Direct CP, T, and/or CPT violations in the K^0-$\overline{K^0}$ system: Implications of the recent KTeV results on 2π decays

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The recent results on the CP violating parameters $\mathrm{Re}(\varepsilon'/\varepsilon)$ and $\Delta\phi \equiv \phi_{00}-\phi_{+-}$ reported by the KTeV Collaboration are analyzed with a view to constrain CP, T, and CPT violations in decay processes. Combined with some relevant data compiled by the Particle Data Group, we find $\mathrm{Re}(\varepsilon_2-\varepsilon_0)=(0.85\pm 3.11)\times 10^{-4}$ and $\mathrm{Im}(\varepsilon_2-\varepsilon_0)=(3.2\pm 0.7)\times 10^{-4}$, where $\mathrm{Re}(\varepsilon_I)$ and $\mathrm{Im}(\varepsilon_I)$ represent respectively CP/CPT and CP/T violations in the decay of K^0 and $\overline{K^0}$ into a 2π state with isospin I.

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Although it has been well established since 1964 [1] that CP symmetry is violated in the K^0-$\overline{K^0}$ system, the origin or mechanism of CP violation is not well understood yet, on the one hand, and no evidence of CP violation has been established in any other systems or processes, on the other hand. Experimental, phenomenological, and theoretical studies of this and related (i.e., T and CPT) symmetries need to be continued with much effort.

The KTeV Collaboration [2] recently reported

$$\mathrm{Re}(\varepsilon'/\varepsilon)=(2.80\pm 0.41)\times 10^{-3}, \tag{1a}$$

$$\Delta\phi=(0.09\pm 0.46)^\circ, \tag{1b}$$

and claimed that $\mathrm{Re}(\varepsilon'/\varepsilon)\neq 0$ definitively established the existence of CP violation in decay processes. In the present Brief Report, we will analyze in detail what the KTeV results imply and see in particular how well CPT symmetry is tested compared to T symmetry.

The K^0-$\overline{K^0}$ mixing and 2π decays. Let $|K^0\rangle$ and $|\overline{K^0}\rangle$ be eigenstates of the strong interaction with strangeness $S=+1$ and -1, related to each other by (CP) and (CPT) operations as [3,4]

$$(CP)|K^0\rangle = e^{i\alpha_K}|\overline{K^0}\rangle, \quad (CPT)|K^0\rangle = e^{i\beta_K}|\overline{K^0}\rangle, \tag{2}$$

where α_K and β_K are arbitrary real parameters. When the weak interaction H_w is switched on, K^0 and $\overline{K^0}$ decay into other states, generically denoted as n, and get mixed. The states with definite mass ($m_{S,L}$) and width ($\gamma_{S,L}$; $\gamma_S>\gamma_L$ by definition) are linear combinations of K^0 and $\overline{K^0}$:

$$|K_S\rangle = \frac{1}{\sqrt{|p_S|^2+|q_S|^2}}(p_S|K^0\rangle + q_S|\overline{K^0}\rangle), \tag{3a}$$

$$|K_L\rangle = \frac{1}{\sqrt{|p_L|^2+|q_L|^2}}(p_L|K^0\rangle - q_L|\overline{K^0}\rangle). \tag{3b}$$

The ratios of the mixing parameters, $q_{S,L}/p_{S,L}$, as well as $\lambda_{S,L}\equiv m_{S,L}-i\gamma_{S,L}/2$, are related to H_w; the explicit expressions can be found in the literature [3,5]. We are interested in 2π decays and specifically in the following quantities:

$$\eta_{+-}=|\eta_{+-}|e^{i\phi_{+-}}\equiv \frac{\langle \pi^+\pi^-,\text{outgoing}|H_\mathrm{w}|K_L\rangle}{\langle \pi^+\pi^-,\text{outgoing}|H_\mathrm{w}|K_S\rangle}, \tag{4a}$$

$$\eta_{00}=|\eta_{00}|e^{i\phi_{00}}\equiv \frac{\langle \pi^0\pi^0,\text{outgoing}|H_\mathrm{w}|K_L\rangle}{\langle \pi^0\pi^0,\text{outgoing}|H_\mathrm{w}|K_S\rangle}, \tag{4b}$$

$$r\equiv \frac{\gamma_S(\pi^+\pi^-)-2\gamma_S(\pi^0\pi^0)}{\gamma_S(\pi^+\pi^-)+\gamma_S(\pi^0\pi^0)}, \tag{5}$$

where $\gamma_{S,L}(n)$ denotes the partial width for $K_{S,L}$ to decay into the final state n.

Parametrization and conditions imposed by CP, T, and CPT symmetries. We shall parametrize q_S/p_S and q_L/p_L as [3]

$$\frac{q_S}{p_S}=e^{i\alpha_K}\frac{1-\varepsilon-\delta}{1+\varepsilon+\delta}, \tag{6a}$$

$$\frac{q_L}{p_L}=e^{i\alpha_K}\frac{1-\varepsilon+\delta}{1+\varepsilon-\delta}, \tag{6b}$$

and the amplitudes for K^0 and $\overline{K^0}$ to decay into 2π states with isospin $I=0$ or 2 as [3,6]

$$\langle(2\pi)_I|H_\mathrm{w}|K^0\rangle = F_I(1+\varepsilon_I)e^{i\alpha_K/2}, \tag{7a}$$

$$\langle(2\pi)_I|H_\mathrm{w}|\overline{K^0}\rangle = F_I(1-\varepsilon_I)e^{-i\alpha_K/2}. \tag{7b}$$

Our parametrization is very unique in that it is invariant under rephasing of the initial states, $|K^0\rangle$ and $|\overline{K^0}\rangle$. It is, however, not invariant under rephasing of the final states, $|(2\pi)_I\rangle$. By making use of the phase ambiguity, one may, without loss of generality, set [6]

$$\mathrm{Im}(F_I)=0. \tag{8}$$

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One can readily verify [3,6] that CP, T, and CPT symmetries impose such conditions as

$$CP \text{ symmetry: } \varepsilon=0, \ \delta=0, \ \varepsilon_I=0,$$

$$T \text{ symmetry: } \varepsilon=0, \ \mathrm{Im}(\varepsilon_I)=0, \tag{9}$$

$$CPT \text{ symmetry: } \delta=0, \ \mathrm{Re}(\varepsilon_I)=0.$$

Observed and expected smallness of symmetry violation allows one to treat all these parameters as small.

Formulas relevant for analysis. Defining

$$\eta_I=|\eta_I|e^{i\phi_I}\equiv\frac{\langle(2\pi)_I|H_{\mathrm{w}}|K_L\rangle}{\langle(2\pi)_I|H_{\mathrm{w}}|K_S\rangle}, \tag{10a}$$

$$\omega\equiv\frac{\langle(2\pi)_2|H_{\mathrm{w}}|K_S\rangle}{\langle(2\pi)_0|H_{\mathrm{w}}|K_S\rangle}, \tag{10b}$$

one finds [7,8], from Eqs. (3a),(3b), (6a), (6b), and (7a),(7b),

$$\eta_I=\varepsilon-\delta+\varepsilon_I, \tag{11a}$$

$$\omega=\mathrm{Re}(F_2)/\mathrm{Re}(F_0), \tag{11b}$$

and, by means of isospin decomposition,

$$\eta_{+-}=\eta_0+\varepsilon', \tag{12a}$$

$$\eta_{00}=\eta_0-2\varepsilon', \tag{12b}$$

$$r=4\mathrm{Re}(\omega'), \tag{13}$$

where

$$\varepsilon'\equiv(\eta_2-\eta_0)\omega', \tag{14a}$$

$$\omega'\equiv\frac{1}{\sqrt{2}}\omega e^{i(\delta_2-\delta_0)}, \tag{14b}$$

δ_I being the *S*-wave $\pi\pi$ scattering phase shift for the isospin I state at an energy of the rest mass of K^0. Note that we have treated ω', which is a measure of deviation from the $\Delta I=1/2$ rule, as well as a small quantity. From Eqs. (12a),(12b), it follows that

$$\eta_{00}/\eta_{+-}=1-3\varepsilon'/\eta_0 \tag{15}$$

or

$$\mathrm{Re}(\varepsilon'/\eta_0)=(1/3)(1-|\eta_{00}/\eta_{+-}|), \tag{16a}$$

$$\mathrm{Im}(\varepsilon'/\eta_0)=-(1/3)\Delta\phi, \tag{16b}$$

where

$$\Delta\phi\equiv\phi_{00}-\phi_{+-}. \tag{17}$$

Implications of the KTeV results. With the help of the formulas derived above, we now look into implications of the latest results reported by the KTeV Collaboration [2]. We first note that, since ε in their notation corresponds exactly to η_0 in our notation,[1] their results (1a),(1b) give, either immediately or with the help of Eqs. (16a),(16b),

$$\mathrm{Re}(\varepsilon'/\eta_0)=(2.80\pm0.41)\times10^{-3}, \tag{18a}$$

$$\mathrm{Im}(\varepsilon'/\eta_0)=(-0.52\pm2.68)\times10^{-3}, \tag{18b}$$

$$|\eta_{00}/\eta_{+-}|=0.9916\pm0.0012. \tag{18c}$$

From Eqs. (11a) and (14a), we immediately conclude that $\varepsilon'\neq0$ implies that either ε_0 or ε_2 (or both) is nonvanishing,[2] confirming the assertion that the KTeV result on $\mathrm{Re}(\varepsilon'/\varepsilon)$ established the existence of CP violation in a decay process [2].

To go one step further, we need to know the value of η_0. Since the KTeV collaboration has not yet reported their results on η_{+-} and η_{00} separately, we shall input the Particle Data Group (PDG) [9] values for η_{+-},

$$|\eta_{+-}|=(2.285\pm0.019)\times10^{-3}, \tag{19a}$$

$$\phi_{+-}=(43.5\pm0.6)°, \tag{19b}$$

along with Eqs. (1b) and (18c), into

$$\eta_0\simeq(2/3)\eta_{+-}+(1/3)\eta_{00}, \tag{20}$$

which follows from Eqs. (12a),(12b), to get

$$|\eta_0|=(2.28\pm0.02)\times10^{-3}, \tag{21a}$$

$$\phi_0=(43.53\pm0.94)°. \tag{21b}$$

We shall also use the PDG [9] values for $\gamma_S(\pi^+\pi^-)$ and $\gamma_S(\pi^0\pi^0)$ to get, with the help of Eqs. (5) and (13),

$$\mathrm{Re}(\omega')=(1.46\pm0.16)\times10^{-2}. \tag{22}$$

In order to interpret Eqs.(18a),(18b), we derive from Eqs. (14a),(14b), with the aid of Eqs. (11a),(11b),

$$\varepsilon'/\eta_0=-i\mathrm{Re}(\omega')(\varepsilon_2-\varepsilon_0)e^{-i\Delta\phi'}/[|\eta_0|\cos(\delta_2-\delta_0)] \tag{23}$$

or

$$\varepsilon_2-\varepsilon_0=i(\varepsilon'/\eta_0)|\eta_0|\cos(\delta_2-\delta_0)e^{i\Delta\phi'}/\mathrm{Re}(\omega'), \tag{24}$$

where

$$\Delta\phi'\equiv\phi_0-\delta_2+\delta_0-\pi/2. \tag{25}$$

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[1]For the correspondence between our parametrization and the (more conventional) rephasing-dependent parametrizations, see Refs. [3,8].

[2]Note that the reverse is however not necessarily true; a nonvanishing but equal value for both ε_0 and ε_2 could yield $\varepsilon'=0$.

Inputting Eqs. (18a),(18b), (21a),(21b), and (22), and $\delta_2 - \delta_0$ as well, into Eq. (24), we are able to derive constraints to $\mathrm{Re}(\varepsilon_2-\varepsilon_0)$ and $\mathrm{Im}(\varepsilon_2-\varepsilon_0)$:

$$\mathrm{Re}(\varepsilon_2-\varepsilon_0)=(0.85\pm 3.11)\times 10^{-4}, \tag{26a}$$

$$\mathrm{Im}(\varepsilon_2-\varepsilon_0)=(3.2\pm 0.7)\times 10^{-4}, \tag{26b}$$

where, as $\delta_2-\delta_0$, we have tentatively used the Chell-Olsson value $(-42\pm 4)°$ [10].

Discussion. If, as the value of $\mathrm{Re}(\varepsilon'/\varepsilon)$, one uses, instead of Eq. (1a),

$$\mathrm{Re}(\varepsilon'/\varepsilon)=(2.59\pm 0.36)\times 10^{-3}, \tag{27}$$

which is an average of the KTeV result [2] and the more recent result from the NA48 experiment [11], one will get

$$\mathrm{Re}(\varepsilon_2-\varepsilon_0)=(0.84\pm 3.11)\times 10^{-4}, \tag{28a}$$

$$\mathrm{Im}(\varepsilon_2-\varepsilon_0)=(3.0\pm 0.6)\times 10^{-4}. \tag{28b}$$

Our results (26b) and (28b) indicate that a combination of the parameters which signal direct CP and T violations, $\mathrm{Im}(\varepsilon_2-\varepsilon_0)$, is definitely nonzero and of the order of 10^{-4}. The other results (26a) and (28a) on the other hand indicate that a combination of the parameters which signal direct CP and CPT violations, $\mathrm{Re}(\varepsilon_2-\varepsilon_0)$, is not well determined yet; though consistent with being zero, a value comparable to $\mathrm{Im}(\varepsilon_2-\varepsilon_0)$ is not ruled out.

The procedure of our analysis is rather similar to that done by Dib and Peccei [12], except that they have focused on CPT test and have hesitated to use the value of $\mathrm{Re}(\varepsilon'/\varepsilon)$ as one of inputs, in view of experimental controversy on this quantity at that time. Thus, one of the results they have derived, $\mathrm{Re}(B_2)/\mathrm{Re}(A_2)-\mathrm{Re}(B_0)/\mathrm{Re}(A_0)=(1.3\pm 8.4)\times 10^{-4}$, corresponds exactly to our results Eqs. (26a) and (28a), and a conclusion on direct CP/CPT violation qualitatively similar to ours has already been reached by them.

With the help of the Bell-Steinberger relation [13], one may derive constraints to the indirect and mixed CP, T, and/or CPT violating parameters [7,8,14,15]. It turns out that the values of the direct CP/T violating parameter we have obtained, Eqs. (26b) and (28b), are almost one order smaller than those of the indirect and mixed CP/T violating parameters, $\mathrm{Re}(\varepsilon)$ and $\mathrm{Im}(\varepsilon+\varepsilon_0)$,[3] while the constraints on the direct CP/CPT violating parameter we have found, Eqs. (26a) and (28a), are roughly one order weaker than those on the indirect and mixed CP/CPT violating parameters,[4] $\mathrm{Im}(\delta)$ and $\mathrm{Re}(\delta-\varepsilon_0)$.

To conclude, we recall that the numerical results (26a),(26b), and (28a),(28b) depend much on the value of $\delta_2-\delta_0$, and that this quantity, which features strong interaction effects, is still not well determined.[5] In order to obtain a better constraint on $\varepsilon_2-\varepsilon_0$, a better determination of $\delta_2-\delta_0$, along with a more precise measurement of $\mathrm{Re}(\varepsilon'/\varepsilon)$ and $\Delta\phi$, are required.

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[3] See also Alvalez-Gaume, Kounnas, Lola, and Pavlopoulos [16], in which it is claimed that the recent CPLEAR data allow one to definitively conclude that $\mathrm{Re}(\varepsilon)$ is $\neq 0$ without invoking the Bell-Steinberger relation and that T is violated independent of whether CP and/or CPT are violated or not.

[4] ε_0 and ε_2 (ε and δ) are referred to as a direct (indirect) parameter here. Note that, as emphasized in [3], classification of symmetry-violating parameters into ''direct'' and ''indirect'' ones makes sense only when they are defined in a rephasing-invariant way, i.e., in such a way that they are invariant under rephasing of $|K^0\rangle$ and $|\overline{K^0}\rangle$.

[5] See, for example, Refs. [17,18], and references cited therein for theoretical problems related to determination of $\delta_2-\delta_0$. If, as the value of this quantity, one uses $(-56.7\pm 3.9)°$ quoted in Ref. [17], instead of $(-42\pm 4)°$ used previously, one would find

$$\mathrm{Re}(\varepsilon_2-\varepsilon_0)=(0.01\pm 2.27)\times 10^{-4}, \tag{29a}$$

$$\mathrm{Im}(\varepsilon_2-\varepsilon_0)=(2.44\pm 0.65)\times 10^{-4}, \tag{29b}$$

in place of Eqs. (26a),(26b).

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