# **Electromagnetic decays of heavy baryons**

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The electromagnetic decays of the ground state baryon multiplets with one heavy quark are calculated using heavy hadron chiral perturbation theory. The *M*1 and *E*2 amplitudes for  $S^* \rightarrow S\gamma$ ,  $S^* \rightarrow T\gamma$  and  $S \rightarrow T\gamma$  are separately computed. All *M*1 transitions are calculated up to  $O(1/\Lambda_{\chi}^2)$ . The *E*2 amplitudes contribute at the same order for  $S^* \rightarrow S\gamma$ , while for  $S^* \rightarrow T\gamma$  they first appear at  $O(1/(m_Q \Lambda_{\chi}^2))$  and for  $S \rightarrow T\gamma$  are completely negligible. The renormalization of the chiral loops is discussed and relations among different decay amplitudes are derived. We find that chiral loops involving electromagnetic interactions of the light pseudoscalar mesons provide a sizable enhancement of these decay widths. Furthermore, we obtain an absolute prediction for  $\Gamma(\Xi_c^{O'(*)}) \rightarrow \Xi_c^0 \gamma)$  and  $\Gamma(\Xi_b^{-'(*)}) \rightarrow \Xi_b^{--} \gamma)$ . Our results are compared to other estimates existing in the literature.

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### I. INTRODUCTION

In some kinematical regions, which are not far from the chiral and heavy quark limits, both chiral perturbation [1] and heavy quark effective theories (HQET) [2] can be simultaneously used. In the  $m_Q \rightarrow \infty$  limit, baryons containing a heavy quark can emit and absorb light pseudoscalar mesons without changing its velocity v. In heavy hadron chiral perturbation theory (HHCPT) one constructs an effective Lagrangian whose basic fields are heavy hadrons and light mesons [3–6]. In Ref. [7], the formalism is extended to include also electromagnetism.

We use this hybrid effective Lagrangian to calculate the electromagnetic decay width of the ground state baryons containing a *c* or a *b* quark. We consider the decays  $S^* \rightarrow S\gamma$  and  $S^{(*)} \rightarrow T\gamma$ . For most of these decays the available phase space is small, so that the emission of a pion is suppressed or even forbidden and the electromagnetic process becomes relevant. Some of these decays are starting to be measured [8], which makes it necessary to perform a detailed theoretical analysis.

Some theoretical calculations of these decays can be already found in the literature. The  $\mathcal{O}(1/\Lambda_{\nu})$  amplitudes were first computed in Ref. [7], using HHCPT. A more detailed analysis was presented in Ref. [9], where the widths  $\Gamma(S_c)$  $\rightarrow T_c \gamma$ ) are estimated using heavy-quark and chiral symmetries implemented within the non-relativistic quark model. A similar procedure is followed in Ref. [10], where the heavyquark symmetry is supplemented with light-diquark symmetries to calculate the widths  $\Gamma(\Sigma_c^+ \to \Lambda_c^+ \gamma)$  and  $\Gamma(\Sigma_{c,b}^*$  $\rightarrow \Sigma_{c,b} \gamma$ ). The authors of Ref. [11] apply the relativistic quark model to predict the electromagnetic decays  $\Gamma(S_c^{(*)})$  $\rightarrow T_c \gamma$ ) and  $\Gamma(\Sigma_c^{+*} \rightarrow \Sigma_c^{+} \gamma)$ . In Ref. [12],  $\Gamma(\Sigma_b^{*} \rightarrow \Sigma_b \gamma)$ and  $\Gamma(\Sigma_b^{0(*)} \rightarrow \Lambda_b^0 \gamma)$  are computed with light cone QCD sum rules at leading order in HQET. All these references consider only transitions of the M1 type. Finally, Ref. [13] estimates the ratio of the E2 and M1 amplitudes for  $\Gamma(\Sigma_c^{+*} \rightarrow \Lambda_c^{+} \gamma).$ 

Here, we study all possible  $S^{(*)} \rightarrow T\gamma$  and  $S^* \rightarrow S\gamma$  decays in the context of HHCPT, considering both *M*1 and *E*2

transitions. Section II collects the HHCPT formalism as introduced in Ref. [7]: the effective fields representing S and T baryons, the lowest order chiral Lagrangian and the  $\mathcal{O}(1/m_Q)$  and  $\mathcal{O}(1/\Lambda_{\chi})$  terms. In order to renormalize the resulting chiral loops, the introduction of higher-order operators with unknown couplings is required. In the case of  $S^*$  $\rightarrow S\gamma$ , we calculate all contributions up to  $\mathcal{O}(1/\Lambda_{\nu}^2)$  for M1 and E2 transitions. We find that all divergences and scale dependence can be absorbed in the redefinition of one  $\mathcal{O}(1/\Lambda_{\nu})$  coupling for each type of process (M1,E2). These results are presented in Sec. III. Section IV describes the analogous calculation for  $S^* \rightarrow T\gamma$ ; in this case, the E2 contribution has to be computed up to  $\mathcal{O}(1/m_0 \Lambda_{\gamma}^2)$ , which requires two additional couplings. The decays  $S \rightarrow T \gamma$  are analyzed in Sec. V; as in the previous cases the M1 amplitude is calculated up to  $\mathcal{O}(1/\Lambda_{\chi}^2)$ , while the *E*2 contribution is found to be  $\mathcal{O}(1/m_O^3 \Lambda_{\chi}^2)$  and thus extremely suppressed. In each section we derive relations among amplitudes for different baryons within the same multiplet and between charm and bottom baryons. These relations are valid at lowest order in HHCPT and we prove that they still hold after one-loop chiral corrections are included. Comparing our expectation for the widths to the leading order HHCPT estimate, we find that the infrared effect due to electromagnetic interactions of light pseudoscalar mesons can greatly enhance these widths. This is particularly true for the E2 contributions which are found to be infrared divergent in the exact chiral limit. We also give some comments on results existing in the literature. Finally, Sec. VI summarizes our conclusions.

### **II. HHCPT FORMALISM FOR MAGNETIC MOMENTS**

The light degrees of freedom in the ground state of a baryon with one heavy quark can be either in a  $s_l=0$  or in a  $s_l=1$  configuration. The first one corresponds to  $J^P = \frac{1}{2}^+$  baryons, which are annihilated by  $T_i(v)$  fields transforming as a  $\overline{\mathbf{3}}$  under the chiral subgroup SU(3)<sub>*L*+*R*</sub> and as a doublet under the HQET SU(2)<sub>*v*</sub>. In the second case,  $s_l=1$ , the spin of the heavy quark and the light degrees of freedom combine together to form  $J^P = \frac{3}{2}^+$  and  $J = \frac{1}{2}^+$  baryons, which are de-

generate in mass in the  $m_Q \rightarrow \infty$  limit. The spin- $\frac{3}{2}$  ones are annihilated by the Rarita-Schwinger field  $S^{*ij}_{\mu}(v)$ , while the spin- $\frac{1}{2}$  baryons are destroyed by the Dirac field  $S^{ij}(v)$ . It is very useful to combine both operators into the so-called superfield [14,15]

$$S^{ij}_{\mu}(v) = \sqrt{\frac{1}{3}} (\gamma_{\mu} + v_{\mu}) \gamma^{5} S^{ij}(v) + S^{*ij}_{\mu}(v),$$
  
$$\bar{S}^{\mu}_{ij}(v) = -\sqrt{\frac{1}{3}} \bar{S}_{ij}(v) \gamma^{5} (\gamma^{\mu} + v^{\mu}) + \bar{S}^{*\mu}_{ij}(v),$$
  
(1)

which transforms as a **6** under  $SU(3)_{L+R}$  and as a doublet under  $SU(2)_v$  and is symmetric in the *i*, *j* indices.

The particle assignment for the J = 1/2 charmed baryons of the  $\overline{3}$  and 6 representations is

$$(T_{1}, T_{2}, T_{3}) = (\Xi_{c}^{0}, -\Xi_{c}^{+}, \Lambda_{c}^{+}), \qquad (2)$$

$$S^{ij} = \begin{pmatrix} \Sigma_{c}^{++} & \sqrt{\frac{1}{2}} \Sigma_{c}^{+} & \sqrt{\frac{1}{2}} \Xi_{c}^{+'} \\ \sqrt{\frac{1}{2}} \Sigma_{c}^{+} & \Sigma_{c}^{0} & \sqrt{\frac{1}{2}} \Xi_{c}^{0'} \\ \sqrt{\frac{1}{2}} \Xi_{c}^{+'} & \sqrt{\frac{1}{2}} \Xi_{c}^{0'} & \Omega_{c}^{0} \end{pmatrix}, \qquad (3)$$

and the corresponding bottom baryons are

$$(T_{1}, T_{2}, T_{3}) = (\Xi_{b}^{-}, -\Xi_{b}^{0}, \Lambda_{b}^{0}), \qquad (4)$$

$$\sum_{b}^{ij} \sqrt{\frac{1}{2}} \Sigma_{b}^{0} \sqrt{\frac{1}{2}} \Xi_{b}^{0'} \sqrt{\frac{1}{2}} \Xi_{b}^{0'}$$

$$\sqrt{\frac{1}{2}} \Xi_{b}^{0'} \sqrt{\frac{1}{2}} \Xi_{b}^{0'}$$

The J=3/2 partners of the baryons in Eqs. (3) and (5) have the same SU(3)<sub>V</sub> assignments in  $S^{*ij}_{\mu}$ .

Goldstone bosons are parametrized as

$$\Phi = \begin{pmatrix} \sqrt{\frac{1}{2}}\pi^{0} + \sqrt{\frac{1}{6}}\eta & \pi^{+} & K^{+} \\ \pi^{-} & -\sqrt{\frac{1}{2}}\pi^{0} + \sqrt{\frac{1}{6}}\eta & K^{0} \\ K^{-} & \bar{K}^{0} & -\sqrt{\frac{2}{3}}\eta \end{pmatrix},$$
(6)

and appear in the Lagrangian via the exponential representation  $\xi \equiv \exp(i\Phi/\sqrt{2}f_{\pi})$ , being  $f_{\pi} \sim 93$  MeV the pion decay constant. The lowest-order chiral Lagrangian describing the soft hadronic and electromagnetic interactions of these baryons in the infinite heavy quark mass limit is given by [7]

$$\mathcal{L}^{(0)} = -i\overline{S}^{\mu}_{ij}(v \cdot D)S^{ij}_{\mu} + \Delta_{ST}\overline{S}^{\mu}_{ij}S^{ij}_{\mu} + i\overline{T}^{i}(v \cdot D)T_{i}$$
$$+ig_{2}\varepsilon_{\mu\nu\sigma\lambda}\overline{S}^{\mu}_{ik}v^{\nu}(\xi^{\sigma})^{i}_{j}(S^{\lambda})^{jk} + g_{3}[\epsilon_{ijk}\overline{T}^{i}(\xi^{\mu})^{j}_{l}S^{kl}_{\mu}$$
$$+\epsilon^{ijk}\overline{S}^{\mu}_{kl}(\xi_{\mu})^{l}_{j}T_{i}].$$
(7)

In this formula, the heavy-baryon covariant derivatives are

$$D^{\mu}S^{ij}_{\nu} = \partial^{\mu}S^{ij}_{\nu} + (\Gamma^{\mu})^{i}_{k}S^{kj}_{\nu} + (\Gamma^{\mu})^{j}_{k}S^{ik}_{\nu} - ie\mathcal{A}^{\mu}[\mathcal{Q}_{Q}S^{ij}_{\nu} + Q^{i}_{k}S^{kj}_{\nu} + Q^{j}_{k}S^{ik}_{\nu}],$$

$$D^{\mu}T_{i} = \partial^{\mu}T_{i} - T_{j}(\Gamma^{\mu})_{i}^{J} - ie\mathcal{A}^{\mu}[\mathcal{Q}_{\mathcal{Q}}T_{i} - T_{j}\mathcal{Q}_{i}^{J}], \qquad (8)$$

where  $\mathcal{A}^{\mu}$  is the electromagnetic field,  $Q_Q$  is the heavyquark charge, the light-quark charge matrix Q is given by

$$\mathbf{Q} = \begin{pmatrix} \frac{2}{3} & & \\ & -\frac{1}{3} & \\ & & -\frac{1}{3} \end{pmatrix}, \qquad (9)$$

and the Goldstone mesons appear through axial-vector,  $\xi_{\mu}$ , and vector,  $\Gamma_{\mu}$ , fields

$$\xi_{\mu} = \frac{i}{2} (\xi D_{\mu} \xi^{\dagger} - \xi^{\dagger} D_{\mu} \xi), \quad \Gamma_{\mu} = \frac{1}{2} (\xi D_{\mu} \xi^{\dagger} + \xi^{\dagger} D_{\mu} \xi),$$
(10)

with  $D^{\mu}\xi = \partial^{\mu}\xi - ie\mathcal{A}^{\mu}[Q,\xi].$ 

Because of the different spin configuration of the light degrees of freedom there is an intrinsic mass difference,  $\Delta_{ST} \equiv M_S - M_T$ , between the sextet and triplet baryon multiplets. Notice that a direct coupling of the pseudo-Goldstone bosons to the  $\overline{\mathbf{3}}$  baryons is forbidden at lowest order in  $1/\Lambda_{\chi}$ .

The first contributions to the transitions we are considering come from the following.

(1) The next order (D=5) in the baryon chiral Lagrangian [7]

$$\mathcal{L}^{(long)} = \frac{e}{\Lambda_{\chi}} \{ i c_{S} \text{tr} \left[ \overline{S}_{\mu} Q S_{\nu} + \overline{S}_{\mu} S_{\nu} Q \right] F^{\mu\nu} + c_{ST} [\epsilon_{ijk} \overline{T}^{i} v_{\mu} Q_{l}^{j} S_{\nu}^{kl} + \epsilon^{ijk} \overline{S}_{\nu,kl} v_{\mu} Q_{j}^{l} T_{i} ] \widetilde{F}^{\mu\nu} \},$$

$$(11)$$

where  $c_s$  and  $c_{ST}$  are unknown chiral couplings and  $\tilde{F}^{\mu\nu} = \varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$ . We will take  $\Lambda_{\chi} = 4\pi f_{\pi} \approx 1.2$  GeV, which fixes the normalization of these couplings. A long-distance magnetic moment interaction for just the *T* baryons does not exist, since their light quarks are in a  $s_l = 0$  configuration.

(2) Terms of order  $1/m_Q$  in the heavy quark expansion which break both spin and flavor symmetries [7]



FIG. 1. Meson loops contributing to  $S^* \rightarrow S\gamma$ .

$$\mathcal{L}^{(short)} = -\frac{1}{2m_Q} \overline{S}^{\lambda}_{ij} (iD)^2 S^{ij}_{\lambda} - \frac{eQ_Q}{4m_Q} \overline{S}^{\lambda}_{ij} \sigma_{\mu\nu} S^{ij}_{\lambda} F^{\mu\nu} + \frac{1}{2m_Q} \overline{T}^i (iD)^2 T_i + \frac{eQ_Q}{4m_Q} \overline{T}^i \sigma_{\mu\nu} T_i F^{\mu\nu}.$$
(12)

(3) Chiral loops of Goldstone bosons coupled to photons, as described by the lowest-order Lagrangian.

### III. RESULTS FOR $S^* \rightarrow S\gamma$ DECAYS

We will decompose our results in two different amplitudes

$$\mathcal{A}(B^* \to B\gamma) = A_{M1}\mathcal{O}_{M1} + A_{E2}\mathcal{O}_{E2}, \qquad (13)$$

where the corresponding M1 and E2 operators are defined by

$$\mathcal{O}_{M1} = e\bar{B}\gamma_{\mu}\gamma_{5}B_{\nu}^{*}F^{\mu\nu},$$
  
$$\mathcal{O}_{E2} = ie\bar{B}\gamma_{\mu}\gamma_{5}B_{\nu}^{*}v_{\alpha}(\partial^{\mu}F^{\alpha\nu} + \partial^{\nu}F^{\alpha\mu}).$$
(14)

The leading contributions to M1 transitions come from the light- and heavy-quark magnetic interactions which are of  $\mathcal{O}(1/\Lambda_{\chi})$  and  $\mathcal{O}(1/m_Q)$ , respectively. We have computed the next-to-leading chiral corrections of  $\mathcal{O}(1/\Lambda_{\chi}^2)$ , which originate from the loop diagrams shown in Fig. 1.

The resulting M1 amplitudes can be written as

$$A_{M1}(B^*) = \frac{1}{\sqrt{3}} \left( -\frac{Q_Q}{m_Q} - \frac{2c_s}{3\Lambda_\chi} a_\chi(B^*) + g_2^2 \frac{\Delta_{ST}}{4(4\pi f_\pi)^2} a_{g_2}(B^*) + g_3^2 \frac{m_K}{4\pi f_\pi^2} a_{g_3}(B^*) \right).$$
(15)

In Table I we show the values of the coefficients  $a_i(B^*)$  for the decays of baryons containing one charm or bottom quark. In the table,

$$I_{i} \equiv I(\Delta_{ST}, m_{i}) = 2\left(-2 + \log\frac{m_{i}^{2}}{\mu^{2}}\right) + 2\frac{\sqrt{\Delta_{ST}^{2} - m_{i}^{2}}}{\Delta_{ST}} \log\left(\frac{\Delta_{ST} + \sqrt{\Delta_{ST}^{2} - m_{i}^{2}}}{\Delta_{ST} - \sqrt{\Delta_{ST}^{2} - m_{i}^{2}}}\right).$$
(16)

Due to flavor symmetry, all contributions are equal for charm and bottom baryons, with the only exception being the term proportional to the heavy quark electric charge ( $Q_c =$  $+2/3, Q_b = -1/3$ ). The calculation of the decay amplitudes closely follows the one reported in Ref. [16] for the magnetic moments of the  $S^{(*)}$  baryons. Thus, we list the arguments common to both calculations:

(1) Contributions of  $\mathcal{O}(1/(m_Q \Lambda_{\chi}))$  can be neglected for the *b* baryons. For the *c* baryons, they are expected to be smaller than 15% [16].

(2) The corrections proportional to  $g_2^2$  are obtained performing a one-loop integral (Fig. 1 with an *S* baryon running in the loop) that has to be renormalized. The divergent part of the integral does not depend on the pion or kaon masses and is instead proportional to the mass of the baryon running in the loop. If one considers both pion and kaon loops the divergent part respects the SU(3) structure of the chiral multiplet and can be canceled with an operator of the form

$$i\frac{e}{\Lambda_{\chi}^{2}} \text{tr} \left[\overline{S}_{\mu}(v \cdot DS_{\nu})Q - (v \cdot D\overline{S}_{\mu})S_{\nu}Q\right]F^{\mu\nu}.$$
 (17)

This is the most general dimension-6 chiral- and Lorentzinvariant operator, constructed out of  $S^{ij}_{\mu}$  and  $QF^{\mu\nu}$ , preserving parity and time-reversal invariance, which contributes to the M1 amplitudes. When the equation of motion  $i(v \cdot D)S_{\mu} = \Delta_{ST}S_{\mu}$  is applied, its contribution is of the same form as the term proportional to  $c_s$  in Eq. (11). Thus, the local contribution from the operator in Eq. (17) can be taken into account, together with the lowest-order term in Eq. (11), through an effective coupling  $c_s(\mu)$ . The scale  $\mu$  dependence of the loop integrals is exactly canceled by the corresponding dependence of the coefficient  $c_s(\mu)$ ;

(3) The contribution proportional to  $g_3^2$  involves a loop integral with a baryon of the *T* multiplet running in the loop. Since the Lagrangian does not have any mass term for *T* 

c quark	<i>b</i> quark	$a_{\chi}$	a <sub>g2</sub>	<i>a</i> <sub>g3</sub>
$\overline{\Sigma_c^{++}} \rightarrow \Sigma_c^{++} \gamma$	$\Sigma_{h}^{+*} \rightarrow \Sigma_{h}^{+} \gamma$	2	$I_{\pi} + I_{K}$	$1 + m_{\pi}/m_{K}$
$\Sigma_c^{+*} \rightarrow \Sigma_c^{+} \gamma$	$\Sigma_b^{0*} \rightarrow \Sigma_b^{0} \gamma$	1/2	$I_K/2$	1/2
$\Sigma_c^{0*} \rightarrow \Sigma_c^0 \gamma$	$\Sigma_b^{-*} \rightarrow \Sigma_b^{-} \gamma$	-1	$-I_{\pi}$	$-m_{\pi}/m_{K}$
$\Xi_c^{0'*} \rightarrow \Xi_c^{0'} \gamma$	$\Xi_{b}^{-'*} \rightarrow \Xi_{b}^{-'} \gamma$	-1	$-(I_{\pi}+I_{K})/2$	$-(1+m_{\pi}/m_{K})/2$
$\Xi_{c}^{+'*} \rightarrow \Xi_{c}^{+'} \gamma$	$\Xi_{h}^{0'*} \rightarrow \Xi_{h}^{0'} \gamma$	1/2	$I_{\pi}/2$	$m_{\pi}/(2m_K)$
$\Omega_c^{0*} \rightarrow \Omega_c^0 \gamma$	$\Omega_b^{-*} \rightarrow \Omega_b^{-} \gamma$	-1	$-I_K$	- 1

TABLE I. Contributions to M1 amplitudes for  $S^* \rightarrow S\gamma$ .



FIG. 2. The scaling of the functions  $I(\Delta_{ST},m)$ , Eq. (16), and  $J(\Delta_{ST},m)$ , Eq. (23) as a function of the meson mass *m*. The dashed line is  $I(\Delta_{ST},m)$  and the continuous line is  $J(\Delta_{ST},m)$ . The scale  $\mu$  is fixed at 1 GeV and  $\Delta_{ST} = 168$  MeV.

baryons, the result of the integral is convergent and proportional to the mass of the light mesons.

In order to see the behavior of  $I(\Delta_{ST},m)$  with the meson mass we have plotted it in Fig. 2, for  $\mu = 1$  GeV and  $\Delta_{ST}$  as in Table II. We see that the value of  $I(\Delta_{ST},m)$  raises considerably in the limit of zero Goldstone-boson mass.

From Table I, one can derive the following linearly independent relations for the *M*1 amplitudes of the  $S^* \rightarrow S\gamma$  decays containing a charm quark:

$$A_{M1}(\Sigma_{c}^{++*}) = 2A_{M1}(\Sigma_{c}^{+*}) - A_{M1}(\Sigma_{c}^{0*})$$
$$= 2A_{M1}(\Xi_{c}^{+'*}) - A_{M1}(\Omega_{c}^{0*}),$$

$$A_{M1}(\Sigma_{c}^{++*}) + 2A_{M1}(\Xi_{c}^{0'*}) = A_{M1}(\Sigma_{c}^{0*}) + 2A_{M1}(\Xi_{c}^{+'*})$$
$$= -\frac{2}{\sqrt{3}m_{c}}.$$
(18)

The  $\mathcal{O}(1/\Lambda_{\chi})$  and  $\mathcal{O}(1/\Lambda_{\chi}^2)$  contributions cancel in the sum of the six  $S^* \rightarrow S\gamma M1$  amplitudes. Therefore, the average over the baryon sextet measures the  $O(1/m_Q)$  contribution.

We can write four analogous relations for the bottom baryons:

$$A_{M1}(\Sigma_b^{+*}) = 2A_{M1}(\Sigma_b^{0*}) - A_{M1}(\Sigma_b^{-*})$$
  
=  $2A_{M1}(\Xi_b^{0'*}) - A_{M1}(\Omega_b^{-*}),$ 

TABLE II. Constants used in numerical estimates.

=

$f_{\pi}$	93 MeV
$m_{\pi}$	140 MeV
$m_K$	496.7 MeV
$\Delta_{ST}$	168 MeV
$m_c$	1.3 GeV
$\alpha_{em}(m_{\tau})$	1/133.3
$m_b$	4.8 GeV

$$A_{M1}(\Sigma_{b}^{+*}) + 2A_{M1}(\Xi_{b}^{-'*}) = A_{M1}(\Sigma_{b}^{-*}) + 2A_{M1}(\Xi_{b}^{0'*})$$
$$= \frac{1}{\sqrt{3}m_{b}}.$$
(19)

Two additional equations relate b and c baryons:

$$A(\Sigma_{c}^{+*}) - A(\Sigma_{c}^{+**}) = A(\Sigma_{b}^{0*}) - A(\Sigma_{b}^{+**}),$$
$$A(\Sigma_{c}^{+*}) - A(\Xi_{c}^{+'*}) = A(\Sigma_{b}^{0*}) - A(\Xi_{b}^{0'*}).$$
(20)

The same diagram in Fig. 1 generates the leading contributions to *E*2 transitions. The graph is of  $\mathcal{O}(1/\Lambda_{\chi}^2)$  and one has to include all chiral counterterms up to this order. There is only one operator with these features,

$$\frac{i}{4} \frac{ec_S^{E2}}{\Lambda_{\chi}^2} \text{tr} \left[ \bar{S}_{\mu} Q S_{\nu} + \bar{S}_{\mu} S_{\nu} Q \right] v_{\alpha} (\partial^{\mu} F^{\alpha\nu} + \partial^{\nu} F^{\alpha\mu}), \quad (21)$$

and so only one new unknown constant  $(c_s^{E2})$  appears. The *E*2 amplitudes can be written analogously to the *M*1 case:

$$A_{E2}(B^*) = \frac{1}{6\sqrt{3}} \left( \frac{c_s^{E2}}{\Lambda_\chi^2} b_\chi(B^*) - \frac{g_2^2}{4(4\pi f_\pi)^2} b_{g_2}(B^*) - \frac{g_3^2}{4\pi f_\pi^2} b_{g_3}(B^*) \right).$$
(22)

The coefficients  $b_i$  are shown in Table III, where

$$J_{i} \equiv J(\Delta_{ST}, m_{i}) = \frac{\partial}{\partial \Delta_{ST}} [\Delta_{ST} I(\Delta_{ST}, m_{i})],$$
$$J_{i}^{0} = \lim_{\Delta \to 0} J_{i} = -1 - i \pi + \log m_{i} / \mu.$$
(23)

The scale dependence of  $c_S^{E2}(\mu)$  cancels the one coming from the loop calculation. While the behavior of  $J(\Delta_{ST}, m_i)$ does not change much when one varies the meson mass (see Fig. 2),  $J_i^0$  is infrared divergent in the exact chiral limit. This divergence can be responsible for a considerable enhancement of the electric dipole effects.

The *M*1 and *E*2 amplitudes have identical SU(3) structure. Therefore, we can construct for the *E*2 amplitudes exactly the same relations as in the *M*1 case [Eqs. (18)–(20)]. However, as there are no  $1/m_Q$  terms contributing to *E*2, the last equations in Eqs. (18) and (19) must be replaced by

$$A_{E2}(\Sigma_{c}^{++*}) + 2A_{E2}(\Xi_{c}^{0'*}) = A_{E2}(\Sigma_{c}^{0*}) + 2A_{E2}(\Xi_{c}^{+'*}) = 0,$$
(24)

and

$$A_{E2}(\Sigma_{b}^{+*}) + 2A_{E2}(\Xi_{b}^{-'*}) = A_{E2}(\Sigma_{b}^{-*}) + 2A_{E2}(\Xi_{b}^{0'*}) = 0.$$
(25)

The electromagnetic decay widths are given by

c quark	<i>b</i> quark	$b_{\chi}$	$b_{g_2}$	$b_{g_3}$
$\overline{\Sigma_c^{++} * \to \Sigma_c^{++} \gamma}$	$\Sigma_{h}^{+}* \rightarrow \Sigma_{h}^{+}\gamma$	2	$J_{\pi} + J_{K}$	$J^{0}_{\pi} + J^{0}_{K}$
$\Sigma_c^{+*} \rightarrow \Sigma_c^{+} \gamma$	$\Sigma_b^{0*} \rightarrow \Sigma_b^{0} \gamma$	1/2	$J_K/2$	$J_{K}^{0}/2$
$\Sigma_c^{0*} \rightarrow \Sigma_c^0 \gamma$	$\Sigma_b^{-*} \rightarrow \Sigma_b^{-} \gamma$	-1	$-J_{\pi}$	$-J^0_{\pi}$
$\Xi_c^{0'*} \rightarrow \Xi_c^{0'} \gamma$	$\Xi_{b}^{-'*} \rightarrow \Xi_{b}^{-'} \gamma$	- 1	$-(J_{\pi}+J_{K})/2$	$-(J_{\pi}^{0}+J_{K}^{0})/2$
$\Xi_c^{+'} * \rightarrow \Xi_c^{+'} \gamma$	$\Xi_{b}^{0'*} \rightarrow \Xi_{b}^{0'} \gamma$	1/2	$J_{\pi}/2$	${J}_{\pi}^{0}/2$
$\Omega_c^{0*} \rightarrow \Omega_c^0 \gamma$	$\Omega_b^{-*} \rightarrow \Omega_b^{-} \gamma$	-1	$-J_K$	$-J_K^0$

TABLE III. Contributions to *E*2 amplitudes for  $S^* \rightarrow S\gamma$ .

$$\Gamma(S^* \to S\gamma) = \frac{4\alpha_{em}}{3} \frac{E_{\gamma}^3 M_S}{M_{S^*}} (|A_{M1}|^2 + 3E_{\gamma}^2 |A_{E2}|^2),$$
(26)

where  $M_{S^*}$  and  $M_S$  are the masses of the initial and final baryons and  $E_{\gamma}$  the energy of the outgoing photon.

The *E*2 amplitudes come at higher chiral order with respect to the *M*1 ones. Therefore, the *E*2 contribution to the total width is suppressed by a factor  $(E_{\gamma}/\Lambda_{\chi})^2 \sim 5\%$ . In principle, it should be possible to determine experimentally the ratio  $A_{E2}/A_{M1}$  by studying the angular distribution of photons from the decay of polarized baryons [13,17,18]. The Fermilab E-791 experiment has reported [19] a significant polarization effect on the production of  $\Lambda_c$  baryons, which perhaps could be useful in future measurements of these electromagnetic decays.

In order to provide an absolute theoretical prediction for all the decay widths, it is necessary to have an estimate of the couplings  $c_s$ ,  $g_2$ , and  $g_3$  (we neglect for the moment the small E2 contamination). The couplings  $g_2$  and  $g_3$  have been calculated theoretically [5,20–22]; we report the results of these computations in Table IV.

There exists an experimental measurement of  $g_3$  from CLEO coming from the decay  $\Sigma_c^* \rightarrow \Lambda_c \pi$  [23,24],  $g_3 = 0.99 \pm 0.17$ . The direct measurement of  $g_2$  is not possible at present. However, the quark model relates its value to  $g_3$  [24], yielding  $g_2 = 1.40 \pm 0.25$ .

The constant  $c_s$  is *a priori* unknown and its value should be extracted from the experiment or predicted by some more fundamental model. This coupling appears also in the calculation of the magnetic moments of  $S^{(*)}$  baryons [16]. Thus, the determination of its numerical value via the measurement of any of these electromagnetic decays, would also provide an absolute prediction for the magnetic moments.

Having a numerical determination of the couplings  $g_2$  and  $g_3$ , it is possible to derive a scale independent relation between any couple of M1 (E2) amplitudes. The combinations

$$A_{M1}(B_1^*) - \frac{a_{\chi}(B_1^*)}{a_{\chi}(B_2^*)} A_{M1}(B_2^*), \ A_{E2}(B_1^*) - \frac{b_{\chi}(B_1^*)}{b_{\chi}(B_2^*)} A_{E2}(B_2^*)$$
(27)

are independent of the unknown coupling  $c_S(\mu)$  and can then be predicted. For instance,

$$A_{M1}(\Sigma_{c}^{++*}) + 2A_{M1}(\Sigma_{c}^{0*})$$

$$= \frac{1}{\sqrt{3}} \frac{g_{3}^{2}}{4\pi f_{\pi}^{2}} (m_{K} - m_{\pi}) + \frac{\Delta_{ST}}{4\sqrt{3}} \frac{g_{2}^{2}}{(4\pi f_{\pi})^{2}} (I_{K} - I_{\pi})$$

$$- \frac{2}{\sqrt{3}m_{c}}.$$
(28)

In order to get a numerical estimate of the left-hand side of Eq. (28), we set  $g_2=1.5\pm0.3$ ,  $g_3=0.99\pm0.17$  and the rest of the constants as in Table II. We find then

$$A_{M1}(\Sigma_c^{++*}) + 2A_{M1}(\Sigma_c^{0*}) = 0.57 \pm 0.67 \text{ GeV}^{-1}.$$
 (29)

The analogous relation for b baryons reads

$$A_{M1}(\Sigma_b^{+*}) + 2A_{M1}(\Sigma_b^{-*}) = 1.58 \pm 0.66 \text{ GeV}^{-1}.$$
 (30)

The main contribution to these values corresponds to the chiral loop, with a much smaller correction coming from the  $1/m_Q$  term. These sums would be zero if none of the previous contributions were included. The large errors in Eqs. (29) and (30) come from the present uncertainties on  $g_{2,3}$  (~20%). The same consideration holds for all numerical results in this and in the following sections.

A further comment is now in order. To estimate the importance of the effect of one-loop HHCPT we define the ratio [see Eq. (15)],

TABLE IV. Theoretical estimates of $g_2$ and $g_2$	TABLE IV.	Theoretical	estimates	of	$g_2$	and	8
--	-----------	-------------	-----------	----	-------	-----	---

Model	82	<i>g</i> <sub>3</sub>
Large $N_c$ [20]	1.88	1.53
Quark model [5]	1.5	1.06
Short-distance QCD sum rule [21]	$0.83 \pm 0.23$	$0.67 \pm 0.18$
Light-cone QCD sum rules [22]	$1.56 \pm 0.3 \pm 0.3$	$0.94 \pm 0.06 \pm 0.2$

$$R(B^*) = \frac{3}{2(4\pi f_{\pi})} \frac{g_2^2 \Delta_{ST} \ a_{g2}(B^*)/4 + 4\pi m_K g_3^2 a_{g3}(B^*)}{|c_s(\mu)| a_{\chi}(B^*)}.$$
(31)

We find for  $\Lambda_{\chi}/2 < \mu < \Lambda_{\chi}$ ,

$$R(\Sigma_{c}^{++*}) = R(\Sigma_{b}^{+*}) = R(\Xi_{c}^{0'*}) = R(\Xi_{b}^{-'*}) = \frac{3.2 \pm 1.9}{|c_{S}(\mu)|},$$

$$R(\Sigma_{c}^{+*}) = R(\Sigma_{b}^{0*}) = R(\Omega_{c}^{0*}) = R(\Omega_{b}^{-*}) = \frac{5.5 \pm 2.9}{|c_{S}(\mu)|},$$

$$R(\Sigma_{c}^{0*}) = R(\Sigma_{b}^{-*}) = R(\Xi_{c}^{+'*}) = R(\Xi_{b}^{0'*}) = \frac{1.0 \pm 1.1}{|c_{S}(\mu)|}.$$
(32)

The scale dependence of this result is not very strong and in any case within the errors. Naïvely one expects  $|c_s(\mu)| \sim \mathcal{O}(1)$ . Thus, from Eq. (32), we can deduce that the infrared effect due to the coupling of the photon to light mesons, is large on these electromagnetic decays. This affirmation can be sustained also comparing our results with some estimates existing in the literature (so far there are no experimental data on  $S^* \rightarrow S\gamma$  decays). In Ref. [12] the three decays  $\Sigma_b^* \rightarrow \Sigma_b \gamma$  are predicted, using light cone QCD sum rules; these results respect the HQET and chiral symmetries and agree with the first of our relations in Eq. (19), provided the proper relative signs among the amplitudes are chosen, namely,

$$\frac{\sqrt{\Gamma(\Sigma_b^{*+} \to \Sigma_b^+ \gamma)} - \sqrt{\Gamma(\Sigma_b^{*-} \to \Sigma_b^- \gamma)}}{2\sqrt{\Gamma(\Sigma_b^{*0} \to \Sigma_b^0 \gamma)}} = 0.98.$$
(33)

In order to derive this number from the results of Ref. [12], we have made use of the baryon masses in Table V. However, in Ref. [12] all coupling constants are determined at leading order in HQET. Writing

$$c_{S}(\mu)_{\overline{MS}} = c_{S}^{0} + \frac{(c_{S}^{1}(\mu))_{\overline{MS}}}{\Lambda_{Y}},$$
(34)

we derive [consistently with Eq. (33)] from Ref. [12]

$$-1.6 < c_s^0 < -1.2$$
 or  $1.3 < c_s^0 < 1.7$  (35)

depending on the overall sign of the amplitudes.<sup>1</sup> Thus Ref. [12], obtains the expected order of magnitude of  $c_s^0$ , however, the important chiral effect due to the photon-meson coupling is not taken into account . Thus, choosing the sign between the amplitudes consistently with Eq. (33), it is impossible to deduce Eq. (30) from their calculation. Reference [10] estimates these same decay rates and its results are consistent with Ref. [12] so that the same comments are valid also for this reference. These considerations apply also if one

TABLE V. Masses of charm and bottom baryons. All masses of *b* baryons (except  $\Lambda_b^0$ ) and the ones of  $\Sigma_c^{+*}$ ,  $\Omega_c^{0*}$  have been estimated theoretically in Ref. [25]. The measured masses are taken from [26].

c baryons	M (MeV)	b baryons	M (MeV)
$\Xi_c^0$	$2470.3 \pm 1.8$	$\Xi_{b}^{-}$	5805.7±8.1
$\Xi_c^+$	$2465.6 \pm 1.4$	$\Xi_b^0$	$5805.7 \pm 8.1$
$\Lambda_c^+$	$2284.9 \pm 0.6$	$\Lambda_b^0$	$5624 \pm 9$
$\Sigma_c^{++}$	$2452.8 \pm 0.6$	$\Sigma_{b}^{+}$	$5824.2 \pm 9.0$
$\Sigma_c^+$	$2453.6 \pm 0.9$	$\Sigma_b^0$	$5824.2 \pm 9.0$
$\Sigma_c^0$	$2452.2 \pm 0.6$	$\Sigma_b^{-}$	$5824.2 \pm 9.0$
$\Xi_c^{0'}$	$2577.3 \pm 3.2$	$\Xi_{b}^{-\prime}$	$5950.9 \pm 8.5$
$\Xi_{c}^{+\prime}$	$2573.4 \pm 3.1$	$\Xi_{h}^{0'}$	$5950.9 \pm 8.5$
$\Omega_c^{0}$	$2704.0 \pm 4.0$	$\Omega_b^{-}$	$6068.7 \pm 11.1$
$\Sigma_c^{++*}$	$2519.4 \pm 1.5$	$\Sigma_b^{+*}$	$5840.0 \pm 8.8$
$\Sigma_c^{+*}$	$2518.6 \pm 2.2$	$\Sigma_b^{0*}$	$5840.0 \pm 8.8$
$\Sigma_c^{0*}$	$2517.5 \pm 1.4$	$\Sigma_b^{-*}$	$5840.0 \pm 8.8$
$\Xi_c^{0'*}$	$2643.8 \pm 1.8$	$\Xi_{b}^{-'*}$	$5966.1 \pm 8.3$
$\Xi_c^{+'*}$	$2644.6 \pm 2.1$	$\Xi_{b}^{0'*}$	$5966.1 \pm 8.3$
$\Omega_c^{0*}$	$2760.5 \pm 4.9$	$\Omega_b^{-*}$	$6083.2 \pm 11.0$

considers the computation of the decays  $\Sigma_c^* \rightarrow \Sigma_c \gamma$  of Ref. [24]. In this case the first of our relations in Eq. (18) is exactly fulfilled and we can derive  $|c_s^0| = 1 \pm 1$ . We note, however, that the predictions of Refs. [10,11] and Ref. [24] for  $\Gamma(\Sigma_c^{*+} \rightarrow \Sigma_c^+ \gamma)$  are in desagreement as a much higher rate is predicted in the first two references.

## IV. RESULTS FOR $S^* \rightarrow T\gamma$ DECAYS

The *M*1 and *E*2 operators for these decays are defined as in Eq. (14). Similarly to what we have done in the previous paragraph, we write the *M*1 amplitude for  $S^* \rightarrow T\gamma$  decays as

$$A_{M1}(B^*) = -\sqrt{2} \frac{c_{ST}}{\Lambda_{\chi}} a_{\chi}(B^*) + g_2 g_3 \frac{\Delta_{ST}}{2\sqrt{2}(4\pi f_{\pi})^2} a_g(B^*).$$
(36)

The values of the coefficients  $a_i$  are written in Table VI.

The first term in Eq. (36) comes from the Lagrangian (11), while the second one corresponds to the diagram of Fig. 3. As in the case of  $S^* \rightarrow S\gamma$ , when all Goldstone boson loops are included, the scale  $\mu$  dependence of the loop diagram is canceled by the corresponding dependence of an effective  $c_{ST}(\mu)$ . After applying the equations of motion, the

TABLE VI. Contributions to *M*1 amplitudes for  $S^* \rightarrow T\gamma$ .

c quark	<i>b</i> quark	$a_{\chi}$	$a_g$
$\overline{\Sigma_c^+ *} \rightarrow \Lambda_c^+ \gamma$	$\Sigma_b^{0*} \rightarrow \Lambda_b^0 \gamma$	1	$2I_{\pi} + I_{K}/2$
$\Xi_c^{+'*} \rightarrow \Xi_c^+ \gamma$	$\Xi_b^{0'*} { ightarrow} \Xi_b^0 \gamma$	1	$I_{\pi}/2 + 2I_{K}$
$\Xi_c^{0'*} \rightarrow \Xi_c^0 \gamma$	$\Xi_b^{-'*} {\rightarrow} \Xi_b^{-} \gamma$	0	$-I_{\pi}/2 + I_{K}/2$

<sup>&</sup>lt;sup>1</sup>The difference in the absolute value of positive and negative interval in Eq. (35) is due to the heavy quark term in Eq. (15).



FIG. 3. Meson loops contributing to  $S^{(*)} \rightarrow T\gamma$ .

effective coupling  $c_{ST}(\mu)$  contains all contributions to the M1 amplitude coming from  $\mathcal{O}(1/\Lambda_{\gamma}^2)$  counterterms, namely

$$i\epsilon_{ijk}Q_{l}^{j}(\overline{T}^{i}(v\cdot DS_{\nu}^{kl}) - (v\cdot D\overline{S}_{\nu}^{kl})T^{i})v_{\mu}\widetilde{F}^{\mu\nu},$$
$$i\epsilon_{ijk}Q_{l}^{j}(\overline{T}^{i}(D_{\mu}S_{\nu}^{kl}) - (D_{\mu}\overline{S}_{\nu}^{kl})T^{i})\widetilde{F}^{\mu\nu}.$$
(37)

Our result in Eq. (36) does not depend on the heavy quark charge or mass. We thus obtain the same predictions for charm and bottom baryons. All constants can be eliminated in the relations

$$A_{M1}(\Sigma_{c}^{+*}) - A_{M1}(\Xi_{c}^{+'*}) = -3A_{M1}(\Xi_{c}^{0'*}),$$
$$A_{M1}(\Sigma_{b}^{0*}) - A_{M1}(\Xi_{b}^{0'*}) = -3A_{M1}(\Xi_{b}^{-'*}).$$
(38)

It is interesting to notice that  $A_{M1}(\Xi_c^{0'*})$  does not depend on  $c_{ST}$ . Since at  $\mathcal{O}(1/\Lambda_{\chi}^2)$  this decay does not get any contribution from local terms, its M1 amplitude results from a *finite* chiral loop calculation (it cannot be divergent because there is no possible counter-term to renormalize it), so that we have an absolute prediction for its value in terms of  $g_2$ and  $g_3$ . Using for  $g_2$  and  $g_3$  the same values as in Eq. (29), we find

$$\Gamma_{M1}(\Xi_c^{0'*}) = 5.1 \pm 2.7 \text{ keV}.$$
 (39)

Lower estimates of this decay width are reported in Ref. [11],  $\Gamma_{M1}(\Xi_c^{0'*})=0.68\pm0.04$  keV, and in Ref. [24],  $\Gamma_{M1}(\Xi_c^{0'*})=1.1$  keV. These authors do not consider chiral corrections to their result which cannot be neglected. In particular, the result of Ref. [24] is worth a further comment. The effective coupling to the *M*1 operator in this decay is estimated using the nonrelativistic quark model. This coupling is found to be proportional to  $1/M_d - 1/M_s$ , where  $M_{d,s}$  are the constituent quark mass of the down and strange quarks, respectively. However,

$$\frac{1}{M_d} - \frac{1}{M_s} = \frac{M_s - M_d}{M_s M_d} \sim \mathcal{O}\left(\frac{m_K^2 - m_\pi^2}{\Lambda_\chi^3}\right). \tag{40}$$

Thus, the effect they calculate represents a higher-order correction to our result.

The corresponding decay for *b* baryons,  $\Xi_b^{-'*} \rightarrow \Xi_b^- \gamma$  can also be predicted, using the existing estimates for the masses of these baryons (see Table V),

$$\Gamma_{M1}(\Xi_b^{-'*}) = 4.2 \pm 2.4 \text{ keV.}$$
 (41)

The dominant error of Eqs. (39) and (41) comes from the determination of the couplings  $g_{2,3}$ .

The E2 amplitude in  $S^* \rightarrow T\gamma$  is suppressed by an extra power of  $1/m_Q$ . The first non-zero contributions come at  $\mathcal{O}(1/m_Q\Lambda_{\gamma}^2)$ . At this order we find

(i) a divergent contribution [13] arising from the lowestorder Lagrangian (7), through the loop in Fig. 3, which is proportional to the mass splitting between S and  $S^*$  baryons [27],

$$\Delta M_Q = 3 \frac{\lambda_{2S}}{m_Q}; \tag{42}$$

(ii) a spin symmetry-breaking operator of  $\mathcal{O}(1/m_0)$ ,

$$\mathcal{L}' = i \frac{g'}{m_Q} [\epsilon_{ijk} \overline{T}^i \sigma^{\mu\nu} (\xi_\mu)^j_l S^{kl}_\nu + \epsilon^{ijk} \overline{S}^{\mu}_{kl} \sigma_{\mu\nu} (\xi^\nu)^l_j T_i],$$
(43)

which gives rise to divergent loop diagrams, as the one in Fig. 3, where one of the vertices is proportional to g';

(iii) further, there are finite contributions of the same order coming from

$$-i\frac{c_T^{E2}}{m_Q\Lambda_\chi^2}\epsilon_{ijk}\overline{T}^i\sigma_{\mu\nu}Q_l^jS_\alpha^{kl}\partial^\alpha\widetilde{F}^{\mu\nu}.$$
(44)

We could also include the operator

$$i\epsilon_{ijk}\overline{T}^{i}\sigma_{\mu\nu}Q^{j}_{l}S^{kl}_{\alpha}\partial^{\nu}\widetilde{F}^{\mu\alpha},\qquad(45)$$

but its contribution is proportional to that in Eq. (44) up to higher order corrections.

Finally, the E2 amplitude can be written as

$$A_{E2}(B^*) = -\frac{1}{\sqrt{2}} \frac{c_T^{E2}}{m_Q \Lambda_\chi^2} b_\chi(B^*) -\frac{1}{24\sqrt{2}} \frac{g' g_2}{m_Q (4\pi f_\pi)^2} b_{g'}(B^*) +\frac{\lambda_{2S}}{24\sqrt{2}} \frac{g_2 g_3}{m_Q (4\pi f_\pi)^2} b_g(B^*).$$
(46)

The values of the different contributions are collected in Table VII, where

$$G_i = \frac{\partial J_i}{\partial \Delta_{ST}} \Big|_{\Delta_{ST} = 0} = \frac{2\pi}{m_i}.$$
(47)

We underline the infrared divergent behavior of this term. Neither the interaction in Eq. (43) nor the local term Eq. (44) have been taken into account in the literature. An estimate of E2 for  $\Sigma_c^{+*} \rightarrow \Lambda_c^+ \gamma$  is provided in Ref. [13], considering only the  $b_g$  contribution.

c quark	b quark	$b_{\chi}$	b <sub>g'</sub>	b <sub>g</sub>
$\overline{\Sigma_c^{+*} \rightarrow \Lambda_c^{+} \gamma}$	$\Sigma_{h}^{0*} \rightarrow \Lambda_{h}^{0} \gamma$	1	$2J_{\pi} + J_{K}/2$	$2G_{\pi} + G_{K}/2$
$\Xi_c^{+*} \rightarrow \Xi_c^{+} \gamma$	$\Xi_{b}^{0*} \rightarrow \Xi_{b}^{0} \gamma$	1	$J_{\pi}/2 + 2J_K$	$G_{\pi}/2 + 2G_K$
$\Xi_c^{0*} \rightarrow \Xi_c^{0} \gamma$	$\Xi_b^- * \to \Xi_b^- \gamma$	0	$-J_{\pi}/2 + J_{K}/2$	$-G_{\pi}/2+G_{K}/2$

TABLE VII. Contributions to *E*2 amplitudes for  $S^* \rightarrow T\gamma$ .

By eliminating the unknown coupling constants, one can deduce the relation

$$A_{E2}(\Sigma_{c}^{+*}) - A_{E2}(\Xi_{c}^{+'*}) = -3A_{E2}(\Xi_{c}^{0'*}).$$
(48)

The same relation holds for the corresponding b baryons, since

$$A_{E2}(B_b^*) = \frac{m_c}{m_b} A_{E2}(B_c^*).$$
(49)

The decays  $\Xi_c^{0*} \to \Xi_c^0 \gamma$  and  $\Xi_b^{-*} \to \Xi_b^- \gamma$  do not get any contribution from the local term proportional to  $c_T^{E2}$ ; their  $O(1/m_Q \Lambda_{\chi}^2) E2$  amplitude is also given by a finite loop calculation. Unfortunately, since the coupling g' is not known, there is no absolute prediction in this case. An experimental measurement of these E2 amplitudes would provide a direct estimate of g'.

## V. RESULTS FOR $S \rightarrow T \gamma$

The calculation of the M1 amplitude for  $S \rightarrow T\gamma$  decays is analogous to that of the previous section. Now the M1 operator is defined as

$$\mathcal{O}_{M1} = ie\bar{B}_T \sigma_{\mu\nu} B_S F^{\mu\nu} \tag{50}$$

and the corresponding amplitude can be written in the form

$$A_{M1}(B) = \frac{1}{\sqrt{6}} \frac{c_{ST}}{\Lambda_{\chi}} a_{\chi}(B) - g_2 g_3 \frac{\Delta_{ST}}{4\sqrt{6}(4\pi f_{\pi})^2} a_g(B),$$
(51)

where the coefficients satisfy

$$a_{\chi}(B) = a_{\chi}(B^*), \quad a_g(B) = a_g(B^*).$$
 (52)

Therefore, relation (38) is also valid in this case. The widths of the decays  $\Xi_c^{0'} \rightarrow \Xi_c^0 \gamma$  and  $\Xi_b^{-'} \rightarrow \Xi_b^- \gamma$  can be predicted through a finite loop calculation. From

$$\Gamma(S \to T\gamma) = 16\alpha_{em} \frac{E_{\gamma}^3 M_T}{M_S} |A_{M1}|^2, \qquad (53)$$

we find

$$\Gamma(\Xi_c^{0'}) = (1.2 \pm 0.7) \text{ keV},$$
  
 $\Gamma(\Xi_b^{-'}) = (3.1 \pm 1.8) \text{ keV}.$  (54)

Again the dominant error in Eq. (54) is given by the uncertainty of  $g_{2,3}$ .

As in Sec. III in order to estimate the importance of chiral corrections we use the ratios [see Eqs. (36) and (51)]

$$R(B^{(*)}) = \frac{g_2 g_3 \Delta_{ST} a_g(B^{(*)})}{16\pi f_\pi a_\chi(B^{(*)}) |c_{ST}(\mu)|}.$$
(55)

We find (we consider  $\Lambda_{\chi}/2 < \mu < \Lambda_{\chi}$ )

$$R(\Sigma_{c}^{+(*)}) = R(\Sigma_{b}^{0(*)}) = -(1.6 \pm 0.6) / |c_{ST}(\mu)|,$$
  
$$R(\Xi_{c}^{+'(*)}) = R(\Xi_{b}^{0'(*)}) = -(2.4 \pm 0.8) / |c_{ST}(\mu)|.$$
(56)

Therefore, the one loop chiral contribution cannot be neglected for  $|c_{ST}(\mu)| \sim \mathcal{O}(1)$ . In Refs. [11] and [9,24], numerical values for all  $S_c^{(*)} \rightarrow T\gamma$  at  $\mathcal{O}(1/\Lambda_{\chi})$  are given using, respectively the relativistic three quark model and the constituent quark-model. As in Sec. III we can define

$$c_{ST}(\mu)_{\overline{MS}} = c_{ST}^0 + \frac{(c_{ST}^1(\mu))_{\overline{MS}}}{\Lambda_{\chi}}.$$
 (57)

From Refs. [9,11,24] we find

$$0.83 < |c_{ST}^0| < 1.6. \tag{58}$$

Our results in Eq. (54) can be compared with other estimates existing in the literature. Reference [11] reports  $\Gamma(\Xi_c^{0'}) = 0.17 \pm 0.02$  keV, Ref. [28] reports  $\Gamma(\Xi_c^{0'}) \sim 0.2$  keV, while Ref. [9] quotes  $\Gamma(\Xi_c^{0'}) = 0.3$  keV. Further, the same argument as in Sec. IV can also be used now to understand these low values obtained in the constituent quark model [9].

For these decays the *E*2 amplitude is further suppressed than in the previous cases. The lowest-order contribution appears at  $O(1/m_O^3 \Lambda_{\chi}^2)$  and, therefore, can be neglected.

### **VI. CONCLUSIONS**

We have calculated the electromagnetic one-photon decays  $S^* \rightarrow S\gamma$  and  $S^{(*)} \rightarrow T\gamma$  using heavy hadron chiral perturbation theory. For each of these decays we have provided an estimate of both the *M*1 and *E*2 amplitudes. The computation of the *M*1 amplitudes up to  $O(1/\Lambda_{\chi}^2)$  involves the introduction of the unknown constants  $c_S$  for  $S^* \rightarrow S\gamma$  and  $c_{ST}$  for  $S^{(*)} \rightarrow T\gamma$ . Eliminating these couplings we derive relations among different amplitudes. Moreover, since charm and bottom baryons are described by the same arbitrary constants, we can connect the amplitudes of the two kinds of hadrons. The *E*2 contributions appear at different higher orders for the three kinds of decays:  $\mathcal{O}(1/\Lambda_{\chi}^2)$  for  $S^* \rightarrow S\gamma$ ,  $\mathcal{O}(1/m_Q\Lambda_{\chi}^2)$  for  $S^* \rightarrow T\gamma$  and  $\mathcal{O}(1/m_Q^2\Lambda_{\chi}^2)$  for  $S \rightarrow T\gamma$ . They introduce additional unknown constants:  $c_S^{E2}$  for  $S^* \rightarrow S\gamma$ ;  $c_T^{E2}$  and g' for  $S^* \rightarrow T\gamma$  (the *E*2 amplitude for  $S \rightarrow T\gamma$  is completely negligible). The *E*2 effects can be strongly enhanced by a term which is infrared divergent in the exact chiral limit. The possibility of measuring the ratio  $A_{E2}/A_{M1}$ , using polarized initial baryons, has been suggested in Ref. [13] and could be performed with an analysis of the photon distribution.

Furthermore, we obtain an absolute prediction for  $\Gamma(\Xi_c^{0'(*)} \to \Xi_c^0 \gamma)$  and  $\Gamma(\Xi_b^{-'(*)} \to \Xi_b^- \gamma)$ . At  $\mathcal{O}(1/\Lambda_{\chi}^2)$ , these decay widths do not get any contribution from local terms in the Lagrangian and, therefore, their values are fixed by a finite chiral loop calculation.

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Finally, we have shown that chiral loops involving photon-meson coupling cannot be neglected in the computation of the amplitudes of these decays. These interactions generate the dominant contribution to the electromagnetic decays of heavy baryons.

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- S. Weinberg, Physica A 96A, 327 (1979); J. Gasser and H. Leutwyler, Nucl. Phys. B250, 456 (1985); B250, 517 (1985);
   B250, 539 (1985); A. Pich, Rep. Prog. Phys. 58, 563 (1995), and references therein.
- [2] For reviews, see, for instance, H. Georgi, HUTP-91-A039, Published in Boulder TASI 91:0589-630; B. Grinstein, Annu. Rev. Nucl. Part. Sci. 42, 101 (1992); M. Neubert, in *Heavy flavors II*, edited by A.J. Buras and M. Linder (World Scientific, Singapore, 1997), p. 239; Phys. Rep. 245, 259 (1994); M.B. Wise, hep-ph/9805468; A. Pich, in *Probing the Standard Model of Particle Interactions*, Proceedings of Les Houches Summer School, 1997, edited by R. Gupta *et al.* (North-Holland, Amsterdam, 1999), Vol. II, p. 949, and references therein.
- [3] M. Wise, Phys. Rev. D 45, R2188 (1992).
- [4] G. Burdman and J. Donoghue, Phys. Lett. B 280, 287 (1992).
- [5] T.M. Yanet al., Phys. Rev. D 46, 1148 (1992); 55, 5851 (1997).
- [6] P. Cho, Phys. Lett. B 285, 145 (1992).
- [7] P. Cho and H. Georgi, Phys. Lett. B 296, 408 (1992); 300, 410(E) (1993).
- [8] CLEO Collaboration, C.P. Jessop *et al.*, Phys. Rev. Lett. 82, 492 (1999).
- [9] H.-Y. Cheng et al., Phys. Rev. D 47, 1030 (1993).
- [10] S. Tawfiq, J.G. Körner, and P.J. O'Donnell, hep-ph/9909444.
- [11] M.A. Ivanov et al., Phys. Rev. D 60, 094002 (1999).

- [12] S.-L. Zhu and Y.-B. Dai, Phys. Rev. D 59, 114015 (1999).
- [13] M.J. Savage, Phys. Lett. B 345, 61 (1995).
- [14] H. Georgi, Nucl. Phys. B348, 293 (1991).
- [15] A. Falk, Nucl. Phys. **B378**, 79 (1992).
- [16] M.C. Bañuls et al., Phys. Rev. D 61, 074007 (2000).
- [17] H.J. Rose and D.M. Brink, Rev. Mod. Phys. 39, 306 (1967).
- [18] M.N. Butler *et al.*, Phys. Lett. B **304**, 353 (1993); **314**, 122 (1993).
- [19] E-791 Collaboration, E.M. Aitala *et al.*, Phys. Lett. B **379**, 292 (1996).
- [20] Z. Guralnik, M. Luke, and A.V. Manohar, Nucl. Phys. B390, 474 (1993).
- [21] A. Grozin and O.I. Yakovlev, Eur. Phys. J. C 2, 721 (1998).
- [22] S.-L. Zhu and Y.-B. Dai, Phys. Lett. B 429, 72 (1998).
- [23] CLEO Collaboration, G. Brandenburg *et al.*, Phys. Rev. Lett. 78, 2304 (1997).
- [24] H.-Y. Cheng, Phys. Lett. B 399, 281 (1997).
- [25] E. Jenkins, Phys. Rev. D 55, 10 (1997); 54, 4515 (1996); see, also, A. Zalewska and I. Zalewski, Acta Phys. Pol. 27, 1935 (1996).
- [26] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C 3, 1 (1998), and 1999 partial update for edition 2000 [http://pdg.lbl.gov].
- [27] See, e.g., P. Cho, Nucl. Phys. B396, 183 (1993); B421, 683(E) (1994); E. Jenkins, Phys. Rev. D 54, 4515 (1996).
- [28] J. Dey, M. Dey, V. Shevchenko, and P. Volkovitsky, Phys. Lett. B 337, 185 (1994).