String theoretic axion coupling and the evolution of cosmic structures

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We examine the effects of the axion coupling to $R\tilde{R}$ on the evolution of cosmic structures. It is shown that the evolutions of the scalar- and vector-type perturbations are not affected by this axion coupling. However, the axion coupling causes an asymmetric evolution of the two polarization states of the tensor-type perturbation, which may lead to a sizable polarization asymmetry in the cosmological gravitational wave if inflation involves a period in which the axion coupling is important. The polarization asymmetry produced during inflation is conserved over the subsequent evolution as long as the scales remain in the large-scale limit, and thus this may lead to an observable trace in the cosmic microwave background radiation.

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String or M theory has received much attention as the best candidate for the unified theory of all fundamental forces [1]. In view of the fact that its typical energy scale is too high to be probed by laboratory experiments,[1] cosmology can be one of the best testing grounds for string or M theory. In this regard, it is important to search for a realistic inflation model within the framework of string or M theory. The next step would be to find out whether this inflation model allows successful structure formation. As is well known, due to inflation microscopic quantum fluctuations can be magnified to macroscopic classical structures which can evolve into the large-scale structures we observe today, such as the galaxy distribution and the temperature fluctuations in the cosmic microwave background radiation (CMBR). According to this paradigm, the amplitude and spectrum of the large angular scale fluctuations in the CMBR can work as a window to the early universe, and particularly to the inflation era. Such studies in the context of low-energy effective action of string theories have been done before [3,4], some of which include the effects of higher-order corrections in the expansion in either the inverse string tension or the string coupling [5]. We may mention, however, that the pre-big-bang scenario [3], which is an inflation mechanism based on the low-energy effective action of the string theory, in fact, has several difficulties: difficulties in the smooth connection between the contracting and expanding phases [6], in getting a large smooth universe from a small inhomogeneous one [7], in the fine-tuning problem [8], and in making the large-scale structures [9]. Further studies concerning the role of higher derivative terms in string cosmology can be found in [10].

One of the few model-independent predictions of string or M theory is the existence of axions which couple to $F\tilde{F} = \eta^{abcd}F_{ab}F_{cd}$ and/or to $R\tilde{R} \equiv \eta^{abcd}R_{ab}{}^{ef}R_{cdef}$ [11,12], where η^{abcd} is a (totally antisymmetric) Levi-Civita tensor density. It is rather easy to see that for the spatially homogeneous and isotropic background the axion coupling to $F\tilde{F}$ affects *neither* the evolution of the background *nor* the evolution of perturbations in the linear approximation [13]. The axion coupling to $R\tilde{R}$ does not affect the evolution of background also, however, it can affect the evolution of perturbations in the early universe [14].

Compared to the Ricci scalar curvature term, $R\tilde{R}$ is higher order in the dimensionful gravitational coupling $\kappa^2 = 8\pi G_N$. Although suppressed by κ^2, it may lead to sizable effects in some string inflation models (pre-big-bang as an example) which encounter a curvature singularity when the cosmological evolution is governed by the lowest order effective action [3]. In such inflation models, it is expected [6,4] that higher dimensional terms of the curvature tensor regulate the curvature singularity to the period of a large but finite curvature $\sim 1/\kappa^2$ which would smoothly evolve into the standard radiation-dominated flat universe. Clearly in the high curvature period, higher dimensional terms of curvature tensor such as the axion coupling to $R\tilde{R}$ and/or the moduli coupling to the Gauss-Bonnet combination [5] can be as important as the Ricci scalar term [15–18].

We may emphasize that the pre-big-bang scenario is not the only (or major) framework in which $R\tilde{R}$ can be relevant for the structure formation. The axion (a) coupling to $R\tilde{R}$ can provide a significant P-violating environment to the gravity sector only when $(da/dt)/M_P^2$ is not so small. Such a large axion velocity can be compatible with inflation only when the inflation takes place near the Planckian time. This is hard to be realized in the conventional inflation model in which the inflation takes place far after the Planckian time, and this is the main reason why we mention the pre-big-bang scenario as an example despite its many difficulties mentioned above.

[1]Recently, it has been noted that some string theories allow the Kaluza-Klein scale or the string scale to be far below the Planck scale, even as low as TeV [2].

Recently, authors of [14] considered the effect of the axion-coupled $R\tilde{R}$ term on gravitational waves. They derived the gravitational wave equation in the Minkowsky background and showed that the $R\tilde{R}$ coupling term leads to asymmetric equations for the left- and right-handed polarization states of graviational waves.[2] In this paper, we wish to generalize the study of [14] by discussing the effects of the axion coupling on the evolution of cosmological structures of all types in the context of evolving cosmological background. It turns out that the evolutions of scalar- and vector-type structures are not affected by the axion coupling. However, as noted in [14] it causes an asymmetric evolution of the two polarization states of tensor-type perturbation. This may lead to a sizable polarization asymmetry in tensor-type perturbation if inflation involves a period during which the axion coupling is important, such as the high curvature period in the pre-big-bang model. Moreover, in the large-scale limit, the amplitudes of both polarization states are separately conserved, as can be seen in Eq. (18). Thus once the polarization asymmetry in tensor-type perturbation was generated during inflation, it will be preserved over the subsequent evolution stage as long as the scale remains in the large-scale limit. This then may lead to an observable trace in the large angular scale polarization asymmetry in the CMBR [14].

Our starting point is an effective action including the axion coupling to $R\tilde{R}$, which may correspond to the low-energy effective action of string or M theory [1]:

$$S=\int d^4x\sqrt{-g}\bigg[\frac{1}{2\kappa^2}R-\frac{1}{2}\omega(\phi)\phi^{;c}\phi_{,c}-V(\phi)+\frac{1}{8}\nu(\phi)R\tilde{R}+L_m\bigg], \tag{1}$$

where $\kappa^2=8\pi G_N$ is the reduced Newton constant, R is the scalar curvature, $V(\phi)$, $\omega(\phi)$, and $\nu(\phi)$ are some generic functions[3] of the axion field ϕ, $R\tilde{R}\equiv\eta^{abcd}R_{ab}{}^{ef}R_{cdef}$, and L_m is the Lagrangian including the fields other than the axion field ϕ. Here we are interested in how the evolution of the axion background affects the evolution of cosmic structures through the coupling to $R\tilde{R}$, and thus other axion couplings, e.g., the axion coupling to moduli, are ignored for the sake of simplicity. Generically, string or M theory predicts many axions [11,12], and then ϕ may be identified as the axion combination which couples to $R\tilde{R}$. The gravitational field equation is given by

$$R_{ab}-\frac{1}{2}Rg_{ab}=\kappa^2\bigg[\omega(\phi)\bigg(\phi_{,a}\phi_{,b}-\frac{1}{2}\phi^{;c}\phi_{,c}g_{ab}\bigg)-V(\phi)g_{ab}+\tilde{T}_{ab}+T^{(m)}_{ab}\bigg],$$

$$\tilde{T}_{ab}\equiv\eta_{(a}{}^{cde}(R^f{}_{b)cd}\nu_{,e;f}-2R_{b)c;d}\nu_{,e}), \tag{2}$$

where $X_{,a}$ and $X_{;a}$ denote the normal and covariant derivatives of X, respectively, $X_{(ab)}\equiv\frac{1}{2}(X_{ab}+X_{ba})$, and $T^{(m)}_{ab}$ is the additional energy-momentum tensor arising from L_m which would include the contribution from cosmic fluid.

As the spacetime metric, we consider a spatially flat, homogeneous, and an isotropic background including the most general form of space-time dependent perturbations:

$$ds^2=-a^2(1+2\alpha)d\eta^2-a^2(\beta_{,\alpha}+B_\alpha)d\eta dx^\alpha+a^2[\delta_{\alpha\beta}(1+2\varphi)+2\gamma_{,\alpha\beta}+2C_{\alpha,\beta}+2C_{\alpha\beta}]dx^\alpha dx^\beta, \tag{3}$$

where $a(t)$ is the cosmic scale factor with $dt\equiv ad\eta$. Here $\alpha(\mathbf{x},t)$, $\beta(\mathbf{x},t)$, $\varphi(\mathbf{x},t)$, and $\gamma(\mathbf{x},t)$ characterize the scalar-type perturbations, $B_\alpha(\mathbf{x},t)$ and $C_\alpha(\mathbf{x},t)$ are the transverse vector-type perturbation, and finally $C_{\alpha\beta}(\mathbf{x},t)$ stands for the tracefree, transverse tensor-type perturbation. The spatial indices are based on the metric $\delta_{\alpha\beta}$. We also decompose the energy-momentum tensor and the axion field as

$$T^a_b(\mathbf{x},t)=\overline{T^a_b}(t)+\delta T^a_b(\mathbf{x},t),\quad \phi(\mathbf{x},t)=\overline{\phi}(t)+\delta\phi(\mathbf{x},t). \tag{4}$$

In the following, the overbar in the background configuration will be omitted unless necessary. Since the three types of perturbations decouple from each other due to the symmetry of the background and also the assumed linearity of the structures, we can handle them individually.

It is well known that $\nu(\phi)R\tilde{R}$ does not affect the equation for background [4]. After some calculation we can show that for the scalar-type perturbation we have $\tilde{T}_{ab}=0$, and as a result the axion coupling does not affect the evolution of the scalar-type perturbation as well. This can be explained by that we cannot form a scalar ($\tilde{T}_{00}$) or a vector ($\tilde{T}_{0\alpha}$) or a symmetric tensor ($\tilde{T}_{\alpha\beta}$) which contains η^{abcd} with the derivatives of the scalar and $\delta_{\alpha\beta}$ only. Thus the evolution of scalar-type perturbation is the same as that in the case without the axion coupling which has been studied in [18].

We have nontrivial contributions from the axion coupling $\nu(\phi)R\tilde{R}$ only in the equations for vector- and tensor-type perturbations. (For explicit calculations, the Appendices of [20,21] will be useful.) For the vector-type perturbation, Eq. (2) leads to

$$\frac{k^2}{2a^3}(a\Psi_\alpha-\kappa^2\dot{\nu}\epsilon_\alpha{}^{\gamma\delta}\Psi_{\gamma,\delta})=\kappa^2\delta T^{(m)0}_\alpha, \tag{5}$$

$$\frac{1}{a}\frac{\partial}{\partial t}[a(a\Psi_{(\alpha}-\kappa^2\dot{\nu}\Psi_{\gamma,\delta}\epsilon_{(\alpha}{}^{\gamma\delta})_{,\beta)}]=\kappa^2\delta T^{(m)}_{\alpha\beta}, \tag{6}$$

[2]The analogous effect for photons due to the optical activity of a spacetime varying Nambu-Goldstone axion field with $aF\tilde{F}$ coupling, i.e., rotation of the polarization plane of the light as it propagates through an axion domain wall, was studied in [19].

[3]Here we use the effective action written in the Einstein frame. The axion dependence of the Kähler metric ω and the potential V can arise from nonperturbative effects in string theory.

where we have introduced

$$\epsilon^{\alpha\beta\gamma} \equiv a^4 \overline{\eta}^{0\alpha\beta\gamma}, \quad \Psi_\alpha \equiv B_\alpha + a\dot{C}_\alpha \tag{7}$$

with the overdot denoting the derivative with respect to t. When expressed in terms of the following notation based on the vector-type harmonic function $Y^{(v)}_\alpha$ with $Y^{(v)}_{\alpha\beta} \equiv -k^{-1} Y^{(v)}_{(\alpha,\beta)}$ [21]:

$$\delta T^{(m)0}_\alpha \equiv (\mu + p) v_\omega Y^{(v)}_\alpha, \quad \delta T^{(m)}_{\alpha\beta} \equiv \pi^{(v)} Y^{(v)}_{\alpha\beta}. \tag{8}$$

Equations (5) and (6) lead to

$$\frac{1}{a^4}\frac{\partial}{\partial t}[a^4(\mu+p)\times v_\omega] = -\frac{k}{2a}\pi^{(v)}, \tag{9}$$

where v_ω is related to the rotational velocity of vector-type fluid perturbations and $\pi^{(v)}$ is the anisotropic stress of the fluid which can work as the sink or source of the rotation [21]. If we ignore the anisotropic stress, the angular momentum of the *fluid* whose energy momentum tensor is given by $T^{(m)}_{ab}$ is conserved as

$$a^4(\mu+p)\times v_\omega(\mathbf{x},t) \sim L(\mathbf{x}). \tag{10}$$

Therefore, the presence of the axion coupling does *not* affect the rotational-type perturbation of the fluid component which is described again by the angular momentum conservation, although the evolution of the associated metric components Ψ_α is affected by the axion coupling as is apparent in Eq. (5).

In fact, it is the tensor-type perturbation that may receive an observable impact from the axion coupling. Equation (2) gives the following equation for the evolution of the tensor-type perturbation:

$$D_{\alpha\beta} - \frac{2\kappa^2}{a}\epsilon_{(\alpha}{}^{\gamma\delta}[(\ddot{\nu} - H\dot{\nu})\dot{C}_{\beta)\gamma} + \dot{\nu}D_{\beta)\gamma}]_{,\delta} = \kappa^2 \delta T^{(m)}_{\alpha\beta}, \tag{11}$$

where

$$D_{\alpha\beta} \equiv \ddot{C}_{\alpha\beta} + 3H\dot{C}_{\alpha\beta} - \frac{1}{a^2}\Delta C_{\alpha\beta}. \tag{12}$$

(Here Δ denotes the spatial Laplacian.) A similar equation of the gravitational wave in the presence of axion coupling was derived in [14], but only for the Minkowski background. Let us expand the tensor perturbation [22] as

$$C_{\alpha\beta}(\mathbf{x},t) \equiv \sqrt{\mathrm{Vol}} \int \frac{d^3k}{(2\pi)^3} \sum_l e^{(l)}_{\alpha\beta}(\mathbf{k}) h_{l\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{13}$$

where $e^{(l)}_{\alpha\beta}$ is the circular polarization tensor ($l = L,R$) with the property $ik_\gamma \epsilon_\alpha{}^{\gamma\delta} e^{(l)}_{\beta\delta} = k\lambda_l e^{(l)}_{\alpha\beta}$ ($\lambda_L = -1$ and $\lambda_R = +1$) where $k \equiv |\mathbf{k}|$. Then, ignoring the anisotropic stress of the additional fluid, Eq. (11) becomes

$$\frac{1}{a^3(1+2\lambda_l\kappa^2\dot{\nu}k/a)}\frac{\partial}{\partial t}[a^3(1+2\lambda_l\kappa^2\dot{\nu}k/a)\dot{h}_{l\mathbf{k}}] + \frac{k^2}{a^2}h_{l\mathbf{k}} = 0. \tag{14}$$

If we set $\psi_{l\mathbf{k}} \equiv z_l h_{l\mathbf{k}}$ with $z_l \equiv a\sqrt{1+2\lambda_l\kappa^2\dot{\nu}k/a}$, we can easily show

$$\psi''_{l\mathbf{k}} + (k^2 - z''_l/z_l)\psi_{l\mathbf{k}} = 0, \tag{15}$$

where the prime denotes the derivative with respect to the conformal time η.

In many prototype inflation models, $z''_l/z_l = n_l/\eta^2$ with $n_l =$ constant provides a good approximation [22], and then the solution of Eq. (15) is given by

$$h_{l\mathbf{k}}(t) = \frac{\sqrt{|\eta|}}{a\sqrt{1+2\lambda_l\kappa^2\dot{\nu}k/a}}[\tilde{c}_{1l}(\mathbf{k})H^{(1)}_{\nu_l}(k|\eta|) + \tilde{c}_{2l}(\mathbf{k})H^{(2)}_{\nu_l}(k|\eta|)], \tag{16}$$

where $H^{(1)}_{\nu_l}$ and $H^{(2)}_{\nu_l}$ are Hankel functions of the first and second kinds with $\nu_l \equiv \sqrt{n_l + 1/4}$. Even for the generic form of z_l, we can derive *asymptotic* solutions of Eq. (15). In the small-scale limit where k^2 term dominates, we have a general solution given by

$$h_{l\mathbf{k}}(t) = \frac{1}{a\sqrt{1+2\lambda_l\kappa^2\dot{\nu}k/a}}[c_{1l}(\mathbf{k})e^{ik\eta} + c_{2l}(\mathbf{k})e^{-ik\eta}], \tag{17}$$

where c_{1l} and c_{2l} are integration constants of the left and right traveling waves. In the opposite limit of negligible k^2 term, which we call the large-scale limit, we have a general solution of the form

$$h_{l\mathbf{k}}(t) = C_l(\mathbf{k}) - D_l(\mathbf{k})\int_0^t \frac{dt}{a^3(1+2\lambda_l\kappa^2\dot{\nu}k/a)}, \tag{18}$$

where C_l and D_l are the coefficients of relatively growing and decaying modes, respectively. Ignoring the transient solution, the above solution manifestly shows that the amplitudes of both polarization states of the tensor-type perturbation are conserved in the large scale limit. Notice that the asymptotic solutions in Eqs. (17) and (18) are valid for generic forms of time varying $V(\phi)$, $\omega(\phi)$, and $\nu(\phi)$.

Remarkably, the conservation properties in Eqs. (10) and (18) are valid for generic forms of the axion potential $V(\phi)$, the axion Kähler metric $\omega(\phi)$, and also the axion coupling $\nu(\phi)$ to $R\tilde{R}$. The large-scale conservation property of tensor-type perturbation is particularly relevant in the inflationary scenario. During the transition period from inflation to ordinary radiation era, the observationally relevant scales stay in the superhorizon size for which the large-scale condition may apply. As long as the scale remains in the large-scale limit, the conservation property of the tensor-type perturbation is valid independently of how the axion field (and also other scalar fields which may have nonminimal coupling to gravity) is settled down to its vacuum expectation value during the transition period.

The inflation stage in the early universe can generate the stochastic gravitational wave by rapidly stretching the quantum vacuum fluctuations of the perturbed metric to the superhorizon scale. Although we need a specific inflation model to derive the generated power spectrum of the gravitational wave, in the following we will describe the process of deriving the power spectrum. In order to handle the quantum-mechanical generation of the gravitational wave, instead of the classical metric perturbation $C_{\alpha\beta}$ we consider the Hilbert space operator $\hat{C}_{\alpha\beta}$ and its mode function expansion defined in Eqs. (21),(23) of [22]:

$$\hat{h}_l(\mathbf{x},t) \equiv \frac{1}{2}\int \frac{d^3k}{(2\pi)^{3/2}} \hat{C}_{\alpha\beta}(\mathbf{x},t;\mathbf{k}) e^{(l)\alpha\beta}(\mathbf{k})$$

$$\equiv \int \frac{d^3k}{(2\pi)^{3/2}} [e^{i\mathbf{k}\cdot\mathbf{x}} \tilde{h}_{l\mathbf{k}}(t) \hat{a}_{l\mathbf{k}} + \text{H.c.}], \tag{19}$$

where the creation and annihilation operators of each polarization state follow $[\hat{a}_{l\mathbf{k}}, \hat{a}^\dagger_{l'\mathbf{k}'}] = \delta_{ll'}\delta^3(\mathbf{k}-\mathbf{k}')$, and zero otherwise. The gravitational wave part of the action in Eq. (1) expanded to second order in $\hat{h}_l$ can be derived as

$$\delta^2 S_g = \int a^3 \sum_l F_l \left(\dot{\hat{h}}_l^2 - \frac{1}{a^2} \hat{h}_l{}^{|\gamma} \hat{h}_{l,\gamma} \right) dt d^3x,$$

$$F_l \equiv \frac{1}{\kappa^2} \left(1 + 2\lambda_l \kappa^2 \frac{k}{a} \dot{\nu} \right). \tag{20}$$

The conjugate momenta are $\delta\hat{\pi}_{h_l}(\mathbf{x},t) = \partial\mathcal{L}/\partial\dot{\hat{h}}_l = 2a^3 F_l \dot{\hat{h}}_l$, and from the equal time commutation relation between $\hat{h}_l$ and $\delta\hat{\pi}_{h_l}$ we have

$$\tilde{h}_{l\mathbf{k}}(t)\dot{\tilde{h}}^*_{l\mathbf{k}}(t) - \tilde{h}^*_{l\mathbf{k}}(t)\dot{\tilde{h}}_{l\mathbf{k}}(t) = \frac{i}{2a^3 F_l}. \tag{21}$$

When we derive Eq. (16) we mentioned that many conventional inflation models satisfy n_l=const. In such a case the mode function has a solution

$$\tilde{h}_{l\mathbf{k}}(\eta) = \frac{\sqrt{\pi|\eta|}}{2a} [c_{l1}(\mathbf{k}) H^{(1)}_{\nu_l}(k|\eta|) + c_{l2}(\mathbf{k}) H^{(2)}_{\nu_l}(k|\eta|)] \sqrt{\frac{1}{2F_l}}, \tag{22}$$

where according to the normalization condition in Eq. (21) the coefficients $c_{l1}(\mathbf{k})$ and $c_{l2}(\mathbf{k})$ follow $|c_{l2}(\mathbf{k})|^2 - |c_{l1}(\mathbf{k})|^2 = 1$. The power spectrum of the Hilbert space gravitational wave operator based on the vacuum expectation value is introduced as

$$\mathcal{P}_{\hat{C}_{\alpha\beta}}(\mathbf{k},t) \equiv \frac{k^3}{2\pi^2} \int \langle \hat{C}_{\alpha\beta}(\mathbf{x}+\mathbf{r},t) \hat{C}^{\alpha\beta}(\mathbf{x},t) \rangle_{\text{vac}} e^{-i\mathbf{k}\cdot\mathbf{r}} d^3r, \tag{23}$$

where $\langle\rangle_{\text{vac}} \equiv \langle \text{vac}||\text{vac}\rangle$ is a vacuum expectation value with $\hat{a}_{l\mathbf{k}}|\text{vac}\rangle \equiv 0$ for all $\mathbf{k}$. We can show

$$\mathcal{P}_{\hat{C}_{\alpha\beta}}(\mathbf{k},t) = 2\sum_l \mathcal{P}_{\hat{h}_l}(\mathbf{k},t) = 2\sum_l \frac{k^3}{2\pi^2} |\tilde{h}_{l\mathbf{k}}(t)|^2, \tag{24}$$

which is the desired power spectrum (summed over each polarization state) expressed in a generic form. Applications to several known inflation models were made in [22].

Since it distinguishes the states with different polarization as in Eq. (14), the axion coupling generically leads to polarization asymmetry in tensor-type perturbations. In particular, if inflation involves a period in which $\dot{\nu}k/a \sim 1/\kappa^2$, the resulting polarization asymmetry can be sizable. Of course, this would not take place if the cosmic evolution during the whole inflation period is determined by the dynamics at energy scales significantly below the Planck scale $M_P \equiv 1/\sqrt{\kappa}$. However, this can be a possibility in the string cosmology scenario which encounters a curvature singularity during inflation at the lowest order approximation. In such inflation models, higher dimensional terms are expected to regulate the curvature singularity to a value comparable to $1/\kappa^2$, and then the axion coupling to $R\tilde{R}$ can give a non-negligible effect although it is suppressed by κ^2. The evolution of the tensor-type perturbation after inflation is mainly described by the large-scale solution (18) with conserved amplitudes. Thus once the polarization asymmetry was generated during inflation, it will be preserved over the subsequent evolution as long as the scale remains in the large-scale limit. This then may lead to an observable trace in the large angular scale polarization asymmetry in the CMBR [14]. For a more quantitative study of this problem, we will need a specific inflation model for which our results can be easily applied.

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[1] M. Green, J. Schwarz, and E. Witten, *Superstring Theory*, (Cambridge University Press, Cambridge, England, 1987), Vols. 1 and 2; J. Polchinski, *String Theory*, Vols. I and II (Cambridge University Press, Cambridge, England, 1998).

[2] I. Antoniadis, Phys. Lett. B **246**, 377 (1990); I. Antoniadis, C. Munoz, and M. Quiros, Nucl. Phys. **B397**, 515 (1993); J. D. Lykken, Phys. Rev. D **54**, 3693 (1996); N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, *ibid.* **436**, 257 (1998); C. P. Burgess, L. E. Ibanez, and F.

Quevedo, *ibid.* **447**, 257 (1999).
[3] G. Veneziano, Phys. Lett. B **265**, 287 (1991); R. Brustein, hep-th/9801008.
[4] I. Antoniadis, J. Rizos, and K. Tamvakis, Nucl. Phys. **B415**, 497 (1994); R. Brustein and R. Madden, J. High. Energy Phys. **07**, 006 (1999).
[5] C. G. Callan, D. Friedan, E. J. Martinec, and M. J. Perry, Nucl. Phys. **B262**, 593 (1985); B. Zwiebach, Phys. Lett. **156B**, 315 (1985); S. Deser and A. N. Redlich, Phys. Lett. B **176**, 350 (1986); D. J. Gross and J. H. Sloan, Nucl. Phys. **B291**, 41 (1987).
[6] N. Kaloper, R. Madden, and K. A. Olive, Phys. Lett. B **371**, 34 (1996); S.-J. Rey, Phys. Rev. Lett. **77**, 1929 (1996).
[7] N. Kaloper, A. Linde, and R. Bousso, Phys. Rev. D **59**, 043 508 (1999).
[8] M. S. Turner and E. J. Weinberg, Phys. Rev. D **56**, 4604 (1997).
[9] R. Brustein, M. Gasperini, M. Giovannini, V. Mukhanov, and G. Veneziano, Phys. Rev. D **51**, 6744 (1995); J. Hwang, Astropart. Phys. **8**, 201 (1998).
[10] M. C. Bento and O. Bertolami, Phys. Lett. B **368**, 198 (1996); J. Ellis, K. Kaloper, K. A. Olive, and J. Yokoyama, Phys. Rev. D **59**, 103 503 (1999); M. C. Bento, Phys. Lett. B **452**, 240 (1999).
[11] E. Witten, Phys. Lett. **149B**, 351 (1984); K. Choi and J. E. Kim, *ibid.* **154B**, 393 (1985); **165B**, 71 (1985).
[12] T. Banks and M. Dine, Nucl. Phys. **B479**, 173 (1996); K. Choi, Phys. Rev. D **56**, 6588 (1997).
[13] For cosmological implications of the axion coupling to $F\tilde{F}$, see, R. Brustein and D. H. Oaknin, Phys. Rev. Lett. **82**, 2628 (1999); Phys. Rev. D **60**, 023508 (1999).
[14] A. Lue, L. Wang, and M. Kamionkowski, Phys. Rev. Lett. **83**, 1506 (1999).
[15] R. Easther and K. Maeda, Phys. Rev. D **54**, 7252 (1996); P. Kanti, J. Rizos, and K. Tamvakis, *ibid.* **59**, 083 512 (1999).
[16] M. Gasperini, Phys. Rev. D **56**, 4815 (1997).
[17] S. Kawai, M. Sakagami, and J. Soda, Phys. Lett. B **437**, 284 (1998).
[18] J. Hwang, Phys. Rev. D **53**, 762 (1996); J. Hwang and H. Noh, *ibid.* **61**, 043511 (2000).
[19] P. Sikivie, Phys. Lett. **137B**, 353 (1984); M. C. Huang and P. Sikivie, Phys. Rev. D **32**, 1560 (1985); D. Harari and P. Sikivie, Phys. Lett. B **289**, 67 (1992).
[20] H. Noh and J. Hwang, Phys. Rev. D **55**, 5222 (1997).
[21] J. Hwang and H. Noh, Phys. Rev. D **57**, 2617 (1998).
[22] J. Hwang, Class. Quantum Grav. **15**, 1401 (1998).