Neutrinos on Earth and in the heavens

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Recent data suggest a simple and intriguing form of the neutrino mass matrix. We show how the data may constrain solar neutrino oscillations to be nearly maximal [and rule out the small-angle Mikheyev-Smirnov-Wolfenstein (MSW) explanation of solar neutrino observations] if relic neutrinos comprise at least three percent of the critical mass density of the universe.

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Cosmologists differ on whether or not neutrinos play an essential role in the evolution of the large-scale structure of the universe [1]. In this paper, we assume that they do, and that the sum of their masses is several electron volts so that relic neutrinos comprise several percent of the critical mass density. Under this hypothesis, we demonstrate how a wide range of observations and deductions relating to neutrinos can be explained in terms of a specific effective neutrino mass matrix $\mathcal{M}$ involving a suggestive pattern of neutrino masses and mixings. Although many of our arguments may be found elsewhere in part or in other contexts [2], a cogent synthesis may be useful.

The literature is rife with both experimental and theoretical claims regarding neutrino properties, many of them in conflict with one another. Below is a somewhat arbitrary selection of neutrino ''facts'' we shall accept and describe. They are consistent with one another and are suggested by current data, but they are not decisively established. We do not intend to argue that these particular facts should be accepted as true [3], but rather that, if true, they constrain the neutrino mass matrix to have a form that we find both fascinating and a bit bizarre [4].

Fact 1. There exist precisely three chiral neutrino states with Majorana masses, m_1, m_2, and m_3 (taken to be real and non-negative). In particular we do not consider the existence of additional neutrinos, sterile or otherwise.

Fact 2. Atmospheric neutrinos rarely oscillate into electron neutrinos. This is a plausible, but not inescapable [5], interpretation of recent data from the Super-Kamiokande Collaboration [6] and from CHOOZ [7].

Fact 3. Atmospheric muon neutrinos suffer maximal, or nearly maximal, two-flavor oscillations into tau neutrinos,

$$\sin^2 2\theta > 0.82, \tag{1}$$

and the required neutrino mass-squared difference Δ_a satisfies [6]. The relevant mixing angle satisfies [8]

$$10^{-3}\ \text{eV}^2 < \Delta a < 7\times 10^{-3}\ \text{eV}^2. \tag{2}$$

Fact 4. Oscillations are needed to resolve the discrepancy between the observed and computed solar neutrino fluxes [9,10]. A relevant neutrino squared-mass different in the range

$$6\times 10^{-11}\ \text{eV}^2 < \Delta_s < 2\times 10^{-5}\ \text{eV}^2 \tag{3}$$

provide Mikheyev-Smirnov-Wolfenstein (MSW) explanations for larger values of Δ_s, and just-so explanations for smaller values. It has been suggested [11] that the solar neutrino deficit may result from maximal time-averaged vacuum oscillations. If so, the bound $\Delta_s < 10^{-3}\ \text{eV}^2$ is obtained from reactor experiments [7]. It is premature and unnecessary for us to choose among these proposed solutions to the solar neutrino puzzle.

Fact 5. Here we assume that neutrino masses are large enough to play a significant cosmological role and take

$$\frac{m_1+m_2+m_3}{3} \equiv M \sim 2\ \text{eV}. \tag{4}$$

This is the least well established fact in our list, but it is crucial to our discussion.

Fact 6. Careful studies of many nuclear species have failed to detect neutrinoless double beta decay. These experiments provide bounds on $\mathcal{M}_{ee}$, the *ee* component of the Majorana neutrino mass matrix in the charged lepton flavor basis—a weighted average of neutrino masses. In this paper, we adopt the strongest published bound, $\mathcal{M}_{ee} < B = 0.46$ eV [12].

These ''facts'' dramatically constrain the form of the neutrino mass matrix. In addition, they compel solar neutrino oscillations to be maximal, thereby ruling out the small-angle MSW solution of the solar neutrino puzzle. We begin by considering the well-known implications of fact 1. The most general mass matrix involving three chiral neutrinos is a 3×3 complex symmetric matrix $\mathcal{M}$. It may be written

$$\mathcal{M} = e^{i\eta} U_0^* D_0 U_0^\dagger, \tag{5}$$

where U_0 is an element of SU(3) and D is a diagonal matrix with real non-negative entries m_i. The mass matrix would be real were *CP* conserved, but it is not. Consequently $\mathcal{M}$ involves nine convention-independent parameters. Judicious choice of the phases of the flavor eigenstates allows us to rewrite Eq. (5) as

$$\mathcal{M} = U^* D U^\dagger, \tag{6}$$

where U is a unitary ''Kobayashi-Maskawa'' matrix (involving three angles θ_i and a complex phase δ) expressing flavor eigenstates in terms of mass eigenstates. In standard notation,

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$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} c_2c_3 & c_2s_3 & s_2e^{-i\delta} \\ -c_1s_3-s_1s_2c_3e^{i\delta} & +c_1c_3-s_1s_2s_3e^{i\delta} & s_1c_2 \\ +s_1s_3-c_1s_2c_3e^{i\delta} & -s_1c_3-c_1s_2s_3e^{i\delta} & c_1c_2 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \tag{7}$$

with s_i and c_i standing for sines and cosines of θ_i. The remaining five parameters appear in the diagonal matrix D, which may be written

$$D = \begin{pmatrix} m_1e^{i\phi} & 0 & 0 \\ 0 & m_2e^{i\phi'} & 0 \\ 0 & 0 & m_3 \end{pmatrix}. \tag{8}$$

Each of the phase factors $(e^{i\delta}, e^{i\phi}, e^{i\phi'})$, if not real, is *CP* violating.

The amplitude for atmospheric muon neutrinos with energy E_a to oscillate into ν_e over a distance R_a is

$$\sum_j U_{\mu j}U_{ej}^* e^{im_j^2 R_a/2E_a}. \tag{9}$$

According to fact 2, this amplitude must be small over the range of R_a and E_a relevant to atmospheric neutrinos, around $2E_a/R_a \approx 10^{-3}$ eV^2. It follows that $|m_j^2 - m_k^2|R_a/2E_a$ must be small for some pair of neutrino mass eigenstates j and k. To prove this, we assume the contrary. It follows that the amplitude (9) is small for a range R_a and E_a if and only if $U_{\mu j}U_{ej}^*$ is small for each j. But fact 3 requires that ν_μ is not close to a mass eigenstate. Thus $U_{\mu j}U_{ej}^*$ can be small for each j only if ν_e is close to a mass eigenstate. This would lead us to the so-called small-angle MSW solution that requires $\Delta_s < 2\times 10^{-5}$ eV^2: small compared to $2E_a/R_a$ and contrary to the hypothesis—QED.

Thus the neutrino mass eigenstates associated with atmospheric oscillations must have a squared-mass difference $\Delta_a > 5\times 10^{-4}$ eV^2, while those associated with solar oscillations must have a much smaller squared-mass difference, $\Delta_s \ll \Delta_a$. Without loss of generality, we take $\Delta_a \equiv |m_3^2 - m_2^2|$ and $\Delta_s \equiv |m_2^2 - m_1^2|$. This may all sound rather obvious, and indeed it is the standard wisdom. But notice that the argument of the previous paragraph rules out the possibility of the time-averaged solar neutrino solution with Δ_s near the CHOOZ bound.

We assume that all differences of squares of neutrino masses are less than 7×10^{-3} eV^2. Yet according to fact 5 [and Eq. (4) in particular] the sum of the neutrino masses must be several eV.[1] Thus the three neutrinos must have equal mass to a precision of at least 10^{-3} if they are to play a role in large-scale structure formation. From the point of view of particle theory, this is truly a bizarre result and not at all what one would expect from the simplest seesaw mechanisms. It is nonetheless an immediate consequence of the facts we have accepted. We now proceed to a more detailed discussion of the mass matrix (6), from which additional constraints can be found.

We may express the *in vacua* energy-dependent survival probabilities for solar and atmospheric neutrinos in terms of the parameters so defined. Because the path length R_s of a solar neutrino is roughly an astronomical unit, Eq. (2) yields $\Delta_a R_s/E \gg 1$. Using this relation, we obtain

$$P(\nu_e \to \nu_e)|_{\text{solar}} \simeq 1 - \frac{\sin^2 2\theta_2}{2} - \cos^4\theta_2 \sin^2 2\theta_3 \sin^2(\Delta_s R_s/4E). \tag{10}$$

Because the path length of an atmospheric neutrino R_a can be no greater than Earth's diameter, Eq. (3) yields $\Delta_s R_a/E \ll 1$. Using this relation we obtain

$$P(\nu_\mu \to \nu_\mu)|_{\text{atmospheric}} = 1 - 4\sin^2\theta_1\cos^2\theta_2(1 - \sin^2\theta_1\cos^2\theta_2)\sin^2(\Delta_a R_a/4E). \tag{11}$$

These oscillations produce electron or tau neutrinos in the ratio

$$\left.\frac{P(\nu_\mu \to \nu_e)}{P(\nu_\mu \to \nu_r)}\right|_{\text{atmospheric}} \simeq \frac{\tan^2\theta_2}{\cos^2\theta_1}. \tag{12}$$

Note that none of Eqs. (10), (11), and (12) involve the *CP*-violating parameter δ.

We turn to the consequences of our other tentatively accepted facts. Fact 2 and Eq. (12) yield

$$\theta_2 \simeq 0, \tag{13}$$

expressing the absence of oscillations of atmospheric oscillations into electron neutrinos. This result greatly simplifies Eqs. (10) and (11), which become

$$P|_{\text{solar}} \simeq 1 - \sin^2 2\theta_3 \sin^2(\Delta_s R_s/4E), \qquad P|_{\text{atmospheric}} \simeq 1 - \sin^2 2\theta_1 \sin^2(\Delta_a R_a/4E). \tag{14}$$

Thus θ_1 is the parameter controlling atmospheric neutrino oscillations, and we conclude from fact 3 and Eq. (1) that

$$\sin 2\theta_1 \simeq 1, \tag{15}$$

expressing the observation that these oscillations are nearly maximal.

[1]The bound $M < 4.4$ eV follows from a recent measurement of the tritium beta spectrum [13].

Finally, we must address fact 6, the observed suppression of neutrinoless double beta decay. The amplitude for this process is proportional to the quantity $\mathcal{M}_{ee}$, on which the bound is

$$\mathcal{M}_{ee} \equiv |m_1 c_2^2 c_3^2 e^{i\phi} + m_2 c_2^2 s_3^2 e^{i\phi'} + m_3 s_2^2 e^{i2\delta}| < B. \quad (16)$$

We have seen that fact 5 requires neutrino masses to be equal to a precision sufficient to neglect their differences in Eq. (16). Furthermore, Eq. (13) lets us put Eq. (16) into the simple form

$$|\cos^2\theta_3 e^{i\phi} + \sin^2\theta_3 e^{i\phi'}| < B/M. \quad (17)$$

If we set $M = 2$ eV and $B = 0.46$ eV (the current experimental upper bound), we find that Eq. (16) can be satisfied if $\phi + \phi' \simeq \pi$ and $|\cos 2\theta_3| < 0.23$, or

$$\sin^2 2\theta_3 > 0.95. \quad (18)$$

If $M \geqslant 2$ eV, our six facts are mutually consistent if and only if solar neutrino oscillations are nearly maximal. Somewhat stronger bounds on neutrinoless double beta decay could strengthen Eq. (18) enough to leave just-so oscillations [14] as the only viable explanation of the solar neutrino data [10]. Conversely, if the small angle MSW description of solar neutrino oscillations is correct, the sum of the neutrino masses is bounded above by $3B \simeq 1.4$ eV. In this case, future double beta-decay experiments may exclude the cosmological relevance of relic neutrinos.

The neutrino matrix we are led to has approximately the following form:

$$\mathcal{M} = M \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}. \quad (19)$$

This has several intriguing properties: $\mathcal{M}\mathcal{M}^\dagger$ is approximately a multiple of the unit matrix and $\mathcal{M}_{ee} \simeq 0$. To the extent that these relations are satisfied, the following processes are forbidden at all orders in perturbation theory: neutrinoless double beta decay; muon decay into $e + \gamma$ or into $e + e + \bar{e}$; muonium-antimuonium transitions; muon-electron conversion via capture; the induction of an electron electric dipole moment. This point is academic because the detection of any of the above processes (except neutrinosless double beta decay) would require a radical revision of the standard model.

Neutrino astronomy is a new science. Future observations of neutrinos from nearby supernovae, or among ultrahigh energy cosmic rays, are likely sources of new information about particles and the universe. These neutrinos, having traversed great distances, will experience time-averaged oscillations so that their composition at detection will not coincide with their composition at birth. Let D_l be the number of detected neutrinos with identities ν_l, and B_l their numbers at birth. With the above mixing parameters we find

$$\begin{pmatrix} D_e \\ D_\mu \\ D_\tau \end{pmatrix} = \frac{1}{8} \begin{pmatrix} 4 & 2 & 2 \\ 2 & 3 & 3 \\ 2 & 3 & 3 \end{pmatrix} \begin{pmatrix} B_e \\ B_\mu \\ B_\tau \end{pmatrix}. \quad (20)$$

Half of the ν_e burst from a supernovae reach Earth as ν_e, while cosmic ν_μ's are seen as 25% ν_e's and 37.5% ν_τ's.

Let us summarize our results. There are nine parameters in the neutrino mass matrix, all but one of which are severely constrained by the facts we have accepted. The three neutrino masses are nearly (but not quite) the same. The angles relating flavor and mass eigenstates take the following simple values: $\theta_1 \simeq \theta_3 \simeq \pi/4$ and $\theta_2 \simeq 0$. These simple relations may indicate a deeper underlying truth.

In this connection, note that just one of the three *a priori CP*-violating parameters in $\mathcal{M}$ is unconstrained by our analysis: We must have $\phi - \phi' \simeq \pi$ to suppress neutrinoless double beta decay and the parameter δ is *hors de combat* because it always occurs multiplied by $\sin\theta_2$, which nearly vanishes.

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