CP conserving constraints on supersymmetric *CP* violation in the MSSM

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We address the following question. Take the constrained minimal supersymmetric standard model (CMSSM) with the two CP violating supersymmetry (SUSY) phases different from zero, and neglect the bound coming from the electric dipole moment (EDM) of the neutron: is it possible to fully account for CP violation in the kaon and B systems using only the SUSY contributions with a vanishing CKM phase? We show that the BR($B\to X_s\gamma$) constraint, though CP conserving, forces a negative answer to the above question. This implies that even in the regions of the CMSSM, where a cancellation of different contributions to the EDM allows for large SUSY phases, it is not possible to exploit the SUSY phases to fully account for observable CP violation. Hence to have sizable SUSY contributions to CP violation, one needs new flavor structures in the sfermion mass matrices beyond the usual CKM matrix.

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I. INTRODUCTION

Since the initial work of Kobayashi and Maskawa, the standard model (SM) of electroweak interactions is known to be able to accommodate the experimentally observed CP violation through a unique phase δ_{CKM} in the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. However, the available experimental information, namely ε_K and ε'/ε, is not enough to establish this phase as the only source of CP violation.

Most of the extensions of the SM include new observable phases that may significantly modify the pattern of CP violation. Supersymmetry (SUSY) is, without a doubt, one of the most popular extensions of the SM. Indeed, in the minimal supersymmetric extension of the SM (MSSM), there are additional phases which can cause deviations from the predictions of the SM. After all possible rephasings of the parameters and fields, there remain at least two new physical phases in the MSSM Lagrangian. These phases can be chosen to be the phases of the Higgsino Dirac mass parameter ($\phi_\mu=\mathrm{Arg}[\mu]$) and the trilinear sfermion coupling to the Higgs ($\phi_{A_0}=\mathrm{Arg}[A_0]$) [1]. In fact, in the so-called constrained minimal supersymmetric standard model (CMSSM), with strict universality at the grand unification scale, these are the only new phases present.

It was soon realized that for most of the CMSSM parameter space, the experimental bounds on the electric dipole moments of the electron and neutron constrained $\phi_{A_0,\mu}$ to be at most $\mathcal{O}(10^{-2})$. Consequently these new supersymmetric phases have been taken to vanish exactly in most studies of CMSSM.

However, in the last few years, the possibility of having nonzero SUSY phases has again attracted a great deal of attention. Several new mechanisms have been proposed to suppress electric dipole moments (EDMs) below the experimental bounds while allowing SUSY phases $\mathcal{O}(1)$. Methods of suppressing the EDMs consist of cancellation of various SUSY contributions among themselves [2], nonuniversality of the soft breaking parameters at the unification scale [3], and approximately degenerate heavy sfermions for the first two generations [4]. In a recent work [5], we showed that, in a model with heavy sfermions of the first two generations and in the large $\tan\beta$ regime, ε_K and ε_B could receive very sizable contributions from these new SUSY phases. Similar studies [6], including a larger set of experimental constraints, have reported the impossibility of such large supersymmetric contributions.[1]

In this work, we are going to complete our previous analysis with the inclusion of all the relevant constraints in a CMSSM scenario. In doing so we adopt a different perspective. We will assume from the very beginning that both supersymmetric phases are $\mathcal{O}(1)$, ignoring for the moment EDM bounds.[2] In these conditions, and taking into account other CP-conserving constraints, we will analyze the effects on the low energy CP violation observables, especially ε_K and ε_B. It should be noted that the model used in [5] can be easily obtained as a limit of the CMSSM by decoupling the first two generations of squarks and neglecting the intergeneration mixing in the sfermion mass matrices. Hence, our

[1]In this paper we restrict our discussions to the CMSSM. If one relaxes some of the constraints of this model, for instance by allowing for large gluino mediated CP violation with nonuniversal soft SUSY breaking terms, then it might still be possible to have fully supersymmetric ε and ε'/ε [7].

[2]EDM cancellations may be obtained through nontrivial relative phases in the gaugino mass parameters (see for instance the third paper in Ref. [2]). However, for the discussion of the present paper, no explicit mechanism for such a cancellation is needed.

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results in the more general CMSSM will include this model as a limiting case.

In the next section we study the new sources of flavor mixing present at the electroweak scale in any supersymmetric model. In Sec. III we are going to analyze neutral meson mixing, i.e., $K^0-\overline{K^0}$ and $B^0-\overline{B^0}$ mixings, with large supersymmetric phases. Section IV will be devoted to the study of the branching ratio of the decay $b \to s\gamma$. In Sec. V we will show the impact of the measured $b \to s\gamma$ branching ratio on the supersymmetric contributions to ε_K and ε_B. Section VI will present our conclusions, and in Appendices A and B we collect, respectively, the formulas for the integration of relevant renormalization group equations (RGE's) and the different loop functions appearing in the text.

II. FLAVOR MIXING IN THE CMSSM

The issue of flavor changing neutral current (FCNC) interactions in the CMSSM has been widely investigated in the literature. For the completeness of the discussion, we briefly recall those properties which will be relevant for our analysis.

The CMSSM is the simplest supersymmetric structure we can build from the SM particle content. This model is completely defined once we specify the soft-supersymmetry breaking terms. These are taken to be strictly universal at some ultra-high energy scale, which we take to be the grand unification scale (M_{GUT}):

$$(m_Q^2)_{ij}=(m_U^2)_{ij}=(m_D^2)_{ij}=(m_L^2)_{ij}=(m_E^2)_{ij}=m_0^2\ \ \delta_{ij}, \tag{1}$$

$$m_{H_1}^2=m_{H_2}^2=m_0^2,$$

$$m_{\tilde g}=m_{\tilde W}=m_{\tilde B}=m_{1/2},$$

$$(A_U)_{ij}=A_0e^{i\phi_A}(Y_U)_{ij}, \quad (A_D)_{ij}=A_0e^{i\phi_A}(Y_D)_{ij},$$

$$(A_E)_{ij}=A_0e^{i\phi_A}(Y_E)_{ij}.$$

That is, there is a common mass for all the scalars, m_0^2, a single gaugino mass, $m_{1/2}$, and all the trilinear soft-breaking terms are directly proportional to the corresponding Yukawa couplings in the superpotential with a proportionality constant $A_0e^{i\phi_A}$.

Now, with the use of the renormalization group equations (RGE) of the MSSM, as explained in Appendix A, we can obtain the whole supersymmetric spectrum at the electroweak scale. All the supersymmetric masses and mixings are then a function of m_0^2, $m_{1/2}$, A_0, ϕ_A, ϕ_μ, and $\tan\beta$. We require radiative symmetry breaking to fix $|\mu|$ and $|B\mu|$ [8,9] with tree-level Higgs potential.[3]

[3]The RGE's of the MSSM have received a vast amount of attention in the literature. However, in most of the previous analyses the SUSY phases ϕ_A and ϕ_μ are switched off. For this reason we prefer to give the relevant RGE's with nonvanishing SUSY phases in Appendix A.

It is important to notice that, even in a model with universal soft-breaking terms at some high energy scale as the CMSSM, some off-diagonality in the squark mass matrices appears at the electroweak scale. Working on the basis of squarks rotated parallel to the quarks, the so-called super CKM basis (SCKM), we find that the squark mass matrix is not flavor diagonal at M_W. This is due to the fact that at M_{GUT} there exist two nontrivial flavor structures, namely the two Yukawa matrices for the up and down quarks, which are not simultaneously diagonalizable. This implies that through RGE evolution some flavor mixing leaks into the sfermion mass matrices. In a general supersymmetric model, the presence of new flavor structures in the soft-breaking terms would generate large flavor mixing in the sfermion mass matrices. However, in the CMSSM the two Yukawa matrices are the only source of flavor change. As always in the SCKM basis, any off-diagonal entry in the sfermion mass matrices at M_W will be necessarily proportional to a product of Yukawa couplings. The RGE's for the soft-breaking terms are sets of linear equations, and thus, to match the correct quirality of the coupling, Yukawa couplings or trilinear soft terms must enter the RGE in pairs, as we can see in Eqs. (A1)–(A3) in Appendix A.

In fact, in the up (down) squark mass matrix the up (down) Yukawas will also be diagonalized and so will mainly contribute to diagonal entries while off-diagonal entries will be due to the down (up) Yukawa matrix. This means, for instance, that in this model the off-diagonality in the $M_{LL}^{(d)\ 2}$ matrix will roughly be $c\cdot Y_u Y_u^\dagger$, with c a proportionality factor that typically is

$$c \simeq \frac{1}{(4\pi)^2}\log\left(\frac{M_{\mathrm{GUT}}}{M_W}\right) \simeq 0.20, \tag{2}$$

as expected from the loop factor and the running from M_{GUT} to M_W. Nevertheless, we have to keep in mind that this is simply a typical estimate and the final value of c can suffer a sizable variation depending on many other factors not present in this simple estimate.

On the other hand, this has clear implications on the $\tan\beta$ dependence of these off-diagonal entries of the sfermion mass matrices. In the basis where the down Yukawa matrix is diagonal, we can write the up and down Yukawas as

$$Y_U(M_Z)=\frac{g}{\sqrt{2}M_W\sin\beta}V_{CKM}^\dagger M_u,$$

$$Y_D(M_Z)=\frac{g}{\sqrt{2}M_W\cos\beta}M_d \tag{3}$$

with V_{CKM} the Cabibbo-Kobayashi-Maskawa mixing matrix and $M_{u,d}$ the diagonalized mass matrices for the quarks. We can see in this equation that for $\tan\beta \gtrsim 1$ the up Yukawa matrix will maintain similar values when going to large $\tan\beta$. Hence, the off-diagonal entries in the down squarks mass matrix will be roughly stable with $\tan\beta$. In the up squark mass matrix we have the opposite situation and the $\tan\beta$ dependence is very strong. In this case the off-diagonal entries depend on the down Yukawa matrix that grows linearly with tan β for large tan β. This means that we can expect the flavor change in the up and down squark mass matrix to be similar when $\tan\beta\simeq m_t/m_b\simeq 40$, while for $\tan\beta\simeq 2$ the flavor change in the up mass matrix will be approximately $(\tan\beta=40)^2/(\tan\beta=2)^2=400$ times smaller (see Appendix A for details). These points also apply to the left-right submatrices where again flavor changing entries will be due to the opposite isospin Yukawa matrix. In fact, this left-right sfermion mixing only appears after electroweak symmetry breaking. The expression for these matrices in the SCKM basis is

$$M_{LR}^{(u)2}=\left(\frac{v_2}{\sqrt{2}}V_{CKM}A_U^*(M_Z)-\left|\mu(M_Z)\right|e^{i\phi_\mu}\cot\beta\, M_u\right), \tag{4}$$

$$M_{LR}^{(d)2}=\frac{v_1}{\sqrt{2}}A_D^*(M_Z)-|\mu(M_Z)|e^{i\phi_\mu}\tan\beta\, M_d\,. \tag{5}$$

Then, these left-right mixings will have an additional suppression proportional to the mass of the corresponding right-handed quark (remember that $A_U v_1\approx A_0 M_U$). This is always true for all the generation changing entries that are produced by the A matrices. However, in the down mass matrix, this suppression can be partially compensated by a large value of tan β in the diagonal terms proportional to μ. These are all well-known facts in the different studies of FCNC processes in the framework of the CMSSM [9,10] and imply that flavor mixing is still dominantly given by the usual CKM mixing matrix in W bosons, charged Higgs bosons, and chargino vertices.

In this work, we are especially interested in CP violating observables. Then we must also consider the presence of observable phases in the sfermion mass matrices. In the following we will take the CKM matrix exactly real to isolate pure effects of the new supersymmetric phases [11]. The sfermion mass matrices contain several physical phases that give rise to CP violation phenomena. In particular, before RGE evolution, these phases (ϕ_A, ϕ_μ) are confined to the left-right part of the sfermion mass matrix while both the left-left, m_Q^2, and right-right $m_{U,D}^2$, matrices are real and diagonal. However this is not true anymore at M_W; ϕ_A leaks into the off-diagonal elements of these Hermitian matrices through RGE evolution. From the explicit RGE in the MSSM, Eq. (A1), it is clear that this phase only enters the $(m_Q^2)_{ij}$ evolution through the combinations $(A_U A_U^\dagger)_{ij}$ or $(A_D A_D^\dagger)_{ij}$. At $M_{\rm GUT}$ these matrices have a common phase, and so the combination $(AA^\dagger)$ is exactly real. So to the extent that the A matrices keep a uniform phase during RGE evolution, no phase will leak into the m_Q^2 matrices. However, we can easily see from Eqs. (A2) and (A3) that this is not the case, and different elements of the A matrices are renormalized differently. In this equation, we can see that only the terms involving two Yukawa and one A matrix can produce a mismatch in the phases. Moreover, these terms will only be important when there are no small Yukawas involved. Then, we can expect a mismatch only on the off-diagonal elements involving the third generation. Keeping this in mind, the general form of the m_Q^2 matrix at M_W in terms of the initial conditions is

$$m_Q^2(M_W)=\eta_Q^{(m)}m_0^2+\eta_Q^{(A)}A_0^2+\eta_Q^{(g)}m_{1/2}^2+(\eta_Q^{(gA)}e^{i\phi_A}+\eta_Q^{(gA)\,T}e^{-i\phi_A})A_0 m_{1/2}, \tag{6}$$

where the coefficients η are 3×3 matrices with real numerical entries. In this expression we can see that the presence of imaginary parts will be linked to the nonsymmetric part of the $\eta_Q^{(gA)}$ matrices. As is clear from the mass matrices in Appendix A [Eqs. (A5)–(A7) and (A12)–(A14)], these nonsymmetric parts of m_Q^2 are always more that three orders of magnitude smaller than the corresponding symmetric parts. This means that in the SCKM basis, the imaginary parts of any mass insertion are present only in one part per $2-3\times 10^3$, and are always associated with (3,i) MI, as in Eqs. (A8)–(A11) and (A15)–(A18). A very similar situation was also found by Bertolini and Vissani in the CMSSM with vanishing SUSY phases for the leakage of $\delta_{\rm CKM}$ [12,3]. So we conclude that in the processes we will consider, we can take both $M^{(u)2}_{LL}$ and $M^{(d)2}_{LL}$ as real to a very good approximation.

In the following we will analyze the new effects of this model on indirect CP violation in K and B systems. In doing so, we will use both the exact vertex mixing method and the mass insertion (MI) approximation [13]. Notice that the MI approximation is extremely good in the case of the CMSSM where all the off-diagonal entries are sufficiently small. The size of these off-diagonal entries directly gives, in the MI approximation, the amount of flavor changing induced by the sfermion mass matrices. A possible exception may arise in the stop squark and sbottom sectors that, in any case, could be diagonalized to ensure the validity of the MI approximation [14]. As we will see in the next section, this is frequently useful to understand the exact results obtained in the vertex mixing method.

III. INDIRECT CP VIOLATION IN THE CMSSM

In the SM, neutral meson mixing arises at one loop through the well-known W box. However, in the CMSSM, there are new contributions to $\Delta F=2$ processes coming from boxes mediated by supersymmetric particles. These are charged Higgs boxes ($H^\pm$), chargino boxes ($\chi^\pm$), and gluino-neutralino boxes ($\tilde{g}$, χ^0). The amount of the indirect CP violation in the neutral meson $\mathcal{M}$ system is measured by the well-known $\varepsilon_{\mathcal{M}}$ parameter

$$\varepsilon_{\mathcal{M}}=\frac{1}{\sqrt{2}}\frac{\mathrm{Im}\langle\mathcal{M}^0|\mathcal{H}_{eff}^{\Delta F=2}|\overline{\mathcal{M}}^0\rangle}{\Delta M_{\mathcal{M}}}, \tag{7}$$

where $\Delta M_{\mathcal{M}}$ is the $\mathcal{M}-\overline{\mathcal{M}}$ mass splitting. $\varepsilon_{\mathcal{M}}$ depends on the matrix elements of the $\Delta F=2$ Hamiltonian, $\mathcal{H}_{eff}^{\Delta F=2}$, which can be decomposed as

$$\mathcal{H}_{eff}^{\Delta F=2}=-\frac{G_F^2M_W^2}{(2\pi)^2}(V_{td}^*V_{tq})^2[C_1(\mu)Q_1(\mu)+C_2(\mu)Q_2(\mu)+C_3(\mu)Q_3(\mu)], \tag{8}$$

where the relevant four-fermion operators are given by

$$Q_1=\overline{d}_L^\alpha\gamma^\mu q_L^\alpha\cdot\overline{d}_L^\beta\gamma_\mu q_L^\beta,$$

$$Q_2=\overline{d}_L^\alpha q_R^\alpha\cdot\overline{d}_L^\beta q_R^\beta,$$

$$Q_3=\overline{d}_L^\alpha q_R^\beta\cdot\overline{d}_L^\beta q_R^\alpha \tag{9}$$

with $q=s,b$ for the K and B systems, respectively, and α,β as color indices. In the CMSSM, these are the only three operators present in the limit of vanishing m_d.

At this point, we are going to divide our discussion into two parts. We analyze separately the effective operator Q_1 that preserves chirality along the fermionic line, and the operators Q_2 and Q_3 that change chirality along the fermionic line. As we will see below, the flavor mixing in the sfermion mass matrix and the experimental constraints on both kinds of operators are very different.

A. Chirality conserving transitions

In Eq. (8), Q_1 is the only operator present that does not involve a chirality change in the fermionic line. With respect to the associated sfermion, no chirality change in the sfermion propagator will be needed, and so the suppression associated with left-right sfermion mixing can be avoided. In general, $C_1(\mu_0)$ can be decomposed as follows

$$C_1(\mu_0)=C_1^W(M_W)+C_1^H(M_W)+C_1^{\tilde{g},\chi^0}(M_W)+C_1^\chi(M_W). \tag{10}$$

The usual SM W box, where all the couplings are purely left-handed, can only contribute to this effective operator. However, with $\delta_{\rm CKM}=0$, C_1^W does not contain any complex phase and hence cannot contribute to the imaginary part in $\varepsilon_{\mathcal{M}}$. In any case, it will always be, in the CMSSM, the dominant contribution to $\Delta M_{\mathcal{M}}$. Similarly, the charged Higgs contribution, C_1^H depends on the same combination of CKM elements with no other CP violating phase [9]. So it will not contribute to our CP violating observable.

Gluino and neutralino contributions to $C_1^{\tilde{g},\chi^0}$ are specifically supersymmetric. They involve the superpartners of quarks and gauge bosons. Here the source of flavor mixing is not directly the usual CKM matrix. It is the presence of off-diagonal elements in the sfermion mass matrices, as discussed in the previous section. From the point of view of CP violation, we will always need a complex Wilson coefficient. In the SCKM basis all gluino vertices are flavor diagonal and real. This means that in the MI approximation we need a complex mass insertion in one of the sfermion lines. As explained in the previous section, these MI are proportional to Yukawa couplings and real up to 1 part in 2×10^3. The complete expressions for the gluino contributions to $\Delta F=2$ processes in the MI approximation can be found in [15]. The bounds obtained there for the real and imaginary parts of the mass insertions required to saturate ΔM_K and ε_K are

$$\sqrt{|\mathrm{Re}(\delta_{12}^d)_{LL}^2|}<4\times10^{-2}, \tag{11}$$

$$|(\delta_{12}^d)_{LL}|\sin(2\phi_{LL})<3\times10^{-3},$$

$$(\delta_{ij}^d)_{AB}=\frac{(M_{AB}^2)_{ij}}{\tilde{M}},$$

where $\tilde{M}$ is an average squark mass.

In the CMSSM, as we can see in Appendix A, these mass insertions are much smaller. In particular, the fact that the bound on ΔM_K, the real part of the MI, is satisfied implies that the imaginary parts are at least two orders of magnitude below the required value to saturate ε_K. Hence, no sizable contributions to ε_K from gluino boxes are possible. The situation in $B^0-\overline{B}^0$ mixing is completely analogous; assuming that the minimum phase from the mixing observable in the B factories is around 0.1 radian, we would need an imaginary contribution not more than one order of magnitude below the real one. With the arguments given above, this is clearly out of reach for gluino boxes in the CMSSM. Neutralino contributions are generally smaller than gluino due to smaller couplings with the same source of flavor mixing. In fact, although neutralino vertices in the SCKM basis also involve the complex neutralino mixings, any imaginary part on this operator will only be due to a complex mass insertion. This can be seen in the explicit expressions in [9] where all neutralino mixings in this operator appear in pairs with its complex conjugate counterpart.

Finally, the charginos also contribute to $C_1(M_W)^\chi$. In this case, flavor mixing comes explicitly from the CKM mixing matrix, although off-diagonality in the sfermion mass matrix introduces a small additional source of flavor mixing:

$$C_1^\chi(M_W)=\sum_{i,j=1}^{2}\sum_{k,l=1}^{6}\sum_{\alpha\gamma\alpha'\gamma'}\frac{V_{\alpha'd}^*V_{\alpha q}V_{\gamma'd}^*V_{\gamma q}}{(V_{td}^*V_{tq})^2}[G^{(\alpha,k)i}G^{(\alpha',k)j*}G^{(\gamma',l)i*}G^{(\gamma,l)j}\;\;Y_1(z_k,z_l,s_i,s_j)], \tag{12}$$

where $z_k = M^2_{\tilde{u}_k}/M^2_W$, $s_i = M^2_{\tilde{\chi}_i}/M^2_W$, and $V_{\alpha q}\cdot G^{(\alpha,k)i}$ represent the coupling of chargino and squark k to left-handed down quarks

$$G^{(\alpha,k)i} = \left(\Gamma^{\alpha k}_{UL} C^*_{R1i} - \frac{m_\alpha}{\sqrt{2} M_W \sin\beta} \Gamma^{\alpha k}_{UR} C^*_{R2i} \right), \quad (13)$$

where Γ_{UL} and Γ_{UR} are 6×3 matrices such that the 6×6 unitary matrix $\Gamma_U \equiv \{\Gamma_{UL}\Gamma_{UR}\}$ diagonalizes the up-squark mass matrix, $\Gamma_U M^2_U \Gamma^\dagger_U = \mathrm{diag}(M^2_{\tilde{u}_1}, \ldots, M^2_{\tilde{u}_6})$. C_R is one of the matrices that diagonalize the chargino mass matrix through a biunitary transformation $C^\dagger_R M^-_\chi C_L = \mathrm{diag}(M_{\chi^\pm_1}, M_{\chi^\pm_2})$, with

$$M^-_\chi = \begin{pmatrix} \tilde{m}_W & M_W \cos\beta \\ M_W \sin\beta & |\mu| e^{i\phi_\mu} \end{pmatrix}. \quad (14)$$

From these equations it is clear that $G^{(\alpha,k)i}$ will in general be complex, as both ϕ_μ and ϕ_A are present in the different mixing matrices. The loop function $Y_1(a,b,c,d)$ is given in Eq. (B1) of Appendix B.

The main part of C^χ_1 in Eq. (12) will be given by pure CKM flavor mixing, neglecting the additional flavor mixing in the squark mass matrix [10,16]. In this case, $\alpha = \alpha'$ and $\gamma = \gamma'$, we have

$$C^{(0)\chi}_1(M_W) = \sum_{i,j=1}^{2} \sum_{k,l=1}^{6} \sum_{\alpha\gamma} \frac{V^*_{\alpha d} V_{\alpha q} V^*_{\gamma d} V_{\gamma q}}{(V^*_{td} V_{tq})^2} [G^{(\alpha,k)i} G^{(\alpha,k)j*} G^{(\gamma,l)i*} G^{(\gamma,l)j} \; Y_1(z_k, z_l, s_i, s_j)]. \quad (15)$$

But, taking into account that $Y_1(a,b,c,d)$ is symmetric under the exchange of any pair of arguments we have

$$\begin{aligned} & G^{(\alpha,k)i} G^{(\alpha,k)j*} G^{(\gamma,l)i*} G^{(\gamma,l)j} \; Y_1(z_k, z_l, s_i, s_j) \\ &= \frac{1}{2} (G^{(\alpha,k)i} G^{(\alpha,k)j*} G^{(\gamma,l)i*} G^{(\gamma,l)j} \\ &\quad + G^{(\alpha,k)i*} G^{(\alpha,k)j} G^{(\gamma,l)i} G^{(\gamma,l)j*}) Y_1(z_k, z_l, s_i, s_j), \end{aligned} \quad (16)$$

and so $C^{(0)\chi}_1$ is exactly real [5]. This is not exactly true in the CMSSM, where there is additional flavor change in the sfermion mass matrices. Here some imaginary parts appear in the C^χ_1 in Eq. (12). Being associated to the size of intergenerational sfermion mixings, these imaginary parts will be maximal for large tan β. In Fig. 1 we show in a scatter plot the size of imaginary and real parts of C^χ_1 in the K system for a fixed value of tan β=40. The region of SUSY parameters explored in this and all of the following scatter plots is 50 GeV$\leq m_0, m_{1/2}$, $A_0 \leq 500$ GeV, and $0 \leq \phi_A, \phi_\mu \leq 2\pi$. With these initial conditions we impose that all squarks are heavier than 100 GeV with the exception of the tops squarks that, as the charginos, are only required to be above 80 GeV. Furthermore we impose the constraint from the $b \to s\gamma$ decay. Notice that, as we will see later, this is a conservative attitude in the sense that other constraints that we do not impose could only make our conclusions stronger. Under these conditions, we can see here that in the CMSSM this Wilson coefficient is always real up to a part in 10^5. Figure 2 is the equivalent plot for the case of B^0-$\bar{B}^0$ mixing. Here, imaginary parts are relatively larger but, in any case, out of reach for the foreseen B factories.

Taking this into account, from the point of view of experimental interest, we will always neglect imaginary parts in the Wilson coefficient C_1 within the CMSSM. Notice that this would not apply in a general model with nonuniversality at the grand unified theory (GUT) scale [15] and each particular model should be considered separately.

B. Chirality changing transitions

From the point of view of flavor change and CP violation, operators Q_2 and Q_3 are different from Q_1. These two operators always involve a change in the chirality of the external quarks and consequently also a change of the chirality of the associated squarks or gauginos. In particular, this

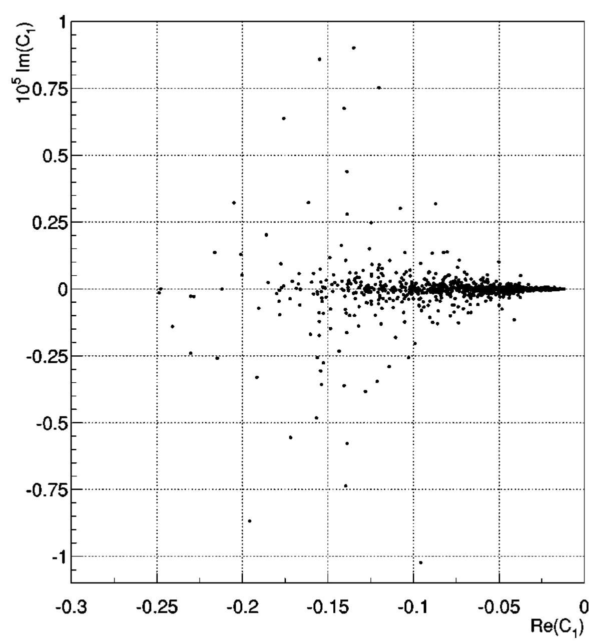

FIG. 1. Imaginary and real parts of the Wilson coefficient C^χ_1 in kaon mixing.

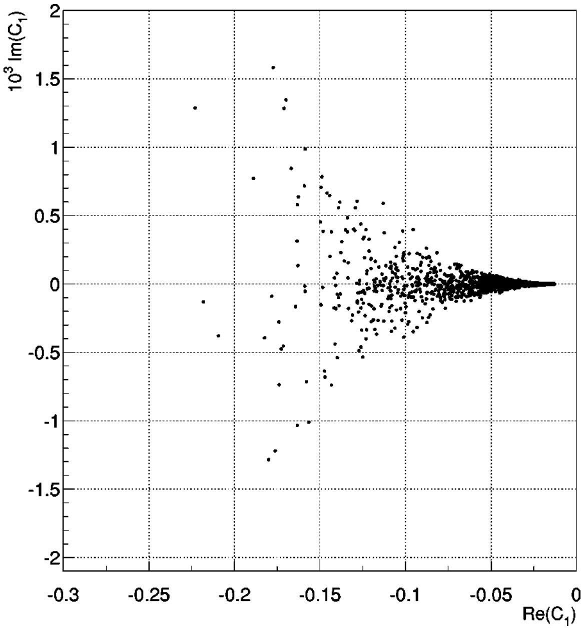

FIG. 2. Imaginary and real parts of the Wilson coefficient C_1^χ in B mixing.

implies the direct involvement of the supersymmetric phases. On the other hand, these operators are suppressed by the presence of down quark Yukawa couplings, and so can only be relevant in the region of large $\tan\beta$ [5]. We can write the different contributions to C_2 and C_3 as

$$C_2(M_W)=C_2^H(M_W)+C_2^{\tilde{g}}(M_W), \tag{17}$$

$$C_3(M_W)=C_3^{\tilde{g},\chi^0}(M_W)+C_3^{\chi}(M_W).$$

In first place, the charged Higgs boson contributes only to C_2 but, parallel to the discussion for $C_1^{W,H}$, the absence of phases prevents it from contributing to $\varepsilon_{\mathcal{M}}$.

Gluino and neutralino boxes contribute both to Q_2 and Q_3. However flavor change will be given in this case by an off-diagonal left-right mass insertion. In the CMSSM these MI are always proportional to the mass and are never enhanced by large $\tan\beta$ values [see Eq. (4)] of the right-handed squark. This implies that these left-right flavor transitions from gluino will always be smaller in the CMSSM than the corresponding chargino contributions, where flavor change is directly given by the CKM matrix. In fact, this is already well-known for the case of $b\to s\gamma$ decay [17], which is completely equivalent from the point of view of flavor change.

Hence, the most important contribution, especially for light stop and chargino, will be the chargino box. Before the inclusion of QCD effects, it contributes solely to the coefficient C_3,

$$C_3^\chi(M_W)=\sum_{i,j=1}^{2}\sum_{k,l=1}^{6}\sum_{\alpha\gamma\alpha'\gamma'}\frac{V^*_{\alpha' d}V_{\alpha q}V^*_{\gamma' d}V_{\gamma q}}{(V^*_{td}V_{tq})^2}\frac{m_q^2}{2M_W^2\cos^2\beta}\times H^{(\alpha,k)i}G^{(\alpha',k)j*}G^{(\gamma',l)i*}H^{(\gamma,l)j}Y_2(z_k,z_l,s_i,s_j), \tag{18}$$

where $m_q/(\sqrt{2}M_W\cos\beta)\cdot V_{\alpha q}\cdot H^{(\alpha,k)i}$ represents the coupling of chargino and squark to the right-handed down quark q with,

$$H^{(\alpha,k)i}=C^*_{L2i}\Gamma_{UL}^{\alpha k}, \tag{19}$$

and $Y_2(a,b,c,d)$ given in Eq. (B2). Unlike the C_1^χ Wilson coefficient, C_3^χ is complex even in the absence of intergenerational mixing in the sfermion mass matrices [5]. In fact, the presence of flavor violating entries in the up-squark mass matrix hardly modifies the results obtained in their absence [10,16]. So in these conditions we have

$$C_3^\chi(M_W)=\sum_{i,j=1}^{2}\sum_{k,l=3,6}[F_s(3,k,3,l,i,j)-2F_s(3,k,1,1,i,j)+F_s(1,1,1,1,i,j)], \tag{20}$$

$$F_s(\alpha,k,\gamma,l,i,j)=\frac{m_q^2}{2M_W^2\cos^2\beta}H^{(\alpha,k)i}G^{(\alpha,k)j*}G^{(\gamma,l)i*}\times H^{(\gamma,l)j}Y_2(z_k,z_l,s_i,s_j),$$

where we have used CKM unitarity and degeneracy of the first two generations of squarks. Due to the differences between H and G couplings, this contribution is always complex in the presence of SUSY phases. The most relevant feature of Eqs. (18) and (20) is the explicit presence of the external quark Yukawa coupling squared, $m_q^2/(2M_W^2\cos^2\beta)$. This is the reason why this contribution is usually neglected in the literature [6,9,16]. However, as we showed in [5], this contribution could be relevant in the large $\tan\beta$ regime. For instance, in B^0-$\overline{B^0}$ mixing we have $m_b^2/(2M_W^2\cos^2\beta)$ that for $\tan\beta\gtrsim 25$ is larger than 1 and so it is not suppressed at all when compared with the C_1^χ Wilson coefficient. This means that this contribution can be very important in the large $\tan\beta$ regime [5] and could have observable effects in CP violation experiments in the new B factories. However, in our previous work [5], we did not include the additional constraints coming from $b\to s\gamma$ decay. In the next sections we will analyze the relation of $\varepsilon_{\mathcal{M}}$ with this decay, and the constraints imposed by its experimental measure.

IV. $B\to S\gamma$ IN THE CMSSM

The decay $b\to s\gamma$ has already been extensively studied in the context of the CMSSM with vanishing SUSY phases [17]. Because the branching ratio is a CP conserving observable, the presence of new phases will not modify the main features found in [17] concerning the relative importance of the different contributions. However, in the presence of the new SUSY phases, these contributions will have different phases and will be observable through the interference. As we will see next, the experimental constraints will also have a large impact on the imaginary parts of the decay amplitudes.

This decay is described by the following $\Delta F=1$ effective Hamiltonian:

$$\mathcal{H}_{eff}^{\Delta F=1}=-\frac{4G_F}{\sqrt{2}}V_{ts}^{*}V_{tb}\sum_{i=2,7,8}\mathcal{C}_i\mathcal{Q}_i, \tag{21}$$

where the relevant operators are given by

$$\mathcal{Q}_2=\overline{s}_L\gamma_\mu c_L\overline{c}_L\gamma^\mu b_L, \tag{22}$$

$$\mathcal{Q}_7=\frac{em_b}{16\pi^2}\overline{s}_L\sigma^{\mu\nu}F_{\mu\nu}b_R, \tag{23}$$

$$\mathcal{Q}_8=\frac{g_s m_b}{16\pi^2}\overline{s}_L\sigma^{\mu\nu}G_{\mu\nu}b_R. \tag{24}$$

Here $\mathcal{C}_2(\mu_0)=1$, and the Wilson coefficients $\mathcal{C}_{7,8}$ can be decomposed accordingly with the particles in the loop,

$$\mathcal{C}_7(M_W)=\mathcal{C}_7^W(M_W)+\mathcal{C}_7^H(M_W)+\mathcal{C}_7^{\chi^\pm}(M_W)+\mathcal{C}_7^{\tilde{g}\chi^0}(M_W), \tag{25}$$

$$\mathcal{C}_8(M_W)=\mathcal{C}_8^W(M_W)+\mathcal{C}_8^H(M_W)+\mathcal{C}_8^{\chi^\pm}(M_W)+\mathcal{C}_8^{\tilde{g}\chi^0}(M_W).$$

Among these contributions, the *W* penguin diagram is exactly the same as in the SM and it does not depend on any supersymmetric parameters; it is simply a function of SM couplings and masses. This contribution is [9]

$$\mathcal{C}_7^W(M_W)=-\frac{3}{2}x_t[Q_U F_1(x_t)+F_2(x_t)], \tag{26}$$

$$\mathcal{C}_8^W(M_W)=-\frac{3}{2}x_t F_1(x_t),$$

with $x_t=m_t^2/M_W^2$ and Q_U the charge of the up quarks. Similarly, in the charged Higgs penguins all the variables are known with the exception of M_h. Again this contribution is unchanged by the inclusion of the new SUSY phases,

$$\mathcal{C}_7^H(M_W)=-\frac{x_t}{2x_h}\{\cot^2\beta[Q_U F_1(x_t/x_h)+F_2(x_t/x_h)] + Q_U F_3(x_t/x_h)+F_4(x_t/x_h)\}, \tag{27}$$

$$\mathcal{C}_8^H(M_W)=-\frac{x_t}{2x_h}[\cot^2\beta F_1(x_t/x_h)+F_3(x_t/x_h)],$$

where $x_h=M_h^2/M_W^2$. This contribution gives a sizable correction to the $b\to s\gamma$ decay that constrains the mass of the charged Higgs in two Higgs doublet models or in the MSSM with low tan β. However, in the case of moderate-large tan β, chargino contributions may partially compensate this charged Higgs contribution relaxing the constraints [17].

In addition to the $W^\pm$ and charged Higgs contributions analyzed above, there are three specifically supersymmetric contributions mediated by gluino, neutralino, and chargino. In gluino or neutralino diagrams flavor change is due to the off-diagonality in the sdown mass matrix. Being left-right flavor off-diagonal transitions, they are suppressed by the mass of the *b* quark. Indeed, smallness of gluino and neutralino contributions has already been established in [17], where it was shown that in the CMSSM, such contributions are roughly one order of magnitude smaller than the chargino contribution.

Together with the $W^\pm$ and charged Higgs, the most important supersymmetric contribution will be, especially in the large-moderate tan β regime, the chargino contribution. In the *W* and charged Higgs contributions, the necessary chirality flip for the dipole amplitude is always proportional to m_b. However, in the chargino penguin the chirality flip can be made either through a chargino mass insertion in the loop or through an external leg mass insertion proportional to m_b. In fact, as pointed out in [9], this enhancement due to m_{χ^i}/m_b is partially compensated by the presence of the *b* Yukawa coupling. Nevertheless, this compensation is only effective for low values of tan β. In terms of the chargino-quark-squark couplings used in the previous section, these contributions are

$$\mathcal{C}_7^{\chi^\pm}(M_W)=\sum_{k=1}^{6}\sum_{i=1}^{2}\sum_{\alpha,\beta=u,c,t}\frac{V_{\alpha b}V_{\beta s}^{*}}{V_{tb}V_{ts}^{*}} \times\left(G^{(\alpha,k)i}G^{*(\beta,k)i}F_L^7(z_k,s_i) + \frac{m_b}{\sqrt{2}M_W\cos\beta}H^{(\alpha,k)i}G^{*(\beta,k)i}\frac{M_{\chi^i}}{m_b}F_R^7(z_k,s_i)\right),$$

$$\mathcal{C}_8^{\chi^\pm}(M_W)=\sum_{k=1}^{6}\sum_{i=1}^{2}\sum_{\alpha,\beta=u,c,t}\frac{V_{\alpha b}V_{\beta s}^{*}}{V_{tb}V_{ts}^{*}} \times\left(G^{(\alpha,k)i}G^{*(\beta,k)i}F_L^8(z_k,s_i) + \frac{m_b}{\sqrt{2}M_W\cos\beta}H^{(\alpha,k)i}G^{*(\beta,k)i}\frac{M_{\chi^i}}{m_b}F_R^8(z_k,s_i)\right) \tag{28}$$

with the loop functions defined in Appendix B. Similarly to the situation for the Wilson coefficient C_3, we can, to a very good approximation, neglect the presence of intergenerational mixing in the up-squark mass matrix [9,10], then,

$$\mathcal{C}_7^{\chi^\pm}(M_W)=\sum_{k=3,6}\sum_{i=1}^{2}\left(G^{(3,k)i}G^{*(3,k)i}F_L^7(z_k,s_i) - G^{(1,1)i}G^{*(1,1)i}F_L^7(z_1,s_i) + \frac{m_{\chi^i}}{m_b}\frac{m_b}{\sqrt{2}M_W\cos\beta}[H^{(3,k)i}G^{*(3,k)i}F_R^7(z_k,s_i) - H^{(1,1)i}G^{*(1,1)i}F_R^7(z_1,s_i)]\right)$$

$$\mathcal{C}_8^{\chi^\pm}(M_W)=\sum_{k=3,6}\sum_{i=1}^{2}\Bigg(G^{(3,k)i}G^{*(3,k)i}F_L^8(z_k,s_i)-G^{(1,1)i}G^{*(1,1)i}F_L^8(z_1,s_i)+\frac{m_{\chi^i}}{m_b}\frac{m_b}{\sqrt{2}M_W\cos\beta}(H^{(3,k)i}G^{*(3,k)i}F_R^8(z_k,s_i)-H^{(1,1)i}G^{*(1,1)i}F_R^8(z_1,s_i))\Bigg) \tag{29}$$

where, once more, we use CKM unitarity and degeneracy of the first two generations of squarks.

The second term in $\mathcal{C}_{7,8}$ in Eq. (30), which corresponds to the chargino mass insertion in the loop, is dominant in the large $\tan\beta$ regime. Notice that both $G^{*(\alpha,k)i}$ and $H^{(\alpha,k)i}$ are products of the squark and chargino mixing matrices that can be $\mathcal{O}(1)$ (in the case of flavor-diagonal stop mixings). Then for stop and chargino masses around the electroweak scale, this term has an extra enhancement of $1/\cos\beta$. This means that for large $\tan\beta$ we can approximate these Wilson coefficients as

$$\mathcal{C}_7^{\chi^\pm}(M_W)=\sum_{k=3,6}\sum_{i=1}^{2}\frac{m_{\chi^i}}{m_b}\frac{m_b}{\sqrt{2}M_W\cos\beta}[H^{(3,k)i}G^{*(3,k)i}F_R^7(z_k,s_i)-H^{(1,1)i}G^{*(1,1)i}F_R^7(z_1,s_i)],$$

$$\mathcal{C}_8^{\chi^\pm}(M_W)=\sum_{k=3,6}\sum_{i=1}^{2}\frac{m_{\chi^i}}{m_b}\frac{m_b}{\sqrt{2}M_W\cos\beta}[H^{(3,k)i}G^{*(3,k)i}F_R^8(z_k,s_i)-H^{(1,1)i}G^{*(1,1)i}F_R^8(z_1,s_i)]. \tag{30}$$

V. $B\to S\gamma$ AND $\varepsilon_{\mathcal{M}}$: CORRELATED ANALYSIS

As we have seen in Sec. III, chargino contribution to the C_3 Wilson coefficient, Eq. (20), is the main contribution to indirect CP violation of the new supersymmetric phases for large values of $\tan\beta$. However, if we compare this Wilson coefficient with the chargino contribution to the decay $b\to s\gamma$, Eqs. (20) and (30), we can see that both chargino contributions are deeply related. In fact, if we make a rough approximation and assume that the two different loop functions involved are of the same order, i.e.,

$$Y_2(z_k,z_l,s_i,s_j)\approx\sqrt{s_is_j}\;\;F_R^7(z_k,s_i)\;\;F_R^7(z_l,s_j), \tag{31}$$

we would obtain

$$C_3(M_W)=(\mathcal{C}_7(M_W))^2\frac{m_q^2}{M_W^2}. \tag{32}$$

Of course, this cannot be considered as a good approximation. As we can see from their explicit expressions in Appendix B, the loop functions are clearly different. Anyway, they can be expected to give results of the same order of magnitude. So the order of magnitude of C_3 is determined by the allowed values of $\mathcal{C}_7$, as we will explicitly show below.

To reach this goal, we will follow [18], where they constrain in a model-independent way new physics contributions to the Wilson coefficients involved in the $b\to s\gamma$ decay. In terms of these Wilson coefficients, the branching ratio $\mathrm{BR}(B\to X_s\gamma)$ is

$$\mathrm{BR}(B\to X_s\gamma)\simeq 1.258+0.382|\xi_7|^2+0.015|\xi_8|^2+1.395\,\mathrm{Re}[\xi_7]+0.161\,\mathrm{Re}[\xi_8]+0.083\,\mathrm{Re}[\xi_7\xi_8^*], \tag{33}$$

where $\xi_a=\mathcal{C}_a(M_W)/\mathcal{C}_a^{W^\pm}(M_W)$. The different coefficients appearing in Eq. (33) are the SM renormalization group evolved contributions that must be recovered in the limit $\xi_a=1$. The numerical values are taken from [18]. We have not taken into account the errors associated with the choice of the scale and the restrictions on the photon energy that do not modify our conclusions. Now using the experimental measure, $\mathrm{BR}(B\to X_s\gamma)=(3.14\pm0.48)\times10^{-4}$, we can constrain the allowed values of the complex variables ξ_7 and ξ_8. In fact, we can already see from Eq. (33) that in the approximation $\xi_7\approx\xi_8$ this is simply the equation of an ellipse in the $\mathrm{Re}[\xi_7]-\mathrm{Im}[\xi_7]$ plane. In the case of supersymmetry with large $\tan\beta$, the new physics contribution to ξ_7 and ξ_8 will be mainly due to the chargino. The allowed values of ξ_7 directly constrain then the chargino contributions to $\mathcal{C}_7(M_W)$ and indirectly constrain the values of $C_3(M_W)$.

In Fig. 3, we show a scatter plot of the allowed values of $\mathrm{Re}(\mathcal{C}_7)$ versus $\mathrm{Im}(\mathcal{C}_7)$ in the CMSSM for a fixed value of $\tan\beta$ with the constraints from Eq. (33). Notice that a relatively large value of $\tan\beta$, for example, $\tan\beta\gtrsim10$, is needed to compensate the W and charged Higgs contributions and cover the whole allowed area with positive and negative values. However, the shape of the plot is clearly independent of $\tan\beta$; only the number of allowed points and its location in the allowed area depend on the value considered. In this figure we take $\tan\beta=40$ because only a large value could give rise to observable CP violation [5]. The values of $\mathcal{C}_7$ and $\mathcal{C}_8$ used here are the values obtained in the CMSSM for

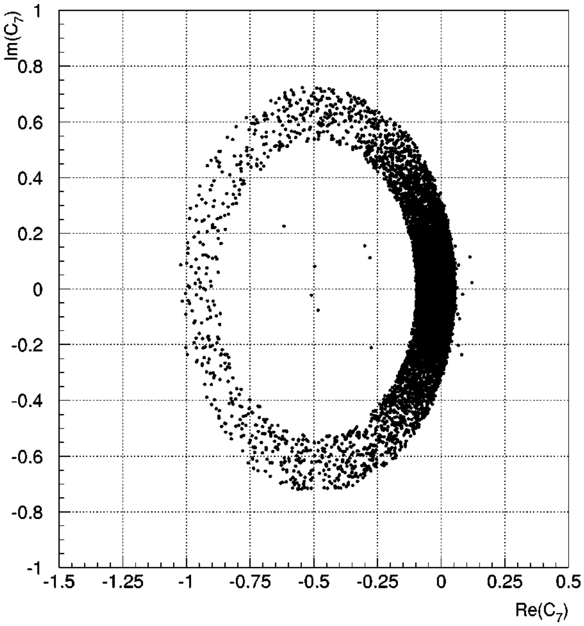

FIG. 3. Experimental constraints on the Wilson coefficient $\mathcal{C}_7$.

a given set of initial conditions. Although we do not use the approximation $\xi_7 \approx \xi_8$ this does not modify the elliptic shape of the plot.

Figure 4 shows the allowed values for a rescaled Wilson coefficient $\overline{C}_3(M_W) = M_W^2/m_q^2 C_3(M_W)$ corresponding to the same allowed points of the SUSY parameter space in Fig. 3. As we anticipated previously, the allowed values for $\overline{C}_3$ are close to the square of the values of $\mathcal{C}_7$ in Fig. 3 slightly scaled by different values of the loop functions. This is the proof of the importance of the $b \to s\gamma$ constraint on the chargino contributions to indirect CP violation.

We can immediately translate this result to a constraint on the size of the chargino contributions to $\varepsilon_{\mathcal{M}}$:

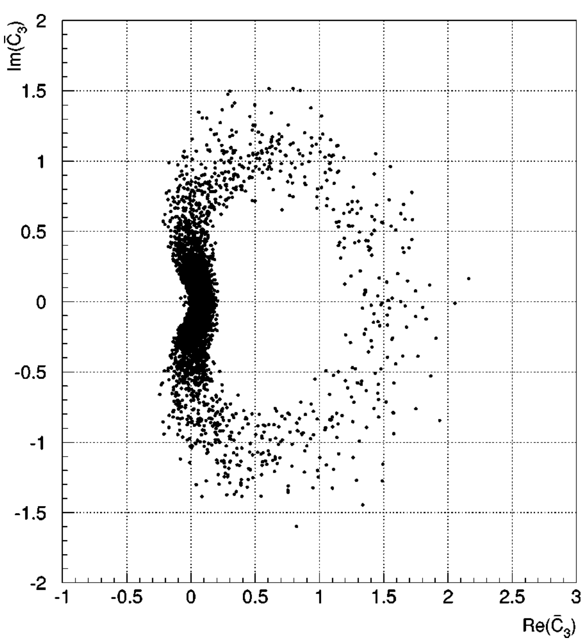

FIG. 4. Allowed values for the rescaled WC $\overline{C}_3$.

$$\varepsilon_{\mathcal{M}} = \frac{G_F^2 M_W^2}{4\pi^2\sqrt{2}\ \ \Delta M_{\mathcal{M}}} \frac{(V_{td}V_{tq})^2}{24} F_{\mathcal{M}}^2 M_{\mathcal{M}} \eta_3(\mu) B_3(\mu) \times \frac{M_{\mathcal{M}}^2}{m_q^2(\mu) + m_d^2(\mu)} \mathrm{Im}[C_3]. \tag{34}$$

In this expression $M_{\mathcal{M}}$, $\Delta M_{\mathcal{M}}$, and $F_{\mathcal{M}}$ denote the mass, mass difference, and decay constant of the neutral meson $\mathcal{M}^0$. The coefficient $\eta_3(\mu) = 2.93$ [19] includes the RGE effects from M_W to the meson mass scale, μ, and $B_3(\mu)$, the B parameter associated with the matrix element of the Q_3 operator [19].

Then for the K system, using the experimentally measured value of ΔM_K, we obtain

$$\varepsilon_K^{\chi} = 1.7 \times 10^{-2} \frac{m_s^2}{M_W^2} \mathrm{Im}[\overline{C}_3] \approx 0.4 \times 10^{-7} \mathrm{Im}[\overline{C}_3]. \tag{35}$$

Given the allowed values of $\overline{C}_3$ in Fig. 4, this means that in the CMSSM, even with large SUSY phases, chargino cannot produce a sizable contribution to ε_K. We have seen in Sec. III that gluino and neutralino also give negligible contributions in the CMSSM or in a model without off-diagonal soft-breaking terms at the GUT scale. Hence indirect CP violation in the kaon system will be mainly given by the usual SM box and the presence of a CP violating phase in the CKM matrix, δ_{CKM}, is still needed.

The case of B^0-$\overline{B}^0$ mixing has a particular interest due to the arrival of new data from the B factories. In fact, as explained at the end of Sec. III and in [5], in the large $\tan\beta$ regime chargino contributions to indirect CP violation can be very important. However, for any value of $\tan\beta$ we must satisfy the bounds from the $b \to s\gamma$ decay. So if we apply these constraints to the $B^0 - \overline{B}^0$ mixing,

$$\varepsilon_B^{\chi} = 0.17 \frac{m_b^2}{M_W^2} \mathrm{Im}[\overline{C}_3] \approx 0.5 \times 10^{-3} \mathrm{Im}[\overline{C}_3] \tag{36}$$

where once again, with the allowed values of Fig. 4, we get a very small contribution to CP violation in the mixing. We must take into account that the mixing-induced CP phase, θ_M, measurable in B^0 CP asymmetries, is related to ε_B by $\theta_M = \arcsin\{2\sqrt{2} \cdot \varepsilon_B\}$. The expected sensitivities on the CP phases at the B factories are around ± 0.1 radians, so this supersymmetric chargino contribution will be completely out of reach. Gluino and neutralino contributions to indirect CP violation can also be discarded in the CMSSM. Once again we have to conclude that no new contributions to indirect CP violation from the new SUSY phases will be observable in B^0 CP asymmetries in the framework of the CMSSM. Recently, the Collider Detector at Fermilab (CDF) [20] has provided preliminary indications that $\sin 2\beta$ is in agreement with the SM predictions. Clearly, from the above result [Eq. (36)], it appears that the CMSSM contribution is too small by itself to account for this result.

VI. CONCLUSIONS

In this work, the effects of nonvanishing supersymmetric phases on indirect CP violation in K and B systems have been analyzed within the CMSSM. We have found that operators involving only left-handed external quarks are not sensitive to these new phases at an observable level. This is due to the absence of intergenerational mixings beyond those originated from the CKM matrix. On the contrary, operators involving both right- and left-handed quarks are in general complex, even in the absence of $\delta_{\rm CKM}$, and could be relevant in the large $\tan\beta$ regime. However, we have shown that these contributions are deeply related with the BR($B \to X_s \gamma$) decay. So, taking into account the constraints coming from this decay, these contributions also turn out to be too small to be measured experimentally.

Although these conclusions are specific for indirect CP violation, they could also be implemented for chargino mediated direct CP violation in the decays. Again, in these decays the same chargino-quark-squark couplings are involved and we can also expect a big impact of the $b \to s\gamma$ constrain. In fact, the conclusions reached in this paper are far more general. The correlation between $b \to s\gamma$ and SUSY induced indirect CP violation exists in any supersymmetric model with sufficiently small intergenerational mixings in the sfermion mass matrices. This would include specifically all the models without new flavor structures beyond the usual CKM matrix at the GUT scale and simplified models as the one the authors used in [5].

In summary, concerning the simpler supersymmetric models, like CMSSM, the constraints coming from BR($B \to X_s \gamma$) decay are sufficient to rule out pure supersymmetric indirect CP violation in K and B systems, even in the absence of any electric dipole moment constraints. This has very important consequences for the supergravity induced models where a cancellation between different supersymmetric contributions allows large supersymmetric phases while respecting EDM bounds [2]. In these models, even in the regions of parameter space where this cancellation occurs, no observable effect of the large SUSY phases will appear on indirect CP violation experiments. However, as pointed out by Baek and Ko [6], these phases would still be observable in CP asymmetries in the $b \to s\gamma$ decay.

All this means that the presence of large SUSY phases is not sufficient to produce observable effects at the low energy experiments. In particular, new sources of flavor change beyond the usual CKM matrix are needed. And so, any deviation from the SM expectations at indirect CP violation experiments due to supersymmetry should be taken as a sign of nonuniversality of the soft-breaking terms. In this context one recalls the recent studies on superstring compactifications with nonuniversal gaugino masses [21].

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APPENDIX A: INTEGRATION OF RGE'S IN CMSSM

In this Appendix we describe the new features of the integration of RGE's in the CMSSM with nonvanishing SUSY phases relevant to our analysis. The complete matrix form of the RG equations can be found in [9]. Using their notation and conventions, with the only change of $A_q = m Y_q^A$, we will mainly concentrate on the left-left scalar-quark mass matrix and the trilinear soft-breaking coupling evolution:

$$\frac{dm_Q^2}{dt} = (\tfrac{16}{3}\tilde{\alpha}_3 M_3^2 + 3\tilde{\alpha}_2 M_2^2 + \tfrac{1}{9}\tilde{\alpha}_1 M_1^2)\mathbf{1} - \tfrac{1}{2}[\tilde{Y}_U\tilde{Y}_U^\dagger m_Q^2 + m_Q^2\tilde{Y}_U\tilde{Y}_U^\dagger + 2(\tilde{Y}_U m_U^2 \tilde{Y}_U^\dagger + \overline{\mu_2^2}\tilde{Y}_U\tilde{Y}_U^\dagger + \tilde{A}_U\tilde{A}_U^\dagger)] + \tfrac{1}{2}[\tilde{Y}_D\tilde{Y}_D^\dagger m_Q^2 + m_Q^2\tilde{Y}_D\tilde{Y}_D^\dagger + 2(\tilde{Y}_D m_D^2\tilde{Y}_D^\dagger + \overline{\mu_1^2}\tilde{Y}_D\tilde{Y}_D^\dagger + \tilde{A}_D\tilde{A}_D^\dagger)], \tag{A1}$$

$$\frac{d\tilde{A}_U}{dt} = \tfrac{1}{2}(\tfrac{16}{3}\tilde{\alpha}_3 + 3\tilde{\alpha}_2 + \tfrac{1}{9}\tilde{\alpha}_1)\tilde{A}_U - (\tfrac{16}{3}\tilde{\alpha}_3 M_3 + 3\tilde{\alpha}_2 M_2 + \tfrac{1}{9}\tilde{\alpha}_1 M_1)\tilde{Y}_U - \tfrac{1}{2}[4\tilde{A}_U\tilde{Y}_U^\dagger\tilde{Y}_U + 6\,\mathrm{Tr}(\tilde{A}_U\tilde{Y}_U^\dagger)\tilde{Y}_U + 5\tilde{Y}_U\tilde{Y}_U^\dagger\tilde{A}_U + 3\,\mathrm{Tr}(\tilde{Y}_U\tilde{Y}_U^\dagger)\tilde{A}_U + 2\tilde{A}_D\tilde{Y}_D^\dagger\tilde{Y}_U + \tilde{Y}_D\tilde{Y}_D^\dagger\tilde{A}_U], \tag{A2}$$

$$\frac{d\tilde{A}_D}{dt} = \tfrac{1}{2}(\tfrac{16}{3}\tilde{\alpha}_3 + 3\tilde{\alpha}_2 + \tfrac{1}{9}\tilde{\alpha}_1)\tilde{A}_D - (\tfrac{16}{3}\tilde{\alpha}_3 M_3 + 3\tilde{\alpha}_2 M_2 + \tfrac{1}{9}\tilde{\alpha}_1 M_1)\tilde{Y}_D - \tfrac{1}{2}[4\tilde{A}_D\tilde{Y}_D^\dagger\tilde{Y}_D + 6\mathrm{Tr}(\tilde{A}_D\tilde{Y}_D^\dagger)\tilde{Y}_D + 5\tilde{Y}_D\tilde{Y}_D^\dagger\tilde{A}_D + 3\mathrm{Tr}(\tilde{Y}_D\tilde{Y}_D^\dagger)\tilde{A}_D + 2\tilde{A}_U\tilde{Y}_U^\dagger\tilde{Y}_D + \tilde{Y}_U\tilde{Y}_U^\dagger\tilde{A}_D + 2\mathrm{Tr}(\tilde{A}_E\tilde{Y}_E^\dagger)\tilde{Y}_D + Tr(\tilde{Y}_E\tilde{Y}_E^\dagger)\tilde{A}_D]. \tag{A3}$$

Except for the Yukawa coupling matrices, the RGE's of all other quantities are linear [9]. This means, in particular, that RGE's of all soft masses, though coupled, can be solved as a linear combination of the GUT-scale parameters m_0, $A_0 e^{i\phi_A}$, and

$M_{1/2}$ at any scale below M_G. However, one notices that the initial conditions on the trilinear couplings require the knowledge of the particular Yukawa texture at the unification scale. To do this, we numerically integrate the Yukawa RGE with a given value of $\tan\beta$ and in terms of the fermion masses and the CKM matrix. Specifying the GUT-scale initial conditions in this way, it is straightforward to compute all soft masses at M_W for arbitrary values of m_0, $A_0 e^{i\phi_A}$, and $M_{1/2}$. Thanks to the linearity of the corresponding RGE's, the soft masses at M_W admit the following expansion

$$A_{U,D}(M_Z) = \alpha^A_{U,D} A_0 e^{i\phi_A} + \alpha^g_{U,D} M_{1/2}, \tag{A4}$$

$$m^2_{Q,U,D}(M_Z) = \eta^m_{Q,U,D}\, m_0^2 + \eta^A_{Q,U,D} A_0^2 + \eta^g_{Q,U,D} M^2_{1/2} + (\eta^{(g\,A)}_{Q,U,D} e^{i\phi_A} + \eta^{(g\,A)\,T}_{Q,U,D} e^{-i\phi_A}) A_0 M_{1/2},$$

where the coefficients α and η are 3×3 matrices with real numerical entries. One notices that the matrices $m^2_{Q,U,D}(M_Z)$ would be completely real were it not for the nonsymmetric terms in the matrix $\eta^{g\,A}_{Q,U,D}$. However, it will be seen from the specific examples that this matrix remains nearly symmetric and, thus, CP violating entries $m^2_{Q,U,D}(M_Z)$ are extremely suppressed. Moreover, one notices that $A_{U,D}(M_Z)$ carries, in general, large CP violating phases; however, these terms are effective only for intragenerational LR-type mixings. Hence, this particular observation shows the importance of chargino contributions for CP violation in FCNC processes, as explained in Sec. II.

As mentioned before, due to the nonlinearity of the RGE's for Yukawa matrices, it is not possible to give a fully analytic solution for the soft mass parameters. Nevertheless, once we fix $\tan\beta$, we can numerically integrate the Yukawa RGE. Therefore, below we give semianalytic solutions of RGE's for $\tan\beta=2$ and $\tan\beta=40$ to illustrate the small and large $\tan\beta$ regimes.

Fixing $\tan\beta=2$, we get for the relevant η matrices in Eq. (A4),

$$\eta^g_Q = \begin{pmatrix} 7.07 & 2.79\times10^{-4} & -7.02\times10^{-3} \\ 2.79\times10^{-4} & 7.07 & 4.92\times10^{-2} \\ -7.02\times10^{-3} & 4.92\times10^{-2} & 5.74 \end{pmatrix}, \tag{A5}$$

$$\tfrac{1}{2}(\eta^{(g\,A)}_Q + \eta^{(g\,A)\,T}_Q) = \begin{pmatrix} 5.34\times10^{-6} & -3.44\times10^{-5} & 7.90\times10^{-4} \\ -3.44\times10^{-5} & 2.29\times10^{-4} & -5.52\times10^{-3} \\ 7.90\times10^{-4} & -5.52\times10^{-3} & 0.15 \end{pmatrix}, \tag{A6}$$

$$\tfrac{1}{2}(\eta^{(g\,A)}_Q - \eta^{(g\,A)\,T}_Q) = \begin{pmatrix} 0 & 0 & 1.34\times10^{-8} \\ 0 & 0 & -8.55\times10^{-8} \\ -1.34\times10^{-8} & 8.55\times10^{-8} & 0 \end{pmatrix}, \tag{A7}$$

where the vanishing off-diagonal entries in the last matrix mean values smaller than 10^{-10} in absolute magnitude. Among the matrices involved in Eq. (A4), η^g is always the largest one for similar values of $M_{1/2}$ and m_0. So it sets the scale of the matrix element while $\eta^{(gA)}$ is the only one that can produce an imaginary part. Hence, we do not specify the other η matrices, which area not important for our discussion.

Once we obtain the $m_Q(M_W)$ matrix with the help of Eq. (A4) we can get the values of the $M^{(u)2}_{LL}$ and $M^{(d)2}_{LL}$ in the SCKM basis that give the size of flavor change in the squark mass matrices compared with the diagonal elements. For $\tan\beta=2$, those elements of the squark mass-squared matrix causing LL transitions between first and second, as well as second and third generations, are given by

$$(M^{(u)2}_{LL})_{12} = -2.79\times10^{-7}\, m_0^2 - 9.30\times10^{-8}\, A_0^2 - 1.17\times10^{-6}\, M^2_{1/2} + 8.15\times10^{-7}\, A_0 M_{1/2} \cos\phi_A, \tag{A8}$$

$$(M^{(u)2}_{LL})_{23} = -4.07\times10^{-5}\, m_0^2 - 1.15\times10^{-5}\, A_0^2 - 1.61\times10^{-4}\, M^2_{1/2} + 1\times10^{-4}\, A_0 M_{1/2} \cos\phi_A - 1.71\times10^{-7}\, A_0 M_{1/2}\; i\sin\phi_A, \tag{A9}$$

$$(M^{(d)2}_{LL})_{12} = 9.38\times10^{-5}\, m_0^2 + 3.75\times10^{-6}\, A_0^2 + 2.79\times10^{-4}\, M^2_{1/2} + 6.87\times10^{-5}\, A_0 M_{1/2} \cos\phi_A, \tag{A10}$$

$$(M^{(d)2}_{LL})_{23} = 1.67\times10^{-2}\, m_0^2 + 5.32\times10^{-4}\, A_0^2 + 4.91\times10^{-2}\, M^2_{1/2} - 1.1\times10^{-2}\, A_0 M_{1/2} \cos\phi_A - 1.70\times10^{-7}\, A_0 M_{1/2}\; i\sin\phi_A. \tag{A11}$$

Now, we repeat the same quantities above for $\tan\beta=40$:

$$\eta_Q^g = \begin{pmatrix} 7.07 & 2.44\times 10^{-4} & -5.80\times 10^{-3} \\ 2.44\times 10^{-4} & 7.07 & 4.06\times 10^{-2} \\ -5.80\times 10^{-3} & 4.06\times 10^{-2} & 4.97 \end{pmatrix}, \tag{A12}$$

$$\frac{1}{2}(\eta_Q^{(g\,A)} + \eta_Q^{(g\,A)\,T}) = \begin{pmatrix} 8.32\times 10^{-6} & -4.57\times 10^{-5} & 7.82\times 10^{-4} \\ -4.57\times 10^{-5} & 5.20\times 10^{-4} & -5.47\times 10^{-3} \\ 7.82\times 10^{-4} & -5.47\times 10^{-3} & 0.22 \end{pmatrix}, \tag{A13}$$

$$\frac{1}{2}(\eta_Q^{(g\,A)} - \eta_Q^{(g\,A)\,T}) = \begin{pmatrix} 0 & 0 & -1.64\times 10^{-6} \\ 0 & 0 & 1.14\times 10^{-5} \\ 1.64\times 10^{-6} & -1.14\times 10^{-5} & 0 \end{pmatrix}, \tag{A14}$$

$$(M_{LL}^{2\,(u)})_{12} = -8.77\times 10^{-5}\, m_0^2 - 2.77\times 10^{-5}\, A_0^2 - 3.0\times 10^{-4}\, M_{1/2}^2 + 1.21\times 10^{-4}\, A_0 M_{1/2} \cos\phi_A + i\, 1.1\times 10^{-10}\, A_0 M_{1/2} \sin\phi_A, \tag{A15}$$

$$(M_{LL}^{2\,(u)})_{23} = -1.28\times 10^{-2}\, m_0^2 - 2.70\times 10^{-3}\, A_0^2 - 3.77\times 10^{-2}\, M_{1/2}^2 + 5.67\times 10^{-3}\, A_0 M_{1/2} \cos\phi_A + i\, 2.30 \times 10^{-5}\, A_0 M_{1/2} \sin\phi_A, \tag{A16}$$

$$(M_{LL}^{2\,(d)})_{12} = 7.51\times 10^{-5}\, m_0^2 + 7.74\times 10^{-6}\, A_0^2 + 2.44\times 10^{-4}\, M_{1/2}^2 - 9.13\times 10^{-5}\, A_0 M_{1/2} \cos\phi_A, \tag{A17}$$

$$(M_{LL}^{2\,(d)})_{23} = 1.34\times 10^{-2}\, m_0^2 + 7.84\times 10^{-4}\, A_0^2 + 4.05\times 10^{-2}\, M_{1/2}^2 - 1.1\times 10^{-2}\, A_0 M_{1/2} \cos\phi_A + i\, 2.28\times 10^{-5}\, A_0 M_{1/2} \sin\phi_A. \tag{A18}$$

A comparison of the corresponding quantities in $\tan\beta=2$ and $\tan\beta=40$ cases reveals the sensitivity of the results on $\tan\beta$. As explained in Sec. II, $Y_U(M_Z)$ remains nearly unchanged while $Y_D(M_Z)$ assumes an order of magnitude enhancement as $\tan\beta$ varies from 2 to 40. This change in $Y_D(M_Z)$ affects various quantities as dictated by the differential equations (A1)–(A3).

APPENDIX B: LOOP FUNCTIONS

In this appendix we collect the different loop function used in the text. The functions Y_1 and Y_2 entering $B-\overline{B}$ and $K-\overline{K}$ mixings are given by

$$Y_1(a,b,c,d) = \frac{a^2}{(b-a)(c-a)(d-a)}\ln a + \frac{b^2}{(a-b)(c-b)(d-b)}\ln b + \frac{c^2}{(a-c)(b-c)(d-c)}\ln c + \frac{d^2}{(a-d)(b-d)(c-d)}\ln d \tag{B1}$$

and

$$Y_2(a,b,c,d) = \sqrt{4cd}\left[\frac{a}{(b-a)(c-a)(d-a)}\ln a + \frac{b}{(a-b)(c-b)(d-b)}\ln b + \frac{c}{(a-c)(b-c)(d-c)}\ln c + \frac{d}{(a-d)(b-d)(c-d)}\ln d\right]. \tag{B2}$$

For the analysis of $b\to s\gamma$ branching ratio the following loop functions are relevant:

$$F_1(x) = \frac{1}{12(x-1)^4}(x^3 - 6x^2 + 3x + 2 + 6x\ln x), \tag{B3}$$

$$F_2(x) = \frac{1}{12(x-1)^4}(2x^3 + 3x^2 - 6x + 1 - 6x^2 \ln x), \tag{B4}$$

$$F_3(x) = \frac{1}{2(x-1)^3}(x^2 - 4x + 3 + 2\ln x), \tag{B5}$$

$$F_4(x)=\frac{1}{2(x-1)^3}(x^2-1-2x\ln x), \tag{B6}$$

$$F_L^7(x,y)=\frac{1}{x}[Q_U F_2(y/x)+F_1(y/x)], \tag{B7}$$

$$F_R^7(x,y)=\frac{1}{x}[Q_U F_4(y/x)+F_3(y/x)], \tag{B8}$$

$$F_L^8(x,y)=\frac{1}{x}F_2(y/x), \tag{B9}$$

$$F_R^8(x,y)=\frac{1}{x}F_4(y/x). \tag{B10}$$

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