Semiclassical charged black holes with a quantized massive scalar field

Brett E. Taylor* and William A. Hiscock†
Department of Physics, Montana State University, Bozeman, Montana 59717

Paul R. Anderson‡
Department of Physics, Wake Forest University, Winston-Salem, North Carolina 27109
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Semiclassical perturbations to the Reissner-Nordström metric caused by the presence of a quantized massive scalar field with arbitrary curvature coupling are found to first order in $\epsilon = \hbar/M^2$. The DeWitt-Schwinger approximation is used to determine the vacuum stress-energy tensor of the massive scalar field. When the semiclassical perturbation are taken into account, we find extreme black holes will have a charge-to-mass ratio that exceeds unity, as measured at infinity. The effects of the perturbations on the black hole temperature (surface gravity) are studied in detail, with particular emphasis on near extreme ''bare'' states that might become precisely zero temperature ''dressed'' semiclassical black hole states. We find that for minimally or conformally coupled scalar fields there are *no* zero temperature solutions among the perturbed black holes.

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I. INTRODUCTION

The back reaction of quantized fields on the spacetime geometry of a black hole can have very significant and important implications. For example back reaction from the particle production that occurs in the Hawking effect [1] causes the black hole to gradually become smaller in size. As it does so its temperature becomes higher and its entropy lower. If a black hole is placed in thermal equilibrium with radiation in a cavity, the back reaction of quantized fields can again alter its temperature and entropy [2–4].

Because of the difficulty in computing the stress-energy tensor for quantized fields in black hole spacetimes, various approximations have been used for all back-reaction calculations that have been done so far. One extremely useful approximation is to calculate (either exactly or within some approximation scheme) $\langle T^{\mu}{}_{\nu}\rangle$ in a classical black hole geometry, and then compute semiclassical corrections to the metric as linear perturbations. This works particularly well for static solutions such as occur for a zero temperature black hole or a black hole in thermal equilibrium with radiation in a cavity. The fact that the solutions are static makes the problem much more tractable. An advantage of this approach is that it gives direct information about how semiclassical effects alter the geometry from that of the corresending classical solution to Einstein's equations. Even though such models have so far only been applied to the static case, they are relevant to the issue of the end point of black hole evaporation, because the perturbations give information about how quantum effects alter the temperature of a black hole. For example, if uncharged zero temperature solutions were found which possessed nonzero mass, these could be potential endpoint ''remnants'' of black hole evaporation.

To date the linearized semiclassical back-reaction equations have only been solved in the case of an initially Schwarzschild black hole in thermal equilibrium with massless radiation in a cavity. York considered the perturbation due to a conformally coupled quantized scalar field [2]. Hochberg, Kephart, and York extended this work to include the effects of massless quantized spinor and vector fields [3]. Anderson *et al.* [4] studied the perturbations due to the vacuum stress-energy of a quantized massless scalar field with arbitrary curvature coupling. In all these cases, the stress-energy tensor of the quantized field was treated using analytic approximations developed by Page, Brown, and Ottewill [5–7] and Anderson, Hiscock, and Samuel [8].

In this paper we investigate charged and uncharged black holes which interact with an uncharged quantized massive scalar field of arbitrary curvature coupling. The general ''bare'' spacetime is described by the Reissner-Nordström metric

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2, \quad (1)$$

where Q is the charge on the black hole and M is its mass. We treat the vacuum stress-energy of the quantized massive scalar field as a perturbation on the ''bare'' Reissner-Nordström spacetime, solving the semiclassical Einstein equations to find the first-order-in-$\hbar$ semiclassical corrections to the Reissner-Nordström metric. We consider a situation analogous to that of Ref. [4], where the black hole is in thermal equilibrium with the quantized field, imposing microcanonical boundary conditions on a spherical boundary surface surrounding the black hole. The vacuum stress-energy is analytically approximated using the DeWitt-

*Present address: Department of Chemistry and Physics, Radford University, Radford, VA 24142. Electronic address: brett@peloton.runet.edu

†Electronic address: billh@orion.physics.montana.edu

‡Electronic address: paul@planck.phy.wfu.edu

Schwinger approximation; previous exact, numerical work by Anderson, Hiscock, and Samuel [8] (AHS) has demonstrated that the DeWitt-Schwinger approximation to the vacuum stress-energy is quite good (1% or better) in the Reissner-Nordström spacetime when $mM \geqslant 2$.

The perturbation caused by the quantized field will change the temperature of the black hole; we examine in detail the sign and size of this effect. We are particularly interested in situations where the perturbed black hole has precisely zero temperature. Within the context of a perturbative approach, this means the unperturbed Reissner-Nordström spacetime must be nearly extreme. The ''bare'' Reissner-Nordström spacetime that results in a ''dressed'' zero temperature black hole could be either a nearly extreme Reissner-Nordström black hole or possibly a Reissner-Nordström naked singularity, with $|Q|$ slightly greater than M. We are able to handle the case of ''bare'' naked singularities because the DeWitt-Schwinger approximation for $\langle T^{\mu}{}_{\nu}\rangle$ is purely local.

Section II describes the approximate vacuum stress-energy tensor and the semiclassical linearized Einstein equations. The metric perturbations are also derived and displayed in this section. In Sec. III the temperature perturbations are determined, and we search for zero temperature solutions. We find that there are no zero temperature solutions for plausible values of the scalar field's curvature coupling; specifically, there are none for minimally or conformally coupled scalar fields. Section IV summarizes our conclusions. Throughout this paper we use units such that $\hbar = G = c = k_B = 1$. The sign conventions are those of Misner, Thorne, and Wheeler [9].

II. SEMICLASSICAL PERTURBATION METHOD

In semiclassical gravity, one quantizes the matter fields but not the spacetime geometry. This modifies the right hand side of the Einstein field equations, replacing the classical stress-energy tensor with the expectation value of the quantum stress-energy operator. The semiclassical Einstein equations then take the form

$$G^{\mu}{}_{\nu} = 8\pi\langle T^{\mu}_{\nu}\rangle. \tag{2}$$

In examining the semiclassical perturbations of the Reissner-Nordström metric caused by the vacuum energy of a quantized scalar field, we will continue to consider the electromagnetic field to be classical; for the Reissner-Nordström geometry, then, the semiclassical equations will contain both classical and quantum stress-energy contributions:

$$G^{\mu}{}_{\nu} = 8\pi[\langle T^{\mu}{}_{\nu}\rangle + T^{\mu}{}_{\nu}]. \tag{3}$$

The classical stress-energy term on the right-hand side of Eq. (3) represents the electromagnetic field's stress-energy in the Reissner-Nordström geometry; it will vanish in the Schwarzschild limit as $Q \to 0$.

The exact calculation of the expectation value for the stress-energy of a quantized field in a curved spacetime is a non-trivial exercise. Anderson, Hiscock, and Samuel [8] have developed a method for numerically calculating the vacuum stress-energy tensor of both massive and massless quantized scalar fields with arbitrary curvature coupling in a general static, spherically symmetric spacetime. As part of this method, they also developed an analytic approximation to $\langle T^{\mu}{}_{\nu}\rangle$ for massive scalar fields based on the DeWitt-Schwinger expansion in inverse powers of the field mass. This approximation is state independent and entirely local, depending at each point only on the values of the curvature and its derivatives. One expects, *a priori*, the approximation to become increasingly accurate as the ratio of the Compton wavelength of the field to the local radius of curvature approaches zero, i.e., in the limit $Mm \gg 1$.

They then applied these techniques to the Reissner-Nordström spacetime, obtaining exact numerical values for the vacuum stress-energy for both massive and massless fields. Comparing the exact values of $\langle T^{\mu}{}_{\nu}\rangle$ to those provided by the DeWitt-Schwinger approximation, they showed that in the Reissner-Nordström spacetime, the approximate values are quite good (within a few percent of the exact values) near the horizon if the field mass is chosen to satisfy $mM \geqslant 2$. One might expect the DeWitt-Schwinger approximation to also fail to be adequate for black holes with nonzero temperature, at large values of r where temperature-dependent (hence, state-dependent) terms associated with the gas of produced particles dominate $\langle T^{\mu}{}_{\nu}\rangle$. However, for a massive field, there will be essentially no particles created by the hole if the temperature is substantially less than the mass of the field, $T \ll m$. For any Reissner-Nordström black hole, the temperature satisfies $T < (4\pi M)^{-1}$, so the temperature will be substantially less than the field mass as long as $(4\pi)^{-1} \ll Mm$. This condition is adequately satisfied when the previously mentioned criterion for the validity of the DeWitt-Schwinger approximation, $mM \geqslant 2$, holds. Hence, for any Reissner-Nordström black hole and massive scalar field combination satisfying $mM \geqslant 2$, we will assume the DeWitt-Schwinger approximation is valid throughout the region exterior to the horizon.[1]

Using the results of AHS for the case of a quantized massive scalar field in the Reissner-Nordström spacetime the following values for the expectation value of the stress-energy tensor are obtained:

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[1]Strictly speaking the expectation value of the stress-energy tensor for a massive field in a thermal state must approach a nonzero constant in the limit $r \to \infty$. However, in the case when $mM \gg 1$ there will be very few ''realizations'' of the quantum field theory in which any particles are present. Another way to think about this is that in a nonstatic state it would take a very long time on average before a particle is produced. Thus to a good approximation it should be adequate to use the DeWitt-Schwinger approximation out to arbitrarily large values of r. Of course for the zero temperature case one expects the DeWitt-Schwinger approximation to be valid throughout the region exterior to the event horizon as long as $mM \gg 1$ is satisfied.

$$\langle T^t{}_t\rangle=\frac{\epsilon}{\pi^2 m^2}\left[\frac{1237M^5}{5040r^9}-\frac{25M^4}{224r^8}-\frac{1369M^5r_+}{1008r^{10}}+\frac{41M^4r_+}{105r^9}+\frac{3M^3r_+}{56r^8}+\frac{613M^5r_+^2}{210r^{11}}-\frac{73M^4r_+^2}{3360r^{10}}-\frac{41M^3r_+^2}{210r^9}-\frac{3M^2r_+^2}{112r^8}\right.$$
$$-\frac{2327M^5r_+^3}{1260r^{12}}-\frac{613M^4r_+^3}{210r^{11}}+\frac{883M^3r_+^3}{1260r^{10}}+\frac{2327M^4r_+^4}{840r^{12}}+\frac{613M^3r_+^4}{840r^{11}}-\frac{883M^2r_+^4}{5040r^{10}}-\frac{2327M^3r_+^5}{1680r^{12}}+\frac{2327M^2r_+^6}{10080r^{12}}$$
$$+\xi\left(\frac{-11M^5}{10r^9}+\frac{M^4}{2r^8}+\frac{217M^5r_+}{30r^{10}}-\frac{14M^4r_+}{5r^9}-\frac{226M^5r_+^2}{15r^{11}}+\frac{77M^4r_+^2}{180r^{10}}+\frac{7M^3r_+^2}{5r^9}+\frac{91M^5r_+^3}{10r^{12}}+\frac{226M^4r_+^3}{15r^{11}}\right.$$
$$\left.\left.-\frac{182M^3r_+^3}{45r^{10}}-\frac{273M^4r_+^4}{20r^{12}}-\frac{113M^3r_+^4}{30r^{11}}+\frac{91M^2r_+^4}{90r^{10}}+\frac{273M^3r_+^5}{40r^{12}}-\frac{91M^2r_+^6}{80r^{12}}\right)\right], \tag{4}$$

$$\langle T^r{}_r\rangle=\frac{\epsilon}{\pi^2 m^2}\left[\frac{-47M^5}{720r^9}+\frac{7M^4}{160r^4}+\frac{2081M^5r_+}{5040r^{10}}-\frac{16M^4r_+}{63r^9}+\frac{3M^3r_+}{280r^8}-\frac{13M^5r_+^2}{18r^{11}}+\frac{983M^4r_+^2}{10080r^{10}}+\frac{8M^3r_+^2}{63r^9}-\frac{3M^2r_+^2}{560r^8}\right.$$
$$+\frac{421M^5r_+^3}{1260r^{12}}+\frac{13M^4r_+^3}{18r^{11}}-\frac{383M^3r_+^3}{1260r^{10}}-\frac{421M^4r_+^4}{840r^{12}}-\frac{13M^3r_+^4}{72r^{11}}+\frac{383M^2r_+^4}{5040r^{10}}+\frac{421M^3r_+^5}{1680r^{12}}-\frac{421M^2r_+^6}{10080r^{12}}$$
$$+\xi\left(\frac{3M^5}{10r^9}-\frac{M^4}{5r^8}-\frac{49M^5r_+}{30r^{10}}+\frac{14M^4r_+}{15r^9}+\frac{14M^5r_+^2}{5r^{11}}-\frac{61M^4r_+^2}{180r^{10}}-\frac{7M^3r_+^2}{15r^9}-\frac{13M^5r_+^3}{10r^{12}}-\frac{14M^4r_+^3}{5r^{11}}\right.$$
$$\left.\left.+\frac{52M^3r_+^3}{45r^{10}}+\frac{39M^4r_+^4}{20r^{12}}+\frac{7M^3r_+^4}{10r^{11}}-\frac{13M^2r_+^4}{45r^{10}}-\frac{39M^3r_+^5}{40r^{12}}+\frac{13M^2r_+^6}{80r^{12}}\right)\right], \tag{5}$$

where $r_+=M+\sqrt{M^2-Q^2}$ is the radius of the unperturbed event horizon, $\epsilon=1/M^2$ is our expansion parameter for the perturbation (in conventional units, $\epsilon=M_{\text{Planck}}^2/M^2$) and ξ is the curvature coupling for the field. We do not display the value of $\langle T^\theta{}_\theta\rangle$, as it not needed here. Knowledge of the two components shown above is sufficient to completely solve the perturbed semiclassical Einstein equations.

The semiclassical Einstein equations may be more easily solved if a coordinate transformation is made to ingoing Eddington-Finklestein coordinates. The transformation is described by

$$\frac{\partial t}{\partial v}=1, \tag{6}$$

$$\frac{\partial t}{\partial \tilde{r}}=-\left(1-\frac{2M}{\tilde{r}}+\frac{Q^2}{\tilde{r}^2}\right)^{-1}, \tag{7}$$

$$\frac{\partial r}{\partial v}=0, \tag{8}$$

$$\frac{\partial r}{\partial \tilde{r}}=1. \tag{9}$$

The Reissner-Nordström metric takes the following form in ingoing Eddington-Finklestein coordinates:

$$ds^2=-\left(1-\frac{2M}{\tilde{r}}+\frac{Q^2}{\tilde{r}^2}\right)dv^2+2dv\,d\tilde{r}+\tilde{r}^2d\Omega^2. \tag{10}$$

The components of the expectation value of the stress-energy tensor in these coordinates are

$$\langle T^v{}_v\rangle=\langle T^t{}_t\rangle, \tag{11}$$

$$\langle T^{\tilde{r}}{}_{\tilde{r}}\rangle=\langle T^r{}_r\rangle, \tag{12}$$

$$\langle T^v{}_{\tilde{r}}\rangle=\left(1-\frac{2M}{\tilde{r}}+\frac{Q^2}{\tilde{r}^2}\right)^{-1}[\langle T^r{}_r\rangle-\langle T^t{}_t\rangle]. \tag{13}$$

Setting $\tilde{r}=r$, one can write the metric for a general static spherically symmetric spacetime as

$$ds^2=-e^{2\psi(r)}\left(1-\frac{2m(r)}{r}+\frac{Q^2}{r^2}\right)dv^2+2e^{\psi(r)}dv\,dr+r^2d\Omega^2. \tag{14}$$

The perturbations to the Reissner-Nordström metric can be introduced by an expansion of the e^ψ and $m(r)$ metric functions to first order in the parameter ϵ:

$$e^{\psi(r)}=1+\epsilon\rho(r), \tag{15a}$$

$$m(r)=M[1+\epsilon\mu(r)]. \tag{15b}$$

The components of the Einstein tensor can now be calculated in the metric given by Eq. (14) with the expansions given in Eqs. (15a),(15b). These can be substituted into Eq. (2) along with the classical background stress-energy and the approximate stress-energy of the quantized field from Eqs. (11)–(13). This yields two first order differential equations for $\mu(r)$ and $\rho(r)$:

$$\frac{d\mu}{dr}=-\frac{4\pi r^2}{M\epsilon}\langle T^t{}_t\rangle, \tag{16}$$

$$\frac{d\rho}{dr}=\frac{4\pi r}{\epsilon}\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)^{-1}[\langle T^r{}_r\rangle-\langle T^t{}_t\rangle]. \tag{17}$$

Here $\langle T^t{}_t\rangle$ and $\langle T^r{}_r\rangle$ are given by Eq. (4) and Eq. (5). The ϵ factors in the denominator of the leading terms in both Eq. (16) and Eq. (17) are exactly canceled by the overall factor of ϵ in the expressions for $\langle T^t{}_t\rangle$ and $\langle T^r{}_r\rangle$. These differential equations can be integrated to find the general solutions for μ and ρ. They are

$$\begin{aligned}\mu=C_1+\frac{1}{\pi m^2}\Bigg[&\frac{1237M^4}{7560r^6}-\frac{5M^3}{56r^5}-\frac{4169M^4}{158760r_+^6}+\frac{461M^3}{6615r_+^5}-\frac{6607M^2}{105840r_+^4}+\frac{3007M}{158760r_+^3}-\frac{1369M^4r_+}{1764r^7}+\frac{82M^3r_+}{315r^6}\\&+\frac{3M^2r_+}{70r^5}+\frac{613M^4r_+^2}{420r^8}-\frac{73M^3r_+^2}{5880r^7}-\frac{41M^2r_+^2}{315r^6}-\frac{3Mr_+^2}{140r^5}-\frac{2327M^4r_+^3}{2835r^9}-\frac{613M^3r_+^3}{420r^8}+\frac{883M^2r_+^3}{2205r^7}\\&+\frac{2327M^3r_+^4}{1890r^9}+\frac{613M^2r_+^4}{1680r^8}-\frac{883Mr_+^4}{8820r^7}-\frac{2327M^2r_+^5}{3780r^9}+\frac{2327Mr_+^6}{22680r^9}+\xi\Bigg(-\frac{11M^4}{15r^6}+\frac{2M^3}{5r^5}+\frac{4M^4}{45r_+^6}-\frac{11M^3}{45r_+^5}\\&+\frac{41M^2}{180r_+^4}-\frac{13M}{180r_+^3}+\frac{62M^4r_+}{15r^7}-\frac{28M^3r_+}{15r^6}-\frac{113M^4r_+^2}{15r^8}+\frac{11M^3r_+^2}{45r^7}+\frac{14M^2r_+^2}{15r^6}+\frac{182M^4r_+^3}{45r^9}+\frac{113M^3r_+^3}{15r^8}\\&-\frac{104M^2r_+^3}{45r^7}-\frac{91M^3r_+^4}{15r^9}-\frac{113M^2r_+^4}{60r^8}+\frac{26Mr_+^4}{45r^7}+\frac{91M^2r_+^5}{30r^9}-\frac{91Mr_+^6}{180r^9}\Bigg)\Bigg],\end{aligned} \tag{18}$$

and

$$\begin{aligned}\rho=C_2+\frac{1}{\pi m^2}\Bigg[&-\frac{29M^4}{280r^6}+\frac{817M^4}{3528r_+^6}-\frac{3221M^3}{8820r_+^5}+\frac{253M^2}{1680r_+^4}+\frac{184M^4r_+}{441r^7}+\frac{M^3r_+}{35r^6}-\frac{229M^4r_+^2}{420r^8}-\frac{92M^3r_+^2}{441r^7}-\frac{M^2r_+^2}{70r^6}+\frac{229M^3r_+^3}{420r^8}\\&-\frac{229M^2r_+^4}{1680r^8}+\xi\Bigg(\frac{7M^4}{15r^6}-\frac{14M^4}{15r_+^6}+\frac{23M^3}{15r_+^5}-\frac{13M^2}{20r_+^4}-\frac{32M^4r_+}{15r^7}+\frac{13M^4r_+^2}{5r^8}+\frac{16M^3r_+^2}{15r^7}-\frac{13M^3r_+^3}{5r^8}+\frac{13M^2r_+^4}{20r^8}\Bigg)\Bigg].\end{aligned} \tag{19}$$

The integration constants for both μ and ρ have been chosen so that $\mu(r_+)=C_1$ and $\rho(r_+)=C_2$. The perturbed spacetime is now defined to first order in ϵ to within the two integration constants C_1 and C_2.

The horizon radius is no longer located at r_+ due to the perturbation from the presence of the quantized field. Its radius is now defined implicitly as the solution to the equation

$$r_h=m(r_h)+\sqrt{m(r_h)^2-Q^2}. \tag{20}$$

We can utilize the horizon location to define the perturbed mass of the black hole,

$$M_{BH}=m(r_h)=M[1+\epsilon\mu(r_+)], \tag{21}$$

to first order in ϵ; r_h has been changed to r_+ in the final expression on the right, as the difference would be of order ϵ^2. This physical, or dressed, mass, M_{BH}, is a function of the bare, and unmeasurable mass, M, plus a small perturbation:

$$M_{BH}=M+\epsilon MC_1. \tag{22}$$

The horizon radius is then expressed in terms of the dressed mass of the black hole:

$$r_h=M_{BH}+\sqrt{M_{BH}^2-Q^2}. \tag{23}$$

The bare mass, M, and the mass perturbation, δM, cannot be measured independently; only the dressed mass M_{BH} has physical meaning. We will hereafter only refer to the dressed mass, M_{BH}, defined implicitly in Eq. (23). The arbitrary but physically unmeasureable integration constant C_1 is then absorbed into the definition of M, as in Ref. [4]. The perturbed metric's mass function now takes the form

$$m(r)=M_{BH}[1+\epsilon\tilde{\mu}(r)], \tag{24}$$

where

$$\tilde{\mu}(r)=\mu(r)|_{C_1=0}. \tag{25}$$

The metric can now be rewritten in terms of the dressed mass M_{BH} and $\tilde{\mu}(r)$. Because these quantities are those that can be physically measured, the BH subscript and the tilde will now be dropped, writing only M and μ respectively. In addition, we will denote r_h by r_+ henceforth, since they have the same definition once the mass has been renormalized. It is now convenient to transform from Eddington-Finklestein coordinates back to (t,r,θ,ϕ) coordinates. Doing so one finds that the perturbed metric takes the form

$$ds^2=-[1+2\epsilon\rho(r)]\left(1-\frac{2m(r)}{r}+\frac{Q^2}{r^2}\right)dt^2 + \left(1-\frac{2m(r)}{r}+\frac{Q^2}{r^2}\right)^{-1}dr^2+r^2d\Omega^2. \tag{26}$$

The remaining integration constant, C_2, was fixed in the case of the massless field studied in Ref. [4] by enclosing the black hole in a cavity and imposing microcanonical boundary conditions. However, as discussed above there is no gas of massive scalar particles surrounding the black hole, because the DeWitt-Schwinger approximation is state (and hence temperature) independent. In the domain where the DeWitt-Schwinger approximation is valid (i.e., $Mm>2$), the temperature of the hole is so low that it creates a negligible number of such particles. The stress-energy associated with the massive scalar field in this limit is essentially the result of vacuum polarization, not particle production. Hence, placing the hole in a cavity to allow thermal equilibrium is neither appropriate nor necessary.

Instead, the integration constant C_2 may be fixed by requiring that g_{tt} in the perturbed metric of Eq. (26) approach the usual value of -1 as $r\rightarrow\infty$. This implies that C_2 simply determines the normalization of the time coordinate at infinity. For g_{tt} to approach -1 as a limiting value requires that $\rho(\infty)=0$, and therefore that

$$C_2=-\tilde{\rho}(r)|_{r\rightarrow\infty}, \tag{27}$$

where

$$\tilde{\rho}(r)=\rho(r)|_{C_2=0}. \tag{28}$$

It is worth noting that this condition is identical to the microcanonical boundary condition in the limit that the cavity radius approaches infinity. With the fixing of C_2 the perturbed spacetime is now completely defined; C_1 and C_2 no longer appear in the perturbed metric, having been fixed; the mass M now refers to the ''dressed'' black hole mass, defined implicitly by the horizon radius through Eq. (23).

III. RESULTS

In this section we will concentrate on the examination of two properties of the perturbed black hole metric: first, the relation of the mass M defined by the horizon radius to the mass that would be measured at infinity by a Keplerian orbit, M_∞; second, the effect of the semiclassical perturbation on the temperature of the black hole.

With the integration constant C_1 absorbed into the horizon-defined mass M as described in Eq. (24) above, the horizon radius keeps its simple, Reissner-Nordström form, as seen in Eq. (23). However, the price paid is that now the mass M is not the mass that would be measured for the perturbed black hole by an observer at infinity, say by observing the properties of an orbiting test mass at large r. The mass of the black hole at infinity will be

$$M_\infty=M[1+\epsilon\mu(r)|_{r\rightarrow\infty}]=M+\delta M. \tag{29}$$

The difference between the mass measured at infinity and the horizon defined mass is then

$$\delta M=\frac{\epsilon M}{\pi m^2}\left[-\frac{4169M^4}{158760r_+^6}+\frac{461M^3}{6615r_+^5}-\frac{6607M^2}{105840r_+^4} +\frac{3007M}{158760r_+^3}+\xi\left(\frac{4M^4}{45r_+^6}-\frac{11M^3}{45r_+^5}+\frac{41M^2}{180r_+^4}-\frac{13M}{180r_+^3}\right)\right]. \tag{30}$$

Depending on the value of the scalar field's curvature coupling, ξ, M_∞ can be either larger ($\delta M>0$) or smaller ($\delta M<0$) than M. Examination of Eq. (30) shows that for a conformally coupled field ($\xi=1/6$), $\delta M>0$ for all values of $Q^2/M^2\lesssim 0.954463$, while for a minimally coupled field ($\xi=0$), $\delta M>0$ for all $Q^2/M^2\lesssim 0.998701$. Interestingly, in the extreme Reissner-Nordström limit, for which $Q^2/M^2=1$, δM becomes negative and independent of ξ:

$$\delta M_{ERN}=\frac{-17\epsilon}{317520\pi m^2 M}. \tag{31}$$

This implies that an extreme black hole, perturbed semiclassically by a massive quantized scalar field, will have a charge-to-mass ratio (as measured at infinity) greater than unity.

In order to determine the effect the presence of the quantized scalar field has on the temperature, the surface gravity of the black hole must be calculated. For the perturbed metric in Eq. (26) the surface gravity to first order in ϵ is

$$\kappa=\frac{\sqrt{M^2-Q^2}}{r_+^2}(1+\epsilon C_2)+4\pi r_+\langle T^t_t\rangle, \tag{32}$$

which reduces to the usual Reissner-Nordström surface gravity as $\epsilon\rightarrow 0$. The perturbation in the surface gravity is given by

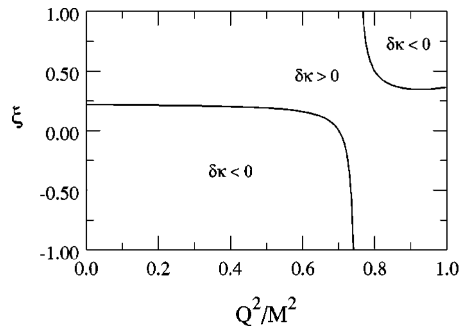

FIG. 1. The curves represent semiclassical black hole solutions for which the change in temperature is zero for particular values of the charge and curvature coupling constant.

$$\delta\kappa=\kappa-\frac{\sqrt{M^2-Q^2}}{r_+^2} \tag{33}$$

or, explicitly,

$$\delta\kappa=\frac{\epsilon M^2}{35280\pi m^2 r_+^8}[2458M^3-5766M^2r_+ +4617Mr_+^2-1239r_+^3+\xi(-9408M^3+22736M^2r_+ -18620Mr_+^2+5096r_+^3)]. \tag{34}$$

The perturbation in the surface gravity depends on the value of the curvature coupling constant, ξ, which appears in both the expression for C_2 and μ. In addition the surface gravity will also depend on the field mass m and the overall size of the perturbation, ϵ. Both of these are multiplicative factors that only affect the overall size of the perturbation to the surface gravity. Our attention here is focused on (1) the sign of $\delta\kappa$ for various combinations of black hole state and field curvature coupling, and (2) which black holes states can conceivably have the total surface gravity (κ, in the perturbed state), and hence temperature, equal to zero.

Since the expression for $\delta\kappa$ is linear in ξ, it is a simple matter to find the value of ξ for each value of Q^2/M^2 that will result in $\delta\kappa=0$. The domain of allowed black hole states may then be divided into regions where $\delta\kappa>0$ and regions where $\delta\kappa<0$. Figure 1 illustrates the sign of $\delta\kappa$ as a function of Q^2/M^2 and ξ.

In the Schwarzschild limit, $\delta\kappa$ simplifies to the form

$$\delta\kappa_{\mathrm{Sch}}=\epsilon\left(\frac{-37+168\xi}{645120\pi m^2M^3}\right), \tag{35}$$

which is negative for both the minimal and conformally coupled massive scalar field, as well as for any field with $\xi<37/168$. Thus, the likely effect of a semiclassical perturbation from a massive quantized scalar field on a Schwarzschild black hole is to lower its temperature.

In the extreme Reissner-Nordström limit, $\delta\kappa$ becomes

$$\delta\kappa_{\mathrm{ERN}}=\epsilon\left(\frac{5-14\xi}{2520\pi m^2M^3}\right), \tag{36}$$

which is positive for both the minimal and conformally coupled field, as well as for any field with $\xi<5/14$. Thus, for a Reissner-Nordström black hole with a semiclassical perturbation provided by a massive quantized scalar field, unless the curvature coupling takes on apparently unnatural values ($\xi>5/14$), there will be *no* zero temperature solution. The extreme black hole (now defined as the black hole with maximum possible charge-to-mass ratio, beyond which are naked singularity solutions) will have a nonzero temperature.

Given the somewhat surprising result that zero temperature solutions to the linearized semiclassical back-reaction equations do not exist for realistic values of the curvature coupling constant ξ, it is perhaps useful to ask whether zero temperature solutions to the full nonlinear semiclassical back-reaction equations exist. Although it is difficult to find solutions to the nonlinear equations everywhere outside the event horizon (even with the use of the DeWitt-Schwinger approximation) it is possible to solve the equations near the event horizon. To do so one can simply expand the metric functions, the Einstein tensor, and the stress-energy tensor in powers of $(r-r_h)$ and solve the equations order by order in $(r-r_h)$. Utilizing this approach with the full nonlinear equations, we have found that zero temperature solutions of the extreme Reissner-Nordström form near the event horizon exist. These solutions have a slightly different ratio between the charge Q and the radius r_h of the event horizon than do the classical Reissner-Nordström solutions.

However, the existence of zero temperature local solutions to the full nonlinear equations is probably irrelevant from a physical point of view. The reason is that the DeWitt-Schwinger approximation contains terms with up to six derivatives of the metric. These higher derivatives lead to many locally (i.e., near the horizon) sensible solutions to the equations that are an artifact of the approximation since the exact stress-energy tensor contains terms with up to only four derivatives of the metric. Even in the case when the exact stress-energy tensor is used in the semiclassical back-reaction equations it has been argued that the higher derivatives here also lead to physically unacceptable solutions [10]. Thus the most likely situation is that the only solutions to the nonlinear equations that are physically acceptable when the DeWitt-Schwinger approximation is used are those that reduce to the solutions to the linearized equations in the limit $mM\to\infty$. In this case we have already seen that zero temperature solutions do not exist for reasonable values of ξ.

IV. DISCUSSION

We have investigated the effect the vacuum stress-energy of a quantized massive scalar field has on the geometry of a charged black hole, within the context of linear perturbation theory. We have found the metric functions that describe such a semiclassically perturbed black hole, to first order in $\epsilon=\hbar/M^2$. We have shown that the mass of such a black hole, as measured at infinity, will differ from the mass defined in terms of the horizon radius. For an extreme black hole, the mass at infinity will always be less than the mass defined by the horizon radius. The charge-to-mass ratio of an extreme black hole, as measured at infinity, will exceed unity for all values of the massive field scalar curvature coupling. We have also examined the effect of the semiclassical perturbation on the surface gravity of the black hole. For reasonable values of the scalar field curvature coupling, the perturbation lowers the temperature of a Schwarzschild black hole. For an extreme black hole, the temperature is raised by the perturbation for any scalar field with $\xi < 5/14$, including the physically interesting cases of minimal and conformal coupling. Thus, within the context of Reissner-Nordström black holes semiclassically perturbed by the vacuum energy of a massive scalar field, there are no plausible zero-temperature solutions.

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