Axion model in extra dimensions with TeV scale gravity

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A simple axion model is proposed in the scenario of large extra dimensions where the gravity scale is as low as 1 TeV. To obtain an intermediate-scale decay constant of the axion, the axion is assumed to live in a subspacetime (brane) of the whole bulk. In this model there appear Kaluza-Klein modes of the axion which have a stronger interaction than those of the graviton. The axion brane plays the role of an absorber of the graviton Kaluza-Klein modes. For these reasons, the phenomenology and cosmology of the axion become highly nontrivial. We discuss various cosmological constraints as well as astrophysical ones and show that the model is viable for certain choices of the dimensionality of the axion brane. The structure of the model proposed here provides a viable realization of the fat brane idea to relax the otherwise very severe cosmological constraints.

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I. INTRODUCTION

It has been suggested that the fundamental scale of nature can be as low as TeV, whereas the largeness of the effective Planck scale or the weakness of the gravity at a long distance can be explained by introducing large extra dimensions [1–4]. When there exist n of such extra dimensions, the relation between the gravitational constant $8\pi G_N = 1/M_{pl(4)}^2$ in 4 dimensions and the fundamental scale $M_{pl(4+n)}$ in $(4+n)$ dimensions is given by

$$M_{pl(4)}^2 \sim V_n M_{pl(4+n)}^{n+2}, \tag{1}$$

where V_n is the volume of the extra-dimensional space. For $M_{pl(4+n)} \sim 1$ TeV, the size of the extra dimensions r_n is computed as

$$r_n \sim V_n^{1/n} \sim 10^{32/n-6} M_{\rm TeV}^{-2/n-1} \mathrm{MeV}^{-1} \sim 10^{32/n-17} M_{\rm TeV}^{-2/n-1} \mathrm{cm}, \tag{2}$$

where $M_{\rm TeV} \equiv M_{pl(4+n)}/\mathrm{TeV}$. The case $n=1$ is excluded because the gravitational law would change at a macroscopic level, but the cases $n \geqslant 2$ are allowed by gravity experiments.

There are also astrophysical and cosmological bounds [3] because there is a tower of graviton Kaluza-Klein (KK) modes which contribute to the supernova cooling [5], the total mass of the universe, and the late-time photon production [6], etc. We may avoid some of these problems by assuming an extremely low reheating temperature of the early universe.

To have an inflation model with such a low temperature is already a tough subject, especially with the cosmological moduli problem [7]. Furthermore, we need baryon asymmetry after the inflation while the temperature is quite low and we knew that the process which generates baryon asymmetry should not induce proton decay. It is better to have a higher reheating temperature for many reasons, but it is very hard to avoid known restrictions.

There is another problem in TeV scale gravity. The simplest model of this type would not provide intermediate scales which are necessary to explain phenomenological issues such as the apparent gauge coupling unification [8], small neutrino masses, and the strong CP problem. It was pointed out [3] that particles dwelling in extra dimensions, other than a graviton, can have similar effective interaction terms in four-dimensional physics. This type of interaction is suppressed as gravity and thus gives extremely weak interaction between bulk matter and normal matters. For the neutrino, there have been many discussions and much research about how to extend the minimal setting to obtain the small neutrino masses and mixing [9–11], but the axion as a solution of the strong CP solution has not been studied thoroughly in this context. In this paper we will propose an axion model in TeV scale gravity for various numbers of the extra dimensions, with special emphasis on cosmological constraints: the addition of the axion in the large-extra-dimension model substantially alters the cosmology. In the following sections, we will use the convention $M_{pl} \equiv M_{pl(4)}$ = reduced Planck mass and $M_* \equiv M_{pl(4+n)}$ = fundamental scale.

II. PQ SCALE IN EXTRA DIMENSIONS

If an axion is a boundary field confined on 4 dimensions, the Peccei-Quinn (PQ) scale $f_{\rm PQ}$ is bounded by $M_* \sim 1$ TeV. To obtain a higher PQ scale, the axion field has to be inevitably a bulk field. If it lives in the whole $(4+n)$-dimensional bulk, the PQ scale will be $f_{\rm PQ} \sim M_{pl}$.

However, the damped coherent oscillation of the axion vacuum at the early universe with $f_{\rm PQ} \sim M_{pl}$ would overclose the universe. A conventional argument gives an upper bound $f_{\rm PQ} \leqslant 10^{12}$ GeV. Even when an entropy production takes place after the QCD phase transition (e.g., the reheating temperature is smaller than ~ 1 GeV), $f_{\rm PQ}$ cannot be much larger than 10^{15} GeV.

In Ref. [12], it was shown that the late thermal inflation can raise the bound of the PQ scale up to 10^{15} GeV. This argument can be applied to our case, provided that the coherent oscillation of the inflaton is followed by the reheating process. Then the bound on $f_{\rm PQ}$ is

$$f_{\rm PQ} < 10^{15}\ \mathrm{GeV} \left(\frac{h}{0.7}\right) \left(\frac{\pi/2}{\theta}\right) \left(\frac{\mathrm{MeV}}{T_R}\right)^{1/2}, \tag{3}$$

TABLE I. PQ scales and lifetimes of axion and graviton KK modes for $M_*=1$ TeV, where $M_{100}\equiv m_A/100$ MeV.

(m,n)	r_n^{-1}[MeV]	f_{PQ}[GeV]	τ_g[sec]	τ_a[sec]
(1,2)	4×10^{-10}	5×10^{10}	$2\times10^7 M_{100}^{-4}$	$6\times10^7 M_{100}^{-3}$
(2,3)	6×10^{-5}	2×10^{13}	$1\times10^6 M_{100}^{-5}$	$8\times10^{12} M_{100}^{-3}$
(2,4)	2×10^{-2}	5×10^{10}	$2\times10^{11} M_{100}^{-5}$	$6\times10^7 M_{100}^{-3}$
(3,4)		3×10^{14}	$3\times10^7 M_{100}^{-6}$	$3\times10^{15} M_{100}^{-3}$
(2,5)	0.7	1×10^9	$2\times10^{14} M_{100}^{-5}$	$5\times10^4 M_{100}^{-3}$
(3,5)		2×10^{12}	$1\times10^{12} M_{100}^{-6}$	$7\times10^{10} M_{100}^{-3}$
(4,5)		2×10^{15}	$9\times10^9 M_{100}^{-7}$	$1\times10^{17} M_{100}^{-3}$
(3,6)	7	5×10^{10}	$2\times10^{15} M_{100}^{-6}$	$6\times10^7 M_{100}^{-3}$
(4,6)		2×10^{13}	$1\times10^{14} M_{100}^{-7}$	$8\times10^{12} M_{100}^{-3}$

TABLE II. PQ scales and lifetimes of axion and graviton KK modes for $M_*=10$ TeV.

(m,n)	r_n^{-1}[MeV]	f_{PQ}[GeV]	τ_g[sec]	τ_a[sec]
(1,2)	4×10^{-8}	2×10^{11}	$2\times10^9 M_{100}^{-4}$	$6\times10^8 M_{100}^{-3}$
(2,3)	3×10^{-3}	4×10^{13}	$3\times10^9 M_{100}^{-5}$	$4\times10^{13} M_{100}^{-3}$
(2,4)	0.6	2×10^{11}	$2\times10^{14} M_{100}^{-5}$	$6\times10^8 M_{100}^{-3}$
(3,4)		6×10^{14}	$1\times10^{12} M_{100}^{-6}$	$9\times10^{15} M_{100}^{-3}$
(2,5)	20	6×10^9	$1\times10^{17} M_{100}^{-5}$	$8\times10^5 M_{100}^{-3}$
(3,5)		4×10^{12}	$2\times10^{16} M_{100}^{-6}$	$5\times10^{11} M_{100}^{-3}$
(4,5)		3×10^{15}	$4\times10^{15} M_{100}^{-7}$	$3\times10^{17} M_{100}^{-3}$
(3,6)	160	2×10^{11}	$2\times10^{19} M_{100}^{-6}$	$6\times10^8 M_{100}^{-3}$
(4,6)		4×10^{13}	$3\times10^{19} M_{100}^{-7}$	$4\times10^{13} M_{100}^{-3}$

where h is from Hubble constant in units of 100 km sec^{-1}Mpc^{-1}, θ is the initial value of PQ vacuum angle, and T_R is the reheating temperature after inflation. Thus the axion model suffers from the overclosure problem if there are no other scales than M_* and M_{pl}.

In this paper, we propose that a natural way to realize an intermediate scale axion is to make it live in a $(4+m)$-dimensional subspacetime [$(3+m)$ brane] $(m<n)$ of the whole $(4+n)$-dimensional bulk. The idea of using subspacetime to realize an intermediate scale already appeared in Ref. [10] in the context of neutrinos.

Let χ be a complex scalar field which contains PQ axion in $4+m$ dimension; $\tilde{a}$. If the axion field lives only on $(4+m)$-dimensional subspacetime where $m<n$ and the volume of extra dimension is V_m,

$$\mathcal{L}_\chi=\int dx^{4+m}\partial^M\chi^*\partial_M\chi+\int dx^4\frac{\tilde{a}(x^A=0)}{\langle\chi\rangle}F\tilde{F}, \qquad (4)$$

where x^A means an extra-dimension coordinate. Assuming that the vacuum expectation value of the χ field does not depend on the extra-dimension coordinates, we obtain

$$f_{\rm PQ}\sim\sqrt{V_m}\langle\chi\rangle\sim r_n^{m/2}M_*^{1+m/2}\sim M_*\left(\frac{M_{pl}}{M_*}\right)^{m/n}$$
$$\sim 10^{3(1+5m/n)}M_{\rm TeV}^{1-m/n}\ \text{GeV}. \qquad (5)$$

Here we have defined the 4D axion field as $a=\sqrt{V_m}\tilde{a}(x^A=0)$ and assumed that the size r_n is common for all extra dimensions.

A lower bound of $f_{\rm PQ}$ comes from astrophysical observations, e.g., red giant and supernova cooling by axion emission. It is known that $f_{\rm PQ}$ should be larger than 10^9 GeV. In extra dimension physics, Kaluza-Klein (KK) modes also contribute to supernova cooling if their masses are smaller than the core temperature (~30 MeV). We discuss this in the next section.

To have 10^9 GeV $<f_{\rm PQ}\leq10^{15}$ GeV, we need $2/5<m/n\leq4/5$. Possible sets of (m,n) with $f_{\rm PQ}$ where $M_*=1$ and 10 TeV can be found in Tables I and II.

III. LABORATORY AND ASTROPHYSICAL CONSTRAINTS

Experiments on detecting an axion from the nuclear reactor and the sun give bounds on PQ scale $<10^4$ GeV (lab), 10^4 GeV $<f_{\rm PQ}<10^6$ GeV (Sun). However, this limit strongly depends on the photon axion coupling and the method of detecting the axion. Furthermore the laboratory bound is much weaker than the astrophysical bound.

The strongest bound from astrophysics is supernova cooling. In SN 1987A observations, it was calculated that

$$\frac{1}{f_{\rm PQ}^2}<10^{-18}\ \text{GeV}^{-2}. \qquad (6)$$

Since the axion KK modes interact exactly the same way as the conventional axion, the effective interaction of the KK modes at the core temperature $T\simeq30$ MeV is

$$\frac{1}{f_{\rm PQ}^2}\times(Tr_n)^m\sim\frac{T^m}{M_*^{m+2}}<10^{-18}\ \text{GeV}^{-2}. \qquad (7)$$

This gives a bound on the fundamental scale:

$$M_*>(10^{18}\times0.03^m)^{\frac{1}{m+2}}\ \text{GeV}. \qquad (8)$$

For $m=1$, $M_*>300$ TeV and for $m=2$, $M_*>5$ TeV.[1] $M_*\sim1$ TeV is allowed only if $m>2$. This suggests that the number of extra dimensions should be at least 3 to include PQ mechanism as a solution of the strong *CP* problem in TeV scale gravity model.

In the near future, high energy accelerator experiments may probe the axion KK mode emission. The graviton KK mode signals in a collider were discussed in detail recently [14]. The scattering cross section of the graviton KK mode emission from high energy scattering with center of mass frame energy $\sqrt{s}$ is

[1]It was argued that there arise large quantum corrections in the case of one extra dimension in general, which would destabilize the gauge hierarchy [13]. This argument would exclude the $m=1$ case.

$$\sigma \propto \frac{1}{M_{pl}^2}(\sqrt{s}r_n)^n \sim \left(\frac{\sqrt{s}}{M_*}\right)^{n+2}\frac{1}{s}, \tag{9}$$

while the KK axion production is

$$\sigma \propto \frac{1}{f_{PQ}^2}(\sqrt{s}r_n)^m \sim \left(\frac{\sqrt{s}}{M_*}\right)^{m+2}\frac{1}{s}. \tag{10}$$

Since the energy dependence of the axion KK mode cross section is different from the graviton KK mode cross section, it might be possible to detect this difference at TeV scale collider experiments.

IV. THERMAL PRODUCTION OF AXION KK MODE

Since the axions dwell in the extra-dimensional brane, their masses are proportional to r_n^{-1} and they have stronger couplings than the graviton KK modes to the normal matters in general cases. If there is no hidden particle which couples to the axion or if the mass of the KK mode axion is lower than the sum of three pion masses ~ 500 MeV, the main decay channel of light KK axion is to two photons. The lifetime of each KK mode for a given (n,m) can be found in Tables I and II:

$$\Gamma_{a_{KK}\to 2\gamma} \simeq \frac{C_{a\gamma}^2}{64\pi}\left(\frac{\alpha}{\pi}\right)^2 \frac{m_A^3}{f_{\rm PQ}^2} \sim 3\times 10^{-8} C_{a\gamma}^2 \frac{m_A^3}{f_{\rm PQ}^2}, \tag{11}$$

where m_A is the mass of the axion KK mode and $C_{a\gamma}$ is the model dependent axion-photon coupling which is usually within 0.1 to 1. This decay can be cosmologically dangerous. For instance, for $f_{\rm PQ}=10^{12}$ GeV and $m_A=1$ MeV, the lifetime of the KK mode is $\tau_A \sim 10^{17}$ sec, which is about the age of the universe.

The graviton KK modes have similar cosmological problems because they can overclose our universe or decay into the photons at a late stage of cosmological evolution. Originally it was suggested that a ''fat brane'' [3] can solve the cosmological problems by absorbing most of the decay products of the KK modes. However, massless particles in the higher-dimensional brane *are not* massless in our four-dimensional universe since they have momenta in the extra dimensions which appear masses in our universe [5]. Another way to avoid this difficulty is assuming a large number of four-dimensional lattices in the bulk. But we need at least 10^6 *empty* universes. Or we should assume the existence of a four-dimensional hidden sector which has 10^6 times more degrees of freedom than those of the standard model, while they should not be produced significantly by the reheating process after inflation.

If we add an axion as a ''brane particle'' in this model, the graviton ''bulk particle'' will decay to the ''brane'' axion more efficiently, since its decay width will be enhanced by a factor $(M_G r_n)^m$. The graviton KK mode with mass m_G will have decay width to the axion

$$\Gamma(g_{KK}\to 2a) \sim \mathcal{O}(10^{-3})\frac{m_G^3}{M_{pl}^2}\times (m_G r_n)^m. \tag{12}$$

After some period, instead of the massive graviton KK fields, we will have the axion KK fields with about same masses. Since the massive axion KK mode can decay into the photon pairs and the lifetime of the graviton KK mode becomes much shorter, the primordial graviton KK mode will not overclose the universe. Instead, it will contribute to the cosmological background radiation. This can be a severe constraint to the axion model. Also axions can be produced thermally during the reheating process. For these reasons, we have to check whether the axion model can survive the cosmological constraints.

To estimate the constraints for various cases, we calculate the amount of the thermal axion produced at the reheating temperature T_R and the axion KK modes from the graviton KK modes decay. In Appendix A, we derived the Boltzmann equation for the yield $Y=\rho/s$ for four different sources of the axion KK modes: (I) decay of graviton KK modes from the inverse decay $(2\gamma, e^+e^-, \overline{\nu}\nu \to g_{KK})$; the energy of the KK modes are concentrated on $m_A \sim T_R$; (II) decay of graviton KK modes produced from the scattering $(e\gamma \to e g_{KK}, e^+e^- \to \gamma g_{KK})$; this contribution is significant only if $m_A < T_R$; (III) the axion KK mode from the pion scattering $(\pi\pi \to \pi a_{KK})$; this process dominates if $T_R > 10$ MeV; (IV) the axion KK mode from the two photon inverse decay $(2\gamma \to a_{KK})$; this gives a significant contribution when $m_A \sim T_R$.

For each case,

$$Y_1 \simeq 3\times 10^{-23}\left(\frac{T_R}{100\ \text{MeV}}\right) A_1, \tag{13}$$

$$Y_2 \simeq 2\times 10^{-23}\left(\frac{T_R}{100\ \text{MeV}}\right) A_2, \tag{14}$$

$$Y_3 \simeq 6\times 10^{-10}\left(\frac{10^{12}\ \text{GeV}}{f_{\rm PQ}}\right)^2 \left(\frac{T_R}{100\ \text{MeV}}\right)^3 A_3, \tag{15}$$

$$Y_4 \simeq 2\times 10^{-16}\left(\frac{10^{12}\ \text{GeV}}{f_{PQ}}\right)^2 \left(\frac{T_R}{100\ \text{MeV}}\right) A_4, \tag{16}$$

where

$$A_1 = \left(\frac{10}{g_*(T_R)}\right)^{3/2}\left(\frac{m_A}{T_R}\right)^3, \tag{17}$$

$$A_2 = \left(\frac{10}{g_*(T_R)}\right)^{3/2}\left[\ln\left(\frac{T_R^3}{m_A^2 m_e}\right) - 0.8\right], \tag{18}$$

$$A_3 = C_{a\pi}^2\left(\frac{10}{g_*(T_R)}\right)^{3/2}\left(\frac{I(T_R)}{1000}\right), \tag{19}$$

$$A_4 = C_{a\gamma}^2 \left(\frac{10}{g_*(T_R)} \right)^{3/2} \left(\frac{m_A}{T_R} \right)^3. \tag{20}$$

For the details of these calculations and the definition of function $I(T)$, see Appendix A [we present the numerical plot of $I(T)$ in Fig. 5].

V. COSMOLOGICAL CONSTRAINTS

In this section we would like to discuss various cosmological constraints on the model for a given (m,n) set. Notice that in Tables I and II, $n \geqslant 5$ in both the $M_* = 1$ and 10 TeV cases and $n=4$ in the 1 TeV case are cosmologically safe if T_R is low enough (~ 1 MeV). If the minimal KK mode mass is greater than 1 MeV, the KK modes are not generated in the thermal bath of such a low reheating temperature. On the other hand $m \leqslant 2$ in $M_* = 1$ TeV and $m=1$ in $M_* = 10$ TeV is forbidden by the astrophysical bound. The cases $n=4, m=3$ at $M_* = 1$ TeV and $n=3, m=2$ at $M_* = 10$ TeV are not trivially allowed or ruled out by the cosmological constraints. Details on these cases are discussed in Appendix B.

A. Big bang nucleosynthesis

At the temperature of the universe around 1 MeV, it is required that there should not be additional particles which contribute to the energy density significantly. Otherwise ^{4}He would be produced more than what is observed now because the universe should expand faster than the standard scenario.

We apply a rather loose bound that the energy density contribution by the KK mode should be smaller than one neutrino energy density at $T=1$ MeV. At high T_R (>10 MeV), axion KK mode production dominated by process III. In case III, one may practically have maximal mass

$$m_1 \equiv \max\{m_\pi, T_R\} \tag{21}$$

for the KK mode which is produced in the thermal bath.

Thus the bound from big bang nucleosynthesis (BBN) is

$$\begin{aligned} \frac{\rho_A}{s}\Big|_{\rm BBN} &\simeq D \int_{m_0}^{m_1} dm_A \times (m_A r_n)^m Y_A \simeq m_1 Y_3 (m_1 r_n)^m \\ &\simeq 2\times 10^{-3} \frac{M_{pl} T_R^3 m_1^{m+1}}{M_*^{m+2} f_\pi^2} A_3 < 0.1 \ \text{MeV}, \end{aligned} \tag{22}$$

where D is normalization constant which is equal to m in torus compactification with universal distance r_n. In our calculations we restrict ourselves to this case.

If $T_R \sim m_\pi$, approximately

$$M_* > 10^{20/(m+2)} \times m_\pi. \tag{23}$$

For $m=1$, this reads $M_* > 600$ TeV, and for $m=2$, $M_* > 15$ TeV. But if the reheating temperature is as low as 10 MeV, this bound is not important since $A_3(T_R)$ is suppressed exponentially.

B. Overclosure of Universe

The total energy of the axion KK modes at present must not exceed the critical density:

$$\rho_A < \rho_c = 3\times 10^{-6} s_0 h^2 \ \text{MeV}, \tag{24}$$

where $s_0 \simeq 3000$ cm^{-3} is the entropy of the present universe.

For the case that the KK modes decay into some relativistic particles, we can divide the bound in two parts; decay before the present time and do not decay until now:

$$\begin{aligned} \frac{\rho_A}{s_0} &\simeq D \int_{m_0}^{m_2} dm_A (m_A r_n)^m Y_A + D \int_{m_2}^{m_1} dm_A (m_A r_n)^m \frac{Y_A T_0}{T(m_A)} \\ &< 3\times 10^{-6} h^2 \ \text{MeV}, \end{aligned} \tag{25}$$

where the axion KK mode with mass m_2 decays at the present time, and m_1 is defined in Eq. (21).

C. Light element destruction

Energetic photons from heavy axion KK modes which decay after 10^4 sec can destroy the light elements made during the nucleosynthesis. Therefore there are several bounds on the density of the KK modes which weigh 10 MeV. They are [15]

$$\frac{\rho_A}{s} \leqslant 10^{-12} \ \text{GeV} \tag{26}$$

for $\tau_A \geqslant 10^7$ sec,

$$\frac{\rho_A}{s} \leqslant 10^{-6} - 10^{-10} \ \text{GeV} \tag{27}$$

for $10^4 < \tau_A \leqslant 10^7$ sec. Usually these bounds are less important than cosmological microwave background bound given below.

D. Cosmological microwave background radiation

If the massive KK modes decay after $\tau_A \geqslant 10^6$ sec but before the recombination era, the produced photons may give a distortion of the cosmological microwave background radiation (CMBR). The Cosmic Background Explorer (COBE) observations give a bound [16]

$$\frac{\Delta \rho_\gamma}{s} \leqslant 2.5\times 10^{-5} T_D, \tag{28}$$

where T_D is the temperature at the KK mode decay.

E. Diffuse photon background

Observations of diffuse photon backgrounds at the present universe give upper bounds on additional contributions to the photon spectrum. For example, for the energy range 800 keV $< E <$ 30 MeV [17]

$$\frac{d\mathcal{F}}{d\Omega} \simeq E \times A (E/E_0)^{-\alpha} \simeq 78 \left(\frac{E}{1 \ \text{keV}} \right)^{-1.4}. \tag{29}$$

Constraints on other ranges of the photon energy can be found, e.g., in Ref. [18].

The theoretical prediction is

$$\frac{d\mathcal{F}}{d\Omega}=\frac{n_A c}{4\pi}\times \mathrm{Br} \tag{30}$$

for the lifetime of the KK mode shorter than the age of the universe, and

$$\frac{d\mathcal{F}}{d\Omega}\sim \mathrm{Br}\times\frac{n_A c}{4\pi}\frac{\Gamma_{a_{KK}\to 2\gamma}}{\mathrm{Br}H_0}\left(\frac{2E}{m_A c^2}\right)^{3/2}(m_A r_n)^m \tag{31}$$

for its lifetime longer than the age of the universe [19]. Here we have introduced Br as a branching ratio of axion decay into two photons.

In our brane picture, there is *a priori* no reason that the axion brane contains only our four-dimensional wall. Rather it will be natural to imagine that there is a parallel universe(s) or another four-dimensional wall in the brane. Or one can just imagine that there are some unknown particles on our wall itself. Then one can consider the situation that thermally produced KK modes of graviton will mainly decay into the axion in the brane and axions both from the graviton decay and from the thermal production will decay into the parallel wall if this wall has some kind of QCD and/or U(1) type of interactions. [Or it can decay into some hidden QCD/U(1) fields in our universe.] Since the axion decay width is highly suppressed by $(\alpha/\pi)^2$ with α being the fine structure constant, it is easy to get a low branching ratio to decay into the photon; in other words, most of the axions decay into the other wall (invisible section) if the coupling constant, the color factor of the other gauge interaction, and/or the number of the fermions with PQ charges in the decay loop diagram in the other wall (invisible section) are large enough.

F. Results

Here we summarize the results we obtained. The reader should refer to Appendix B for more detail. (We approximate $h=0.7$.)

I. $n=4$, $m=3$ and $M_*=1$ TeV case

(1) BBN bound

$$T_R<90\,A_3^{-1/3}\ \mathrm{MeV}\simeq 80\ \mathrm{MeV}. \tag{32}$$

(2) Overclosure bound

$$T_R<12\left(\frac{C_{a\gamma}^2}{A_3^2\mathrm{Br}}\right)^{1/6}\mathrm{MeV}\simeq 30\ \mathrm{MeV}\quad(\text{for}\quad \mathrm{Br}=1). \tag{33}$$

(3) CMBR bound (for $m_A\geqslant 100$ MeV and $T_R>10$ MeV)

$$T_R<2\times 10^{-2}\left(\frac{C_{a\gamma}^2}{A_3^2\mathrm{Br}^3}\right)^{1/6}\ \mathrm{MeV}. \tag{34}$$

(4) Diffused photon bound, for $T_R>10$ MeV,

$$T_R<2\times 10^{-2}\left(\frac{\mathrm{Br}}{C_{a\gamma}}\right)^{-0.63}\left(\frac{m_A}{10\ \mathrm{MeV}}\right)^{-0.53}A_3^{-1/3}\ \mathrm{MeV}, \tag{35}$$

for $T_R<10$ MeV,

$$T_R<0.3\,\mathrm{Br}^{-0.73}\ \mathrm{MeV}, \tag{36}$$

where $\mathrm{Br}<\Gamma_{a\to 2\gamma}/H_0$, and

$$T_R<5C_{a\gamma}^{-0.48}\ \mathrm{MeV}, \tag{37}$$

where $\mathrm{Br}>\Gamma_{a\to 2\gamma}/H_0$.

II. $n=3$, $m=2$ and $M_*=10$ TeV case

(1) BBN bound

$$T_R<100A_3^{-1/3}\ \mathrm{MeV}\simeq 90\ \mathrm{MeV}. \tag{38}$$

(2) Overclosure bound

$$T_R<28\left(\frac{C_{a\gamma}^2}{A_3^2\mathrm{Br}}\right)^{1/6}\ \mathrm{MeV}\simeq 40\ \mathrm{MeV}\quad(\text{for}\quad \mathrm{Br}=1). \tag{39}$$

(3) CMBR bound

$$T_R<4\times 10^{-2}\left(\frac{C_{a\gamma}^2}{A_3^2\mathrm{Br}^3}\right)^{1/6}\ \mathrm{MeV}. \tag{40}$$

(4) Diffused photon bound for $T_R>10$ MeV

$$T_R<3\times 10^{-2}\left(\frac{\mathrm{Br}}{C_{a\gamma}}\right)^{-0.63}\left(\frac{m_A}{10\ \mathrm{MeV}}\right)^{-0.2}A_3^{-1/3}\ \mathrm{MeV}, \tag{41}$$

for $T_R<10$ MeV,

$$T_R<0.1\,\mathrm{Br}^{-1.2}\ \mathrm{MeV}, \tag{42}$$

where $\mathrm{Br}<\Gamma_{a\to 2\gamma}/H_0$, and

$$T_R<3\ \mathrm{MeV}, \tag{43}$$

where $\mathrm{Br}>\Gamma_{a\to 2\gamma}/H_0$. In both cases ($n=4$, $m=3$, and $M_*=1$ TeV and $n=3$, $m=2$, and $M_*=10$ TeV), lifetimes of axion KK modes up to $m_A\sim 1$ GeV are quite long and so the constraint from the light element destruction will not give any further bound.

We also performed computer calculations on both the $n=4$, $m=3$, and $M_*=1$ TeV and the $n=3$, $m=2$, and $M_*=10$ TeV cases. Figures 1 and 2 show that the diffused photon background radiation (DPBR) is the stringent bound if the branching ratio is small. But in an extremely small branching ratio case, CMBR is dominant. This is because the lifetime of the axion KK modes becomes shorter than 10^{13} sec, and so the produced photons will disturb the CMBR spectrum. One can see that BBN is independent of the branching ratio. Note that these behaviors are consistent with what we observed from the analytical computations given above.

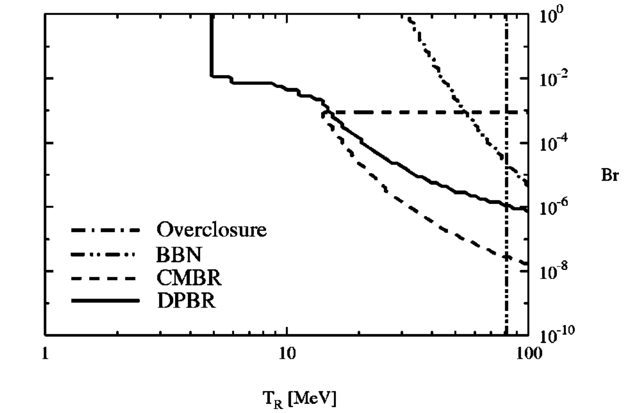

FIG. 1. The bound of T_R and Br for $M_* = 1$ TeV and $n=4,m=3$, The upper and right side of each line is excluded region.

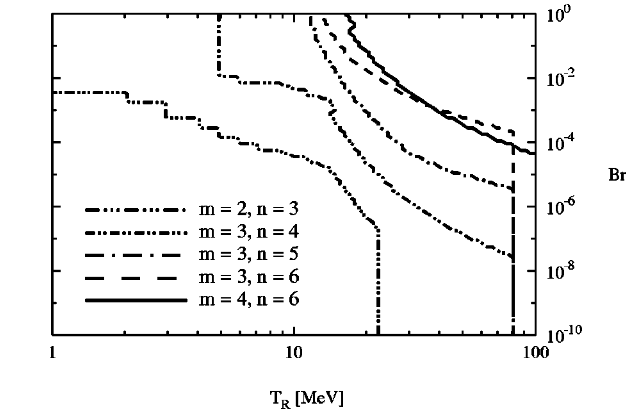

FIG. 3. The bound of T_R and Br for $M_* = 1$ TeV. The upper and right side of each line is the excluded region.

In Figs. 3 and 4, we present combined cosmological limits for all possible combinations of n,m in Tables I and II. We find that the reheating temperature is allowed to be significantly large if the branching ratio to the photon is small enough.

VI. CONCLUSIONS

In this paper, we discussed the axion model in the extra dimensions whose PQ scale lies in an intermediate scale $f_{\rm PQ} \lesssim 10^{15}$ GeV. This intermediate scale can be obtained by introducing a $(3+m)$-dimensional brane in the $(4+n)$-dimensional bulk.

If we include the axion as a brane particle, it will change the phenomenology of the extra dimension physics, especially cosmology. Since the graviton KK mode will decay into the axion KK mode, the overclosure problem is not as serious as the original model of Arkani-Hamed *et al.* On the other hand, the argument from stars and supernova cooling will give a more strict bound on the axion production. Among other things, the most severe cosmological bound comes from photon emission through the decays of the KK modes of the axion. We found that the astrophysical argument restricts the number of the dimensionality of the subspacetime where the axion lives: $m>2$ for $M_* = 1$ TeV and $m>1$ for $M_* = 10$ TeV. The latter cosmological argument requires quite a low reheating temperature after the inflation.

To lift this bound, we can introduce the hidden matter/gauge fields to another four-dimensional wall (or even to our wall itself) which has much stronger coupling to axion and/or much more generations of particles (or maybe much lower QCD phase transition scale, etc). This can significantly lower the branch ratio of the axion KK mode decay into photons.

The whole picture can be used to improve the original fat-brane idea. This higher-dimensional object plays a role of an absorber of the KK graviton modes. If the fat brane couples to four-dimensional wall(s) with interaction stronger than gravity, the produced particles in the fat brane may then decay into relativistic particles on the four-dimensional wall(s). This mechanism can solve the problem of the overclosure of the universe by the KK modes. Note that the fat-brane particles are not necessarily the axions as we discussed, but can be any other weakly interacting particles living in a higher-dimensional brane. Moreover if the particles produced by the fat-brane particle decays do not contain photons or any other cosmologically dangerous particles, then we can avoid other cosmological problems such as the ones related to the cosmic photon backgrounds. It is

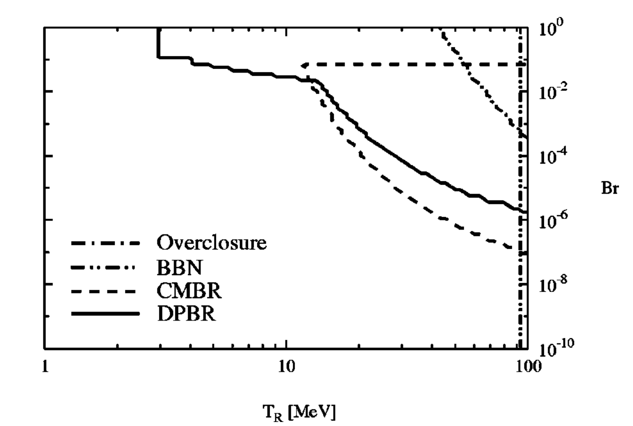

FIG. 2. The bound of T_R and Br for $M_* = 10$ TeV and $n=3,m=2$. The excluded region is the same as in Fig. 1.

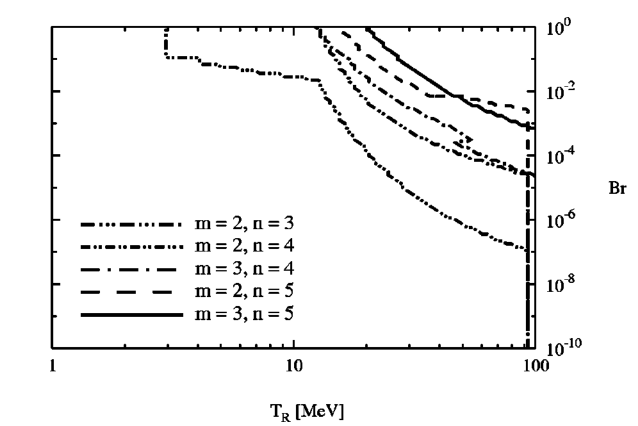

FIG. 4. The bound of T_R and Br for $M_* = 10$ TeV. The excluded region is the same as in Fig. 3.

worth mentioning that some mechanisms of generating small neutrino masses have a similar structure, which will be discussed elsewhere [20].

In this paper, we did not consider an alternative solution to the strong CP problem such as a spontaneous CP breaking model. This typically requires very heavy quarks. If we want to keep all particles carrying standard-model gauge charges in four dimensions, the maximum scale we can have is the fundamental scale, i.e., around TeV. For this reason, this type of model is not favorable in the extra-dimension scheme.

ACKNOWLEDGMENTS

This work was supported in part by the Grant-in-Aid for Scientific Research from the Ministry of Education, Science, Sports, and Culture of Japan, on Priority Area 707 ''Supersymmetry and Unified Theory of Elementary Particles,'' and by the Grant-in-Aid No. 11640246 and No. 98270. S.C. thanks the Japan Society for the Promotion of Science for financial support.

APPENDIX A: ESTIMATION OF THE PRIMORDIAL AXION KK MODE DENSITY

The production rate of the axion KK mode from initial particles i and j can be calculated from the Boltzmann equation:

$$\dot{n}_A+3H(t)n_A=\sum_{ij}\langle\sigma v\rangle_{ij}n_i n_j, \tag{A1}$$

where $n_A, H(t), \sigma, n_i$ represent axion KK mode number density, Hubble constant, scattering cross section, and number density of initial particle i.

Using the relation $t=0.5H(T)^{-1}$, we can convert the time parameter to inverse temperature $x\equiv m/T$, where $H(T)\simeq(g_*/10)^{1/2}T^2/M_{pl}$. If we assume that the KK modes are produced by the particles in equilibrium, we can rewrite Eq. (A1) with the yield $Y\equiv n_A/s$. (s is entropy density $s=2\pi^2/45 g_{*s}T^3$, where $g_{*s}\simeq g_*$ is approximately 10 for 1 MeV $<T<100$ MeV.)

$$\frac{dY}{dx}=\frac{x}{H(m)}\Gamma_A Y_{\rm eq}, \tag{A2}$$

where

$$\Gamma_A=n_{\rm eq}\sum_{ij}\langle\sigma v\rangle_{ij}. \tag{A3}$$

A similar equation can be derived for the inverse decay case:

$$\frac{dY}{dx}=\frac{x}{H(m)}\Gamma(a_{KK}\to{\rm ALL})\left\langle\frac{m_A}{E_A}\right\rangle Y_{\rm eq}. \tag{A4}$$

The yield of the KK mode at the equilibrium $Y_{\rm eq}$ is about $0.28/g_{*s}$ when initial particles are relativistic [or proportional to $\exp(-x)$ if they are nonrelativistic]. After integrating Eqs. (A2) and (A4) from the reheating temperature T_R to present temperature, we will get a result with the form

$$Y\simeq\frac{\Gamma}{H(T_R)}Y_{\rm eq}(T_R)\sim 3\times10^{-2}\left(\frac{10}{g_*(T_R)}\right)^{3/2}\frac{M_{pl}\Gamma}{T_R^2}, \tag{A5}$$

which can be used in most calculations reliably.

Let us estimate the sources of the axion KK mode production. The KK modes of axion can be produced from either the thermal graviton KK mode decay or initial thermal bath. We will classify four relevant cases.

Class I. The KK mode of graviton which has mass around the reheating temperature $m_G\simeq T_R$ generated dominantly through the inverse decay $\gamma\gamma\to g_{KK}$, $\bar{\nu}\nu\to g_{KK}$, and $e^+e^-\to g_{KK}$ [14]

$$\Gamma_{g_{KK}\to2\gamma}=\frac{m_G^3}{80\pi M_{pl}^2}, \tag{A6}$$

$$\Gamma_{g_{KK}\to f\bar{f}}=\frac{m_G^3}{160\pi M_{pl}^2} \tag{A7}$$

for initial spin averaged. For the tensor mode of graviton KK mode we should multiply 5 to Γ. This will generate

$$Y_1\simeq6\times10^{-4}\frac{T_R}{M_{pl}}A_1\simeq3\times10^{-23}\left(\frac{T_R}{100\ \ {\rm MeV}}\right)A_1, \tag{A8}$$

where

$$A_1=\left(\frac{10}{g_*(T_R)}\right)^{3/2}\left(\frac{m_A}{T_R}\right)^3. \tag{A9}$$

Here we approximated that $m_A\simeq m_G$ after the graviton decay. This mode is most abundant at $m_A=T_R$ and decreases quickly if $m_A\ll T_R$.

Class II. The KK modes which have much less mass than $m_G<T_R$ will be produced dominantly by the scattering processes $e^\pm\gamma\to g_{KK}e^\pm$, $e^+e^-\to g_{KK}\gamma$. If we choose a limit that the KK mode mass is less than T_R but greater than the electron mass, we can calculate the interaction rate from the amplitude presented in Ref. [14]:

$$\Gamma_{e^\pm\gamma\to g_{KK}e^\pm}\simeq\langle\sigma v\rangle n_{EQ}\simeq\frac{\alpha}{M_{pl}^2}\left(\ln\frac{T_R^3}{m_G^2 m_e}-\frac{7}{8}\right)\times(0.3)T_R^3, \tag{A10}$$

$$\Gamma_{e^+e^-\to g_{KK}\gamma}\simeq\frac{\alpha}{6M_{pl}^2}\times(0.3)T_R^3. \tag{A11}$$

The yield from scattering is

$$Y_2\simeq2\times10^{-23}\left(\frac{T_R}{100\ \ {\rm MeV}}\right)A_2, \tag{A12}$$

where

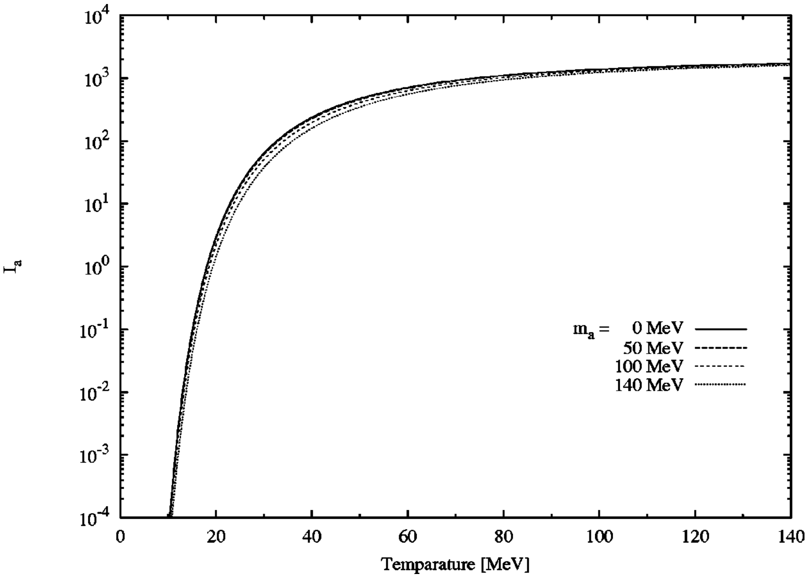

FIG. 5. The function $I(T)$ for various m_A in the KK mode yield III.

$$A_2=\left(\frac{10}{g_*(T_R)}\right)^{3/2}\left[\ln\left(\frac{T_R^3}{m_A^2 m_e}\right)-0.8\right], \tag{A13}$$

for each KK mode with mass m_A. This bound is valid only if A_2 is positive, i.e., the reheating temperature is significantly higher than the KK mode mass.

Class III. Thermal axion produced mainly by pion-pion scattering $\pi^\pm\pi^0\to\pi^\pm a$, $\pi^+\pi^-\to\pi^0 a$, for $T_R>10$ MeV.

$$\Gamma_{2\pi\to a\pi}Y_{EQ}=\frac{3}{1024\pi^5}\frac{C_{a\pi}^2}{f_{PQ}^2 f_\pi^2}T^5\left(\frac{45}{2\pi^2 g_{*s}}\right)\times I(T), \tag{A14}$$

where $C_{a\pi}$ is $(1-z)/3(1+z)$ with $z=m_u/m_d$. In the limit of $m_A=0$, we can use the temperature-dependent function $I(T)$ in Ref. [21]

$$I(T)\equiv\int dx_1 dx_2\frac{x_1^2x_2^2}{y_1y_2}f(y_1)f(y_2)\times\int_{-1}^{1}d\omega\frac{(s-m_\pi^2)^3(5s-2m_\pi^2)}{s^2T^4}, \tag{A15}$$

where $f(y)=1/(e^y-1)$, $x_i=|\vec{p}_i|/T$, $y_i=E_i/T$ $(i=1,2)$, and $s=2[m_\pi^2+T^2(y_1y_2-x_1x_2\omega)]$. We justify using the $m_A=0$ limit by presenting the plot produced by computer (Fig. 5), which shows that the mass dependence of $I(T)$ is indeed small.

$I(T)$ is around 10^3 for $T>50$ MeV, and is suppressed exponentially at $T<10$ MeV. For $T\ll m_\pi$, we can approximate the function $I(T)$:

$$I(T)=\frac{\pi}{8}\left(\frac{3m_\pi}{T}\right)^5\exp\left(-2\frac{m_\pi}{T}\right). \tag{A16}$$

We can estimate thermal axion KK mode yield from pion scattering,

$$Y_3\simeq 2\times10^{-3}\frac{M_{pl}T_R^3}{f_{PQ}^2f_\pi^2}A_3\simeq 6\times10^{-10}\left(\frac{T_R}{100\ \mathrm{MeV}}\right)^3\left(\frac{10^{12}\mathrm{GeV}}{f_{\mathrm{PQ}}}\right)^2A_3, \tag{A17}$$

where

$$A_3=\left(\frac{10}{g_*(T_R)}\right)^{3/2}C_{a\pi}^2\left(\frac{I(T_R)}{1000}\right). \tag{A18}$$

Class IV. Thermal axion can be generated through the two photon inverse decay,

$$\Gamma_{a_{KK}\to 2\gamma}\simeq\frac{C_{a\gamma}^2}{64\pi}\left(\frac{\alpha}{\pi}\right)^2\frac{m_A^3}{f_{PQ}^2}\simeq 2.7\times10^{-8}C_{a\gamma}^2\frac{m_A^3}{f_{PQ}^2}, \tag{A19}$$

which leads to

$$Y_4\simeq 8\times10^{-10}\frac{M_{pl}T_R}{f_{PQ}^2}A_4\simeq 2\times10^{-16}\left(\frac{10^{12}\ \mathrm{GeV}}{f_{\mathrm{PQ}}}\right)^2\left(\frac{T_R}{100\ \mathrm{MeV}}\right)A_4, \tag{A20}$$

where

$$A_4=C_{a\gamma}^2\left(\frac{10}{g_*(T_R)}\right)^{3/2}\left(\frac{m_A}{T_R}\right)^3. \tag{A21}$$

This shows that the axion produced from the inverse decay will dominate over the axion from the pion scattering III, if $T_R<10$ MeV. The thermal axion can be produced with photon electron scattering process II; we will ignore this axion unless the reheating temperature is very low (compared with the process IV, it should be less than a few MeV).

To estimate the energy density of the axion KK mode for a given T, multiply the entropy density at the temperature $s(T)$ and the mass of axion KK mode to the yield. We should count the number of KK modes for the allowed energy range, $(Er_n)^N$ where E is typically T_R. But for case III, $T_R=\max\{m_\pi,T_R\}$. N is the dimension of the bulk where the produced particle exists. $N=n$ and $N=m$ for case {I, II} and {III, IV}, respectively.

APPENDIX B: CALCULATION OF THE COSMOLOGICAL BOUNDS

In this section we describe the details of these calculations for two nontrivial cases, $n=4,m=3$ at $M_*=1$ TeV and $n=3,m=2$ at $M_*=10$ TeV, though we have calculated with a computer all relevant cosmological bounds with allowed sets of m and n and presented these results in Figs. 3 and 4.

This is the case with relatively low $T_R\leqslant\mathcal{O}(100)$ MeV, because of large $f_{\rm PQ}\sim10^{14}$ GeV. Since the lifetime of the axion KK mode is quite long for $m_A<100$ MeV and CMBR is a stronger bound than the light element breaking bound, the bound from light element breaking is not relevant in these cases.

1. Big bang nucleosynthesis bound

The axion KK mode with mass below 100 MeV cannot decay before 1 sec and will be restricted by BBN bound on neutrino species. Let us assume that $T_R>10$ MeV; then

$$\begin{aligned}\left.\frac{\rho_A}{s}\right|_{\rm BBN} &\simeq D\int_{m_0}^{m_1} dm_A\ \ (m_A r_n)^m Y_A\\ &\simeq m_1 Y_3 (m_1 r_n)^m \sim 2\times10^{-3}\frac{M_{pl}T_R^3 m_1^{m+1}}{M_*^{m+2}f_\pi^2}A_3\\ &<0.1\ \ \text{MeV},\end{aligned}\tag{B1}$$

where $m_1=\max\{m_\pi,T_R\}$ and D is a normalization constant defined as

$$\begin{aligned}&D\int_{E_1}^{E_2} dm_A\times m_A^{m-1}r_n^m\\ &\qquad\equiv\text{No. of KK modes between } E_1 \text{ and } E_2.\end{aligned}$$

We used the torus compactification with uniform distance approximation $V_n=r_n^n, V_m=r_n^m$ so that $D=m$. This gives a bound for $n=4,m=3$ at 1 TeV

$$T_R<90A_3^{-1/3}\ \ \text{MeV}\simeq80\ \ \text{MeV}\tag{B2}$$

and for $n=3,m=2$ at 10 TeV

$$T_R<100A_3^{-1/3}\ \ \text{MeV}\simeq90\ \ \text{MeV}.\tag{B3}$$

2. Overclosure bound

We can divide this bound by the KK mass m_2. If $m_A>m_2$, its lifetime is shorter than the age of the universe; otherwise it will remain as cold dark matter. For $T_R<10$ MeV, the overclosure is not a problem in our region of interest. Therefore Y_3 is most dominant source of axion KK mode in this case. The total cold dark matter density will be

$$\frac{\rho_A}{s_0}\simeq D\int_{m_0}^{m_2} dm_A\ \ (m_A r_n)^m Y_3<3\times10^{-6}h^2\ \ \text{MeV},\tag{B4}$$

where $m_2\simeq10$ MeV, $m_0=r_n^{-1}\simeq30$ keV for $n=4,m=3$ at 1 TeV and $m_2\simeq1$ MeV, $m_0\simeq5$ keV for $n=3,m=2$ at 10 TeV. Using Eq. (A17), Eq. (B4) becomes

$$2\times10^{-3}m_2^{m+1}\frac{M_{pl}T_R^3}{f_\pi^2 M_*^{m+2}}A_3<3\times10^{-6}h^2\ \ \text{MeV}.\tag{B5}$$

We can get the bound on T_R for $n=4,m=3$ at 1 TeV,

$$T_R<70\left(\frac{h}{0.7}\right)^{2/3}A_3^{-1/3}\ \ \text{MeV}\tag{B6}$$

and for $n=3,m=2$ at 10 TeV

$$T_R<350\left(\frac{h}{0.7}\right)^{2/3}A_3^{-1/3}\ \ \text{MeV}.\tag{B7}$$

We can consider another situation in which the axion KK mode can decay into some relativistic dark matter X dominantly. In this case,

$$\frac{\rho_A}{s_0}\simeq D\int_{m_2}^{m_1} dm_A\,(m_A r_n)^m Y_3\frac{T_0}{T(m_A)}<3\times10^{-6}h^2\ \ \text{MeV},\tag{B8}$$

where $T_0\sim2\times10^{-13}$ GeV is the current temperature of the universe and

$$T(m_A)\simeq\sqrt{\Gamma_{a_{KK}\to X}M_{pl}}\sim2\times10^{-4}C_{a\gamma}\text{Br}^{-1/2}\frac{m_A^{3/2}M_{pl}^{1/2}}{f_{PQ}}\tag{B9}$$

is the temperature when axion KK mode with mass m_A decays, where Br is the branching ratio for

$$\text{Br}=\frac{\Gamma_{a_{KK}\to2\gamma}}{\Gamma_{a_{KK}\to X}+\Gamma_{a_{KK}\to2\gamma}}\simeq\frac{\Gamma_{a_{KK}\to2\gamma}}{\Gamma_{a_{KK}\to X}},\tag{B10}$$

which is the case where the majority of axion KK modes decays into the invisible relativistic dark matter X. Then Eq. (B8) becomes

$$\begin{aligned}&2\times10^{-9}\text{Br}^{1/2}\frac{f_{\rm PQ}}{C_{a\gamma}}\frac{M_{pl}^{1/2}T_R^3 m_1^{m-1/2}}{M_*^{m+2}f_\pi^2}A_3\ \ \text{MeV}<3\\ &\qquad\times10^{-6}h^2\ \ \text{MeV}.\end{aligned}\tag{B11}$$

For the reasonable range of Br, we can set $m_1=m_\pi$. For $n=4,m=3$ at 1 TeV,

$$T_R < 12\,\mathrm{Br}^{-1/6}\left(\frac{h}{0.7}\right)^{2/3} C_{a\gamma}^{1/3} A_3^{-1/3}\mathrm{MeV}$$
$$\simeq 30\ \mathrm{MeV}\ (\text{for}\ \ \mathrm{Br}=1), \tag{B12}$$

and for $n=3, m=2$ at 10 TeV,

$$T_R < 28\,\mathrm{Br}^{-1/6}\left(\frac{h}{0.7}\right)^{2/3} C_{a\gamma}^{1/3} A_3^{-1/3}\mathrm{MeV}$$
$$\simeq 40\ \mathrm{MeV}\ (\text{for}\ \ \mathrm{Br}=1). \tag{B13}$$

3. Cosmological microwave background radiation

CMBR bound is the most severe during the time period $10^6 < \tau_A < 10^{12}$ sec. The bound

$$\frac{\Delta\rho_\gamma}{s} \leqslant 2.5\times 10^{-5} T_D \tag{B14}$$

is actually weaker than other constraints if the branching ratio Br is large. In this case, the reheating temperature should be relatively small $T_R < 10$ MeV. But if Br is very small, it gives a stronger bound than other cosmological constraints. T_D is the same as Eq. (B9). If we assume $T_R > 10$ MeV, and set $\Delta\rho_\gamma \simeq \mathrm{Br}\times\rho_A(T_D)$, Eq. (B14) becomes

$$2\times 10^{-3}\mathrm{Br}\, m_A^{m+1}\frac{M_{pl}T_R^3}{M_*^{m+2}f_\pi^2}A_3$$
$$\leqslant 5\times 10^{-9} C_{a\gamma}\mathrm{Br}^{-1/2}\frac{M_{pl}^{1/2}m_A^{3/2}}{f_{\mathrm{PQ}}}. \tag{B15}$$

If $T_R \leqslant m_\pi$, the maximal value of KK mode mass is around m_π. This leads to the bound, for $n=4, m=3$ at 1 TeV,

$$T_R < 2\times 10^{-2}\mathrm{Br}^{-1/2}A_3^{-1/3}C_{a\gamma}^{1/3}\ \ \mathrm{MeV}, \tag{B16}$$

and for $n=3, m=2$ at 10 TeV,

$$T_R < 4\times 10^{-2}\mathrm{Br}^{-1/2}A_3^{-1/3}C_{a\gamma}^{1/3}\ \ \mathrm{MeV}. \tag{B17}$$

To have $T_R > 100$ MeV, we need a very small branching ratio $\sim 10^{-7}$. (This bound is not valid if $T_R < 10$ MeV.)

4. Diffused photon background

Since the lifetime of the axion KK mode is longer than 10^{14} sec in the allowed region, we obtain the strongest bound on the reheating temperature from the diffused photon background. Let us consider three cases.

Case 1. $T_R > 10$ MeV: In this case, the majority of KK mode will decay before the present time. The observed bound when 800 keV $< E <$ 30 MeV is

$$\frac{d\mathcal{F}}{d\Omega} < 78\left(\frac{E}{1\,\mathrm{keV}}\right)^{-1.4}\ \ \mathrm{cm}^{-2}\,\mathrm{sr}^{-1}\,\mathrm{sec}^{-1} \tag{B18}$$

and corresponds with the theoretical prediction

$$\frac{d\mathcal{F}}{d\Omega} = \frac{n_\gamma c}{4\pi} \simeq \mathrm{Br}\frac{Y_3 s_0 c}{4\pi}(m_A r_n)^m. \tag{B19}$$

Here $s_0 c \simeq 9\times 10^{13}\mathrm{cm}^{-2}\,\mathrm{sec}^{-1}$. This will give the inequality

$$\mathrm{Br}\times Y_3(m_A r_n)^m < 6\times 10^{-16}\left(\frac{E}{\mathrm{MeV}}\right)^{-1.4}, \tag{B20}$$

where $E \simeq m_A/[2(1+z)]$

$$1+z \simeq 4\times 10^{11}(\Omega_0 h^2)^{-1/3}\left(\frac{\tau_D}{\mathrm{sec}}\right)^{-2/3}. \tag{B21}$$

The time of axion KK mode decay into two photons is

$$\tau_D \simeq \mathrm{Br}\Gamma^{-1}_{a_{KK}\to 2\gamma} \simeq 3.7\times 10^7\mathrm{Br}\frac{f_{PQ}^2}{m_A^3}. \tag{B22}$$

Thus the present energy of diffused photon (for $\Omega_0 \simeq 1$) is

$$E \simeq 10^{-12}\left(\frac{\tau_D}{\mathrm{sec}}\right)^{2/3}\left(\frac{h}{0.7}\right)^{2/3} m_A$$
$$\simeq 10^{-18} C_{a\gamma}^{-4/3}\mathrm{Br}^{2/3}\left(\frac{f_{\mathrm{PQ}}}{\mathrm{GeV}}\right)^{4/3}\left(\frac{h}{0.7}\right)^{2/3}\left(\frac{10\ \ \mathrm{MeV}}{m_A}\right)\ \ \mathrm{MeV}. \tag{B23}$$

Inserting this result into Eq. (B20) and setting $h=0.7$ leads to

$$2\times 10^{-3}\mathrm{Br}\frac{M_{pl}m_A^m T_R^3}{f_\pi^2 M_*^{m+2}}A_3$$
$$< 10^{10}\left(\frac{f_{\mathrm{PQ}}}{\mathrm{GeV}}\right)^{-1.9} C_{a\gamma}^{1.9}\mathrm{Br}^{-0.9}\left(\frac{m_A}{10\ \ \mathrm{MeV}}\right)^{1.4} \tag{B24}$$

and it gives a bound for $n=4, m=3$ at 1 TeV,

$$T_R < 2\times 10^{-2}\left(\frac{\mathrm{Br}}{C_{a\gamma}}\right)^{-0.63}\left(\frac{m_A}{10\ \ \mathrm{MeV}}\right)^{-0.53} A_3^{-1/3}\ \ \mathrm{MeV} \tag{B25}$$

and for $n=3, m=2$ at 10 TeV,

$$T_R < 3\times 10^{-2}\left(\frac{Br}{C_{a\gamma}}\right)^{-0.63}\left(\frac{m_A}{10\ \ \mathrm{MeV}}\right)^{-0.2} A_3^{-1/3}\ \ \mathrm{MeV}. \tag{B26}$$

Case 2. $T_R < 10$ MeV, $\mathrm{Br} < \Gamma_{a_{KK}\to 2\gamma}/H_0$. In this case $Y \simeq Y_4$ and the axion KK mode will decay before the present time. $H_0 \simeq 2\times 10^{-42}h$ GeV is the Hubble constant of the present universe. Then the relation with energy and lifetime becomes the same as Eq. (B24),

$$\mathrm{Br}\,Y_4(m_A r_n)^m = 8\times 10^{-10}\mathrm{Br}\frac{M_{pl} m_A^m T_R}{M_*^{m+2}} A_4 < 10^{10}\left(\frac{f_{\rm PQ}}{\rm GeV}\right)^{-1.9} C_{a\gamma}^{1.9}\mathrm{Br}^{-0.9}\left(\frac{m_A}{10\ \ \rm MeV}\right)^{1.4} \tag{B27}$$

and it gives a bound for $n=4,m=3$ at 1 TeV

$$T_R < 0.06\left(\frac{\rm Br}{C_{a\gamma}}\right)^{-1.9}\left(\frac{m_A}{\rm MeV}\right)^{-1.6} A_4^{-1}\ \ \mathrm{MeV} \tag{B28}$$

and in $m_A \sim T_R$ limit,

$$T_R < 0.3\,\mathrm{Br}^{-0.73}\mathrm{MeV}. \tag{B29}$$

For $n=3,m=2$ at 10 TeV

$$T_R < 0.03\left(\frac{\rm Br}{C_{a\gamma}}\right)^{-1.9}\left(\frac{m_A}{\rm MeV}\right)^{-0.6} A_4^{-1}\ \ \mathrm{MeV}, \tag{B30}$$

and in the $m_A \sim T_R$ limit,

$$T_R < 0.1\,\mathrm{Br}^{-1.2}\ \ \mathrm{MeV}. \tag{B31}$$

Case 3. $T_R < 10$ MeV, $\mathrm{Br} > \Gamma_{a_{KK}\to 2\gamma}/H_0$. In this case, axion KK modes will not decay before the present time. Then the theoretical prediction becomes (see references, for instance, Sec. 5.5 in Ref. [19])

$$\frac{d\mathcal{F}}{d\Omega} \simeq \mathrm{Br}\times\frac{Y_4 s_0 c}{4\pi}\frac{\Gamma_{a_{KK}\to 2\gamma}}{\mathrm{Br}H_0}\left(\frac{2E}{m_A}\right)^{3/2}(m_A r_n)^m \tag{B32}$$

and this will lead to

$$2\times 10^{-17} C_{a\gamma}^2 \frac{M_{pl} T_R}{M_*^{m+2} H_0}\frac{m_A^{m+3/2} E^{3/2}}{f_{PQ}^2} A_4 < 6\times 10^{-16}\left(\frac{E}{\rm MeV}\right)^{-1.4}. \tag{B33}$$

Approximately, for $n=4,m=3$ at 1 TeV,

$$T_R\left(\frac{m_A}{\rm MeV}\right)^{9/2} < 2\times 10^6\left(\frac{h}{0.7}\right) C_{a\gamma}^{-2} A_4^{-1}\left(\frac{E}{\rm MeV}\right)^{-2.9}\ \ \mathrm{MeV}. \tag{B34}$$

We can approximate $m_A \sim T_R \sim E$, then $A_4 \sim C_{a\gamma}^2$ and

$$T_R < 5.5\left(\frac{h}{0.7}\right)^{0.12} C_{a\gamma}^{-0.48}\ \ \mathrm{MeV}. \tag{B35}$$

In the case $n=3,m=2$ at 10 TeV, the majority of the thermal axion KK mode with $T_R > 1$ MeV will decay before the present time. The condition $\mathrm{Br} \simeq \Gamma_{a_{KK}\to 2\gamma}/H_0$ and Eq. (B31) will determine the reheating temperature

$$T_R < 2\sim 3\ \ \mathrm{MeV}. \tag{B36}$$

We performed full calculations for all relevant sets of m and n with a computer and got consistent results. For instance, you can see three different regions of DPBR bound in Figs. 1 and 2.

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