F^0-$\overline{F^0}$ mixing and CP violation in the general two Higgs doublet model

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(Received 14 June 1999; published 14 March 2000)

A phenomenological analysis of the general two Higgs doublet model is presented. Possible constraints of the Yukawa couplings result from the K^0-$\overline{K}^0$, B^0-$\overline{B}^0$, and D^0-$\overline{D}^0$ mixings. It is shown that the emerging of various new sources of CP violation in the model could strongly affect the determination of the unitarity triangle. It could be useful to look for a signal of new physics by comparing the extracted angle β from two different ways, such as from the process $B \to J/\psi K_S$ and from fitting the quantities $|V_{ub}|$, Δm_B, and ϵ.

PACS number(s): 11.30.Er, 12.60.Fr

I. INTRODUCTION

In the standard model (SM) of an electroweak $SU(2)_L \times U(1)_Y$ gauge theory with only one Higgs doublet, the only source of CP violation comes from the complex Yukawa coupling between Higgs and fermion fields [1]. Since the Higgs sector of the SM is not well understood yet, many possible extensions of the SM have been proposed [2]. One of the simplest extensions of the SM is to simply add one Higgs doublet. For convenience, in our following discussions we may call such a minimal extension of the standard model, which only adds an extra Higgs doublet, the standard two Higgs doublet model (S2HDM) [3–8] and assume CP violation solely originating from the Higgs potential [9,7,8]. The most general Yukawa coupling and Higgs potential can be written as

$$L_Y = \overline{Q}_L(\Gamma_1^U \tilde{\phi}_1 + \Gamma_2^U \tilde{\phi}_2)U_R + \overline{Q}_L(\Gamma_1^D \phi_1 + \Gamma_2^D \phi_2)D_R \quad (1)$$

and

$$\begin{aligned} V(\phi_1,\phi_2) = & -\mu_1^2 \phi_1^\dagger \phi_1 - \mu_2^2 \phi_2^\dagger \phi_2 - (\mu_{12}^2 \phi_1^\dagger \phi_2 + \text{H.c.}) \\ & + \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1\phi_2^\dagger\phi_2) \\ & + \lambda_4(\phi_1^\dagger\phi_2\phi_2^\dagger\phi_1) + \frac{1}{2}[\lambda_5(\phi_1^\dagger\phi_2)^2 + \text{H.c.}] \\ & + [(\lambda_6\phi_1^\dagger\phi_1 + \lambda_7\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + \text{H.c.}]. \end{aligned} \quad (2)$$

The major issue with respect to the two Higgs doublet model is that it allows flavor-changing neutral current (FCNC) at the tree level, which must be strongly suppressed in K^0-$\overline{K}^0$ and B^0-$\overline{B}^0$ mixing processes. In order to prevent FCNC from tree level, an *ad hoc* discrete symmetry is often imposed:

$$\phi_1 \to -\phi_1 \quad \text{and} \quad \phi_2 \to \phi_2,$$
$$U_{R_i} \to -U_{R_i} \quad \text{and} \quad D_{R_i} \to \mp D_{R_i}. \quad (3)$$

Thus, one obtains the so-called model I and model II, which depend on whether the up-type and down-type quarks are coupled to the same or a different Higgs doublet, respectively [2]. Once this discrete symmetry is adopted, the factors μ_{12}, λ_6, and λ_7 in Eq. (2) must vanish; as a result no CP violation can occur from $V(\phi)$. Thus the only source of CP violation is the complex Yukawa couplings, which lead to a phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix.

In contrast one can replace the discrete symmetry with an approximate global family symmetry [4,5,7,8], thus the suppression of FCNC can be explained via the smallness of the off-diagonal terms. Furthermore, when abandoning the discrete symmetries, one can obtain rich sources of CP violation from a single relative phase between the two vacuum expectation values of Higgs field after spontaneous symmetry breaking. It has been shown [7,8] that even when the CKM matrix is real, the single phase arising from the spontaneous symmetry breaking can provide enough CP violation to meet the experimental measurments. One particularly important observation is of a new source of CP violation in charged Higgs boson interactions, which is independent of the CKM phase and can lead to a value of ϵ'/ϵ as large as 10^{-3} [7,8]. In the S2HDM, the two Higgs fields have, in general, the vacuum expectation values:

$$\langle \phi_1^0 \rangle = \frac{v}{\sqrt{2}} \cos\beta e^{i\delta},$$
$$\langle \phi_2^0 \rangle = \frac{v}{\sqrt{2}} \sin\beta. \quad (4)$$

It is natural to use a suitable basis,

$$H_1 = \cos\beta\phi_1 e^{-i\delta} + \sin\beta\phi_2,$$
$$H_2 = \sin\beta\phi_1 e^{-i\delta} - \cos\beta\phi_2, \quad (5)$$

such that

$$H_1 = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+\rho) \end{pmatrix},$$

$$H_2=\begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(R+iI) \end{pmatrix}, \tag{6}$$

where H^0, R, and I are real Higgs bosons. The three neutral scalars $\hat{H}_k^0\equiv(R,\rho,I)$ can be rotated to mass eigenstates $h_k^0 \equiv(h,H^0,A)$ via an orthogonal matrix O^H:

$$\hat{H}_k^0=O_{kl}^H H_l^0 . \tag{7}$$

From approximate global family symmetries, we know that the Yukawa coupling matrix Γ_i^F in Eq. (1) has small off-diagonal elements, typically between 0.01 and 0.2 in order to meet the constraint of FCNC from K^0-$\overline{K^0}$, B^0-$\overline{B^0}$ mixing. The Yukawa interaction can be rewritten as [7]

$$L_Y=(L_1+L_2)(\sqrt{2}G_F)^{1/2} \tag{8}$$

with

$$L_1=\sqrt{2}\left(H^+\sum_{i,j}^{3}\ \xi_{d_j}m_{d_j}V_{ij}\overline{u_L^i}d_R^j-H^-\sum_{i,j}^{3}\ \xi_{u_j}m_{u_j}V_{ij}^{\dagger}\overline{d_L^i}u_R^j\right)$$
$$+H^0\sum_{i}^{3}\ (m_{u_i}\overline{u_L^i}u_R^i+m_{d_i}\overline{d_L^i}d_R^i)+(R+iI)\sum_{i}^{3}\ \xi_{d_i}m_{d_i}\overline{d_L^i}d_R^i+(R-iI)\sum_{i}^{3}\ \xi_{u_i}m_{u_i}\overline{u_L^i}u_R^i+\text{H.c.} \tag{9}$$

$$L_2=\sqrt{2}\left(H^+\sum_{i,j'\neq j}^{3}\ V_{ij'}\mu_{j'j}^{d}\overline{u_L^i}d_R^j-H^-\sum_{i,j'\neq j}^{3}\ V_{ij}^{\dagger}\mu_{j'j}^{u}\overline{d_L^i}u_R^j\right)$$
$$+(R+iI)\sum_{i\neq j}^{3}\ \mu_{ij}^{d}\overline{d_L^i}d_R^j+(R-iI)\sum_{i\neq j}^{3}\ \mu_{ij}^{u}\overline{u_L^i}u_R^j+\text{H.c.}, \tag{10}$$

where L_1 has no flavor-changing effects other than that expected for $H^{\pm}$ from the CKM matrix V and L_2 contains the flavor-changing effects for neutral bosons as well as small additional flavor-changing terms for $H^{\pm}$. The factors $\xi_{f_i}m_{f_i}$ and μ_{ij}^f arise primarily from the diagonal and off-diagonal elements of Γ_i^f, respectively.

There are four major sources of CP violation [7,8]: (1) CKM matrix; (2) the phase in factor ξ_{f_i} which provides CP violation in charged-Higgs boson exchange; (3) the phase in μ_{ij}^f which yields CP violation in FCNC; and (4) CP violation in the mixing matrix O^H. One of the most distinctive features of these sources is that the factor ξ_{f_i} can provide CP violation in charged Higgs boson exchange in addition to and independent of the CKM phase. As a consequence, in $\Delta S=1$ transitions its contribution to ϵ'/ϵ could be as large as 10^{-3}. Thus a measurement of ϵ'/ϵ would not necessarily be due to CKM mechanism.

II. CONSTRAINTS FROM K^0-$\overline{K^0}$, B^0-$\overline{B^0}$, AND D^0-$\overline{D^0}$ MIXINGS

In the standard model, it is known that the neutral meson mixings arise from the box diagram through two-W-boson exchange. The extremely small values of the neutral K and B mass differences impose severe constraints on new physics beyond the SM, especially on those with FCNC at tree level. In the S2HDM, additional contributions to the neutral meson mixings can arise from the box diagrams with charged-scalar exchanges and tree diagrams with neutral-scalar exchanges. The mass difference of K_L-K_S is given by

$$\Delta m_K\simeq 2\ \text{Re}\,M_{12}\equiv 2\ \text{Re}\,(M_{12}^{WW}+M_{12}^{HH}+M_{12}^{HW}+M_{12}^{H^0}+M_{12}'), \tag{11}$$

where M_{12}^{WW}, M_{12}^{HH}, and M_{12}^{HW} are the contributions from box diagrams through two-W-boson, two charged-scalar H^+, and one-W-boson and one charged-scalar exchanges, respectively. $M_{12}^{H^0}$ is the one from the FCNC through neutral-scalar exchanges at tree level. M_{12}' presents other possible contributions, such as two-coupled penguin diagrams and nonperturbative effects. They result from the corresponding effective Hamiltonian

$$H_{eff}^{WW}=-\frac{G_F^2}{16\pi^2}m_W^2\sum_{i,j}^{c,t}\ \eta_{ij}\lambda_i\lambda_j\sqrt{x_ix_j}B^{WW}(x_i,x_j)\overline{d}\gamma_\mu(1-\gamma_5)s\overline{d}\gamma^\mu(1-\gamma_5)s, \tag{12}$$

$$H_{eff}^{HH}=-\frac{G_F^2}{16\pi^2}m_W^2\sum_{i,j}^{u,c,t}\eta_{ij}^{HH}\lambda_i\lambda_j\frac{1}{4}\{B_V^{HH}(y_i,y_j)[\sqrt{x_ix_j}\sqrt{y_iy_j}|\xi_i|^2|\xi_j|^2\cdot\overline{d}\gamma_\mu(1-\gamma_5)s\overline{d}\gamma^\mu(1-\gamma_5)s$$
$$+\sqrt{x_sx_d}\sqrt{y_sy_d}\xi_s^2\xi_d^{*2}\overline{d}\gamma_\mu(1+\gamma_5)s\overline{d}\gamma^\mu(1+\gamma_5)s+2\delta_{ij}\sqrt{x_ix_j}\sqrt{y_sy_d}\xi_s\xi_d^*\xi_i\xi_j^*\overline{d}\gamma_\mu(1+\gamma_5)s\overline{d}\gamma^\mu(1-\gamma_5)s]$$
$$+B_S^{HH}(y_i,y_j)\sqrt{x_iy_j}[x_d\xi_d^{*2}\xi_i^*\xi_j^*\overline{d}(1-\gamma_5)s\overline{d}(1-\gamma_5)s+x_s\xi_s^2\xi_i\xi_j\overline{d}(1+\gamma_5)s\overline{d}(1+\gamma_5)s$$
$$+2\sqrt{x_sx_d}\xi_s\xi_d^*\xi_i\xi_j^*\overline{d}(1+\gamma_5)s\overline{d}(1-\gamma_5)s]\},\tag{13}$$

$$H_{eff}^{HW}=-\frac{G}{16\pi^2}m_W^2\sum_{i,j}^{u,c,t}\eta_{ij}^{HW}\lambda_i\lambda_j\{2\sqrt{x_ix_j}\sqrt{y_iy_j}\xi_i\xi_j^*B_V^{HW}(y_i,y_j,y_w)\cdot\overline{d}\gamma_\mu(1-\gamma_5)s\overline{d}\gamma^\mu(1-\gamma_5)s+(y_i+y_j)\sqrt{x_dx_s}\xi_s\xi_d^*$$
$$\times[B_T^{HW}(y_i,y_j,y_w)\cdot\overline{d}\sigma_{\mu\nu}(1-\gamma_5)s\overline{d}\sigma^{\mu\nu}(1+\gamma_5)s+B_S^{HW}(y_i,y_j,y_w)\overline{d}(1-\gamma_5)s\overline{d}(1+\gamma_5)s]\},\tag{14}$$

where the B^{WW}, B_V^{HH}, B_S^{HH}, B_V^{HW}, B_S^{HW}, and B_T^{HW} arise from the loop integrals [8] and are the functions of $x_i=m_i^2/m_W^2$ and $y_i=m_i^2/m_H^2$ with $i=u,c,t,W$. η_{ij}, η_{ij}^{HH}, and η_{ij}^{HW} are the possible QCD corrections and $\lambda_i=V_{is}V_{id}^*$. Note that in obtaining the above results the external momentum of the d and s quark has been neglected. Except for in this approximation, which is reliable as their current mass is small, we keep all the terms. This is because all the couplings λ_i and ξ_i are complex in our model and even if some terms are small, they can still play an important role in CP violation since the observed CP-violating effect in kaon decay is of order 10^{-3}. The contribution of neutral Higgs bosons exchange at tree level can be evaluated by

$$M_{12}^{H^0}=\langle P^0|H_{eff}^{H^0}|\overline{P^0}\rangle$$
$$=\frac{G_F^2}{12\pi^2}f_{P^0}^2\tilde{B}_{P^0}m_{P^0}\left(\sqrt{\frac{m_{f_i}}{m_{f_j}}}\right)^2\left(1+\frac{m_{f_i}}{m_{f_j}}\right)^{-1}m_{f_j'}^2\sum_k$$
$$\times\left(\frac{2\sqrt{3}\pi v m_{P^0}}{m_{H_k^0}m_{f_j'}}\right)^2(Y_{k,ij}^f)^2\tag{15}$$

with

$$(Y_{k,ij}^f)^2=(Z_{k,ij}^f)^2+\frac{1}{2}r_{P^0}S_{k,ij}^fS_{k,ji}^{f*},$$

$$Z_{k,ij}^f=-\frac{i}{2}(S_{k,ij}^f-S_{k,ji}^{f*}).$$

$S_{k,ij}$ is related to μ_{ij}^f through

$$S_{k,ij}^f=(O_{1k}^H+i\sigma_fO_{3k}^H)\frac{\mu_{ij}^f}{\sqrt{m_im_j}},\tag{16}$$

where $\sigma_f=1$ for d type quarks and $\sigma_f=-1$ for u type quarks. The formula is expressed in a form which is convenient in comparison with the one obtained from the box diagram in the standard model. Here $\sqrt{m_{f_i}/m_{f_j}}$ with convention $i<j$ plays the role of the CKM matrix element V_{ij}, and $m_{f_j'}$ is introduced to correspond to the loop-quark mass of the box diagram. Namely f_j' and f_j are the two quarks in the same weak isospin doublet. Note that the result is actually independent of $m_{f_j'}$. Here m_{f_i} are understood to be the current quark masses. In our following numerical estimations we will use $m_u=5.5$ MeV, $m_d=9$ MeV, $m_s=180$ MeV, $m_c=1.4$ GeV, and $m_b=6$ GeV which are defined at a renormalization scale of 1 GeV. f_{P^0} and m_{P^0} are the leptonic decay constant (with normalization $f_\pi=133$ MeV) and the mass of the meson P^0, respectively. $\tilde{B}_{P^0}$ and $\tilde{r}_{P^0}$ are bag parameters defined by

$$\langle P^0|(\overline{f_i}(1\pm\gamma_5)f_j)^2|\overline{P^0}\rangle=-\frac{f_{P^0}m_{P^0}^3}{(m_{f_i}+m_{f_j})^2}\tilde{B}_{P^0},\tag{17}$$

$$1+\tilde{r}_{P^0}=-\frac{\langle P^0|\overline{f_i}(1\pm\gamma_5)f_j\overline{f_i}(1\mp\gamma_5)f_j|\overline{P^0}\rangle}{\langle P^0|\overline{f_i}(1\pm\gamma_5)f_j\overline{f_i}(1\pm\gamma_5)f_j|\overline{P^0}\rangle}.\tag{18}$$

In the vacuum saturation and factorization approximation with the limit of a large number of colors, we have $\tilde{B}_{P^0}\to1$ and $\tilde{r}_{P^0}\to0$, thus $Y_{k,ij}^f=Z_{k,ij}^f$.

It is known that H_{eff}^{WW} contribution to Δm_K is dominated by the c quark exchange and its value is still uncertain due to the large uncertainties of the hadronic matrix element

$$\langle K^0|(\overline{d}\gamma_\mu(1-\gamma_5)s)^2|\overline{K^0}\rangle=-\frac{8}{3}f_K^2m_K^2B_K,\tag{19}$$

where B_K ranges from 1/3 [10] (by the PCAC and SU(3) symmetry), 3/4 [11] (in the limit of a large number of colors) to 1 [12] (by the vacuum insertion approximation). The results from QCD sum rule and lattice calculations lie in this range. For small B_K, the short-distance H_{eff}^{WW} contribution to Δm_K fails badly to account for the measured mass difference.

In general, when neglecting the contribution from top quark which is suppressed by a factor of

$$\left(\frac{V_{td}V_{ts}^*}{V_{cd}V_{cs}^*}\right)^2 \frac{m_t^2}{m_c^2} \sim \mathcal{O}(10^{-2}),$$

we obtain

$$\begin{aligned}\Delta m_K = & \frac{G_F^2}{6\pi^2} f_K^2 B_K m_K m_c^2 \sin^2\theta \\ & \times \Bigg\{ \eta_{cc} B^{WW}(x_c) + \frac{1}{4}\eta_{cc}^{HH} y_c |\xi_c|^4 B_V^{HH}(y_c) \\ & + 2\eta_{cc}^{HW} y_c |\xi_c|^2 B_V^{HW}(y_c, y_w) \\ & + \frac{\tilde{B}_K}{B_K} \sum_k \left(\frac{2\sqrt{3}\pi v m_K}{m_{H_k^0} m_c}\right)^2 Re(Y_{k,12}^d)^2 \Bigg\}, \end{aligned} \tag{20}$$

which is subject to the experimental constraint

$$\Delta m_K = 3.52 \times 10^{-6}\ \text{eV} \simeq \sqrt{2}\frac{G_F^2}{6\pi^2} f_K^2 m_K m_c^2 \sin^2\theta. \tag{21}$$

The effective Hamiltonian for B_d^0-$\overline{B}_d^0$ mixing is calculated with the aid of the box diagrams in full analogy to the treatment of the K^0-$\overline{K}^0$ system. Its explicit expression can be simply read off from the one for K^0-$\overline{K}^0$ system by a corresponding replacement $s \leftrightarrow b$. The ''standard approximation'' made there, namely neglecting the external momenta of the quarks, is also reliable since dominant contributions come from the intermediate top quark. With this analogy, the considerations and discussions on K^0-$\overline{K}^0$ mixing can be applied to the B_d^0-$\overline{B}_d^0$ mixing for the contributions from box diagrams. As it is expected that $|\Gamma_{12}|/2 \ll |M_{12}|$ in the B system (which is different from K system), the mass difference for B_d^0-$\overline{B}_d^0$ system is given by $\Delta m_B \simeq 2|M_{12}|$.

The general form for the mass difference in the B_d^0-$\overline{B}_d^0$ system can be written as

$$\begin{aligned}\Delta m_B \simeq & \frac{G_F^2}{6\pi^2}(f_B\sqrt{B_B\eta_{tt}})^2 m_B m_t^2 |V_{td}|^2 \frac{1}{\eta_{tt}} \\ & \times \Bigg| \Bigg\{ \eta_{tt} B^{WW}(x_t) + \frac{1}{4}\eta_{tt}^{HH} y_t |\xi_t|^4 B_V^{HH}(y_t) \\ & + 2\eta_{tt}^{HW} y_t |\xi_t|^2 B_V^{HW}(y_t, y_w) \\ & + \frac{\tilde{B}_B}{B_B} \sum_k \left(\frac{2\sqrt{3}\pi v m_B}{m_{H_k^0} m_t}\right)^2 \frac{m_d}{m_b} \frac{1}{V_{td}^2} (Y_{k,13}^d)^2 \Bigg\} \Bigg|, \end{aligned} \tag{22}$$

where the top quark dominates over charm quark by a factor of

$$\left(\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}\right)^2 \frac{m_t^2}{m_c^2} \sim \mathcal{O}(10^4).$$

Thus the contribution of charm quark can be safely neglected. Δm_B is subject to the experimental constraint

$$\begin{aligned}\Delta m_B = & (3.6 \pm 0.7) \times 10^{-4}\ \text{eV} \\ \simeq & \frac{G_F^2}{12\pi^2}(135\ \text{MeV})^2 m_B (176\ \text{GeV})^2 (\sin\theta = 0.22)^6. \end{aligned} \tag{23}$$

It is known that in the standard model the short-distance contribution to Δm_D from the box diagram with W-boson exchange is of the order of magnitude $\Delta m_D^{Box} \sim O(10^{-9})$ eV, here the external momentum effects have to be considered and were found to suppress the contribution by two orders of magnitude [13]. This is because of the low mass of the intermediate state. It is not difficult to see that the additional box diagram with charged-scalar gives an even smaller contribution except $|\xi_s|$ is as large as $|\xi_s| \sim 2m_{H^+}/m_s$, which is unreliablly large for the present bound $m_{H^+} > 41$ GeV. It has been shown that dominant contribution to Δm_D may come from the long-distance effect since the intermediate states in the box diagram are d and s quarks. The original estimations were that $\Delta m_D \sim 3 \times 10^{-5}$ eV [14] and $\Delta m_D \sim 1 \times 10^{-6}$ eV [15]. An alternative calculation [16] using the heavy quark effective theory showed that large cancellations among the intermediate states may occur so that the long-distance standard model contribution to Δm_D is only larger by about one order of magnitude than the short-distance contribution, which was also supported in a subsequent calculation [17].

With this in mind, we now consider the contribution to Δm_D from the neutral scalar interaction in our model. It is easy to read off from Eq. (15)

$$\begin{aligned}\Delta m_D^H = 2|M_{12}^H| = & \frac{G_F^2}{6\pi^2} f_D^2 \tilde{B}_D m_D \left(\sqrt{\frac{m_u}{m_c}}\right)^2 m_s^2 \\ & \times \sum_k \left(\frac{2\sqrt{3}\pi v m_D}{m_{H_k^0} m_s}\right)^2 |Y_{k,12}^u|^2, \\ = & 0.6410^{-4}\left(\frac{f_D\sqrt{\tilde{B}_D}}{210\ \text{MeV}}\right)^2 \sum_{k=1}^3 \left(\frac{500\ \text{GeV}}{m_{H_k^0}}\right)^2 |Y_{k,12}^u|^2. \end{aligned} \tag{24}$$

With the above expected values in the second line for various parameters, the predicted value for Δm_D can be close to the current experimental limit $|\Delta m_D| < 1.3 \times 10^{-4}$ eV. This implies that a large D^0-$\overline{D}^0$ mixing which is larger than the standard model prediction does not get excluded. With this analysis, we come to the conclusion that a positive signal of neutral D meson mixing from the future experiments at Fer-

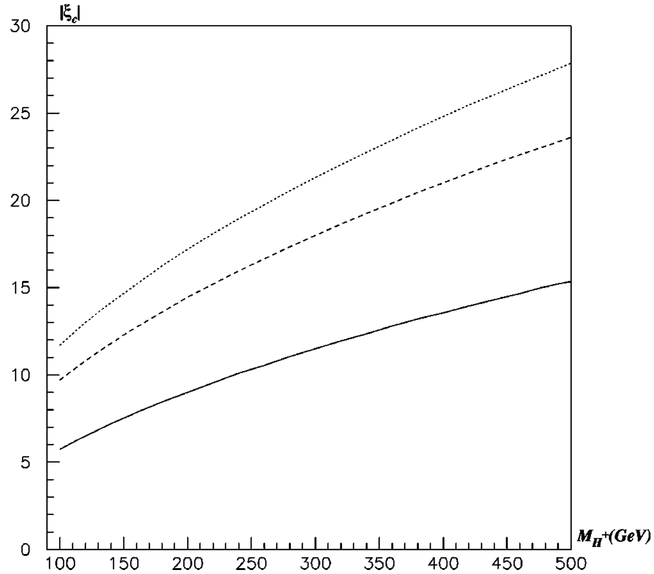

FIG. 1. The upper bound of $|\xi_c|$ with respect to the mass of the charged Higgs scalar in case 1. Three curves correspond to the ratio $(\Delta m_k)_{\rm SM}/(\Delta m_k)_{\rm exp}$ from 0.52 (dotted) and 0.67 (dashed) to 0.91 (solid).

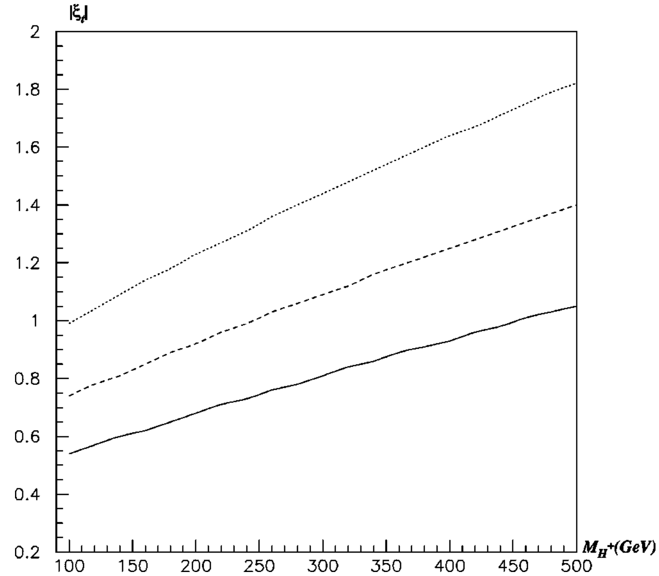

FIG. 2. The value of $|\xi_t|$ with respect to the mass of charged Higgs m_H^+. The three curves corresponding to different ratios of HW and HH box diagrams to the one from the SM are from 2:1 (dotted), 1:1 (dashed), and 0.5:1 (solid).

milab, the Cornell Electron Storage Ring (CESR), and at a τ-charm factory would be in favor of the S2HDM especially when the exotic neutral scalars are not so heavy.

We now proceed to the discussion of the constraints on the parameters of the model. Since the parameters ξ_{f_i},μ_{ij}^f are in general all free parameters, for simplicity we will consider the constraints in two extreme cases.

In case 1, the mass difference is purely explained through the additional box diagrams from two scalar-boson and one-W-boson one scalar-boson. In this case, the parameter ξ_{f_i} is of particular importance. Both its amplitude and phase will play an important role in the neutral meson mass difference and CP violation. It is quite different from the earlier analysis in type 1 and type 2 2HDM [18] in that we do not take ξ_u,ξ_c,ξ_t to be equal, i.e., $\xi_u=\xi_c=\xi_t=\tan\beta$. This is why the constraint from ϵ is much stronger than the one from Δm_K in those models. In the S2HDM, where one has in general $\xi_u\neq\xi_c\neq\xi_t$, there is more freedom to fit ϵ and $\Delta m_K,\Delta m_B$ as well as Δm_D. Since the main contribution to Δm_K comes from the c quark though the loop, the upper bound of ξ_c can be extracted from K^0-$\overline{K}^0$ mixing.

The result is plotted in Fig. 1. In the calculation we take $f_K=161$ MeV and $B_K=0.75$. The range of m_H^+ is from 100 to 1000 GeV. Since the bound of $|\xi_c|$ strongly depends on the SM prediction on Δm_K, three different values of $\Lambda_{\rm QCD}(\Lambda_{\rm QCD}=0.21,0.31,0.41)$ are used and the corresponding ratio to the experimental data $(\Delta m_K)_{exp}$ is 0.52, 0.67, and 0.91 [19].

In the B system, it is of interest to study its relative ratio to the SM, since a large degree of uncertainty can be avoided from CKM matrix $|V_{td}|$ and hardonic matrix elements.

In Fig. 2 we illustrate the relation between $|\xi_t|$ and charged Higgs mass m_H^+ when the ratio of HW and HH box diagram contribution to the one from the WW box in the SM is 2:1, 1:1, and 0.5:1.

As is shown in Fig. 1 and Fig. 2, in general $|\xi_c|$ is much larger than $|\xi_t|$. Even when the HW and HH contribution to Δm_B is twice as much as the SM one, $|\xi_c|$ can still be larger than $|\xi_t|$ by an order of magnitude.

In case 2, the mass difference is fitted through neutral-scalar exchange in the tree level. From Eq. (15) we know that the parameters that arise in M_{12}^0 are $Y_{k,ij}^f$ rather than μ_{ij}^f. If S_{ij}^f is expected to be symmetric under the exchange $i\leftrightarrow j$ and $r_{P^0}=1$ then $Y_{k,ij}^f$ has the following simple form:

$$Y_{k,i,j}^f=O_{1k}^H\frac{\mathrm{Im}\,\mu_{ij}^f}{\sqrt{m_im_j}}+\sigma_fO_{3k}^H\frac{\mathrm{Re}\,\mu_{ij}^f}{\sqrt{m_im_j}}. \tag{25}$$

Hence both the imaginary and real parts of μ_{ij}^f are of importance. Futhermore, the phase in μ_{ij}^f is also a source of CP violation as we have mentioned in the previous section. To simplify the discussion, we assume that one of the scalar bosons, for example, the scalar h, is much lighter than the other two H and A. Here H and A are assumed to be heavier than 500 GeV. The upper bounds can be obtained from K^0-$\overline{K}^0$, B^0-$\overline{B}^0$, and D^0-$\overline{D}^0$ mixing. The present consideration is more general than the one in [21] where all the couplings $Y_{k,ij}^f$ are settled to be equal. As a consequence, the constraints from different meson mixing give different upper bounds upon different $Y_{k,ij}^f$s. The results are shown in Fig. 3. It is seen from Fig. 3 that the upper bound of $Y_{k,12}^u$ is much higher than that from the K^0 and B^0 system. This implies that a larger D^0-$\overline{D}^0$ mixing than the standard model prediction is possible.

III. *CP* VIOLATION AND UNITARITY TRIANGLE

Besides the neutral meson mass difference, the indirect CP violation parameter ϵ_K could also provide constraints on

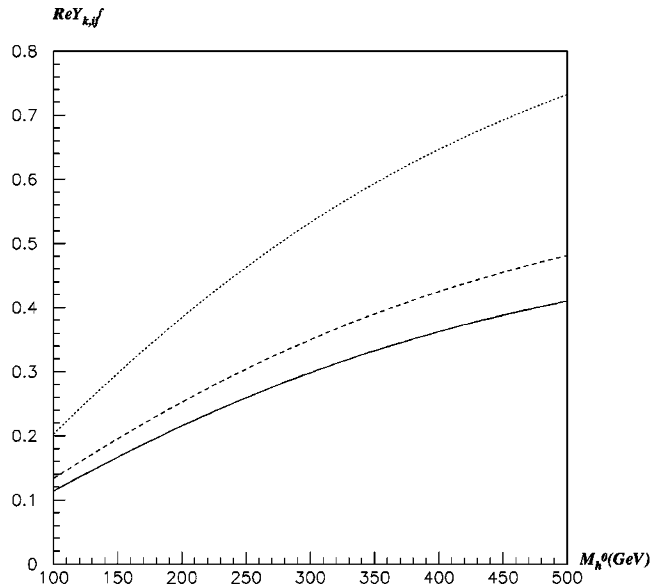

FIG. 3. The m_h^0 dependence of the upper bound of $Y_{k,ij}^f$. Re $Y_{1,12}^d$ from K^0-$\overline{K}^0$ (solid), Re $Y_{1,13}^d$ from B^0-$\overline{B}^0$ (dashed), and Re $Y_{1,12}^u$ from D^0-$\overline{D}^0$ (dotted). The mass of the other scalar m_A^0 is fixed at $m_A^0=500$ GeV.

the values of ξ_{f_i} and μ_{ij}^f. The standard definition of ϵ is

$$\epsilon=\frac{1}{\sqrt{2}}\left(\frac{\text{Im }M_{12}}{2\,\text{Re}\,M_{12}}+\xi_0\right)e^{i\pi/4}, \tag{26}$$

where $\xi_0=\text{Im}\,A_0/\text{Re}\,A_0$ with $|A_0|=(3.314\pm0.004)\times10^{-7}$ GeV is the isospin-zero amplitude of $K\to\pi\pi$ decay. Usually, the ξ_0 term is relatively small as it is proportional to the small direct CP-violating parameter ϵ'.

The first part of contribution to ϵ comes from the box diagram through W-boson and charged-scalar exchange

$$\begin{aligned}\text{Im}\,M_{12}^{Box}&=\text{Im}\,M_{12}^{WW}+\text{Im}\,M_{12}^{HH}+\text{Im}\,M_{12}^{HW}\\&=\frac{G^2}{12\pi^2}f_K^2B_Km_Km_im_j\\&\quad\times\left\{\sum_{i,j}^{c,t}\text{Im}(\lambda_i\lambda_j)\text{Re}\,B_{ij}(m_i,m_j;\xi_i,\xi_j)\right.\\&\quad\left.+\text{Re}(\lambda_i\lambda_j)\text{Im}\,B_{ij}(m_i,m_j;\xi_i,\xi_j)\right\},\end{aligned} \tag{27}$$

where $B_{ij}(m_i,m_j;\xi_i,\xi_j)$ depend on the integral functions of the box diagrams [8]. The imaginary part $\text{Im}\,B_{ij}(m_i,m_j;\xi_i,\xi_j)$ arises from the complex couplings ξ_i.

The second part is due to the flavor-changing neutral-scalar inteactions at tree level

$$\begin{aligned}\text{Im}\,M_{12}^{H^0}&=\frac{G^2}{12\pi^2}f_K^2\tilde{B}_Km_K\left(\sqrt{\frac{m_d}{m_s}}\right)^2m_c^2\\&\quad\times\sum_k\left(\frac{2\sqrt{3}\pi v m_K}{m_{H_k^0}m_c}\right)^2\text{Im}\,(Y_{k,12}^d)^2.\end{aligned} \tag{28}$$

This provides a contribution to ϵ in almost any model which possesses CP-violating flavor-changing neutral-scalar interactions.

In particular, the parameter ϵ could receive large contributions from the long-distance dispersive effects through the π, η, and η' poles [20]. For a quantitative estimate of these effects, we follow the analyses in Refs. [20,22–24]

$$\begin{aligned}(\text{Im}\,M'_{12})_{LD}&=\frac{1}{4m_K}\sum_i^{\pi,\eta,\eta'}\frac{\text{Im}\,(\langle K^0|L_{eff}|i\rangle\langle i|L_{eff}|\overline{K}^0\rangle)}{m_K^2-m_\pi^2}=\frac{1}{4m_K}\frac{2\kappa}{m_K^2-m_\pi^2}(\langle K^0|L_-|\pi^0\rangle\langle\pi^0|L_+|\overline{K}^0\rangle)\\&=\frac{G^2}{12\pi^2}f_K^2B'_Km_K\left(\frac{m_K}{m_s}\right)^2\sin\theta m_s^2\left(\sqrt{\frac{\pi\alpha_s}{2}}\frac{3\kappa A_{K\pi}}{4m_s(m_K^2-m_\pi^2)}\right)\end{aligned} \tag{29}$$

$$\times\sum_i[\text{Im}\,\lambda_i\,\text{Re}\,P_i(m_i,\xi_i)+\text{Re }\lambda_i\,\text{Im}\,P_i(m_i,\xi_i)], \tag{30}$$

where κ is found to be $\kappa\simeq0.15$ when considering the SU(3)-breaking effects in the K-η_8 transition and nonet-symmetry breaking in K-η_o as well as η-η' mixing. We shall not repeat these analyses, and the reader who is interested in them is referred to the paper [24] and references therein. L_- and L_+ are CP-odd and CP-even Lagrangians, respectively (with convention $L_{eff}=L_++iL_-$). The L_- is induced from the gluon-penguin diagram with charged-scalar

$$L_-=f_s\bar{d}\sigma_{\mu\nu}(1+\gamma_5)\lambda^asG_{\mu\nu}^a-f_d\bar{d}\sigma_{\mu\nu}(1-\gamma_5)\lambda^asG_{\mu\nu}^a \tag{31}$$

with

$$f_q=\frac{G}{\sqrt{2}}\frac{g_s}{32\pi^2}m_q\sum_i\text{Im}(\xi_q\xi_i\lambda_i)y_iP_T^H(y_i), \tag{32}$$

where $P_T^H(y_i)$ is the integral function. From f_s and f_d it is not difficult to read off the $\mathrm{Re}\, P_i(m_i,\xi_i)$ and $\mathrm{Im}\ P_i(m_i,\xi_i)$. In obtaining the last expression of the above equation, we have used the result $\langle K^0|L_-|\pi^0\rangle=(f_s-f_d)A_{K\pi}$, where $A_{K\pi}$ has been computed in the MIT bag model and was found [25] to be $A_{K\pi}=0.4\ \mathrm{GeV}^3$ for $\alpha_s=1$, and the convention $\langle\pi^0|L_+|\overline{K^0}\rangle=\frac{1}{2}Gf_K^2B_K'm_K^2(2m_K/m_s)^2\sin\theta$, where B_K' is introduced to fit the experimental value $\langle\pi^0|L_+|\overline{K^0}\rangle=2.58\times10^{-7}\ \mathrm{GeV}^2$, and is found to be $B_K'=1.08$. We then obtain $\sqrt{\pi\alpha_s}3\kappa A_{K\pi}/[4\sqrt{2}m_s(m_K^2-m_\pi^2)]\simeq1.4$.

Neglecting the u-quark contributions and also the terms proportional to m_d in comparison with the terms proportional to m_s, the total contributions to the CP-violating parameter ϵ can be simply calculated from the following formula:

$$\begin{aligned}|\epsilon|={}&3.2\times10^{-3}B_K\left(\frac{|V_{cb}|}{0.04}\right)^2\frac{2|V_{ub}|}{|V_{cb}||V_{us}|}\sin\delta_{KM}\Bigg\{-\frac{1}{4}\left[\eta_{cc}B^{WW}(x_c)+\frac{1}{4}\eta_{cc}^{HH}y_c|\xi_c|^4B_V^{HH}(y_c)+2\,\eta_{cc}^{HW}y_c|\xi_c|^2B_V^{HW}(y_c,y_w)\right]\\&+\left(\frac{|V_{cb}|m_t}{2m_c}\right)^2\left(1-\frac{|V_{ub}|}{|V_{cb}||V_{us}|}\cos\delta_{KM}\right)\left[\eta_{tt}B^{WW}(x_t)+\frac{1}{4}\eta_{tt}^{HH}y_t|\xi_t|^4B_V^{HH}(y_t)+2\,\eta_{tt}^{HW}y_t|\xi_t|^2B_V^{HW}(y_t,y_w)\right]\\&+\frac{m_t}{4m_c}\left[\eta_{ct}B^{WW}(x_c,x_t)+\frac{1}{2}\eta_{ct}^{HH}\sqrt{y_cy_t}|\xi_c|^2|\xi_t|^2B_V^{HH}(y_c,y_t)+4\,\eta_{ct}^{HW}\sqrt{y_cy_t}\ \mathrm{Re}(\xi_c\xi_t)B_V^{HW}(y_c,y_t,y_w)\right]\Bigg\}\\&+2.27\times10^{-3}\frac{\mathrm{Im}(\widetilde{Y}_{k,12}^d)^2}{6.4\times10^{-3}}\widetilde{B}_K\sum_k\left(\frac{10^3\ \mathrm{GeV}}{m_{H_k^0}}l\right)^2+2.27\times10^{-3}\ \mathrm{Im}(\xi_c^*\xi_s^*)^2\frac{6.8\ \mathrm{GeV}^2}{m_{H^+}^2}\widetilde{B}_K\left(\ln\frac{m_{H^+}^2}{m_c^2}-2\right),\\&+2.27\times10^{-3}\ \mathrm{Im}(\xi_c\xi_s)\frac{37\ \mathrm{GeV}^2}{m_{H^+}^2}B_K'\left(\ln\frac{m_{H^+}^2}{m_c^2}-\frac{3}{2}\right)+\frac{\xi_o}{\sqrt{2}},\end{aligned}\tag{33}$$

where we have used the experimental constraint on $2\ \mathrm{Re}\, M_{12}=\Delta m_K^{\mathrm{exp.}}$.

Analogous to Sec. II, we consider the contributions to ϵ in two different cases. In the first one, CP violation is governed by the induced KM mechanism, i.e., the first term of the above equation becomes dominant. In this case, new contributions come from the box diagrams of two charged-scalar and one W-boson-one charged-scalar exchange. Since the expression contains $\mathrm{Re}(\xi_c\xi_t)$ the relative phase θ between ξ_c and ξ_t, $\mathrm{Re}(\xi_c\xi_t)=|\xi_c||\xi_t|\cos\theta$ may play an important role. It is of interest to illustrate how such effects can influence the determination of the unitarity triangle. In Fig. 4 the constraint of vertex A of the unitarity triangle from $|V_{ub}|$, Δm_B, and ϵ is given. Here the new physics effect can change the value of $|V_{td}|$ and the shape of the bounds from ϵ. In the calculation, we take $|\xi_c|=9.8$ and $|\xi_t|=0.54$. The mass of charged Higgs is fixed at $M_H^+=200$ GeV. The relative phase between them is taken to be $\pi/3$ and $2\pi/3$ as two examples. The other input parameters are $B_K=0.75\pm0.15$, $|V_{ub}|/|V_{cb}|=0.08\pm0.02$, and $|V_{cb}|=0.04$. For a comparison, a similar calculation for the standard model with the same parameters is reproduced in Fig. 5. It is found that the shape of the triangle can be largely changed when a different value of the relative phase between ξ_c and ξ_t is taken. The angle β of the triangle may be extremely small when $\cos\theta$ is close to 1.

In the second case, both the charged-scalar and the neutral-scalar exchange contributions to ϵ_K. This case is more important than the one where the neutral-scalar exchange itself is dominant. This is because the relative phase between the charged and neutral-scalar exchange can largely affect the determination of $|V_{td}|$. To illustrate such a phase effect, we choose the ratio of the contribution to Δm_B from charged- and neutral-scalar to be 2:1, and vary the relative phase between them from 0, $\pi/3$, $2\pi/3$ to π. As was pointed out by Soares and Wolfenstein [26] if such a phase emerges, then the unitarity angle extracted from $B\to J/\psi K_S$ will be the total phase ϕ_M rather than β. ϕ_M is defined by

$$M_{12}^{total}=|M_{12}^{SM}+M_{12}^{NEW}|\exp^{2i\phi_M}$$

where the indexes ''SM'' and ''NEW'' indicate the contribution from the standard model and the new physics.

In Fig. 5 the value of V_{td} extracted from Δm_B is plotted in ρ-η plan without considering the uncertainty of $B_K(B_K=0.75)$. The four curves correspond to the above four cases. The figure shows that the additional phase from $Y_{k,ij}^f$ can strongly change the value of V_{td}. Its modulus varies in the interval between 0.7 and 1.2 in this situation.

As an example, the influence on the determination of the unitarity triangle in case 3 (see Fig. 3) is plotted in Fig. 6. Due to the phase effect the bounds from ϵ are also changed to be lower than that from the SM. Since the three bounds from $|V_{ub}|$, Δm_B, and ϵ still have area in common, the triangle remains closed. However, as we have mentioned above, if the angle β is extracted from $B\to J/\psi K_S$ its value will be the total phase ϕ_M which may be much larger. As a consequence, it will cause the unitarity triangle to be

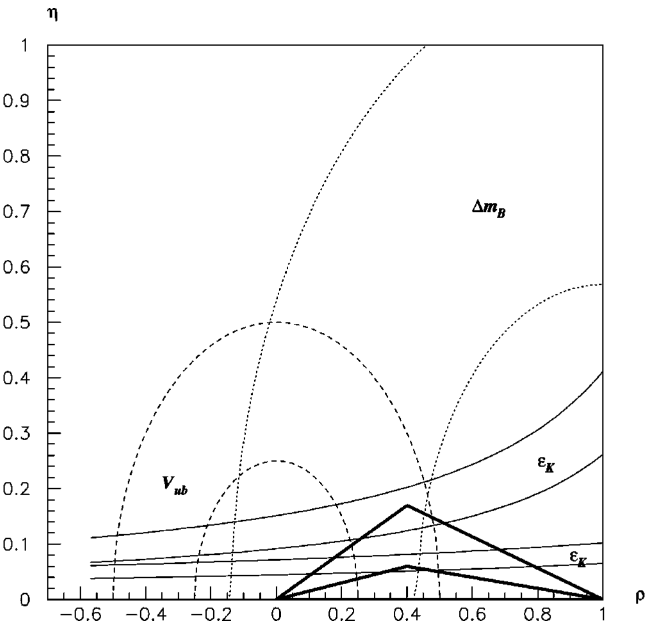

FIG. 4. The constraints on the unitarity triangle in η-ρ plane, the two different triangles corresponding to $\theta=\pi/3$ (a), and $\theta=2\pi/3$ (b). Other parameters are $B_K=0.75\pm0.15$, $|V_{ub}|/|V_{cb}|=0.08\pm0.02$, and $|V_{cb}|=0.04$.

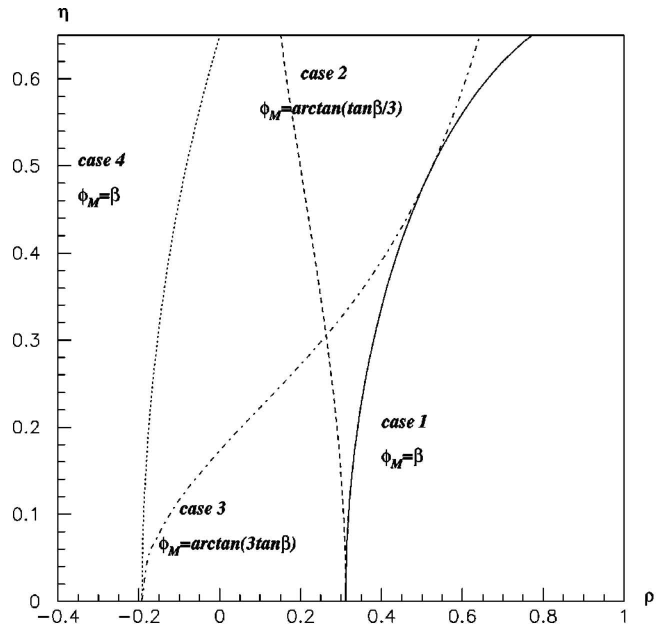

FIG. 6. Constraints on V_{td} from Δm_B. The relative phase between charged and neutral-scalar exchange is taken to be 0 (case 1, solid), $\pi/3$ (case 2, dashed), $2\pi/3$ (case 3, dash-dotted), and π (case 4, dotted).

''open.'' This possibility can be realized in case 3, where $\tan\phi_M$ is three times as large as $\tan\beta$ (see Fig. 7).

IV. CONCLUSIONS

In conclusion, we have studied one of the simplest extentions of the standard model with an extra Higgs doublet, which we have simply labeled as an S2HDM. Some constraints on the parameters in the S2HDM have been obtained from F^0-$\overline{F}^0$ mixing processes. It has been shown that in general $\xi_u\neq\xi_c\neq\xi_t$ and $|\xi_c|\gg|\xi_t|$. Various sources of CP violation have been discussed. Their influence on the determination of the unitarity triangle is studied in detail. We found that angle β of the unitarity triangle could be largely suppressed due to the new contribution from Higgs box diagrams. The phase from neutral Higgs exchange could strongly affect the extraction of β from $B\rightarrow J/\psi K_S$. In some cases, such an effect could be so large that the unitarity triangle cannot remain closed. In particular, it may even result

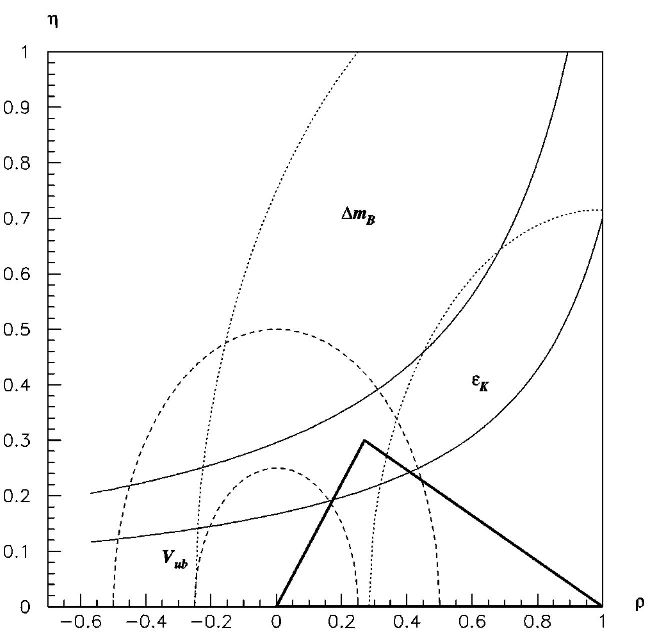

FIG. 5. The constraints on the unitarity triangle from the SM. The parameters B_K, $|V_{ub}|$, $|V_{cb}|$ are the same as in Fig. 4.

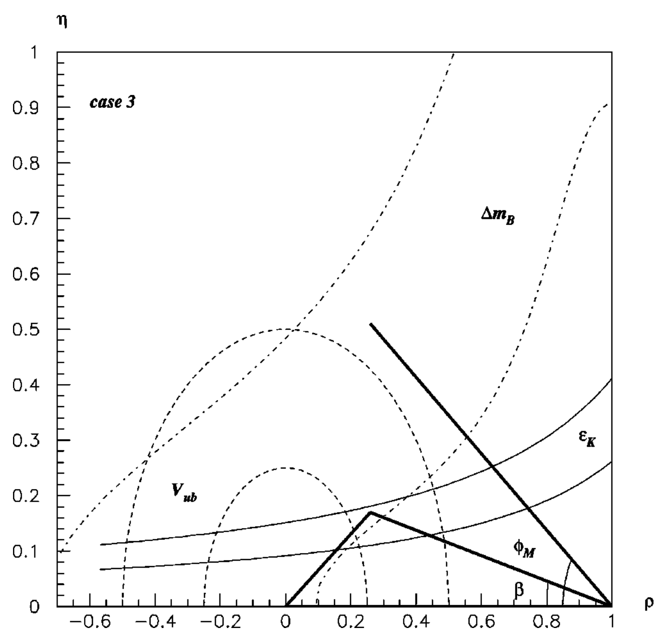

FIG. 7. The constraints on the unitarity triangle in case 3, where $\tan\phi_M$ is three times as large as $\tan\beta$.

in the angle β, which is determined from fitting the quantities $|V_{cb}|$, Δm_K, and ϵ, being different from the one extracted from the decay process $B \to J/\psi K_S$. If this is so, that will be a clear signal of new physics.

ACKNOWLEDGMENTS

This work was supported in part by the NSF of China under Grant No. 19625514.

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