Photon production of axionic cold dark matter

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Using the nonequilibrium quantum field theory, photon production from the coherently oscillating axion field in a flat Robertson-Walker cosmology is reexamined. First neglecting the Debye screening of the baryon plasma to photons, we find that the axions will dissipate into photons via spinodal instability in addition to parametric resonance. As a result of the pseudoscalar nature of the axion-photon coupling, we observe a circular polarization asymmetry in the photons produced. However, these effects are suppressed to an insignificant level in the expanding universe. We then briefly discuss a systematic way of including the plasma effect which can further suppress the photon production. We note that the formalism of the problem can be applied to any pseudoscalar field coupled to a photon in a thermal background in a general curved spacetime.

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I. INTRODUCTION

It is compelling that most of the matter in the universe is in the form of nonbaryonic cold dark matter. If it exists, it would play an important role in the structure formation of the universe [1]. Axions, the pseudo Goldstone bosons, are among the most promising candidates for nonbaryonic cold dark matter. They arise from the spontaneous breaking of the global $U(1)$ symmetry of Peccei and Quinn (PQ), which is introduced to solve the strong CP problem of QCD [2–4]. In standard big-bang cosmology, after the spontaneous breakdown of PQ symmetry, the expectation value of the axion field (i.e., the axionic condensate) takes some random value in the interval $[0,2\pi]$ and is approximately constant over length scales that are smaller than the horizon size [5]. If inflation occurs either after or during PQ symmetry breaking, then the expectation value can be nearly constant throughout the entire universe [6]. At high temperatures above the Λ_{QCD} scale, the axion is massless; however, at low temperatures, the axion develops a mass due to QCD instanton effects [7]. Once the axion mass becomes greater than the universe expansion rate, the expectation value of the axion field begins to oscillate coherently around the minimum of its effective potential that is near the origin. The oscillating axion field then dissipates mainly due to the universe expansion as well as particle production [2,3].

In the original papers [2], simple estimates of the thermal dissipation of the homogeneous axionic condensate were given. These authors considered instabilities arising from the parametric amplification of quantum fluctuations that could pump the energy of the homogeneous axionic condensate into its quantum fluctuations via self-couplings, as well as into quantum fluctuating photon modes via a coupling of the axion to electromagnetism due to the color anomaly of PQ symmetry. This dissipational dynamics via quantum particle production exhibits the features of unstable bands and an exponential growth of the quantum fluctuating modes that are characteristics of parametric resonance. The growth of the modes in the unstable bands translates into profuse particle production. A given unstable mode will grow as long as it lies within the unstable band. However, eventually it will be redshifted out of the band as the universe expands, and then the instabilities of parametric resonance are shut off. In Ref. [2], it has been shown that for the PQ symmetry breaking scale $f_a > 10^{12}$ GeV, because the axion is very weakly coupled, the time it takes to be redshifted out of the unstable band is too short to build up an appreciable growth of the quantum fluctuating modes. Thus, all of these effects are insignificant. The condensate is effectively nondissipative and pressureless. It would survive in the expanding universe, and it behaves like cold dust at the present time. Interestingly, if $f_a \sim 10^{12}$ GeV, it could constitute a major component of the dark matter of the universe.

Recently, the authors of Ref. [8] were motivated by the recent understanding of the important role of spinodal instability and parametric resonance which provide the nonlinear and nonperturbative mechanisms in quantum particle production driven by the large amplitude oscillations of the coherent field [9–13]. They reexamined the issue of the dissipation of the axion field resulting from the production of its quantum fluctuations. They confirmed that the presence of parametric resonance would lead to an explosive growth of quantum fluctuations if the universe was Minkowskian. Taking account of the expansion of the universe, quantum fluctuations of the axion do not become significant. This result confirms the conventional wisdom.

In this paper, we will reexamine the damping dynamics of the axion arising from photon production in an expanding universe in the context of nonequilibrium quantum field theory. The goal of this study is to present a detailed and

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systematic study of the above-mentioned problem using a fully nonequilibrium formalism [9–13]. We will derive the coupled nonperturbative equation for the axion field and the mode equations for the photon field in a flat Robertson-Walker spacetime within the nonperturbative Hartree approximation which is implemented to consistently take the back reaction effects into account. We then try to study both numerically and analytically how the nonperturbative effects of spinodal instability and parametric amplification of quantum fluctuations trigger photon production from the oscillations of the axion field. At this stage, it is worthwhile to mention that our approach can be generalized to any pseudoscalar field coupled to the photon field in a more general curved spacetime. Because of the pseudoscalar nature of the coupling between the axion and the photon, the axion field affects the left- and right-handed circularly polarized photons differently. This leads to production of the two polarized photons in different amounts. This polarization asymmetry, if it survives, may have interesting effects on the polarization of the cosmic microwave background.

To consider the fermionic plasma effect on photon production, one must systematically obtain the nonequilibrium in-medium photon propagators and the off-equilibrium effective vertices between the axion and the photon by integrating out the fermionic field to deal with this problem [12]. In a plasma, the transverse photons are dynamically screened [14]. However, in the literature [2], the arguments stated for including the fermionic plasma effect in support of their conclusions amount to adding by hand the electron plasma frequency into the propagating photon mode equations. This is problematic when we consider propagating photon modes in the presence of a thermal background. In fact, the consequence of the Abelian ward identities reveals that the transverse photons have vanishing static magnetic mass in all orders of perturbation theory [14]. This means that the in-medium transverse photon propagators must be nonlocal in nature, and cannot be approximated by the local propagator as suggested in Ref. [2] even in the low-energy limit [12]. In addition, in a fermionic plasma, the effective coupling between the axion and the photon resulting from integrating out the fermionic thermal loop can be modified at finite temperature as well as out of equilibrium [12]. Therefore, to fully consider the plasma effect, the nonequilibrium in-medium photon propagators as well as the off-equilibrium effective vertices play essential roles. Incorporating these nonequilibrium effects is a challenging task that lies beyond the scope of this paper, but certainly deserves to be taken up in the near future. In the following, we will totally ignore the fermionic plasma effect in order to focus on understanding whether the particle production due to spinodal instability as well as parametric amplification is effective or not in the cosmological context.

In Sec. II, we introduce the axion physics and its coupling to the photon. An effective action of the axion-photon system in the expanding universe is derived. Section III is devoted to the formalism of the problem in terms of nonequilibrium quantum field theory. We obtain the equation of motion for the classical axion field and the photon mode equations. In Sec. IV, we present the numerical results. Section V is our conclusions.

II. EFFECTIVE ACTION OF AXION-PHOTON COUPLING IN AN EXPANDING UNIVERSE

The physics of the axion and its implications for astrophysics and cosmology can be found in the review articles [3,15]. The axion does not couple directly to the photon. However, at tree level, the axion field ϕ has a coupling with the fermionic field ψ,

$$L_{\phi\psi}=g\phi\bar{\psi}\psi. \tag{2.1}$$

Therefore, the axion can couple to the photon via a fermionic loop. As a consequence of the color anomaly of the PQ current, similar to the pion-photon system, the effective Lagrangian density for the axion-photon coupling is

$$L_{\phi A}=c\frac{\phi}{f_a}\epsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu}, \tag{2.2}$$

where $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$. In Eq. (2.2), the scale $f_a \equiv f_{PQ}/N$, where f_{PQ} is the PQ symmetry breaking scale and N is the color anomaly of the PQ symmetry. The coupling constant $c=\alpha(E_{PQ}/N-1.95)/(16\pi)$, where E_{PQ} is the electromagnetic anomaly of the PQ symmetry and α is the fine structure constant. Henceforth, we assume an axion incorporated into the simplest grand unified theories (GUTs) with $E_{PQ}/N=8/3$, such that $c\simeq 1.04\times 10^{-4}$ [16].

Now we write down the effective action for the axion-photon system in the expanding universe:

$$S=\int d^4x\sqrt{g}\left(L_\phi+L_A+\frac{1}{\sqrt{g}}L_{\phi A}\right), \tag{2.3}$$

where $1/\sqrt{g}$ is added in front of $L_{\phi A}$, given by Eq. (2.2), because $\epsilon^{\alpha\beta\mu\nu}$ is a tensor density of weight -1 [17]. In the following, for simplicity, we will assume a flat Robertson-Walker metric:

$$ds^2=-g_{\mu\nu}dx^\mu dx^\nu=dt^2-a^2(t)d\vec{x}^2, \tag{2.4}$$

where the signature is $(-+++)$, and $a(t)$ is the cosmic scale factor. In Eq. (2.3),

$$L_\phi=-\tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-V(\phi,T), \tag{2.5}$$

$$L_A=-\tfrac{1}{4}g^{\alpha\mu}g^{\beta\nu}F_{\alpha\beta}F_{\mu\nu}, \tag{2.6}$$

where the axion potential has a temperature-dependent mass term due to QCD instanton effects, which is of the form [3]

$$V(\phi,T)=m_a^2(T)f_a^2\left(1-\cos\frac{\phi}{f_a}\right), \tag{2.7}$$

$$m_a(T)\simeq 0.1m_{a0}\left(\frac{\Lambda_{QCD}}{T}\right)^{3.7}, \tag{2.8}$$

where m_{a0} is the zero-temperature axion mass, satisfying $m_{a0}f_a \simeq 6.2\times 10^{-3}$ GeV^2. Also, we use $\Lambda_{QCD}\simeq 200$ MeV.

It is well known that the minimal coupling of photons to the metric background is conformally invariant [18]. As such, in the conformally flat metric (2.4), it is convenient to work with the conformal time, $d\eta = a^{-1}(t)dt$. Hence, defining $\phi=\chi/a$, the action (2.3) becomes

$$S=\int d\eta\, d^3\vec{x}\ \mathcal{L}=\int d\eta\, d^3\vec{x}\left[\frac{1}{2}\left(\frac{\partial\chi}{\partial\eta}\right)^2-\frac{1}{2}\left(\frac{\partial\chi}{\partial\vec{x}}\right)^2 + \frac{1}{2a}\frac{d^2a}{d\eta^2}\chi^2 - a^4 V\left(\frac{\chi}{a},T\right)-\frac{1}{4}\eta^{\alpha\mu}\eta^{\beta\nu}F_{\alpha\beta}F_{\mu\nu} + c\frac{\chi}{af_a}\epsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu}\right], \tag{2.9}$$

where $\eta^{\mu\nu}$ is the Minkowski metric. In terms of conformal time, the effective action now has analogy with the effective action in Minkowski spacetime with a time-dependent mass term and interactions.

III. EQUATIONS OF MOTION

The nonequilibrium effective Lagrangian in the closed time path formalism [9–11] is given by

$$\mathcal{L}_{neq}=\mathcal{L}[\chi^+,A_\mu^+]-\mathcal{L}[\chi^-,A_\mu^-], \tag{3.1}$$

where $+$ $(-)$ denotes the forward (backward) time branches. We then decompose $\chi^\pm$ into the axionic mean field and the associated quantum fluctuating fields:

$$\chi^\pm(\vec{x},\eta)=\varphi(\eta)+\psi^\pm(\vec{x},\eta), \tag{3.2}$$

with the tadpole conditions

$$\langle\psi^\pm(\vec{x},\eta)\rangle=0. \tag{3.3}$$

We will implement the tadpole conditions to all orders in the corresponding expansion to obtain the nonequilibrium equations of motion.

To take account of the back reaction effects on the dynamics of the axion field from quantum fluctuating photon modes, we adopt the following Hartree factorization, which is implemented for both $\pm$ components [9–11]:

$$\psi F\tilde{F}\to\psi\langle F\tilde{F}\rangle. \tag{3.4}$$

As seen later, the expectation value can be determined self-consistently. It must be noted that there is no *a priori* justification for such a factorization. However, this approximation provides a nonperturbative framework that allows us to treat photon fluctuations self-consistently [11]. In contrast, we will ignore the quantum fluctuations of the axion, which can be produced via self-couplings, as the study of Ref. [8] has shown that these effects are insignificant.

With Eq. (3.2), we first expand the nonequilibrium Lagrangian density (3.1) in powers of ψ and keep the terms up to linear ψ to ignore its quantum fluctuation effects. Together with Eq. (3.4), the Hartree-factorized Lagrangian then becomes

$$\mathcal{L}[\varphi(\eta)+\psi^+,A_\mu^+]-\mathcal{L}[\varphi(\eta)+\psi^-,A_\mu^-] =\left\{-U(\eta)\psi^+-\frac{1}{4}F^+_{\mu\nu}\tilde{F}^{+\mu\nu}+\frac{c}{af_a}\varphi(\eta)F^+_{\mu\nu}\tilde{F}^{+\mu\nu} + \frac{c}{af_a}\psi^+\langle F^{+\mu\nu}\tilde{F}^+_{\mu\nu}\rangle\right\}-\{+\to -\}, \tag{3.5}$$

where

$$U(\eta)=\ddot{\varphi}(\eta)-\frac{\ddot{a}(\eta)}{a(\eta)}\varphi(\eta)+a^3(\eta)m_a^2(T)f_a\ \sin\left[\frac{\varphi(\eta)}{a(\eta)f_a}\right]. \tag{3.6}$$

The overdot means the time derivative with respect to conformal time.

With the tadpole conditions (3.3), we obtain the following equation of motion for the axionic mean field:

$$\ddot{\theta}(\eta)+2\frac{\dot{a}(\eta)}{a(\eta)}\dot{\theta}(\eta)+a^2(\eta)m_a^2(T)\sin\theta(\eta)-\frac{1}{a^2(\eta)}\left(\frac{c}{f_a^2}\right) \times\langle F\tilde{F}\rangle(\eta)=0, \tag{3.7}$$

where we define the dimensionless field $\theta(\eta)=\varphi(\eta)/[a(\eta)f_a]$.

Within the Hartree approximation, photon production processes do not involve photons in intermediate states [11,13]. To avoid gauge ambiguities, we will work in the Coulomb gauge and concentrate only on the physical transverse gauge field, $\vec{A}_T(\vec{x},\eta)$ [11,13]. Then, the Heisenberg field equation for $\vec{A}_T(\vec{x},\eta)$ can be read off from the quadratic part of the Lagrangian in the form

$$\frac{d^2}{d\eta^2}\vec{A}_T(\vec{x},\eta)-\vec{\nabla}^2\vec{A}_T(\vec{x},\eta)+4c\,\dot{\theta}(\eta)\vec{\nabla}\times\vec{A}_T(\vec{x},\eta)=0. \tag{3.8}$$

It is more convenient to decompose the field $\vec{A}_T(\vec{x},\eta)$ into the Fourier mode functions $V_{\lambda\vec{k}}(\eta)$ in terms of circularly polarized states,

$$\vec{A}_T(\vec{x},\eta)=\int\frac{d^3k}{\sqrt{2(2\pi)^3k}}\vec{A}_T(\vec{k},\eta) =\int\frac{d^3k}{\sqrt{2(2\pi)^3k}}\{[b_{+\vec{k}}V_{1\vec{k}}(\eta)\vec{\epsilon}_{+\vec{k}} + b_{-\vec{k}}V_{2\vec{k}}(\eta)\vec{\epsilon}_{-\vec{k}}]e^{i\vec{k}\cdot\vec{x}}+\text{H.c.}\}, \tag{3.9}$$

where $b_{\pm\vec{k}}$ are destruction operators, and $\vec{\epsilon}_{\pm\vec{k}}$ are circular polarization unit vectors defined in Ref. [11]. Then the mode equations are

$$\frac{d^2V_{1k}(\eta)}{d\eta^2}+k^2V_{1k}(\eta)-4kc\,\dot\theta(\eta)V_{1k}(\eta)=0,$$

$$\frac{d^2V_{2k}(\eta)}{d\eta^2}+k^2V_{2k}(\eta)+4kc\,\dot\theta(\eta)V_{2k}(\eta)=0, \quad (3.10)$$

with the expectation values given by [11]

$$\langle F\tilde F\rangle(\eta)=\frac{1}{\pi^2}\int k^2dk\coth\left[\frac{k}{2T_i}\right]\frac{d}{d\eta}\times(|V_{1k}(\eta)|^2-|V_{2k}(\eta)|^2), \quad (3.11)$$

where we have assumed that at initial time η_i, the photons are in local equilibrium at the initial temperature T_i. Clearly, the photon mode equations (3.10) are decoupled in terms of the circular polarization mode functions. The axion field acts as the time-dependent mass term thats triggers photon production. The effective mass terms have opposite signs for the two polarizations due to the pseudoscalar nature of the axion-photon coupling. This will lead to production of the differently polarized photons in different amounts, resulting in a polarization asymmetry in photon emission. The expectation value of the number operator for the asymptotic photons with momentum $\vec k$ is given by [11]

$$\begin{aligned}\langle \mathbf{N}_k(\eta)\rangle&=\frac{1}{2k}[\dot{\vec A}_T(\vec k,\eta)\cdot\dot{\vec A}_T(-\vec k,\eta)+k^2\vec A_T(\vec k,\eta)\cdot\vec A_T(-\vec k,\eta)]-1\\&=\frac{1}{4k^2}\coth\left[\frac{k}{2T_i}\right][|\dot V_{1k}(\eta)|^2+k^2|V_{1k}(\eta)|^2]-\frac12+\frac{1}{4k^2}\coth\left[\frac{k}{2T_i}\right][|\dot V_{2k}(\eta)|^2+k^2|V_{2k}(\eta)|^2]-\frac12\\&=N_+(k,\eta)+N_-(k,\eta),\end{aligned} \quad (3.12)$$

which is the number of photons with momentum $\vec k$ per unit comoving volume.

IV. NUMERICAL RESULTS

In this section, we will compute the photon production from the axionic condensate with $f_a=10^{12}$ GeV, which has a zero-temperature mass $m_{a0}\simeq 6.2\times10^{-6}$ eV. As we will show below, the photon production takes place mainly during the radiation-dominated epoch. So, we simply assume a radiation-dominated universe.

At temperature T, the Hubble parameter is

$$H\equiv\left(\frac{\dot a}{a}\right)^2=\frac{5}{3}g^{1/2}(T)\frac{T^2}{m_{pl}}, \quad (4.1)$$

where $g(T)$ is the number of effective degrees of freedom at temperature T, m_{pl} is the Planck scale, and the cosmic scale factor is

$$a(\eta)=\frac{\eta}{\eta_1}, \quad (4.2)$$

where η_1 is the time when the axion field starts to oscillate, defined by a temperature T_1 such that $3H(T_1)=m_a(T_1)$. As such, $\eta_1^{-1}=m_a(T_1)/3$. For $f_a=10^{12}$ GeV, from Eqs. (2.8) and (4.1) we find $T_1\simeq0.9$ GeV and $g(T_1)\simeq60$.

Changing the variable η into a, the equations of motion (3.7) and (3.10) become

$$\frac{d^2\theta}{da^2}+\frac{2}{a}\frac{d\theta}{da}+9a^2\frac{m_a^2(T)}{m_a^2(T_1)}\sin\theta-\frac{9c}{m_a^2(T_1)a^2f_a^2}\langle F\tilde F\rangle=0, \quad (4.3)$$

$$\frac{d^2V_{1\xi}}{da^2}+\xi^2V_{1\xi}-4\xi c\frac{d\theta}{da}V_{1\xi}=0,$$

$$\frac{d^2V_{2\xi}}{da^2}+\xi^2V_{2\xi}+4\xi c\frac{d\theta}{da}V_{2\xi}=0, \quad (4.4)$$

where $\xi=k\eta_1$ and

$$\frac{m_a^2(T)}{m_a^2(T_1)}=\begin{cases}\left(\frac{T_1}{T}\right)^{7.4}=a^{7.4}\left[\frac{g(T)}{g(T_1)}\right]^{7.4/4}, & T\gg\Lambda_{QCD},\\ 10^2\left(\frac{T_1}{\Lambda_{QCD}}\right)^{7.4}, & T\ll\Lambda_{QCD}.\end{cases} \quad (4.5)$$

Note that $g(T)$ does not change significantly from T_1 to Λ_{QCD}. Henceforth, we approximate $g(T)\simeq g(T_1)\simeq60$, where $T_1=0.9$ GeV. It is worth pointing out that the mode equations (4.4) have unstable modes via the spinodal instability for sufficiently low-momentum modes with $\xi<4c|d\theta/da|$, where the effective mass becomes negative.

To solve Eqs. (4.3) and (4.4), we have to specify the initial conditions for the axion and photon fields. The amplitude of the axion field is frozen for $\eta\ll\eta_1$, i.e.,

$$\theta=1,\quad \frac{d\theta}{da}=0\quad \text{as}\quad a=a_i\ll1. \quad (4.6)$$

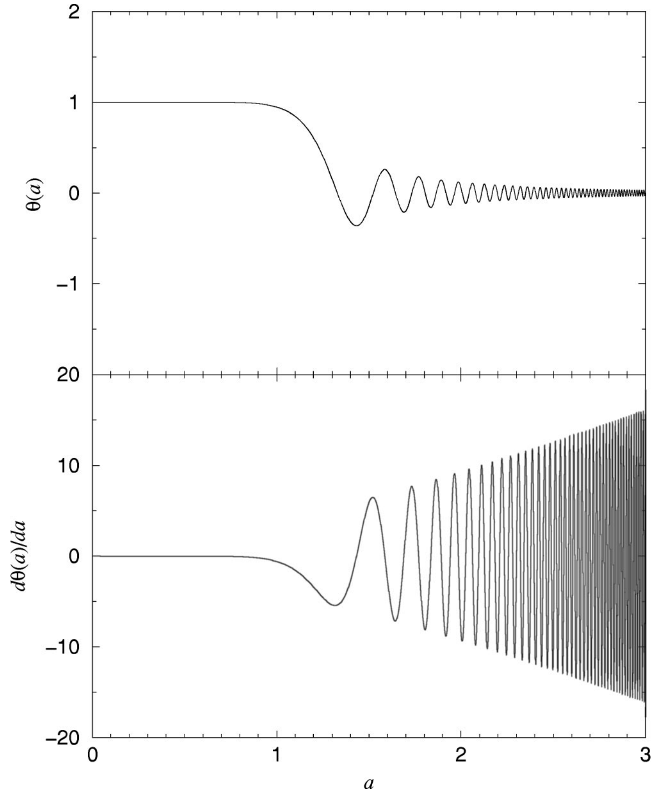

FIG. 1. Time evolution of the axion mean field $\theta(a)$ and its time derivative $d\theta(a)/da$, where a is the cosmic scale factor.

For the photon mode functions, we propose

$$V_{1\xi}=V_{2\xi}=1, \quad \frac{dV_{1\xi}}{da}=\frac{dV_{2\xi}}{da}=-i\xi \quad \text{as} \quad a=a_i\ll 1. \tag{4.7}$$

These initial conditions are physically plausible and simple enough for us to investigate a quantitative description of the dynamics. To evaluate the $\langle F\tilde{F}\rangle$ in Eq. (3.11) and the photon number operator in Eq. (3.12), we approximate the Bose enhancement factor by

$$\coth\left[\frac{k}{2T_i}\right]=\coth\left[\frac{\xi}{2\Gamma}\right]\simeq\frac{2\Gamma}{\xi}, \tag{4.8}$$

where $\Gamma\equiv\eta_1 T_i \geq \eta_1 T_1 \simeq 10^{18}$, and we are interested in $\xi < 100$.

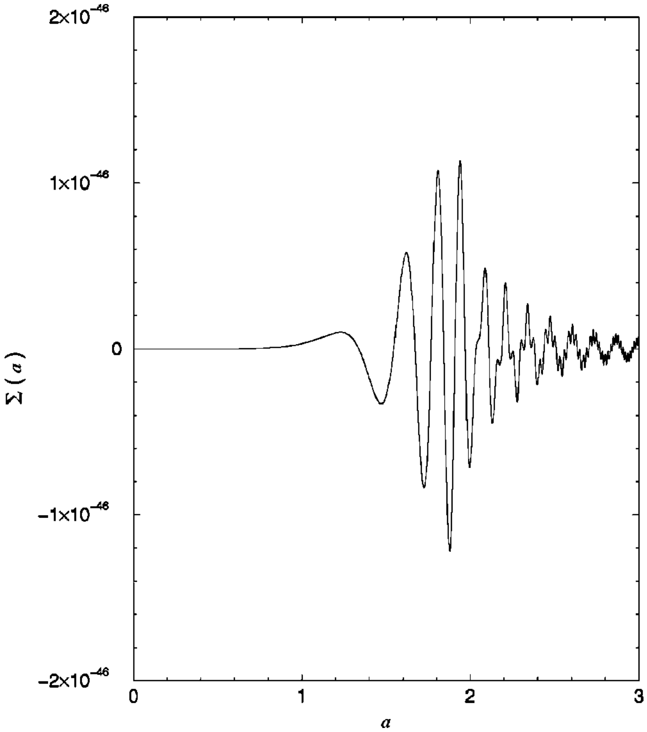

FIG. 2. Time evolution of $\langle F\tilde{F}\rangle$, plotted with the quantity $\Sigma(a)$ given by the last term of Eq. (4.3).

In Fig. 1, we plot the temporal evolution of the axion field and its time derivative by choosing $a_i=0.01$, which corresponds to $T_i=100T_1$. Due to the expansion of the universe, the field amplitude decreases with time. However, the rate of change of the amplitude increases with time. To understand this, we redefine $\theta\equiv\tilde{\theta}/a$ in Eq. (4.3). The $\langle F\tilde{F}\rangle$ term can be neglected, being extremely small as shown in Fig. 2, where we have evaluated the last term of Eq. (4.3) denoted by $\Sigma(a)$. Then the equation of motion for $\tilde{\theta}$ when $\theta\ll 1$ is given by

$$\frac{d^2\tilde{\theta}}{da^2}+9a^2\frac{m_a^2(T)}{m_a^2(T_1)}\tilde{\theta}=0. \tag{4.9}$$

From Eq. (4.5), the solutions for $\tilde{\theta}$ are Bessel functions. By matching the boundary conditions at $a=0$ [Eq. (4.6)] and $a=8.4$ (when $T=\Lambda_{QCD}$), we find that

$$\tilde{\theta}=\begin{cases}1.07a^{0.5}J_{1/11.4}(0.53a^{5.7}), & T\gg\Lambda_{QCD},\\ -0.39a^{0.5}J_{1/4}(3917a^2)-1.76a^{0.5}N_{1/4}(3917a^2), & T\ll\Lambda_{QCD}.\end{cases} \tag{4.10}$$

This approximate solution is plotted in Fig. 3. For $T\gg\Lambda_{QCD}$, asymptotically $\tilde{\theta}\propto\cos(0.53\,a^{5.7})/a^{2.35}$. For $T\ll\Lambda_{QCD}$, $\tilde{\theta}\propto\cos(3917a^2)/a^{0.5}$. The latter shows that the axions behave like nonrelativistic matter.

For photon production, we calculate the spectral photon number density $N(\xi,a)$, equal to $\langle \mathbf{N}_k(\eta)\rangle$ in Eq. (3.12). It is convenient to define a ratio

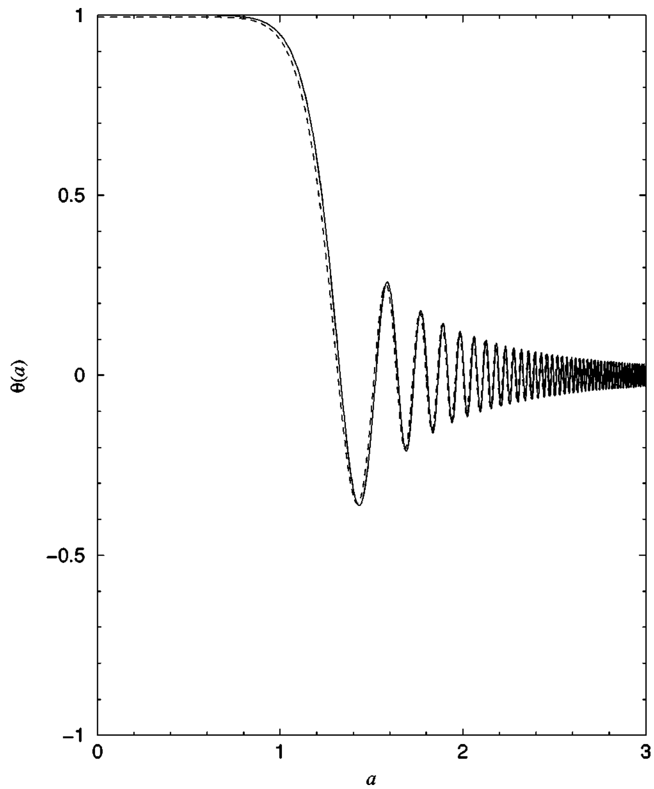

FIG. 3. Approximate solution of $\theta(a)\equiv\tilde{\theta}(a)/a$ from Eq. (4.10) for $0<a<3$ (the dashed line). For comparison, the exact numerical solution from Eq. (4.3) is plotted (the solid line). It can be shown that the approximate solution, Eq. (4.10), is also well fitted to the numerical solution of Eq. (4.3) for $a>3$.

$$n(\xi,a)\equiv\frac{N(\xi,a)-N(\xi,a_i)}{N(\xi,a_i)}, \tag{4.11}$$

which is the excess photons above the thermal background. Three snapshots of the ratio at $a=0.75$, 1.5, and 3 are shown in Fig. 4. It is interesting to see that the production duration of each nonzero mode is short, and higher-momentum modes are produced at later times. This in fact demonstrates a brief exponential growth of the unstable mode due to the parametric resonance instability, which is shut off when the unstable mode has been redshifted out of the unstable band. As a consequence, photon production is limited, with an excess photon ratio typically at a level of 10^{-7}. As we have mentioned above, the spinodal instability happens for low-momentum photon modes. We demonstrate this numerically in Fig. 5, where we have chosen $\xi<0.004$ which lie within the spinodal region where $\xi<4c|d\theta/da|$. The production ratio is also at a level of 10^{-7}, but it is apparent that these low-momentum modes are produced during the first oscillating cycle of the axion field.

The ratio in Fig. 4 can be actually estimated as follows. First, let us find out the approximate form for $d\theta/da$ from Eq. (4.10) and Fig. 1, which is given by

$$\frac{d\theta}{da}\simeq 3.6a^{1.35}\cos(0.53a^{5.7}) \quad \text{for} \quad a_i\leqslant a<8.4, \tag{4.12}$$

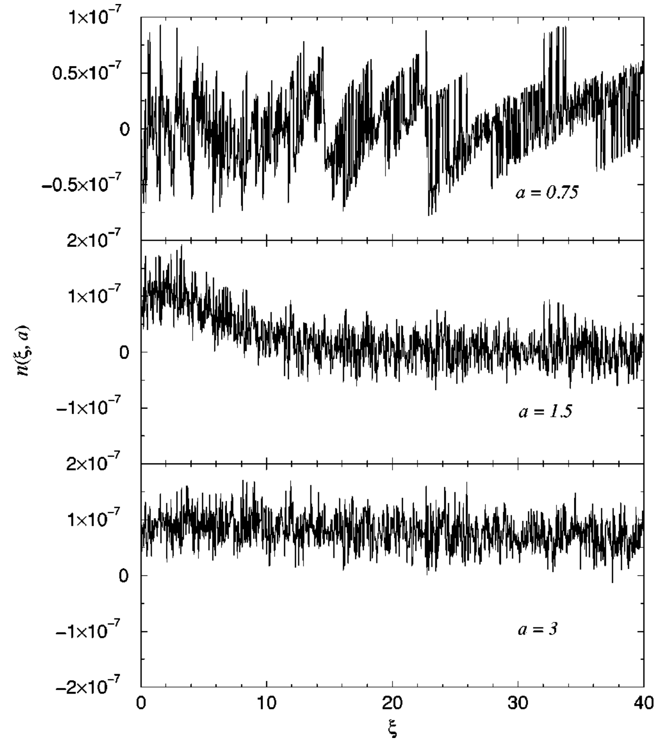

FIG. 4. Three snapshots of the spectral number density ratio $n(\xi,a)$, defined in Eq. (4.11), of the photons produced via parametric amplification at $a=0.75$, 1.5, and 3. The dimensionless quantity $\xi=k\eta_1$, where k is the photon momentum and η_1 is the conformal time when the axion field starts to oscillate.

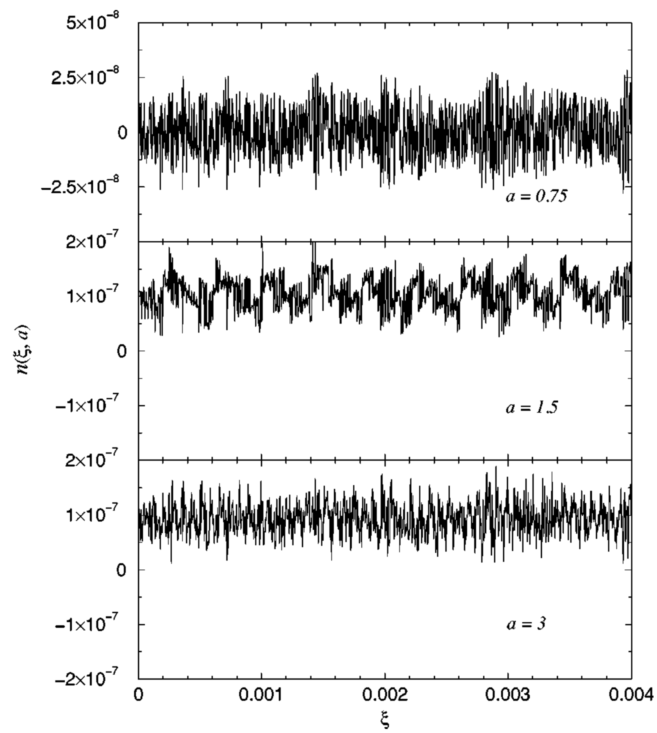

FIG. 5. As in Fig. 4, but for low-momentum photons produced via spinodal instability.

where $a=8.4$ is the scale factor when $T=\Lambda_{QCD}$. At instant a, $d\theta/da$ is oscillating with an effective frequency $\omega=0.53a^{4.7}$. Inserting the approximate form (4.12) with ω treated as a constant into the mode equation of $V_{1\xi}$ in Eq. (4.4), and changing variable to $z=\omega a/2$, we have

$$\frac{d^2V_{1\xi}}{dz^2}+\frac{4\xi^2}{\omega^2}V_{1\xi}-58.1c\frac{\xi}{\omega^2}a^{1.35}\cos(2z)V_{1\xi}=0. \tag{4.13}$$

This is the standard Mathieu equation [19]. The widest and most important instability is the first parametric resonance that occurs at $\xi=\omega/2$ with a narrow bandwidth $\delta\simeq 14.5ca^{1.35}/\omega$. But actually ω is changing with time. As such, the unstable mode will grow exponentially only during a brief period roughly given by

$$\Delta z\simeq\frac{\omega\delta}{\Delta\omega/\Delta z}\simeq\frac{\omega^2\delta}{2\Delta\omega/\Delta a}. \tag{4.14}$$

Consequently, this instability leads to growth of the occupation numbers of the created photons by a growth factor

$$e^{2\mu\Delta z}\simeq 1+2\mu\Delta z, \tag{4.15}$$

where the growth index $\mu\simeq\delta/2$. Hence the mode number density is increased by an amount approximately given by

$$2\mu\Delta z\simeq 42.6c^2a^{-1}\simeq 4.6\times 10^{-7}a^{-1}, \tag{4.16}$$

where a is the scale factor when the mode $V_{1\xi}$ enters into the resonance band. This estimation is of the same order of magnitude as found in the numerical results. Interestingly, the a^{-1} dependence of the mode production can also be seen in Fig. 4. A similar estimation can also be done for the mode function $V_{2\xi}$.

The polarization asymmetry in the photons produced is defined as

$$\Xi(\xi,a)=\frac{N_+(\xi,a)-N_-(\xi,a)}{N_+(\xi,a)+N_-(\xi,a)}. \tag{4.17}$$

We have input the mode solutions for $V_{1\xi}$ and $V_{2\xi}$ to Eq. (3.12) to calculate $\Xi(\xi,a)$ [Eq. (4.17)]. A plot of the asymmetry versus the momentum at $a=3$ is shown in Fig. 6. The numerical result shows that the asymmetry is fluctuating about zero as the photon momentum varies. The fluctuating amplitude is about 10^{-8} of the thermal background. Although the asymmetry averaged out over a wide range of momenta is nearly zero, at certain momenta the photons produced are about 10% circularly polarized. However, subsequently this polarization asymmetry will be damped out by photon-electron scattering in the plasma.

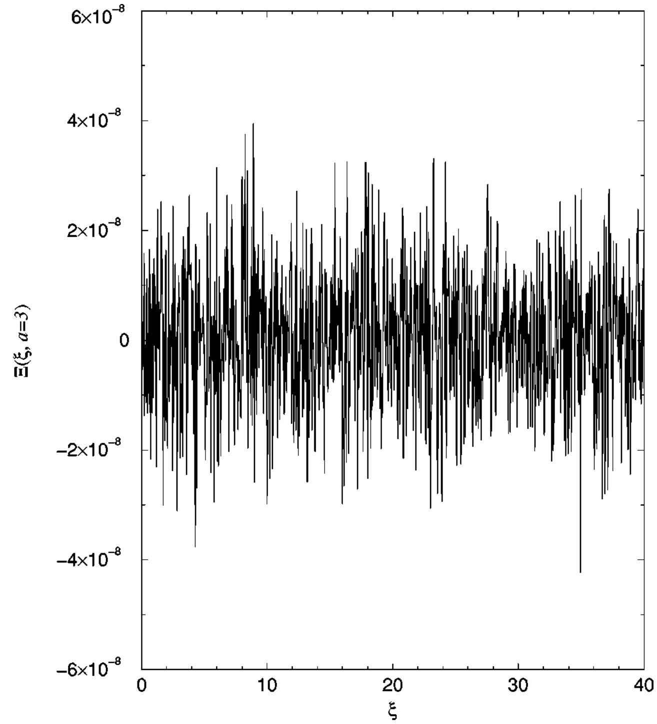

FIG. 6. Circular polarization asymmetry $\Xi(\xi,a)$, defined in Eq. (4.17), of the photons at $a=3$ for $0<\xi<40$.

V. CONCLUSIONS

First we confirm that photon production via parametric resonance is invalidated by the expansion of the universe. We find also that in addition to parametric resonance, for long-wavelength photon modes, a new dissipative channel via spinodal instability is open. This open channel results in long-wavelength fluctuations of the photon modes. Again, this production is suppressed in the expanding universe. We also observe polarization asymmetry in the circularly polarized photons produced as a result of the pseudoscalar nature of the coupling. However, it will be damped out effectively by the plasma. But it would be very interesting to see whether it is possible to generate circular polarization asymmetry in the production of photons from certain pseudoscalar fields such that it may leave an imprint on the polarization of the cosmic microwave background. As to the plasma damping on photon production, we have pointed out the problem in the naive approximation that simply introduces the electron plasma frequency into the photon modes. We have thus proposed a dynamical and nonequilibrium treatment which should be a better approach to consider the plasma effects. The actual calculations are rather difficult, but they certainly deserve further study.

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[1] For a review, see *Critical Dialogues in Cosmology*, edited by N. Turok (World Scientific, Singapore, 1997).
[2] J. Preskill, M. Wise, and F. Wilczek, Phys. Lett. **120B**, 127 (1983); L. Abbott and P. Sikivie, *ibid.* **120B**, 133 (1983); M. Dine and W. Fischler, *ibid.* **120B**, 137 (1983).
[3] See E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, Redwood City, CA, 1990).
[4] R. Peccei and H. Quinn, Phys. Rev. Lett. **38**, 1440 (1977); S. Weinberg, *ibid.* **40**, 223 (1978); F. Wilczek, *ibid.* **40**, 279 (1978).
[5] A. Vilenkin, Phys. Rev. Lett. **48**, 59 (1982).
[6] S.-Y. Pi, Phys. Rev. Lett. **52**, 1725 (1984); D. Seckel and M. S. Turner, Phys. Rev. D **32**, 3178 (1985).
[7] D. Gross, R. Pisarski, and L. Yaffe, Rev. Mod. Phys. **53**, 43 (1981).
[8] E. W. Kolb, A. Singh, and M. Srednicki, Phys. Rev. D **58**, 105004 (1998).
[9] D. Boyanovsky and H. J. de Vega, Phys. Rev. D **47**, 2343 (1993); D. Boyanovsky, D.-S. Lee, and A. Singh, *ibid.* **48**, 800 (1993); D. Boyanovsky, H. J. de Vega, and R. Holman, *ibid.* **51**, 734 (1995); **49**, 2769 (1994); D. Boyanovsky, H. J. de Vega, R. Holman, D.-S. Lee, and A. Singh, *ibid.* **51**, 4419 (1995); D. Boyanovsky, M. D'Attanasio, H. J. de Vega, R. Holman, and D.-S. Lee, *ibid.* **52**, 6805 (1995).
[10] D. Boyanovsky, H. J. de Vega, D.-S. Lee, Y. J. Ng, and S.-Y. Wang, Phys. Rev. D **59**, 125009 (1999); S.-Y. Wang, D. Boyanovsky, H. J. de Vega, D.-S. Lee, and Y. J. Ng, *ibid.* (to be published), hep-ph/9902218.
[11] D. Boyanovsky, H. J. de Vega, R. Holman, and S. Prem Kumar, Phys. Rev. D **56**, 3929 (1997).
[12] D. Boyanovsky, H. J. de Vega, R. Holman, S. Prem Kumar, and R. D. Pisarski, Phys. Rev. D **58**, 125009 (1998); S. Prem Kumar, D. Boyanovsky, H. J. de Vega, and R. Holman, *ibid.* (to be published), hep-ph/9905374, and references therein.
[13] D.-S. Lee and K.-W. Ng, Phys. Lett. B **449**, 274 (1999).
[14] M. Le Bellac, *Thermal Field Theory* (Cambridge University Press, Cambridge, England, 1989).
[15] J.-E. Kim, Phys. Rep. **150**, 1 (1987); H.-Y. Cheng, *ibid.* **158**, 1 (1988); in *Proceedings of the 5th IFT Workshop on Axions*, edited by P. Sikivie [Nucl. Phys. (Proc. Suppl.) **B72** (1999)].
[16] M. Dine, W. Fischler, and M. Srednicki, Phys. Lett. **104B**, 199 (1981); D. Kaplan, Nucl. Phys. **B260**, 215 (1985).
[17] S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972).
[18] See M. S. Turner and L. M. Widrow, Phys. Rev. D **37**, 2743 (1988).
[19] N. W. MacLachlan, *Theory and Application of Mathieu Functions* (Dover, New York, 1961).