. Of course we have three generations of fermions and we expect three light neutrinos and three heavy ones. We assume that the light neutrino mass spectrum is hierarchical as happens for quark and charged lepton mass spectra. We denote by  $m_1, m_2, m_3$  the Dirac masses, by  $M_1, M_2, M_3$ , the heavy neutrino masses, and by  $m_{\nu_1}, m_{\nu_2}, m_{\nu_3}$  the light neutrino masses. From solar and atmospheric neutrino experiments we can infer [5], with some uncertainty, the values of  $m_{\nu_2}$  and  $m_{\nu_3}$ , and of the neutrino mixing matrix  $U$ ,

$$\nu_\alpha = U_{\alpha i} \nu_i, \quad (1)$$

where  $U$  is unitary and connects the mass eigenstates  $\nu_i$  ( $i=1,2,3$ ) to the weak eigenstates  $\nu_\alpha$  ( $\alpha=e,\mu,\tau$ ).

The aim of this paper is to calculate the heavy neutrino masses under simple hypotheses of the Dirac masses of neutrinos and of the matrix

$$V_D = V_\nu^\dagger V_e, \quad (2)$$

which is analogous to the Cabibbo-Kobayashi-Maskawa (CKM) matrix in the Dirac leptonic sector. Namely, we assume  $V_D \sim V_{CKM}$  and  $M_\nu \sim M_u$  or  $M_\nu \sim M_e$ . For  $m_{\nu_1}$  we allow a variation of three orders in the hierarchical regime. We are mostly interested in  $M_3$  (the mass of the heaviest right-handed neutrino), which is related to the new mass scale of the theory, and in  $M_1$  (the mass of the lightest right-handed neutrino), which has some importance in baryogenesis via leptogenesis [6,7]. In grand unified theories (GUTs)  $M_3$  is associated with unification or intermediate scales [8]; thus we match our results with these scales. General considerations on heavy neutrino masses in the seesaw mechanism can be found in Ref. [9]. In the present paper we give a numerical analysis based on the hypotheses above and the experimental data on solar and atmospheric neutrinos.

Let us briefly explain the effect of the seesaw mechanism on leptonic mixing. The part of the Lagrangian we have to consider is

$$\bar{e}_L M_e e_R + \bar{\nu}_L M_\nu \nu_R + g \bar{\nu}_L e_L W + \bar{\nu}_L^c M_R' \nu_R, \quad (3)$$

where  $M_e$  and  $M_\nu$  are the Dirac mass matrices of charged leptons and neutrinos, respectively, and  $M_R'$  is the Majorana mass matrix of right-handed neutrinos. If we assume the elements of  $M_R'$  to be much greater than those of  $M_\nu$ , the seesaw mechanism leads to the effective Lagrangian

$$\bar{e}_L M_e e_R + \bar{\nu}_L M_L' \nu_R^c + g \bar{\nu}_L e_L W + \bar{\nu}_L^c M_R' \nu_R, \quad (4)$$

where

$$M_L' = M_\nu M_R'^{-1} M_\nu^T \quad (5)$$

is the Majorana mass matrix of left-handed neutrinos (in this context the left-handed neutrinos are called light neutrinos and the right-handed neutrinos are called heavy neutrinos). Diagonalization of  $M_e, M_L'$  gives (renaming the fermion fields)

$$\bar{e}_L D_e e_R + \bar{\nu}_L D_L \nu_R^c + g \bar{\nu}_L V_{lep} e_L W + \bar{\nu}_L^c M_R' \nu_R. \quad (6)$$

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Of course we can also diagonalize  $M'_R$  without changing other parts of this Lagrangian. The unitary matrix  $V_{lep}$  [10] describes the weak interactions of light neutrinos with charged leptons. The following steps clarify the structure of  $V_{lep}$ .

If in Eq. (3) we first diagonalize  $M_e$  and  $M_\nu$ , obtaining

$$\bar{e}_L D_e e_R + \bar{\nu}_L D_\nu \nu_R + g \bar{\nu}_L V_D e_L W + \bar{\nu}_L^c M_R \nu_R, \quad (7)$$

then the seesaw mechanism gives

$$\bar{e}_L D_e e_R + \bar{\nu}_L M_L \nu_R^c + g \bar{\nu}_L V_D e_L W + \bar{\nu}_L^c M_R \nu_R, \quad (8)$$

with

$$M_L = D_\nu M_R^{-1} D_\nu. \quad (9)$$

Then, we diagonalize also  $M_L$ ,

$$\bar{e}_L D_e e_R + \bar{\nu}_L D_L \nu_R^c + g \bar{\nu}_L V_s V_D e_L W + \bar{\nu}_L^c M_R \nu_R, \quad (10)$$

and, comparing with Eq. (6), we recognize that

$$V_{lep} = V_s V_D, \quad (11)$$

where

$$V_s M_L V_s^T = D_L. \quad (12)$$

We also understand that

$$V_{lep} = U^\dagger, \quad (13)$$

and point out that  $M'_R(M'_L)$  differs from  $M_R(M_L)$  by a unitary transformation; hence they have the same eigenvalues.

In the Lagrangian (3) it is possible to diagonalize  $M_e$  or  $M_R$  without changing the observable quantities. The same is not true for  $M_\nu$ . Moreover,  $M_e$  and  $M_R$  can be diagonalized simultaneously. In the Lagrangian (4) the following matrices can be diagonalized:  $M_e$ ,  $M_L$ ,  $M_R$ , both  $M_L$  and  $M_R$ , both  $M_e$  and  $M_R$ . When we set  $M_e = D_e$  in Eq. (4) we have  $M_L = U D_L U^T$ , and when we set  $M_L = D_L$  we get  $M_e = U^\dagger D_e U$  if  $M_e$  is chosen to be Hermitian or  $M_e M_e^\dagger = U^\dagger D_e^2 U$  if  $M_e$  contains three zeros [11].

Experimental information on neutrino masses and mixings is increasing rapidly. To be definite we refer to [5], where the matrix  $U$  is written as

$$U = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}c_{23} & c_{12}c_{23} & s_{23} \\ s_{12}s_{23} & -c_{12}s_{23} & c_{23} \end{pmatrix}. \quad (14)$$

There is a zero in position 1-3, although it is only constrained to be much less than 1 [12]. The experimental data on oscillation of atmospheric and solar neutrinos lead to three possible numerical forms for  $U$ , corresponding to the three solutions of the solar neutrino problem: namely, small mixing (SM) and large mixing (LM) Mikheyev-Smirnov-Wolfenstein (MSW) effect [13], and vacuum oscillations

(VOs) [10,14,15]. Choosing the central values of neutrino masses and of  $s_{12}$  and  $s_{23}$  from Ref. [5], we have always  $m_{\nu_3} = 5.7 \times 10^{-11}$  GeV, and

$$U = \begin{pmatrix} 1 & 0.04 & 0 \\ -0.032 & 0.80 & 0.60 \\ 0.024 & -0.60 & 0.80 \end{pmatrix} \equiv U_1, \quad (15)$$

$m_{\nu_2} = 2.8 \times 10^{-12}$  GeV, for the small mixing MSW effect,

$$U = \begin{pmatrix} 0.91 & 0.42 & 0 \\ -0.336 & 0.726 & 0.60 \\ 0.252 & -0.544 & 0.80 \end{pmatrix} \equiv U_2, \quad (16)$$

$m_{\nu_2} = 4.4 \times 10^{-12}$  GeV, for the large mixing MSW effect, and

$$U = \begin{pmatrix} 0.80 & 0.60 & 0 \\ -0.474 & 0.632 & 0.61 \\ 0.366 & -0.488 & 0.79 \end{pmatrix} \equiv U_3, \quad (17)$$

$m_{\nu_2} = 9.2 \times 10^{-15}$  GeV, for vacuum oscillations. We also consider maximal and bimaximal [16] mixing as limiting cases of  $U_1$  and  $U_3$ , respectively:

$$U_m = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}, \quad (18)$$

$$U_b = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 1/\sqrt{2} \\ \frac{1}{2} & -\frac{1}{2} & 1/\sqrt{2} \end{pmatrix}. \quad (19)$$

Experimental data on oscillations only give  $\Delta m_{32}^2$ ,  $\Delta m_{21}^2$ , from which, for hierarchical light neutrino masses, we yield the values of  $m_{\nu_3}$ ,  $m_{\nu_2}$ , because  $\Delta m_{32}^2 \approx m_{\nu_3}^2$ ,  $\Delta m_{21}^2 \approx m_{\nu_2}^2$ . For  $m_{\nu_1}$  we will assume  $m_{\nu_1} \leq 10^{-1} m_{\nu_2}$ . From the unitary matrices written above we see that leptonic mixing between second and third families is large, while the mixing between first and second families may be large or small. It is well known that in the quark sector all mixings are small.

Our determination of  $M_1$ ,  $M_2$ ,  $M_3$  is based on the following assumptions. Looking at Eq. (11) we see that  $V_s$  could be responsible for the enhancement of lepton mixing [17]. Therefore, it is suggestive to assume that the matrix  $V_D$  has just the form of the CKM matrix

$$V_D = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & \lambda^4 \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & \lambda^2 \\ \lambda^3 - \lambda^4 & -\lambda^2 & 1 \end{pmatrix}. \quad (20)$$

This is similar to the form which might originate from GUTs [18],

TABLE I.  $V_D \sim V_{CKM}, M_\nu \sim M_u$ .

	SM MSW	LM MSW	VO	Max	Bimax
$M_1$	$1.3 \times 10^6$	$6.8 \times 10^5$	$2.5 \times 10^6$	$1.1 \times 10^6$	$4.0 \times 10^6$
	$1.6 \times 10^6$	$2.3 \times 10^6$	$3.1 \times 10^9$	$1.2 \times 10^6$	$3.7 \times 10^9$
$M_2$	$3.1 \times 10^{10}$	$1.5 \times 10^{10}$	$1.7 \times 10^{12}$	$3.0 \times 10^{10}$	$1.4 \times 10^{12}$
	$9.4 \times 10^{11}$	$2.0 \times 10^{10}$	$2.0 \times 10^{12}$	$1.1 \times 10^{13}$	$1.5 \times 10^{12}$
$M_3$	$1.7 \times 10^{15}$	$2.8 \times 10^{15}$	$2.2 \times 10^{18}$	$2.2 \times 10^{15}$	$3.7 \times 10^{18}$
	$4.7 \times 10^{16}$	$1.9 \times 10^{18}$	$1.9 \times 10^{21}$	$5.0 \times 10^{15}$	$3.3 \times 10^{21}$

$$V_D = \begin{pmatrix} 1 - \frac{1}{18} \lambda^2 & \frac{1}{3} \lambda & \lambda^4 \\ -\frac{1}{3} \lambda & 1 - \frac{1}{18} \lambda^2 & \lambda^2 \\ \frac{1}{3} \lambda^3 - \lambda^4 & -\lambda^2 & 1 \end{pmatrix}, \quad (21)$$

the difference being in the element  $V_{D12}$ , and this, in turn, to the  $V_D$  which results from the analogy of Ref. [19], where  $V_{D23} \approx 2\lambda^2$ . We also assume  $M_\nu \sim M_u$  or  $M_\nu \sim M_e$ . In particular,

$$M_\nu = D_\nu = (m_\tau/m_b) D_u, \quad (22)$$

where the factor is due to running [20] or

$$M_\nu = D_\nu = D_e. \quad (23)$$

We use quark (and charged lepton) masses at the scale  $M_Z$  as in Ref. [21]. It is important to notice that the values  $M_1, M_2, M_3$  do not depend on the assumption  $M_\nu = D_\nu$ , because  $M_R$  undergoes a unitary transformation. Also  $V_D$  does not change if we rotate  $e_L$  as  $\nu_L$ . In fact, one can always diagonalize  $M_u$  without changing  $V_{CKM}$  [22] and  $M_\nu$  without changing both  $V_D$  and  $M_1, M_2, M_3$ . We vary  $m_{\nu_1}$  by three orders down from  $m_{\nu_1} = 10^{-1} m_{\nu_2}$  to  $m_{\nu_1} = 10^{-4} m_{\nu_2}$ . In the tables we report our results (in GeV). They are obtained in the following way. From Eq. (11) we have

$$V_s = V_{lep} V_D^\dagger; \quad (24)$$

using Eq. (12) we get

$$M_L = V_s^T D_L V_s, \quad (25)$$

and from Eq. (9) we obtain

$$M_R = D_\nu M_L^{-1} D_\nu \quad (26)$$

and then its eigenvalues. We see that in the case  $M_\nu \sim M_u$  the VO solution leads to a scale for  $M_3$  well above the unification scale (around the Planck scale), while the MSW solutions are consistent with this scale.  $M_2$  is around the intermediate scale. In the case  $M_\nu \sim M_e$  the VO solution gives  $M_3$  near the unification scale, while the MSW solutions bring it near an intermediate scale. Also we notice that  $M_1$  may be of the order  $10^6$ , a relatively small value [19,6]. The huge value of  $M_3$  in the VO case is due mainly to the lower values of  $m_{\nu_1}, m_{\nu_2}$  with respect to the MSW case. There is no substantial change if the factor  $m_\tau/m_b$  is erased from Eq.

TABLE II.  $V_D \sim V_{CKM}, M_\nu \sim M_e$ .

	SM MSW	LM MSW	VO	Max	Bimax
$M_1$	$1.7 \times 10^5$	$8.6 \times 10^4$	$2.2 \times 10^5$	$1.4 \times 10^5$	$1.7 \times 10^5$
	$2.0 \times 10^5$	$9.7 \times 10^4$	$1.3 \times 10^7$	$1.6 \times 10^5$	$1.4 \times 10^7$
$M_2$	$2.1 \times 10^9$	$1.0 \times 10^9$	$1.2 \times 10^{11}$	$2.0 \times 10^9$	$9.3 \times 10^{10}$
	$5.4 \times 10^{10}$	$1.3 \times 10^9$	$1.4 \times 10^{11}$	$3.8 \times 10^{11}$	$1.0 \times 10^{11}$
$M_3$	$5.0 \times 10^{11}$	$7.9 \times 10^{11}$	$6.1 \times 10^{14}$	$6.2 \times 10^{11}$	$1.0 \times 10^{15}$
	$1.5 \times 10^{13}$	$5.3 \times 10^{14}$	$5.2 \times 10^{17}$	$2.8 \times 10^{12}$	$9.3 \times 10^{17}$

(22): the numerical results are rescaled by the value 2.8. We have introduced such a factor because the relation  $M_u \sim M_\nu$  is typical of GUTs, where it is true at the unification scale, while the factor  $m_\tau/m_b$  appears at low energy due to running. It can be checked that there is not an essential difference between the results obtained by Eq. (20) ( $V_D \sim V_{CKM}$ ) and those obtained by Eq. (21) ( $V_D \sim V_{GUT}$ ). In fact, numbers differ by no more than one order of magnitude. Moreover, comparing values in the two MSW cases, the effect of changes in  $m_{\nu_1}$  is apparent: in the small mixing solution  $M_3$  varies by one order, in the large mixing solution by three orders. If we want  $M_3$  not to exceed the unification scale, then  $m_{\nu_1}$  cannot be much smaller than  $m_{\nu_2}$ , in the large mixing MSW solutions. Maximal and bimaximal mixings confirm the results obtained for small mixing MSW solutions and vacuum oscillations, respectively.

As a matter of fact  $V_D \sim V_{CKM}$  and  $V_D \sim V_{GUT}$  are not so different from  $V_D \sim I$ . In such a case  $V_{lep} \approx V_s$ . The opposite case is  $V_{lep} \approx V_D$  and then  $V_s \approx I$ ; that is, when  $M_\nu = D_\nu$  also  $M_R = D_R$ . From the seesaw mechanism we obtain  $M_i = m_i^2/m_{\nu_i}$ , which gives  $M_3 \sim 10^{14}$ ,  $M_2 \sim 10^{10}$ ,  $M_1 \sim 10^6 - 10^9$  (MSW),  $M_2 \sim 10^{13}$ ,  $M_1 \sim 10^9 - 10^{12}$  (VO) GeV in the case  $M_\nu \sim M_u$ ;  $M_3 \sim 10^{10}$ ,  $M_2 \sim 10^9$ ,  $M_1 \sim 10^5 - 10^8$  (MSW),  $M_2 \sim 10^{12}$ ,  $M_1 \sim 10^8 - 10^{11}$  (VO) GeV in the case  $M_\nu \sim M_e$ . In the VO solution with  $M_\nu \sim M_e$ ,  $M_2$  exceeds  $M_3$ , and so can  $M_1$ .

Let us now briefly discuss the sensitivity to input mixing angles of the results reported in the first three columns of Tables I and II. By allowing  $s_{12}$  and  $s_{23}$  to vary inside the ranges reported in Ref. [5] we have found that the numerical values of right-handed neutrino masses change by no more than one order of magnitude. The same happens if  $U_{e3}$  is different from zero up to 0.1. Therefore the above considerations on the physical scales do not change.

It is also interesting to match our results, obtained by a hierarchical spectrum, with the degenerate spectrum and democratic mixing [23]:

$$U_d = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{6} & 1/\sqrt{6} & 2/\sqrt{6} \\ 1/\sqrt{3} & -1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix}. \quad (27)$$

Assuming as light neutrino mass  $m_0 = 2$  eV, relevant for hot dark matter, and the demoting mixing  $U = U_d$ , we obtain  $M_1 = 9.2 \times 10^2$ ,  $M_2 = 7.7 \times 10^7$ ,  $M_3 = 5.5 \times 10^{12}$  GeV for  $M_\nu$

$\sim M_u$  and  $M_1=1.2\times 10^2$ ,  $M_2=5.3\times 10^6$ ,  $M_3=1.5\times 10^9$  GeV for  $M_\nu\sim M_e$ , that is,  $M_3$  at the intermediate scale and  $M_1$  even at the electroweak scale. From Eqs. (25),(26) we see that in the case of degenerate masses  $M_R$  is proportional to  $1/m_0$  and one can easily obtain  $M_3$  when  $m_0$  is lowered.

We have calculated the heavy neutrino masses in a seesaw framework, under simple hypotheses of the Dirac sector and using experimental limits on light neutrino masses and mixings. The results have been matched with intermediate

and unification scales. A key result is that the large mixing MSW solution can be reconciled with GUTs. The analysis can be improved when more precise data will be available. Also the effect of phases should be considered [24]. There are several recent studies about the seesaw mechanism [25], based on various forms of mass matrices; a nice review is in Ref. [26]. Instead, in this paper, we work on the matrix  $V_D$  and on  $D_\nu$ , that is, the leptonic quantities which correspond, in the quark sector, to the observable quantities.

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