More on the tensorial central charges in $\mathcal{N}=1$ supersymmetric gauge theories: BPS wall junctions and strings

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We study the central extensions of the $\mathcal{N}=1$ superalgebras relevant to the soliton solutions with the axial geometry—strings, wall junctions, etc. A general expression valid in any four-dimensional gauge theory is obtained. We prove that the only gauge theory admitting BPS strings at *weak coupling* is supersymmetric electrodynamics with the Fayet-Iliopoulos term. The problem of the ambiguity of the (1/2,1/2) central charge in the generalized Wess-Zumino models and gauge theories with matter is addressed and solved. A possibility of existence of the BPS strings at strong coupling in $\mathcal{N}=2$ theories is discussed. A representation of different strings within the brane picture is presented.

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I. INTRODUCTION

In the last several years much has been said about the domain walls in various supersymmetric field theories in four dimensions [1]. The existence of the Bogomol'nyi-Prasad-Sommerfield (BPS) saturated domain walls is in one-to-one correspondence with the central extension of $\mathcal{N}=1$ superalgebra, with the central charge $Z_{\alpha\beta}$ lying in the representation $\{0,1\}$ or $\{1,0\}$ of the Lorentz group [for brevity we will refer to such charges as the (1,0) charges]. In the non-Abelian gauge theories the (1,0) central charge emerges as a quantum anomaly in the superalgebra [2–4]. The possibility of the existence of the tensorial central charges in $\mathcal{N}=1$ superalgebras was noted in the brane context in Ref. [5]. The general theory of the central charges in $\mathcal{N}=1$ superalgebras was revisited recently [6].

In this paper we will discuss, in various theories, the central extensions of $\mathcal{N}=1$ superalgebras with the central charge $Z_{\alpha\dot{\beta}}$ lying in the representation $\{1/2,1/2\}$ of the Lorentz group [to be referred to as the (1/2,1/2) charges]. Such central charges are related to BPS objects with axial geometry, in particular, saturated strings. The fact that they exist is very well known in the context of supersymmetric QED (SQED) with the Fayet-Iliopoulos term; see Refs. [7,8] and especially Ref. [9], specifically devoted to this issue. In Ref. [9] it is shown, in particular, that if the spontaneous breaking of U(1) is due to the superpotential (the so-called *F* model), then the Abrikosov strings cannot be saturated. At the same time, if the spontaneous breaking of U(1) is due to the Fayet-Iliopoulos term (the so-called *D* model, with the vanishing superpotential), then the Abrikosov string is saturated, one-half of supersymmetry is conserved, and the string tension is given by the value of the central charge.[1]

Another physically interesting example where the (1/2,1/2) charges play a role is the wall junction. The fact that generalized Wess-Zumino (GWZ) models with a global symmetry of the U(1) or Z_N type may contain BPS wall junctions was noted in Ref. [10]. Interest in the wall junctions preserving one-quarter of the original supersymmetry was revived recently after the publication of [11,12], discussing such junctions in some GWZ models.

In this work we calculate the central extension of the $\mathcal{N}=1$ superalgebra of the $Z_{\alpha\dot{\beta}}$ type for a generic gauge theory, with or without matter. As will be seen, a spatial integral of a full spatial derivative of the appropriate structure does indeed emerge. It will be explained how the mass of the saturated solitons with axial geometry depends on the combination of the (1,0) and (1/2,1/2) central charges. For the solitons that are pure BPS strings (i.e. they possess axial geometry *and* their energy density is completely localized near some axis) only the (1/2,1/2) charge can contribute. We found that in the Wess-Zumino models, as well as in the gauge theories with matter, the expression for this central charge *per se* contains certain terms with coefficients which are ambiguous. Of critical importance is the ambiguity in the coefficient of the squark term. Using this ambiguity, we will prove that in *weak coupling* the only $\mathcal{N}=1$ gauge model admitting the BPS strings is SQED with the Fayet-Iliopoulos term. We then present some speculative ideas as to the possibility of the BPS strings in the non-Abelian models in *strong coupling*. For the objects of type of the wall junctions the ambiguity mentioned above conspires with a related ambiguity in the (1,0) central charge, so that the resulting

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[1]The statements above refer to $\mathcal{N}=1$ theories. In certain $\mathcal{N}=2$ extensions of QED one finds BPS saturated strings without the Fayet-Iliopoulos term. See Secs. IV and IX C.

TABLE I. General structure of the supercharge anticommutators $\frac{1}{2L}\{\mathcal{Q}\mathcal{Q}\}$.

	$\bar{Q}_{\dot{1}}$	$\bar{Q}_{\dot{2}}$	Q_1	Q_2
Q_1	$\frac{M}{L}-\oint a_k \mathrm{d}x_k$	0	A	0
Q_2	0	$\frac{M}{L}+\oint a_k \mathrm{d}x_k$	0	B
$\bar{Q}_{\dot{1}}$	$\bar{A}$	0	$\frac{M}{L}-\oint a_k \mathrm{d}x_k$	0
$\bar{Q}_{\dot{2}}$	0	$\bar{B}$	0	$\frac{M}{L}+\oint a_k \mathrm{d}x_k$

energy of the wall junction configuration is unambiguous.

II. GENERALITIES

Let $Q_\alpha, \bar{Q}_{\dot{\alpha}}$ be supercharges of the $\mathcal{N}=1$ four-dimensional field theory under consideration. The central charge relevant to strings, $Z_{\alpha\dot{\alpha}}$, appears in the anticommutator

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2P_{\alpha\dot{\alpha}} + 2Z_{\alpha\dot{\alpha}} \equiv 2\left\{P_\mu + \int \mathrm{d}^3x\, \varepsilon_{0\mu\nu\chi}\partial^\nu a^\chi\right\}(\sigma^\mu)_{\alpha\dot{\alpha}}, \quad (1)$$

where P_μ is the momentum operator, and a^ν is an axial vector specific to the theory under consideration. It must be built of dynamical fields of the theory. In other words, the (1/2,1/2) central charge is

$$Z_\mu = \int \mathrm{d}^3x\, \varepsilon_{0\mu\nu\chi}\partial^\nu a^\chi. \quad (2)$$

The corresponding tensor current

$$j_{\rho\mu} = \varepsilon_{\rho\mu\nu\chi}\partial^\nu a^\chi$$

is obviously conserved nondynamically, irrespective of the concrete form of the axial current a^χ.

Assume that the string is aligned along the vector n_μ (it is normalized by the condition $n_\mu n^\mu = -1$), and L is the length of the string (L is assumed to tend to infinity). Then the second term in Eq. (1) can be always represented as

$$Z_\mu = \int \mathrm{d}^3x\, \varepsilon_{0\mu\nu\chi}\partial^\nu a^\chi = TLn_\mu, \quad (3)$$

where T is a parameter of dimension mass squared. The direction of n_μ can always be chosen in such a way as to make T in Eq. (2) positive. We will always assume $T>0$.

In the rest frame of the string lying along the z direction (i.e., $\mathbf{n}=\{0,0,1\}$ or $n_\mu=\{0,0,0,-1\}$) the superalgebra (1) takes the form

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\begin{bmatrix} M-TL & 0 \\ 0 & M+TL \end{bmatrix}_{\alpha\dot{\alpha}}, \quad (4)$$

where M is the total mass of the string. For the saturated strings

$$M = TL; \quad (5)$$

i.e., the mass of the string coincides with the central charge appearing in the $\mathcal{N}=1$ superalgebra (1). The parameter T is then identified with the string tension. If the state of the BPS string is denoted $|\mathrm{str}\rangle$, then

$$Q_1|\mathrm{str}\rangle = \bar{Q}_{\dot{1}}|\mathrm{str}\rangle = 0. \quad (6)$$

In other words, Q_1 and $\bar{Q}_{\dot{1}}$ annihilate the string—this half of supersymmetry is conserved in the saturated string background. The action of Q_2 and $\bar{Q}_{\dot{2}}$ on $|\mathrm{str}\rangle$ produces the fermion zero modes.

Any four-dimensional $\mathcal{N}=1$ theory can be dimensionally reduced to two dimensions, where it becomes $\mathcal{N}=2$ theory. If the latter has topologically stable instantons, elevating the theory back to four dimensions gives us strings. Classical descriptions are totally equivalent. Distinctions occur at the level of quantum corrections, which are to be treated differently in two- and four-dimensional theories. The topological charge of the two-dimensional theory is related to the central charge of the centrally extended algebra (1). This simple observation allows one to use a wealth of information regarding various two-dimensional models in analysis of saturated strings in four dimensions at the classical level.

For the solitons of the wall junction type, which preserve a quarter of the original supersymmetry (more generally, for the BPS solitons with the axial geometry), it is necessary to consider, simultaneously, the (1,0) charge, which appears in the commutator

$$\{Q_\alpha Q_\beta\} = -4i(\vec{\sigma})_{\alpha\beta}\int \mathrm{d}^3x\, \vec{\nabla}\bar{\Sigma}, \quad (7)$$

where $\bar{\Sigma}$ is a scalar operator built of the dynamical fields of the theory, and

$$(\vec{\sigma})_{\alpha\beta} = \{-\tau_3, i, \tau_1\}_{\alpha\beta}. \quad (8)$$

For the BPS strings the (1,0) charge must vanish; however, for the wall junctions and other axial geometry BPS solitons both the (1,0) and (1/2,1/2) charges do not vanish (see Sec. III). In this case the general structure of the super-

charge anticommutators is as in Table I where the integrals above are taken in the plane perpendicular to the axis of the soliton (i.e., in the x,y plane), along a closed path of radius R (it is assumed that $R\to\infty$), dn_k is the element of the length of the curve (see Fig. 1) ($d\vec{n}$ is perpendicular to $d\vec{x}$),

$$A=-2i\int d^2x[(-\partial_1 S_1+\partial_2 S_2)+i(\partial_1 S_2+\partial_2 S_1)],$$

$$B=-2i\int d^2x[(\partial_1 S_1+\partial_2 S_2)+i(-\partial_1 S_2+\partial_2 S_1)],$$

and, finally,

$$\{S_1,S_2\}=\{\mathrm{Re}\Sigma,\mathrm{Im}\Sigma\}, \tag{9}$$

so that

$$\oint a_k dx_k=\int d^2x(\partial_x a_y-\partial_y a_x)$$
$$=\int d^2x[-i\partial_\zeta(a_x+ia_y)+i\partial_{\bar\zeta}(a_x-ia_y)], \tag{10}$$

$$\oint dn_k S_k=\int d^2x[\partial_\zeta\Sigma+\partial_{\bar\zeta}\bar\Sigma], \tag{11}$$

and the complex coordinates $\zeta,\bar\zeta$ are introduced below in Eq. (15).

It is helpful to adopt a phase convention (to be referred to as a *standard convention*). Namely, the phases of ζ and the superpotential will be adjusted in such a way that (i) there is no phase in Eq. (14); (ii) the phase of the superpotential $\mathcal{W}$ on the solution increases in the anti-clockwise direction; (iii) one of the walls in the junction runs along the x axis in the positive direction. Then it is not difficult to show that

$$\oint dn_k S_j\varepsilon_{kj}=0,$$

$B/(-2i)$ is real and positive, and

$$B=-2i\oint dn_k S_k.$$

The BPS bound on the soliton mass is obtained from the requirement of vanishing of the determinant of the above matrix, which implies

$$\frac{M}{L}=-\oint a_k dx_k+2\oint dn_k S_k. \tag{12}$$

For saturated objects the master equation (12) expresses the tensions in terms of two contour integrals over a large circle.

III. GENERALIZED WESS-ZUMINO MODELS

In this section, as a warmup exercise, we will discuss the GWZ models which give rise to the BPS solitons with axial geometry, and derive the (1/2,1/2) central charge in these models. The full expression for the (1,0) central charge was found previously [4]. The Lagrangian has the form

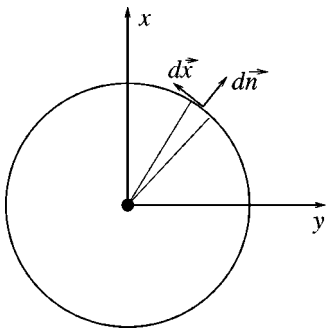

FIG. 1. The integration contour in the x,y plane. The soliton axis (the solid circle) lies perpendicular to this plane.

$$\mathcal{L}=\frac{1}{4}\sum_i\int d^2\theta d^2\bar\theta\bar\Phi_i\Phi_i+\left\{\frac{1}{2}\int d^2\theta\mathcal{W}(\Phi_i)+\mathrm{H.c.}\right\}, \tag{13}$$

where Φ_i is the set of the chiral fields, and the superpotential $\mathcal{W}$ is an analytic function of the fields Φ_i. The original (renormalizable) Wess-Zumino model implies that $\mathcal{W}$ is a cubic polynomial in Φ_i. We shall not limit ourselves to this assumption, keeping in mind that GWZ models with more contrived superpotentials can appear as low-energy limits of some renormalizable microscopic field theories. The case of more general Kähler potential will be considered later.

The equations of the BPS saturation for the solitons with the axial geometry in this model were first derived[2] in Ref. [4]; they have the form

$$\frac{\partial\phi_i}{\partial\zeta}=\frac{1}{2}\frac{\partial\bar{\mathcal{W}}}{\partial\bar\phi_i}, \tag{14}$$

where

$$\zeta=x+iy,\quad \frac{\partial}{\partial\zeta}=\frac{1}{2}\left(\frac{\partial}{\partial x}-i\frac{\partial}{\partial y}\right). \tag{15}$$

The soliton axis is assumed to lie along the z axis, while the soliton profile depends on x,y. Note that it is *not* assumed that the solution of Eq. (14) is analytic in ζ (in fact, one can prove that it must depend on both ζ and $\bar\zeta$ in the general case). A constant phase, which could have appeared on the right-hand side of Eq. (14), is absorbed in ζ.

Given the solution of Eq. (14), one gets two constraints determining the parameter of the residual (conserved) supersymmetry,

[2]See Sec. III D of Ref. [4] entitled, rather awkwardly, ''BPS-saturated strings.'' In fact, the authors meant BPS solitons with the axial geometry.

$$(1+\tau_3)\varepsilon=0,\quad \frac{-i}{2}(1-\tau_3)\varepsilon=\overline{\varepsilon}, \tag{16}$$

where the spinorial indices of $\varepsilon,\overline{\varepsilon}$ are suppressed (both are assumed to be the upper indices), and we follow the notations and conventions collected in [4]. The first constraint implies that ε has only the lower component, which reduces the number of supersymmetries from 4 to 2; the second constraint further reduces the number of the residual supersymmetries to 1.

In order to calculate the (1,0) and (1/2,1/2) central charges one needs the expression for the supercharges. In fact, since we focus on full derivatives, we need to know the supercurrent $J^{\mu}_{\alpha}=(1/2)(\overline{\sigma})^{\dot\beta\beta}J_{\alpha\beta\dot\beta}$, rather than the supercharges *per se*. The corresponding expression is well known (see, e.g. Ref. [4]):

$$J_{\alpha\beta\dot\beta}=2\sqrt{2}\sum\,[(\partial_{\alpha\dot\beta}\overline{\phi})\psi_\beta-i\epsilon_{\beta\alpha}F\overline{\psi}_{\dot\beta}]$$
$$-\frac{\sqrt{2}}{3}\sum\,[\partial_{\alpha\dot\beta}(\psi_\beta\overline{\phi})+\partial_{\beta\dot\beta}(\psi_\alpha\overline{\phi})-3\epsilon_{\beta\alpha}\partial^{\gamma}_{\dot\beta}(\psi_\gamma\overline{\phi})]. \tag{17}$$

The supercharge Q_α is defined as

$$Q_\alpha=\int d^3x J^0_\alpha,\quad J^{\mu}_{\alpha}=\frac{1}{2}(\overline{\sigma}^{\mu})^{\beta\dot\beta}J_{\alpha\beta\dot\beta}. \tag{18}$$

The term in the second line in Eq. (17) is conserved by itself. Moreover, in the supercharge it is represented as an integral over the full derivative. Below we will discuss the impact of deleting this term. We will keep it, however, for the time being, since we want to use the supercurrent which enters in one supermultiplet with the geometric R current [13] (sometimes called the R_0 current). The R_0 current is conserved in conformal theories.

It is not difficult to find the full derivative terms in $\{Q_\alpha\overline{Q}_{\dot\beta}\}$ by computing the canonic commutators of the fields at the tree level [the (1/2,1/2) central charge appears already at the tree level]. The task is facilitated if one observes that in order to get the (1/2,1/2) central charge it is sufficient to keep only terms of mixed symmetry in $\{\overline{Q}_{\dot\alpha}J_{\alpha\beta\dot\beta}\}$, namely, symmetric in α,β and antisymmetric in $\dot\alpha,\dot\beta$ or *vice versa*.

The result of this calculation reduces to Eq. (2) with

$$a^{\mu}=\frac{1}{4}a^{\mu}_{(\psi)}-\frac{1}{6}a^{\mu}_{(\phi)}, \tag{19}$$

where $a^{\mu}_{(\psi)}$ and $a^{\mu}_{(\phi)}$ are the fermion and boson axial currents, respectively;

$$a^{\mu}_{(\psi)}=-\sum\,\psi\sigma^{\mu}\overline{\psi},\quad a^{\mu}_{(\phi)}=-i\sum\,\phi\overleftrightarrow{\partial}^{\mu}\overline{\phi}. \tag{20}$$

The expression for the (1,0) central charge in the GWZ model found previously [4] at the tree level takes the form of Eq. (7) with

$$\overline{\Sigma}=\overline{\mathcal{W}}-\frac{1}{3}\sum\,\overline{\Phi}\frac{\partial\mathcal{W}}{\partial\overline{\Phi}}. \tag{21}$$

One can check that *only* the *combined* contribution of the central charges above correctly reproduces the mass of the BPS solitons with the axial geometry, e.g. the wall junctions. Indeed, Eq. (12) implies that in the model at hand,[3]

$$\frac{M}{L}=\int d^2x\left[\partial_k\overline{\phi}\partial_k\phi+\left|\frac{\partial W}{\partial\phi}\right|^2-\frac{1}{3}\partial_k\partial_k(\overline{\phi}\phi)\right]$$
$$=-2\left(1-\frac{2}{3}\right)\int d^2x[\partial_\zeta\phi\partial_{\overline{\zeta}}\overline{\phi}-\partial_\zeta\overline{\phi}\partial_{\overline{\zeta}}\phi]$$
$$+2\int d^2x\left[\partial_\zeta\left(\mathcal{W}-\frac{1}{3}\phi\frac{\partial\mathcal{W}}{\partial\phi}\right)+\partial_{\overline{\zeta}}\left(\overline{\mathcal{W}}-\frac{1}{3}\overline{\phi}\frac{\partial\overline{\mathcal{W}}}{\partial\overline{\phi}}\right)\right]. \tag{22}$$

On the other hand, for the BPS-saturated solution one can write

$$0=\int d^2x\left[2\partial_\zeta\phi-\frac{\partial\overline{\mathcal{W}}}{\partial\overline{\phi}}\right]\left[2\partial_{\overline{\zeta}}\overline{\phi}-\frac{\partial\mathcal{W}}{\partial\phi}\right]$$
$$=\int d^2x\left[\partial_k\overline{\phi}\partial_k\phi+\left|\frac{\partial\mathcal{W}}{\partial\phi}\right|^2\right]$$
$$+2\int d^2x[\partial_\zeta\phi\partial_{\overline{\zeta}}\overline{\phi}-\partial_\zeta\overline{\phi}\partial_{\overline{\zeta}}\phi]-2\int d^2x[\partial_\zeta\mathcal{W}+\partial_{\overline{\zeta}}\overline{\mathcal{W}}] \tag{23}$$

or

$$\frac{M}{L}=-2\int d^2x[\partial_\zeta\phi\partial_{\overline{\zeta}}\overline{\phi}-\partial_\zeta\overline{\phi}\partial_{\overline{\zeta}}\phi]+2\int d^2x[\partial_\zeta\mathcal{W}+\partial_{\overline{\zeta}}\overline{\mathcal{W}}]. \tag{24}$$

At first sight it might seem that Eqs. (22) and (24) contradict each other, since the axial current contribution to the soliton mass in these two expressions [corresponding to the (1/2,1/2) central charge] has different coefficients [cf. $-2+(4/3)$ in the first case and -2 in the second]. Upon inspection one sees that Eq. (22) has a different expression for the (1,0) central charge too. The difference is

[3]The term $-(1/3)\partial_k\partial_k(\overline{\phi}\phi)$ is irrelevant both for strings and wall junctions, since it vanishes in both cases. It contributes, however, in the energy of the axial geometry solitons of the type discussed in [4]. This term occurs in passing from the canonic energy-momentum tensor

$$\theta^{\rm canonic}_{\mu\nu}=\partial_\mu\overline{\phi}\partial_\nu\phi+\partial_\nu\overline{\phi}\partial_\mu\phi+\text{fermions}-g_{\mu\nu}\mathcal{L}$$

to the one which is traceless in the conformal limit:

$$\theta^{\rm traceless}_{\mu\nu}=\theta^{\rm canonic}_{\mu\nu}+\frac{1}{3}(g_{\mu\nu}\partial^\alpha\partial_\alpha-\partial_\mu\partial_\nu)\overline{\phi}\phi.$$

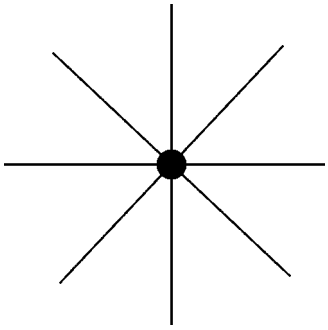

FIG. 2. The domain wall junction in the theory with Z_N symmetry. The ''hub'' is denoted by the closed circle.

$$-\frac{2}{3}\int \mathrm{d}^2x\left[\partial_\zeta\left(\phi\frac{\partial\mathcal{W}}{\partial\phi}\right)+\partial_{\bar\zeta}\left(\bar\phi\frac{\partial\bar{\mathcal{W}}}{\partial\bar\phi}\right)\right].$$

For the BPS saturated solitons satisfying Eq. (14) it is easy to show that

$$-\frac{2}{3}\int \mathrm{d}^2x\left[\partial_\zeta\left(\phi\frac{\partial\mathcal{W}}{\partial\phi}\right)+\partial_{\bar\zeta}\left(\bar\phi\frac{\partial\bar{\mathcal{W}}}{\partial\bar\phi}\right)\right]$$
$$=-\frac{4}{3}\int \mathrm{d}^2x[\partial_\zeta\phi\partial_{\bar\zeta}\bar\phi-\partial_\zeta\bar\phi\partial_{\bar\zeta}\phi]-\frac{1}{3}\int \mathrm{d}^2x\,\partial^\alpha\partial_\alpha\bar\phi\phi. \tag{25}$$

This relation immediately implies the coincidence of the soliton masses ensuing from Eqs. (22) and (24), respectively.

In fact, the superficial difference between them is due to the ambiguity in the choice of supercurrent [the terms with the full derivatives in Eq. (17)] and the corresponding ambiguity in the energy-momentum tensor. Equation (22) is derived on the basis of the supercurrent and the energy-momentum tensor with the properties $\varepsilon^{\alpha\beta}J_{\alpha\beta\dot\beta}=0$, $\theta^\mu_\mu=0$ in the conformal limit. Passing to the minimal supercurrent and the canonic energy-momentum tensor one drops all terms containing the factor 1/3 in Eq. (22) and recovers Eq. (24). The mass of the soliton stays intact due to a reshuffling of contributions due to (1/2,1/2) and (1,0) charges.

To illustrate the point let us consider, for instance, a Z_N model suggested in Ref. [14], with the superpotential

$$\mathcal{W}=N\left\{\Phi-\frac{N}{N+1}\left(\frac{\Phi}{N}\right)^{N+1}\right\}, \tag{26}$$

where Φ is a chiral superfield. The model obviously possesses a Z_N symmetry, and the vacuum manifold corresponds to N points,

$$\phi_k=N\exp\left(\frac{2\pi ik}{N}\right),\quad k=0,1,2,\ldots,N-1, \tag{27}$$

while the vacuum value of the superpotential is

$$\mathcal{W}(\phi_k)=N^2\exp\left(\frac{2\pi ik}{N}\right),\quad N\to\infty. \tag{28}$$

The solution of the BPS saturation equation for an isolated wall exists; it was discussed in [14]. (Here and below N will be assumed large, and only leading terms in N will be kept.) The tension of the minimal wall connecting the neighboring vacua is

$$T=2|\Delta\mathcal{W}|=4\pi N. \tag{29}$$

Consider the BPS wall junctions of the type depicted in Fig. 2. Assuming that there is a solution of Eq. (14), to the leading order in N one can write (at $|\zeta|\to\infty$)

$$\phi=Ne^{i\alpha(\gamma)},\quad \alpha(0)=0,\quad \alpha(2\pi)=2\pi, \tag{30}$$

which entails, in turn,

$$\oint_{|x|=R\to\infty}a_k\mathrm{d}x_k=\frac{N^2}{3}[\alpha(2\pi)-\alpha(0)]=\frac{2\pi}{3}N^2. \tag{31}$$

We also observe that

$$2\oint \mathrm{d}n_k w_k=2N^2R\int \mathrm{d}\gamma\cos(\alpha-\gamma)=4\pi N^2R,$$
$$\{w_1,w_2\}=\{\mathrm{Re}\,\mathcal{W},\mathrm{Im}\,\mathcal{W}\}, \tag{32}$$

which is exactly the mass of N isolated walls inside the contour. Furthermore,

$$2\oint \mathrm{d}n_k S_k=4\pi N^2R-\frac{4\pi}{3}N^2. \tag{33}$$

The total mass of the junction configuration comes out the same from both expressions, Eqs. (22) and (24),

$$\frac{M}{L}=4\pi N^2R-2\pi N^2 \tag{34}$$

(see also [15]). The first term can be interpreted as the mass of the ''spokes'' joined at the origin, while the second as that of the ''hub.''

Let us remark that the stringy (''hub'') contribution to the total mass equals twice the area of the contour on the ϕ plane covered by the solution. Since we consider the junction with N ''minimal'' domain walls connecting the neighboring vacua, the contour is closed. The closeness is nothing but the equilibrium condition at the junction line.

Summarizing, we observe an ambiguity in the (1/2,1/2) central charge. This ambiguity is due to the fact that both the supercurrent and the energy-momentum tensor are not uniquely determined. Both admit certain full derivative terms which are conserved by themselves and, therefore, do not affect the supercharges and the energy-momentum four-vector. They do affect the expressions for the central charges, however. For the soliton solutions of the wall junction type the ambiguity in the (1/2,1/2) central charge combines with another ambiguity, in the (1,0) central charge, to produce an unambiguous expression for the soliton mass. As we will see shortly, the same ambiguity (and a similar conspiracy) takes place in the gauge theories with matter.

Practically, it is more convenient to work with minimal supercurrents (and the canonic energy-momentum tensor). Then, one omits the second line in Eq. (17). The expression for a^μ in the (1/2,1/2) central charge then becomes

$$a^\mu=\frac{1}{4}a^\mu_{(\psi)}-\frac{1}{2}a^\mu_{(\phi)},\quad a^\mu_{(\phi)}=-i\sum\ \phi\overleftrightarrow{\partial^\mu}\overline{\phi}, \tag{35}$$

while $\overline{\Sigma}$ in the (1,0) central charge becomes

$$\overline{\Sigma}=\overline{\mathcal{W}}.$$

IV. SQED WITH THE FAYET-ILIOPOULOS TERM

The simplest theory (and the only one in the class $\mathcal{N}=1$; see below) where saturated strings exist in the weak coupling regime is supersymmetric electrodynamics (SUSY QED, or SQED), with the Fayet-Iliopoulos (FI) term. In superfield notation the Lagrangian of the model has the form

$$\mathcal{L}=\left\{\frac{1}{8e^2}\int \mathrm{d}^2\theta W^2+\mathrm{H.c.}\right\}+\frac{1}{4}\int \mathrm{d}^4\theta(\overline{S}e^V S+\overline{T}e^{-V}T)-\frac{\xi}{4}\int \mathrm{d}^2\theta\mathrm{d}^2\overline{\theta}V(x,\theta,\overline{\theta}), \tag{36}$$

where e is the electric charge, S and T are two chiral superfields with electric charges $+1$ and -1, respectively, and ξ is the coefficient of the Fayet-Iliopoulos term. The model with one chiral superfield is internally anomalous. Topologically stable solutions in this model and its modifications were considered more than once in the past [7–9]. We combine various elements scattered in the literature, with a special emphasis on the algebraic aspect. The supersymmetry of this model is minimal, $\mathcal{N}=1$.

If $\xi\neq 0$, the vacuum state corresponds to the spontaneous breaking of U(1). The spectrum of the model is that of a massive vector supermultiplet (one massive vector field, one real scalar and one Dirac fermion, all of one and the same mass), plus a massless modulus (one chiral superfield) parametrized by the product ST:

$$\Phi=2ST. \tag{37}$$

The vacuum valley is represented by the one-dimensional complex manifold with the Kähler function

$$K(\Phi,\overline{\Phi})=\sqrt{\xi^2+\Phi\overline{\Phi}}. \tag{38}$$

In a generic point nonsingular Abrikosov strings do *not* exist [8]. There is one special point, however, $\Phi=0$, where the theory supports the saturated string.

In components the Lagrangian of SQED, Eq. (36), has the form (in the Wess-Zumino gauge)

$$\mathcal{L}=-\frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu}+(\mathcal{D}_\mu\phi)^\dagger\mathcal{D}^\mu\phi+(\mathcal{D}_\mu\chi)^\dagger\mathcal{D}^\mu\chi-\frac{e^2}{2}(\phi^\dagger\phi-\chi^\dagger\chi-\xi)^2+\text{fermions}, \tag{39}$$

where ϕ and χ are the lowest components of the superfields S and T, respectively, with the electric charges ± 1, e.g.,

$$\mathcal{D}_\mu\phi=\partial_\mu\phi-iA_\mu\phi,\quad [\mathcal{D}_\mu,\mathcal{D}_\nu]\phi=-iF_{\mu\nu}\phi. \tag{40}$$

Without loss of generality we can assume that $\xi>0$.

For static field configurations, assuming in addition that all fields depend only on x and y and $A_0=A_3=0$, one gets an energy functional in the form

$$\begin{aligned}\mathcal{E}&=\int \mathrm{d}x\mathrm{d}y\left\{\frac{1}{2e^2}F_{12}^2+\sum_{i=1,2}(\mathcal{D}_i\phi)^\dagger\mathcal{D}_i\phi+\frac{e^2}{2}(\phi^\dagger\phi-\xi)^2\right\}\\&\equiv\int \mathrm{d}x\mathrm{d}y\left\{\left|\frac{1}{\sqrt{2}e}F_{12}+\frac{e}{\sqrt{2}}(\phi^\dagger\phi-\xi)\right|^2+[(\mathcal{D}_1+i\mathcal{D}_2)\phi]^\dagger(\mathcal{D}_1+i\mathcal{D}_2)\phi\right\}+\mathcal{Q},\end{aligned} \tag{41}$$

where $\mathcal{Q}$ is the surface (topological) term:

$$\mathcal{Q}=\int \mathrm{d}x\mathrm{d}y\left\{\xi F_{12}-\frac{i}{2}\partial_i(\phi^\dagger\overleftrightarrow{\mathcal{D}_j}\phi)\varepsilon^{ij}\right\},\quad i,j=1,2. \tag{42}$$

We will discuss the value of the surface term later.

The saturation equations are

$$F_{12}=-e^2(\phi^\dagger\phi-\xi),$$
$$(\mathcal{D}_1+i\mathcal{D}_2)\phi=0. \tag{43}$$

The ansatz which goes through these equations is

$$\phi=\sqrt{\xi}\eta e^{i\alpha},$$
$$A_i=a\frac{\partial\alpha}{\partial x^i},\quad i=1,2, \tag{44}$$

where

$$\alpha=\mathrm{Arg}\,\zeta,\quad \zeta=x+iy, \tag{45}$$

and η, a are some functions depending on r. This must be supplemented by the standard boundary conditions, namely

$$\eta(r),\ a(r)\to\begin{cases}0 \text{ at } r\to 0,\\ 1 \text{ at } r\to\infty.\end{cases} \tag{46}$$

For the given ansatz the saturation equations (43) degenerate into a system of first-order equations

$$a'=e^2\xi r(\eta^2-1),$$
$$\eta'=-\frac{\eta(1-a)}{r}, \tag{47}$$

where the prime denotes differentiation over r. Its solution is well known.

It is instructive to compare the topological term in Eq. (42) with the central charge of the superalgebra. To derive

the central charge one needs an expression for the supercurrent in SQED, which takes the form (in the spinorial notation)

$$J_{\alpha\beta\dot\beta}=\frac{2}{e^2}(iF_{\beta\alpha}\bar\lambda_{\dot\beta}+\epsilon_{\beta\alpha}D\bar\lambda_{\dot\beta})+2\sqrt{2}\sum(\mathcal{D}_{\alpha\dot\beta}\phi^\dagger)\psi_\beta$$
$$-\frac{\sqrt{2}}{3}\sum[\partial_{\alpha\dot\beta}(\psi_\beta\phi^\dagger)+\partial_{\beta\dot\beta}(\psi_\alpha\phi^\dagger)$$
$$-3\epsilon_{\beta\alpha}\partial^\gamma_{\dot\beta}(\psi_\gamma\phi^\dagger)]. \tag{48}$$

Above it is assumed that there is no superpotential. The expression in the square brackets may or may not be added, at will. [The expression in the square brackets in Eq. (48) is conserved by itself; in the supercharge it presents a full spatial derivative, hence, its contribution vanishes.] The sum runs over various matter supermultiplets, in particular, S and T in the case at hand.

To find the central charge one must compute the anticommutator $\{Q_\alpha,\bar J_{\dot\beta\dot\gamma\delta}\}$. Moreover, we decompose the anticommutator above with respect to irreducible representations of the Lorentz group, by singling out the symmetric and antisymmetric combinations of the indices with and without overdots. The one which is symmetric with respect to both pairs, (α,δ) and $(\dot\beta\dot\gamma)$, is the Lorentz spin 2 (the energy-momentum tensor), which contributes to $P_{\alpha\dot\alpha}$, rather than to the central charge. The combination which is antisymmetric with respect to both pairs, (α,δ) and $(\dot\beta\dot\gamma)$, is Lorentz singlet; it represents the trace terms in the energy-momentum tensor. To single out the central charge we must isolate the terms of the mixed symmetry, i.e., symmetric with respect to (α,δ) and antisymmetric with respect to $(\dot\beta\dot\gamma)$, and *vice versa*.

Keeping in mind this remark, and using the canonic commutation relations and equations of motion for the D field, we get an expression similar to that in the Wess-Zumino model, plus an extra contribution due to the D term:

$$\{Q_\alpha\bar Q_{\dot\alpha}\}=i\xi\int d^3x[F_{\beta\alpha}\varepsilon_{\beta\dot\alpha}-\bar F_{\dot\gamma\dot\alpha}\varepsilon_{\dot\gamma\alpha}]. \tag{49}$$

This implies

$$Z_\mu=\int \mathrm{d}^3x\,\varepsilon_{0\mu\nu\rho}\Big(\xi\partial^\nu A^\rho-\sum\frac{i}{2}\partial^\nu(\bar\phi\overleftrightarrow{\mathcal{D}}^\rho\phi)+\frac{1}{4}\partial^\nu R^\rho$$
$$+\frac{1}{4}\partial^\nu a^\rho_{(\psi)}\Big), \tag{50}$$

where R^ρ is the photino current, while $a^\rho_{(\psi)}$ is that of the electrons:

$$R^\rho=-\frac{1}{e^2}\lambda\sigma^\rho\bar\lambda,\quad a^\mu_{(\psi)}=-\sum\psi\sigma^\mu\bar\psi. \tag{51}$$

Note that the coefficient of the $\bar\phi\overleftrightarrow{\mathcal{D}}^\rho\phi$ (i.e., the selectron axial current) term is ambiguous—it depends on whether the expression in the square brackets in Eq. (48) is included in the definition of the supercurrent. The result quoted above refers to the minimal supercurrent, with the second line in Eq. (48) discarded. Since the (1,0) central charge is irrelevant for the string solution, this ambiguity alone shows that the $\bar\phi\overleftrightarrow{\mathcal{D}}^\rho\phi$ term cannot contribute to the central charge under consideration. It is certainly the case, since $\mathcal{D}^\rho\phi$ falls off sufficiently fast at $r\to\infty$ (where r is the distance to the string axis) for the string solution. At the same time, the photon four-potential A^ρ falls off slowly, as $1/r$. Thus, the (1/2,1/2) central charge is saturated by the ξ term exclusively. The latter is unambiguously fixed in Eq. (50); i.e., it does not depend on the full derivative terms in the supercurrent. The (1/2,1/2) central charge is obviously proportional to ξ and to the magnetic flux of the string,

$$\frac{M}{L}=\xi\mathcal{F}, \tag{52}$$

where

$$\mathcal{F}=\int \mathrm{d}x\mathrm{d}yF_{12}=\oint A_k\mathrm{d}\,x_k\,. \tag{53}$$

Note that the very same saturation equations (43) are obtained in $\mathcal{N}=2$ SQED with the vanishing Fayet-Iliopoulos term and linear superpotential; see Sec. IX C.

V. KÄHLER SIGMA MODELS

In this section we present some arguments concerning strings in the four-dimensional σ models on the Kähler manifolds. The two-dimensional reductions of these models are well studied; in the Euclidean formulation they admit instantons, which are the solutions of the first order self-duality equations. In the supersymmetric version the self-duality equations in two dimensions are reinterpreted as BPS equations in higher-dimensional theories (e.g., [16]). It is obvious that the instantons of the two-dimensional models are BPS strings in four dimensions. Thus, the four-dimensional σ models on the Kähler manifolds do have the BPS strings at the quasiclassical level, at weak coupling. Keeping in mind the assertion we are going to prove later (Sec. VIII) we discuss where the Kähler sigma models stand compared to other models.

Let us start with the CP_1 model. In a sense, this model can be obtained as a limiting case of SQED with a somewhat different matter content compared to that of Sec. IV (see, for instance, [17]). Indeed, assume that the matter superfields S and T have both charges $+1$, rather than ±1. As a quantum theory, it is anomalous, but for the time being we limit ourselves to the classical consideration. The limit to be taken is $e^2\to\infty$. Let us have a closer look at Eq. (39), with the sign of the charge of the χ field reversed [correspondingly, the D term takes the form $D=e^2(\phi^\dagger\phi+\chi^\dagger\chi-\xi)$]. In this limit the photon mass tends to infinity; the photon becomes nondynamical and can be eliminated. It drags with itself two real scalar degrees of freedom. The remaining two scalar degrees of freedom are massless. Their interaction reduces to the

sigma model on a sphere. This is most easily seen from Eq. (39). In the limit $e^2\to\infty$ the D term must vanish, which implies that $\phi^\dagger\phi+\chi^\dagger\chi=\xi$. In fact, gauge freedom allows one to identically eliminate one out of four degrees of freedom residing in ϕ, χ. The remaining three are subject to the constraint, telling us that the radius of the sphere is ξ.

Thus, SQED with the Fayet-Iliopoulos term, in the limit $e^2\to\infty$, gives rise to a model with the action

$$S=\frac{1}{2g^2}\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\ln(1+\bar{\Phi}\Phi), \tag{54}$$

where Φ is a chiral superfield:

$$\Phi(x_L,\theta)=\phi(x_L)+\sqrt{2}\theta^\alpha\psi_\alpha(x_L)+\theta^2F(x_L). \tag{55}$$

The coupling constant $2/g^2$ has the dimension of mass squared and is equal to ξ. The string tension will be proportional to $2/g^2=\xi$. The metric of the sphere in the target space G in this case is

$$G=\frac{2}{g^2}\frac{1}{(1+\bar{\Phi}\Phi)^2}. \tag{56}$$

The energy functional for the stringy solution takes a form which looks exactly as the action in the Euclidean two-dimensional sigma model whose world volume is transverse to the string. It is easy to rewrite it in terms of a topological charge plus a positive definite contribution,

$$\frac{\mathcal{E}}{L}=\int \mathrm{d}^2x\left\{\frac{8}{g^2}\left|\frac{\partial_{\bar{\zeta}}\bar{\phi}}{1+\bar{\phi}\phi}\right|^2+\frac{1}{g^2}\varepsilon_{\mu\nu}\partial_\mu\left(\frac{\bar{\phi}i\overleftrightarrow{\partial}_\nu\phi}{1+\bar{\phi}\phi}\right)\right\}, \tag{57}$$

where the second term, the integral over the full derivative, presents the topological charge and the integral runs in the plane transverse to the string. Instantons saturate the topological charge; since $\pi_2(S_2)=Z$, the saturated solutions are labeled by an integer n, equal to the topological charge. The surface term contribution in Eq. (57) is thus proportional to $g^{-2}n=\xi n$.

In four dimensions the instantons present the BPS saturated strings. These strings are rather peculiar. Since the two-dimensional theory is classically (super)conformally invariant, the two-dimensional instantons can have any size (correspondingly, the cross section of the string in four-dimensional theory can be arbitrary). The larger the transverse size of the string, the smaller the energy density in the string. However, the string tension remains constant proportional to $g^{-2}=\xi$. This is the limiting profile of the Abrikosov string in SQED with the Fayet-Iliopoulos term—the profile it acquires when the vector field mass tends to infinity while the remaining degrees of freedom of the matter fields remain massless.

For our purposes it is important to interpret the surface term contribution in the string tension in terms of the (1/2,1/2) central charge of four-dimensional SQED. Upon inspecting Eq. (50) we conclude that this contribution comes from the first term in Eq. (50). The field A_μ is not dynamical in the limit under consideration, and is expressible in terms of the residual scalars. Since our consideration is quasiclassical, it is not surprising that the current of the matter fermions does not contribute. The second term in Eq. (50) does not contribute either—as was discussed, its coefficient is ambiguous.

The O(3) (or CP_1) model belongs to a more general class of CP_N models. The latter can be derived as the low-energy limit of SQED with the FI term and with $N+1$ chiral matter superfields (all of them have charge $+1$), in the limit $e^2\to\infty$. One can eliminate the nondynamical A_μ field, much in the same way as in CP_1, arriving in this way at a nonlinear sigma model.

One has to introduce complex coordinates $w_i^j=\phi_i/\phi_j$ where $i\neq j$ which can be considered as scalar components of the chiral superfields Φ_i^j. The action can be written in terms of Φ_i^j as follows:

$$S=\frac{1}{2g^2}\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\ln\left(1+\sum_{i,j}\bar{\Phi}_i^j\Phi_i^j\right). \tag{58}$$

The identification $\xi=1/g^2$ is transparent since both parameters determine the size of the target manifold in two formulations.

The general expression for the central charge is [12]

$$Z=\int \mathrm{d}^2x\{\partial_\zeta(K_\phi\partial_{\bar{\zeta}}\bar{\phi}-K_{\bar{\phi}}\partial_{\bar{\zeta}}\bar{\phi})+\partial_{\bar{\zeta}}(K_{\bar{\phi}}\partial_\zeta\bar{\phi}-K_\phi\partial_\zeta\phi)\}, \tag{59}$$

where the complex variable ζ is defined in Eq. (15) and the subscripts ϕ, $\bar{\phi}$ denote the ϕ, $\bar{\phi}$ partial derivatives of the Kähler metric.

More generally, we expect similar strings for all toric varieties which can be presented as low energy limits of gauged linear sigma model. In Sec. IX we shall encounter one more example of the Kähler sigma model coupled to the Abelian gauge field—the low-energy effective action for $\mathcal{N}=2$ SUSY Yang-Mills theory in four dimensions.

VI. SUPERSYMMETRIC GLUODYNAMICS

To begin with, consider the simplest non-Abelian gauge model, SUSY gluodynamics. The Lagrangian is

$$\mathcal{L}=\frac{1}{4g^2}\int \mathrm{d}^2\theta\,\mathrm{Tr}\,W^2+\mathrm{H.c.}, \tag{60}$$

where $W=W^aT^a$, and T^a are the generators of the gauge group G in the fundamental representation. Although the gauge group G can be arbitrary, for definiteness we limit ourselves to SU(N). In components,

$$\mathcal{L}=\frac{1}{g^2}\left\{-\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu}+i\lambda^{a\alpha}\mathcal{D}_{\alpha\dot{\beta}}\bar{\lambda}^{a\dot{\beta}}\right\}. \tag{61}$$

There is a supermultiplet of the classically conserved currents (for a recent review see e.g., [18]),

$$\mathcal{J}_{\alpha\dot\alpha}=-\frac{4}{g^2}\mathrm{Tr}[e^V W_\alpha e^{-V}\overline{W}_{\dot\alpha}]$$
$$=R_{\alpha\dot\alpha}-\{i\theta^\beta J_{\beta\alpha\dot\alpha}+\mathrm{H.c.}\}-2\theta^\beta\overline{\theta}^{\dot\beta}J_{\alpha\dot\alpha\beta\dot\beta}+\cdots, \tag{62}$$

where $R_{\alpha\dot\alpha}$ is the chiral current, $J_{\beta\alpha\dot\alpha}$ is the supercurrent, and $J_{\alpha\dot\alpha\beta\dot\beta}$ is a combination of the energy-momentum tensor $\vartheta_{\alpha\dot\alpha\beta\dot\beta}=(\sigma^\mu)_{\alpha\dot\alpha}(\sigma^\nu)_{\beta\dot\beta}\vartheta_{\mu\nu}$ and a full derivative appearing in the central charge, namely,

$$R_{\alpha\dot\alpha}=-\frac{4}{g^2}\mathrm{Tr}\lambda_\alpha\overline{\lambda}_{\dot\alpha},$$

$$J_{\beta\alpha\dot\alpha}=(\sigma^\mu)_{\alpha\dot\alpha}J_{\mu,\beta}=\frac{4i}{g^2}\mathrm{Tr}\,G_{\alpha\beta}\overline{\lambda}_{\dot\alpha},$$

$$J_{\alpha\dot\alpha\beta\dot\beta}=\vartheta_{\alpha\dot\alpha\beta\dot\beta}-\frac{i}{4}\varepsilon_{\alpha\beta}\partial_{\gamma\{\dot\beta}R^{\gamma}_{\dot\alpha\}}+\frac{i}{4}\varepsilon_{\dot\alpha\dot\beta}\partial_{\dot\gamma\{\beta}R^{\dot\gamma}_{\alpha\}},$$

$$\vartheta_{\alpha\dot\alpha\beta\dot\beta}=\frac{2}{g^2}\mathrm{Tr}[i\lambda_{\{\alpha}\mathcal{D}_{\beta\}\dot\beta}\overline{\lambda}_{\dot\alpha}-i(\mathcal{D}_{\beta\{\dot\beta}\lambda_{\alpha})}\overline{\lambda}_{\dot\alpha\}}+G_{\alpha\beta}\overline{G}_{\dot\alpha\dot\beta}]. \tag{63}$$

The symmetrization over α,β or $\dot\alpha,\dot\beta$ is marked by the braces. In fact, since the chiral current is classically conserved (so far we disregard anomalies), symmetrization in the third line is superfluous: the corresponding expressions are automatically symmetric. To obtain the expression on the right-hand side from $\mathrm{Tr}[e^V W_\alpha e^{-V}\overline{W}_{\dot\alpha}]$ we observe that the expression for $J_{\alpha\dot\alpha\beta\dot\beta}$ has mixed symmetry: the part symmetric in $\{\alpha,\beta\}$ *and* $\{\dot\alpha,\dot\beta\}$ is the (1,1) Lorentz tensor; it represents the (traceless) energy-momentum tensor. The remainder, i.e. the part symmetric in $\{\alpha,\beta\}$ and antisymmetric in $\{\dot\alpha,\dot\beta\}$ or *vice versa*, is the (0,1)+(1,0) Lorentz tensor. The part antisymmetric in both $\{\alpha,\beta\}$ *and* $\{\dot\alpha,\dot\beta\}$ is (0,0). It represents the traces which vanish in the classical approximation. It is quite obvious that the only part relevant for the central charge is (0,1)+(1,0) piece in $J_{\alpha\dot\alpha\beta\dot\beta}$. This means, in particular, that the inclusion of the traces will have no impact on the central charge.

It is easy to see that

$$\{Q_\gamma,\overline{J}_{\dot\beta\dot\alpha\alpha}\}=2J_{\alpha\dot\alpha\gamma\dot\beta}. \tag{64}$$

Combining this equation with the third line in Eq. (63) we conclude that the centrally extended algebra is given by Eq. (1) with

$$a^\nu=\frac{1}{4}R^\nu. \tag{65}$$

Unlike the central extension relevant for the domain walls, which appears [2–4] as a quantum anomaly, in the problem at hand the algebra gets a full-derivative term at the tree level. The presence of the anomaly manifests itself through the fact that the energy-momentum tensor ceases to be traceless, and $\partial_\nu R^\nu$ no longer vanishes. On general grounds it is clear, however, that Eqs. (1), (65) stay intact.

The occurrence of a full-derivative term in the algebra presents a precondition for a nontrivial central extension. Whether or not this term actually vanishes is a dynamical issue which depends on the presence of the string-like solitons. These may be strings or domain-wall junctions, as in Ref. [12]. SUSY gluodynamics is a strongly coupled theory; therefore, one cannot use quasiclassical considerations to search and analyze solitons. The hope is that there is a dual description in terms of effective degrees of freedom, for which quasiclassical analysis may be relevant. Within this dual description the second term in Eq. (1) is mapped onto some relevant operator of the effective theory. It is clear that the second term in Eq. (1) is the necessary but not sufficient condition for the existence of the saturated strings. If it were absent, there would be no hope.

VII. GENERIC NON-ABELIAN MODEL WITH MATTER

The (1/2,1/2) central charge in the generic non-Abelian theory is obtained by combining the expressions we have derived in the previous sections. The operator a_μ in Eq. (1) receives contributions from the gluino term, as in Sec. VI, which is unambiguous, and the contributions from matter (both the scalar and spinor components of matter enter), as in the generalized Wess-Zumino model (Sec. III), whose coefficients are not fixed—they depend on how one defines the supercurrent in those terms that are total derivatives. This ambiguity derives its origin from that in the definition of the supercurrents,

$$J_{\alpha\beta\dot\beta}=\frac{2}{g^2}(iG^a_{\beta\alpha}\overline{\lambda}^a_{\dot\beta}+\epsilon_{\beta\alpha}D^a\overline{\lambda}^a_{\dot\beta})+2\sqrt{2}\sum[(\mathcal{D}_{\alpha\dot\beta}\phi^\dagger)\psi_\beta$$
$$-i\epsilon_{\beta\alpha}F\overline{\psi}_{\dot\beta}]$$
$$-\frac{\sqrt{2}}{3}\sum[\partial_{\alpha\dot\beta}(\psi_\beta\phi^\dagger)+\partial_{\beta\dot\beta}(\psi_\alpha\phi^\dagger)$$
$$-3\epsilon_{\beta\alpha}\partial^\gamma_{\dot\beta}(\psi_\gamma\phi^\dagger)], \tag{66}$$

where the sum runs over all matter supermultiplets, and D^a and F are the corresponding D and F terms. The second line is conserved by itself, nondynamically; the spatial integral of the time-like component reduces to an integral over the total derivative for the second line. Therefore, it may or may not be included in the definition of the supercurrents. This is the supersymmetric analogue of the ambiguity in the energy-momentum tensor in nonsupersymmetric theories with the scalar fields. The ambiguity in the choice of $J_{\alpha\beta\dot\beta}$ leads, with necessity, to the fact that the coefficients of the matter terms in a^μ in Eq. (1), namely, $a^\mu_{(\psi)}$ and $a^\mu_{(\phi)}$, are not uniquely fixed.

Because of this ambiguity, the matter component of a^μ cannot contribute to Z for strings (it could contribute, though, for the wall junctions and other similar object with axial geometry).

VIII. STRINGS CANNOT BE SATURATED IN $\mathcal{N}=1$ NON-ABELIAN GAUGE THEORIES IN WEAK COUPLING

Here we will prove that in the absence of the U(1) factors, even if the theory under consideration does support string-like solitons in the quasiclassical consideration (some examples are discussed e.g., in Ref. [19]), the central charge necessarily vanishes. Therefore, these strings *cannot be saturated.*

In weak coupling (i.e., for the string solitons in the quasiclassical treatment) the (1/2,1/2) central charge must be saturated by the term with the *bosonic* axial current. (We recall that the FI term is absent.) As was explained, the coefficient of this term is not unambiguous—it depends on the definition of the supercurrent (e.g., minimal *versus* conformal). Since we are interested in string solitons, rather than wall junctions, this ambiguity cannot be canceled by that in the (1,0) central charge, since the latter must identically vanish. This is dictated by Lorentz symmetry arguments. This means that the (1/2,1/2) central charge must vanish identically.

The consideration above shows that if BPS objects with axial geometry exist in the quasiclassical limit (in non-Abelian gauge theories), the stringy core must be accompanied by objects with (1,0) charges. In four dimensions domain walls do the job. Within the brane picture it is possible to consider four-dimensional theories as that on the brane embedded in M theory. For instance, the expected domain wall junctions in $\mathcal{N}=1$ Yang-Mills theory—the gauge analogue of the junctions in the GWZ models—can be identified as a junction of M5 branes, so that the definition of the current for the theory on M5 removes any ambiguity.

IX. STRINGS IN THE SEIBERG-WITTEN $\mathcal{N}=2$ MODEL

Here we will speculate on possible BPS strings at strong coupling. As we already know, such strings do not appear in weak coupling. The (1/2,1/2) central charge (appearing in the anticommutator $\{Q,\overline{Q}\}$) is not holomorphic—it need not depend holomorphically on the chiral parameters, in contradistinction with the (1,0) charge. This means that even if both the weak and strong coupling regimes are attainable in one and the same theory, generally speaking, nothing can be said regarding BPS strings in the strong coupling regime from the behavior at weak coupling.

A. Strings in pure $\mathcal{N}=2$ Yang-Mills theory

We turn now to a discussion of the $\mathcal{N}=2$ Yang-Mills theory without matter hypermultiplets. The exact solution for the low-energy effective action, as well as the exact spectrum of the BPS particles, is known [20]. Now we address the issue of possible stringy central charges, besides the standard ones, saturated by particles [21]. From the discussion above we saw that it can be attributed only to the gluino axial current since there is no FI term in the model. Let us restrict ourselves to the SU(2) gauge group.

The key features of the Seiberg-Witten solution can be summarized as follows. The vacuum manifold develops the Coulomb branch which is parametrized by the global coordinate, the order parameter $u=\langle \mathrm{tr}\phi^2\rangle$. At low energies the effective theory becomes Abelian and is described by a single holomorphic function—the prepotential $\mathcal{F}$ which determines the effective coupling constant of the theory $\tau=\partial^2\mathcal{F}/\partial a^2$, as well as the Kähler metric on the Coulomb branch of the moduli space, which appears to be a one-dimensional special Kähler manifold. The Kähler potential can be found from the prepotential as follows:

$$K(a,\overline{a})=\mathrm{Im}\, a_D\overline{a}, \tag{67}$$

where a is the vacuum value of the third component of the scalar field and $a_D=\partial\mathcal{F}/\partial a$. The variable a can be expressed in terms of variable u as follows:

$$a(u)=\int_{-\sqrt{u+\Lambda^2}}^{-\sqrt{u-\Lambda^2}}\frac{x^2dx}{\pi\sqrt{(x^2-u)^2-\Lambda^4}}. \tag{68}$$

Unlike the variable u, the variable a cannot be considered as a global coordinate on the moduli space since the Kähler metric $\mathrm{Im}\tau(a)$ has zeros (here τ is the complexified coupling constant). Therefore, to analyze the complex plane of a, an explicit expression for $a(u)$ is needed. Direct inspection shows that the region of small a is essentially removed from the complex plane so that $|a(u)|>\mathrm{const}\times\Lambda$.

The lower bound on a can be seen also geometrically if we recall that it is just the mass of the W boson, which can be represented in the theory on D3 probes as a pronged string connecting the probe and a split O(7) orientifold [22]. It is clear that the minimal mass of the W boson geometrically is the distance between the 7-branes on the u plane; it is, thus, proportional to Λ. Therefore, we see that π_1 of the scalar field manifold is nontrivial—topologically stable objects with the axial geometry are expected, provided a winds around the ''forbidden'' region.

Whether these objects are strings (i.e., have finite energy per unit length) depends on dynamics, on how fast the volume energy density dies off as we go away from the axis in the perpendicular direction. The convergence could be ensured by an appropriate form of the Kähler metric, as in the sigma models. It is quite obvious that in this case the string tension

$$T=\mathrm{const}\times\Lambda^2. \tag{69}$$

The existence of such stable objects with axial geometry would be a purely strong coupling effect since at the classical level the point $a=0$ is attainable, and, correspondingly, π_1 is trivial. If the strings do exist, they may be BPS saturated provided the term due to the gluino current R^μ in the central charge is nonvanishing. To this end the gluino current must fall off at large distances r from the axis as $1/r$. Finiteness of the string tension would imply then that effective degrees of freedom coupled to R^μ form a U(1) gauge interaction. If the string tension is finite and the gluino current falls off at large distances r from the axis faster than $1/r$, the string is tensionless.

B. Strings in $\mathcal{N}=2$ SQCD

Adding the matter hypermultiplets to the model discussed in Sec. IX A we get $\mathcal{N}=2$ SQCD. Since there is no restoration of the SU(2) gauge symmetry at the generic point at the Coulomb branch, the ''forbidden'' region on the complex a plane exists in the theory with fundamental matter too. The BPS strings may appear on the Coulomb branch, with the tension saturated by the R current of gluinos. The tension now depends on the masses of the fundamental matter and can be determined, in principle, from the explicit expression for $a(u,\Lambda,m)$.

Moreover, the Higgs branch (parametrized by the vacuum expectation values of the fundamentals $\langle Q\rangle,\langle\tilde{Q}\rangle$) is possible, and the question of the BPS strings on the Higgs branch can be addressed. We recall that geometrically the Higgs branch is the hyper-Kähler manifold [23] (for a review see [24]) whose metric can be determined classically. It is not renormalized by quantum corrections. Actually, the Higgs branch for SU(N_c) theory with N_f flavors is the cotangent bundle of the Grassmannian $T^*\mathrm{Gr}_{N_c,N_f}$, with the antisymmetric N_c-form. The metric on this manifold can be found from the Kähler potential

$$K(Q,\tilde{Q})=\mathrm{Tr}\sqrt{k^2+MM^\dagger}, \tag{70}$$

where k is a solution of the equation

$$\det(k1_{N_f}+\sqrt{k^2 1_{N_f}+MM^\dagger})=\det(QQ^\dagger), \tag{71}$$

and $M=Q\tilde{Q}$ is the meson matrix.

Since $\pi_2(\mathrm{Gr}_{n,k})\neq 0$, instantons in the two-dimensional sigma model on $T^*\mathrm{Gr}_{n,k}$ are possible. The arguments presented in Sec. V suggest that these instantons can be interpreted as strings on the Higgs branch. It would be interesting to understand whether a version of the string on the Higgs branch recently found in [25] can be BPS saturated.

The existence of the BPS string on the Higgs branch was recently conjectured within the brane approach [26]. This string was expected to be tensionless at the root of the Higgs branch, which qualitatively agrees with the discussion above.

C. Softly broken $\mathcal{N}=2$ theory (strings in $\mathcal{N}=2$ SQED)

If the softly broken $\mathcal{N}=2$ Yang-Mills theory is considered near the monopole or dyon singularities, the effective low-energy theory which ensues is $\mathcal{N}=2$ dual SQED. This is the famous Seiberg-Witten result. A small mass term of the chiral superfields of the original $\mathcal{N}=2$ non-Abelian theory is translated in a small perturbation of the superpotential for the matter fields in SQED. If the monopole (or dyon) superfields are denoted as M, $\tilde{M}$, the superpotential in the low-energy SQED can be written as

$$\mathcal{W}=\mu u(a_D)+\tilde{M}a_D M, \tag{72}$$

where a_D is a chiral superfield which is the $\mathcal{N}=2$ superpartner of the (dual) vector superfield. The second term in Eq. (72) is fixed by $\mathcal{N}=2$ supersymmetry. The parameter μ in the first term is small. Generically $\mu u(a_D)$ breaks $\mathcal{N}=2$ supersymmetry down to $\mathcal{N}=1$. However, in the linear approximation, when

$$\mathcal{W}=\mu a_D\Lambda+\tilde{M}a_D M, \tag{73}$$

$\mathcal{N}=2$ is unbroken.

Let us forget about the origin of $\mathcal{N}=2$ SQED and discuss this U(1) theory with the superpotential (73) *per se*. Minimization of the potential stemming from Eq. (73) yields the monopole condensation. Abrikosov strings obviously do exist. Their tension is proportional to μ. They were discussed in the literature previously [27,28]. The classical equations for the string reduce to Eq. (43).[4] Thus, the string is saturated. The question is how this could happen given that the (1/2,1/2) central charge must vanish in the absence of the FI term.

The central charge in the anticommutator $\{Q_\alpha\bar{Q}_{\dot\alpha}\}$ is indeed zero. One should not forget however, that SQED with the superpotential (73) is an $\mathcal{N}=2$ theory—there exist two supercharges Q,Q' of the type (1/2,0) and two supercharges $\bar{Q},\bar{Q}'$ of the type (0,1/2). Therefore, one should look for the central extension in the anticommutator of the general form $\{\mathcal{Q}_\alpha\bar{\mathcal{Q}}_{\dot\alpha}\}$ where $\mathcal{Q}$ is a linear combination of Q and Q'. A nonvanishing central term of this type does exist.

If we now return to the original non-Abelian $\mathcal{N}=2$ theory, we conclude that at small μ the string is (approximately) BPS saturated. It becomes exactly saturated in the limit $\mu\to 0$, $\Lambda\to\infty$ with $\mu\Lambda$ fixed [27,28]. The saturation is approximate, rather than exact, since higher order terms in μ [non-linear in a_D terms in the superpotential of the low-energy U(1) theory] break $\mathcal{N}=2$ and return us back to the $\mathcal{N}=1$ theory. In $\mathcal{N}=1$ the extra supercharges Q', $\bar{Q}'$ disappear, while the central charge in the anticommutator $\{Q_\alpha\bar{Q}_{\dot\alpha}\}$ vanishes.

X. BRANE PICTURE: HOW IT CORRESPONDS TO FIELD THEORY

A. Fayet-Iliopoulos string as a membrane

With the brane picture in mind, we can look for the brane configuration corresponding to the BPS strings discussed above. The interpretation of the strings whose tension is proportional to the four-dimensional FI terms is rather simple. Let us consider the brane configuration relevant for the Abelian $\mathcal{N}=2$ Yang-Mills theory in the type IIA picture. It consists of a pair of parallel NS5 branes with world volumes $(x^0,x^1,x^2,x^3,x^4,x^5)$, plus a single D4 brane with world volume (x^0,x^1,x^2,x^3,x^6). The gauge theory is defined on the world volume of the D4 brane, and the distance between the NS5 branes along the x^6 direction plays the role of the inverse coupling in the Abelian theory. Since the four-dimensional FI terms have the meaning of the relative distance between the NS branes in (x^7,x^8,x^9) [29], the ''FI

[4]It should be taken into account that on the solution $|M|=|\tilde{M}|$.

strings'' are nothing but the D2 branes stretched between the NS branes in some of the (x^7,x^8,x^9) directions. The rest of their world volume coordinates coincide with the D4 ones.

This picture gets slightly modified if one considers the Abelian $\mathcal{N}=1$ theory. According to the well-known procedure (see, for instance, [29]), one has then to rotate one of the NS5 branes, which now has $(x^0,x^1,x^2,x^3,x^8,x^9)$ as the world volume. The Fayet-Iliopoulos term now has the meaning of a displacement of the NS5 branes along x^7. The D2 brane stretched between the NS5 branes with world volume (x^0,x^1,x^7) plays the role of the BPS string.

Let us note that the FI string can be elevated smoothly in M theory. Indeed, the NS5 branes and the D4 brane can be identified with the single M5 brane in M theory. The FI string can be considered as an M2 brane stretched between two components of the M5 branes. The tension of the FI string is proportional to the length of the M2 brane along the x^7 direction and, therefore, proportional to the value of the FI parameter ξ, in full agreement with the field theory expectations. Recently a similar picture for the FI strings was discussed in [26].

B. (Conjectured) strong coupling BPS strings via branes

The BPS saturated objects with axial geometry were discussed in the brane picture previously. For instance, the domain wall junction in $\mathcal{N}=1$ supersymmetric gluodynamics which is expected to saturate both the (1/2,1/2) and (1,0) central charges occurs as the intersection of the M5 branes, since the domain walls were identified as M5 branes wrapped on 4-manifold in M theory [30].

Here we would like to add a few remarks on a possible interpretation of strong coupling BPS strings in the brane picture. Previous attempts to recognize BPS tensionless strings in four dimensions, apparently seen within the brane approach [31], were based on the intersection of the M5 branes or the M2 brane stretched between two M5 branes. The BPS string on the Coulomb branch discussed in Sec. IX is nothing but a wrapped M5 brane, since its tension is proportional to the area on the region on the Coulomb branch. However, the explicit geometry of intersection of the M5 branes yielding saturation of the (1/2,1/2) central charge is still to be clarified.

Another possible approach to the brane interpretation of BPS strings in Yang-Mills theories follows from the correspondence between Yang-Mills theories in four dimensions and two-dimensional sigma models. It was recently recognized [32–34] that there is a close relation between two-dimensional CP_N models (which have $\mathcal{N}=2$) and Yang-Mills theories in four dimensions, with $\mathcal{N}=1$ or $\mathcal{N}=2$, with or without fundamental matter. In the latter case the correspondence relies on the coincidence of the spectra of the BPS domain walls in four dimensions and BPS solitons in two.

A more direct relation connects the $\mathcal{N}=2$ theory with N_f flavors at the root of the baryonic branch of the moduli space with the $CP_{2N_c-N_f-1}$ model [33,34]. The translation dictionary between the two models looks as follows: the complex coupling in four dimensions corresponds to a complex parameter combining the two-dimensional FI term with the θ term; twisted mass terms in $d=2$ correspond to the coordinates on the Coulomb branch in the $d=4$ theory; finally, the Riemann surfaces providing the BPS spectra in both theories are the same.

The correspondence above has a rather simple explanation in the brane description of both theories. It appears that the brane configurations for both theories are actually the same. The $d=4$ theory is defined on the world volume of the D4 branes stretched between a pair of the NS5 branes. The coupling constant is just the distance between the NS5 branes. In M theory all branes above are elevated to a pair of M5 branes, one of which is flat and the second is wrapped around the Riemann surface. The configuration is described by the holomorphic embedding into four-dimensional space,

$$(t-\Lambda^N)\left\{t\Lambda^{N-N_f}\prod^{N-N_f}(v-\tilde{m}_i)-\prod^{N}(v-m_i)\right\}=0, \tag{74}$$

where the first factor represents the flat brane, while the second the curved one, and m_i's correspond to the masses of the fundamental hypermultiplets.

Let us add a D2 brane and consider the Abelian gauge theory on its world volume. If the D2 brane is stretched between the same NS5 branes, we arrive at the CP_N model in $d=2$ where it has the extended supersymmetry, $\mathcal{N}=2$. This explains the coincidence between the complexified coupling constant in the four-dimensional theory and the FI term in the two-dimensional theory. Therefore, the picture can be apparently interpreted as follows: the $d=2$ sigma model, with the twisted masses added, is the theory on the brane which is the probe for the $\mathcal{N}=2$ low-energy theory in four dimensions.

In [33,34] it was shown that the spectrum of BPS particles in $\mathcal{N}=2$ theory at the root of the baryonic branch exactly coincides with the spectrum of BPS dyonic kinks in the corresponding CP_N model. Moreover, the brane identification shows that hypermultiplets in $d=4$ and $d=2$ arise essentially in the same way. Therefore, we can use the relation between the models in a twofold way. The existence of instantons in the CP_N model implies that one can expect BPS saturated strings at the root of the baryonic branch. In the opposite direction, the (1/2,1/2) central charge in four dimensions can be mapped onto the central charge of the $\mathcal{N}=2$ two-dimensional theory. Since in the formulation of the sigma model with nondynamical vector field the gauge 2-potential plays the role of the current a^μ in Eq. (1), the central charge is actually mapped onto the Chern number $\int A dx$. Certainly, these issues need further clarification. We hope to discuss them elsewhere.

XI. CONCLUSIONS AND DISCUSSION

In this paper we elaborated on the generic structure of the central charges in supersymmetric gauge theories in four dimensions. The central finding is that the (1/2,1/2) charge is ambiguous in the part related to the matter fields, due to possible total derivative terms in the supercurrents. The part

related to the gauge fields (including gaugino) is unambiguous. That is why in the weak coupling regime the only model admitting BPS strings is SQED with the Fayet-Iliopoulos term. In the non-Abelian theories the Fayet-Iliopoulos term is forbidden; hence, at weak coupling there can be no BPS strings. Even if some strings exist, they are nonsaturated by necessity. These assertions are proved at the theorem level.

The ambiguity we found does not preclude from existence other BPS saturated objects with axial geometry, i.e., the wall junctions. The ambiguity in the (1/2,1/2) charge is combined with that in the (1,0) charge to produce a well-defined answer for the tension of the walls and the ''hub'' in the middle. We presented some examples.

The strong coupling regime is a different story. Since the analyticity argument does not apply to the (1/2,1/2) charge, the existence or non-existence of the BPS strings should be discussed separately at weak and strong couplings—the lessons we learn at weak coupling say nothing about possible scenarios at strong coupling. We speculated on different cases when the BPS-saturated objects with axial geometry may appear in the strong coupling regime. We argued that saturation of the (1/2,1/2) charge at strong coupling can be attributed to the M5 brane intersection. (The Fayet-Iliopoulos BPS strings come from the M2 branes.) The BPS-saturated strings may be expected in the $\mathcal{N}=2$ Yang-Mills theories on the Coulomb branch.

In the $\mathcal{N}=1$ gauge theories an obvious candidate for the BPS saturation is the domain wall junction. One cannot assert at the moment with absolute certainty that the strong coupling junction exists in $\mathcal{N}=1$ supersymmetric gluodynamics, but the *M* theory arguments suggest that such junctions do exist. Additional support in favor of this conclusion is provided by field-theoretic models considered in [15].

A comment is in order regarding the situation in supergravity coupled to Yang-Mills theory. Upon inspecting the (1/2, 1/2) central charge one finds the term $H=dB-K$ in the anticommutator $\{Q,\overline{Q}\}$, where B is the two-form field and $K=AdA-\frac{2}{3}A^3$ is the dual of the Chern-Simons current in the Yang-Mills theory. Therefore, we see that the axial current of gluons enters into the central charge, if gravity degrees of freedom are taken into account. We plan to discuss this point in more detail elsewhere.

Since the $\mathcal{N}=2$ Yang-Mills theory enjoys duality, one can pose a question of the duality partner of the BPS string. Four-dimensional BPS strings can be viewed as objects dual (in the Dirac sense) to localized objects. Indeed, since in d space-time dimensions the p brane is dual to a $(d-p-4)$ brane, the Dirac quantization condition amounts to the observation that the $(d-p-4)$ brane is weakly coupled if the p brane is strongly coupled and *vice versa*. Therefore, one can expect that within the framework of duality the strongly coupled BPS string has something to do with the instantons at weak coupling.

To make a conjecture regarding the central charge dualizing the stringy one, let us observe that there is a contribution in the central charge for $\{Q,\overline{Q}\}$ in six dimensions, saturated by instantonic strings. In five dimensions the instanton presents a particle with mass $1/g^2$, saturating [35] the central charge $\int d^4x\tilde{F}F$. In four dimensions we can expect a remnant of this central charge resulting from dimensional reduction.

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