

S , T , U parameters in an $SU(3)_C \times SU(3)_L \times U(1)$ model with right-handed neutrinos

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(Received 4 March 1999; published 18 February 2000)

The S , T , U parameters in the $SU(3)_C \times SU(3)_L \times U(1)$ model with right-handed neutrinos are calculated. Explicit expressions for the oblique and Z - Z' mixing contributions are obtained. We show that the bilepton oblique contributions to S and T parameters are bounded: $-0.085 \leq S \leq 0.05$, $-0.001 \leq T \leq 0.08$. The Z - Z' mixing contribution is positive and above 10%, but it will increase quickly with a higher Z' mass. The consequent mass splitting of the bilepton is derived to be 15%. The limit on the mass of the *neutral* bilepton in this model is obtained.

PACS number(s): 12.60.Cn, 12.10.Dm, 14.80.Cp

I. INTRODUCTION

Evidence for neutrino oscillation and consequently a non-zero neutrino mass from the SuperKamiokande atmospheric neutrino data is compelling [1]. This is the first experimental measurement that significantly deviates from the standard model (SM), and calls for its extension.

Among the possible extensions, the models based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$ (3 3 1) gauge group [2,3] have the following intriguing features. First, the models are anomaly-free only if the number of families N is a multiple of three. Further, from the condition of QCD asymptotic freedom, which means $N < 5$, it follows that N is equal to 3. The second characteristic is that Peccei-Quinn (PQ) [4] symmetry, a solution of the strong CP problem, naturally occurs in these models [5]. It is worth mentioning that the implementation of PQ symmetry is usually possible only at a classical level (it will be broken by quantum corrections through instanton effects), and there have been a number of attempts to find models that solve the strong CP question. In the 3 3 1 models the PQ symmetry following from the gauge invariant Lagrangian does not have to be imposed. The third interesting feature is that one of the quark families is treated differently from the other two [6,7]. This could lead to a natural explanation of the unbalancing heavy top quarks in the fermion mass hierarchy [7]. Recent analyses have indicated that signals of new particles in this model, bileptons [8] and exotic quarks [9] may be observed at the Fermilab Tevatron and the CERN Large Hadron Collider (LHC).

There are two main versions of the 3 3 1 models: the minimal in which all lepton components $(\nu, l, l^c)_L$ belong to the same lepton triplet and a variant, in which right-handed neutrinos are included, i.e., $(\nu, l, \nu^c)_L$ are in the triplet (hereafter we call it a model with right-handed neutrinos [11,12]). New gauge bosons in the minimal model are bileptons $(Y^\pm, Y^{\pm\pm})$ carrying lepton number $L = \pm 2$ and Z' . Most analyses of the 3 3 1 models have centered on the bileptons [13–16] and Z' [17,18]. In the second model, the bileptons with lepton number $L = \pm 2$ are singly charged $Y^\pm$ and *neutral* gauge bosons $X^0, \bar{X}^0$, and both are responsible for lepton violating interactions. This model is interesting because of the existence of right-handed neutrinos and the neutral bilepton X^0 , the later being a promising candidate in accelerator

experiments [19]. Since the symmetry of the $SU(2)_L$ gauge group is broken, generically the neutral bilepton has a mass M_{X^0} different from that of the singly charged bilepton $M_{Y^\pm}$. However, looking at recent review [15] we see that there is almost no bound on the X^0 mass (the limit given there for M_{X^0} is above 44 GeV).

Heavy particles can be indirectly observable via radiative corrections in the SM-type theories [20]. At present the oblique radiative parameters S , T [21] and U [22] can be used optimally to extract new-physics effects. In the early papers the focus was on fermionic contributions [23]. The aim of this paper is to calculate the S , T , U parameters, and to get a bound on the bilepton masses.

This paper is organized as follows. In Sec. II we briefly introduce necessary elements of the model, and the bilepton mass splitting due to the symmetry breaking is given. Section III is devoted to calculating the new gauge boson contributions to the S , T , U parameters. We make a remark on the minimal model in Sec. IV. A numerical evaluation is presented in Sec. V. We summarize our result and make conclusions in the last section.

II. THE MODEL AND BILEPTON MASS SPLITTING

In this section we first recapitulate the basic elements of the model. Based on the vacuum expectation value (VEV) structure and the muon decay experiment we obtain a bound on the neutral bilepton mass M_{X^0} . The details can be found in Ref. [11]. In the variant of the 3 3 1 model the third member of the lepton triplet is a right-handed (RH) neutrino instead of the antilepton l^c_L :

$$f_L^a = [\nu_L^a, e_L^a, (\nu_L^c)^a]^T \sim (1, 3, -1/3), \quad (2.1)$$

where $a=1,2,3$ is the generation index.

This assignment leads to the electric charge and hypercharge operators which are now defined by

$$Q = \frac{1}{2}\lambda_3 - \frac{1}{2\sqrt{3}}\lambda_8 + N,$$

$$Y = 2N - \lambda_8 / \sqrt{3} \quad [\lambda_8 = \text{diag}(1, 1, -2) / \sqrt{3}].$$

The exotic quarks have charges $2/3$ and $-1/3$ and are $SU(2)_L$ singlets

$$Q_{iL} = \begin{pmatrix} d_{iL} \\ -u_{iL} \\ D_{iL} \end{pmatrix} \sim (3, \bar{3}, 0), \quad D_{iR} \sim (3, 1, -1/3), \quad i=1,2,$$

$$Q_{3L} = \begin{pmatrix} u_{3L} \\ d_{3L} \\ T_L \end{pmatrix} \sim (3, 3, 1/3), \quad T_R \sim (3, 1, 2/3). \quad (2.2)$$

The symmetry breaking can be achieved with three $SU(3)_L$ Higgs triplets

$$\chi = (\chi^0, \chi^-, \chi^{o-})^T, \quad \rho = (\rho^+, \rho^0, \rho^{o+})^T, \quad \eta = (\eta^0, \eta^-, \eta^{o-})^T. \quad (2.3)$$

They acquire the vacuum expectation values (VEVs): $\langle \chi \rangle^T = (0, 0, \omega/\sqrt{2})$, $\langle \rho \rangle^T = (0, u/\sqrt{2}, 0)$, and $\langle \eta \rangle^T = (v/\sqrt{2}, 0, 0)$. The gauge symmetry is broken to the SM gauge symmetry by $\omega \neq 0$. The complex gauge bosons $\sqrt{2}W_\mu^+ = W_\mu^1 - iW_\mu^2$, $\sqrt{2}Y_\mu^- = W_\mu^6 - iW_\mu^7$, $\sqrt{2}X_\mu^0 = W_\mu^4 - iW_\mu^5$ have the following masses:

$$m_W^2 = \frac{1}{4}g^2(u^2 + v^2), \quad M_Y^2 = \frac{1}{4}g^2(v^2 + \omega^2),$$

$$M_X^2 = \frac{1}{4}g^2(u^2 + \omega^2). \quad (2.4)$$

The physical neutral gauge bosons are mixtures of Z, Z' :

$$Z^1 = Z \cos \phi - Z' \sin \phi,$$

$$Z^2 = Z \sin \phi + Z' \cos \phi. \quad (2.5)$$

Here the photon field A_μ and Z, Z' are given by [11]

$$A_\mu = s_W W_\mu^3 + c_W \left(-\frac{t_W}{\sqrt{3}} W_\mu^8 + \sqrt{1 - \frac{t_W^2}{3}} B_\mu \right),$$

$$Z_\mu = c_W W_\mu^3 - s_W \left(-\frac{t_W}{\sqrt{3}} W_\mu^8 + \sqrt{1 - \frac{t_W^2}{3}} B_\mu \right), \quad (2.6)$$

$$Z'_\mu = \sqrt{1 - \frac{t_W^2}{3}} W_\mu^8 + \frac{t_W}{\sqrt{3}} B_\mu,$$

where the usual notation is used: $s_W \equiv \sin \theta_W$. The mixing angle ϕ is given by

$$\tan^2 \phi = \frac{m_Z^2 - m_{Z^1}^2}{M_{Z^2}^2 - m_Z^2}, \quad (2.7)$$

where m_{Z^1} and M_{Z^2} are the *physical* mass eigenvalues with

$$m_Z^2 = \frac{g^2}{4c_W^2}(u^2 + v^2) = \frac{m_W^2}{c_W^2}, \quad (2.8)$$

$$M_{ZZ'}^2 = -\frac{g^2}{4c_W^2 \sqrt{3 - 4s_W^2}}[u^2 - v^2(1 - 2s_W^2)], \quad (2.9)$$

$$M_{Z'}^2 = \frac{g^2}{4(3 - 4s_W^2)} \left[4\omega^2 + \frac{u^2}{c_W^2} + \frac{v^2(1 - 2s_W^2)^2}{c_W^2} \right]. \quad (2.10)$$

One of the Higgs bosons can be identified with the SM Higgs [25].

The lower limit on the singly charged bilepton is obtained by the “wrong” muon decay [24]

$$R = \frac{\Gamma(\mu^- \rightarrow e^- \nu_e \bar{\nu}_\mu)}{\Gamma(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu)} \sim \left(\frac{m_W}{M_Y} \right)^4. \quad (2.11)$$

The observed limit $R < 1.2\%$ (at 90% C.L.) gives $M_Y \geq 230$ GeV.

From Eq. (2.4) we get a bound on the bilepton mass splitting

$$|M_Y^2 - M_X^2| \leq m_W^2. \quad (2.12)$$

Combining Eqs. (2.11) and (2.12) we get the *first* preliminary constraint on the neutral bilepton mass:

$$M_{X^0} \geq 230 \pm 17 \text{ GeV, } 90\% \text{ C.L.} \quad (2.13)$$

In conclusion, the model predicts three kinds of new particles: new gauge bosons $Y^\pm$, X^0 , $\bar{X}^0$, and Z' , new exotic quarks T , D_1 , D_2 and new Higgs scalars. Bileptons ($Y^\pm, X^0$) make an $SU(2)_L$ doublet with hypercharge $Y = 1/2$, while exotic quarks and Z' are $SU(2)_L$ singlets. Because of the VEV structure, the mass splitting of the bileptons is bounded by the SM W boson mass m_W^2 .

III. CONTRIBUTIONS OF NEW PARTICLES TO S , T , U PARAMETERS

Since new quarks are $SU(2)_L$ singlets, they do not enter into the oblique corrections to the S , T , U parameters which are only sensitive to $SU(2)$ breaking. Similarly, Z' will not contribute except through Z - Z' mixing.

A. Effective interaction

We begin by writing the Lagrangian for the bileptonic gauge field Y and its would-be Nambu-Goldstone (WBNG) boson below the $SU(3)_L$ breaking scale. They are $SU(2)_L$ doublets with the hypercharge $Y = \frac{1}{2}$

$$\Phi = \begin{pmatrix} \Phi^+ \\ \Phi_0 \end{pmatrix}, \quad Y_\mu = \begin{pmatrix} Y_\mu^+ \\ X_\mu^0 \end{pmatrix}. \quad (3.1)$$

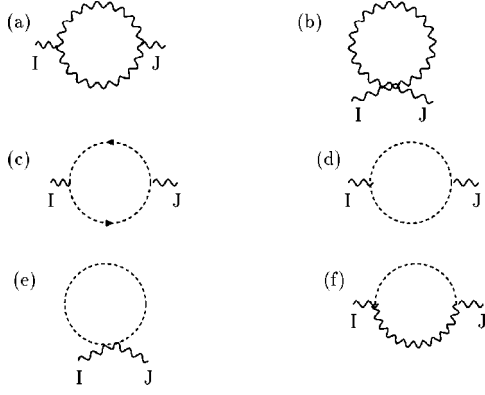


FIG. 1. Feynman diagrams contributing to vacuum polarizations Π_{IJ} ($I, J=1,3,8$). Wavy lines denote bileptons X, Y , dashed lines associated WBNG bosons, and dashed arrows FP ghosts.

The effective Lagrangian is given

$$\mathcal{L}_0 = -\frac{1}{2}(Y_{\mu\nu})^\dagger Y^{\mu\nu} + (D_\mu \Phi - iMY_\mu)^\dagger (D^\mu \Phi - iMY^\mu) - igY_\mu^\dagger F^{\mu\nu}(W)Y_\nu + i\frac{1}{2}g'Y_\mu^\dagger F^{\mu\nu}(B)Y_\nu, \quad (3.2)$$

where M is 2×2 matrix given by

$$M = \begin{pmatrix} M_+ & 0 \\ 0 & M_0 \end{pmatrix}, \quad (3.3)$$

and $D_\mu = \partial_\mu - igW_\mu + i\frac{1}{2}g'B_\mu$ with $g = \sqrt{3}g'$. For the shorthand hereafter we denote $M_{Y^+} \equiv M_+$, $M_{X^0} \equiv M_0$. As in the analysis of the minimal version [14], here we assume that the contribution of new physical Higgs bosons to S and T are small compared with the bilepton ones.

B. Oblique corrections

As it was shown in Ref. [13] contributions from Higgs fields turn on the masses of gauge bosons: the SM, Z' and bileptons. One loop diagrams contributing to vacuum polarizations Π_{IJ} ($I, J=1,3,8$) are shown in Fig. 1. Figure 1(c) is the Faddeev-Popov (FP) ghost contribution and Figs. 1(d)–1(f) are the contributions of the WBNG bosons. The calculations below were done in the 't Hooft-Feynman gauge. For convenience we use the following functions which will arise when a bilepton of one kind and its associated FP ghost and WBNG boson go around the loops:

$$E(q^2, M^2) = q^2 \left[3\Delta + \frac{2}{3} - \bar{F}_0(q^2, M, M) - 12\bar{F}_3(q^2, M, M) - 3 \ln M^2 \right] + M^2(\Delta + 1 - \ln M^2),$$

$$E'(q^2, M^2) = q^2 \left[3\Delta + \frac{2}{3} - \bar{F}_0(q^2, M, M) - 12\bar{F}_3(q^2, M, M) - 3 \ln M^2 \right] - 2M^2(3\Delta + 1 - 3 \ln M^2).$$

Here

$$\Delta \equiv \frac{2}{4-n} - \gamma_E - \ln(\pi),$$

where n is the space-time dimensionality, and γ_E is the Euler-Mascheroni constant.

The vacuum polarizations are then summarized by

$$\Pi_{38} = \frac{1}{64\pi^2\sqrt{3}}[E(q^2, M_0^2) - E(q^2, M_+^2)],$$

$$\Pi_{33} = \frac{1}{64\pi^2}[E'(q^2, M_+^2) + E'(q^2, M_0^2)], \quad (3.4)$$

$$\Pi_{11} = \frac{1}{32\pi^2} \left\{ q^2 \left[3\Delta - \frac{2}{3} - 5\bar{F}_0(q^2, M_+, M_0) + 12\bar{F}_3(q^2, M_+, M_0) - 3 \ln(M_+ M_0) \right] - M_+^2[3\Delta + 1 - \bar{F}_0(q^2, M_+, M_0)] - M_0^2[3\Delta + 1 - \bar{F}_0(q^2, M_+, M_0)] - 10\bar{F}_4(q^2, M_+, M_0) + 5(M_+^2 \ln M_+^2 + M_0^2 \ln M_0^2) - 2(M_+^2 \ln M_0^2 + M_0^2 \ln M_+^2) \right\},$$

where functions $\bar{F}_0(s, M, m)$, $\bar{F}_3(s, M, m)$, and $\bar{F}_4(s, M, m)$ are defined in Ref. [14]. They differ from those F s in Ref. [13] by a term proportional to $\ln(Mm)$.

For later use we write down the mentioned functions at $q^2=0$ and small q^2 behavior:

$$\bar{F}_0(0, M, m) = -\frac{1}{2} \left[2 - \frac{M^2 + m^2}{M^2 - m^2} \ln \frac{M^2}{m^2} \right] = -\frac{\varepsilon^2(M, m)}{4} + O[\varepsilon^3(M, m)], \quad (3.5)$$

$$\begin{aligned}\bar{F}_3(0, M, m) &= -\frac{5}{36} + \frac{M^2 m^2}{3(M^2 - m^2)^2} + \frac{M^2 + m^2}{12(M^2 - m^2)} \\ &\times \left[1 - \frac{2M^2 m^2}{(M^2 - m^2)^2} \right] \ln \frac{M^2}{m^2} \\ &= \frac{\varepsilon^2(M, m)}{12} + O[\varepsilon^3(M, m)],\end{aligned}$$

$$\begin{aligned}\bar{F}_4(0, M, m) &= -\frac{1}{4} \left[M^2 + m^2 - \frac{M^4 + m^4}{M^2 - m^2} \ln \frac{M^2}{m^2} \right] \\ &= m^2 \left[\frac{\varepsilon^2(M, m)}{6} + O[\varepsilon^3(M, m)] \right],\end{aligned}$$

$$\begin{aligned}\bar{F}_A(0, M, m) &= \frac{1}{2(M^2 - m^2)} \left[M^2 + m^2 - \frac{2M^2 m^2}{M^2 - m^2} \ln \frac{M^2}{m^2} \right] \\ &= \frac{\varepsilon(M, m)}{6} \left[1 - \frac{\varepsilon(M, m)}{2} \right] + O[\varepsilon^3(M, m)],\end{aligned}$$

where $\varepsilon(M, m) \equiv (M^2 - m^2)/m^2$. In the case of identical masses $m = M$ we have

$$\begin{aligned}\bar{F}_0(m_Z^2, M, M) &= -\frac{\delta(M)}{6} \left[1 + \frac{\delta(M)}{10} + \frac{\delta^2(M)}{70} \right] \\ &\quad + O[\delta^4(M)], \\ \bar{F}_3(m_Z^2, M, M) &= \frac{1}{6} \left[1 + \frac{2M^2}{m_Z^2} \right] \bar{F}_0(m_Z^2, M, M) + \frac{1}{18},\end{aligned}\tag{3.6}$$

where $\delta(M) \equiv m_Z^2/M^2$. The function $\bar{F}_4$ can be calculated through $\bar{F}_0$ by the following relation:

$$\begin{aligned}\bar{F}_4(m_W^2, M, m) &= \frac{M^2 + m^2}{2} \bar{F}_0(m_W^2, M, m) \\ &\quad - \frac{M^2 - m^2}{2m_W^2} [\bar{F}_0(m_W^2, M, m) - \bar{F}_0(0, M, m)].\end{aligned}\tag{3.7}$$

Other useful formulas are given in Appendix A of Ref. [14].

The contributions to S , T , U from bileptonic gauge bosons coming through the transverse self-energies are given

$$\begin{aligned}S_{\text{pol}} &= -16\pi \text{Re} \frac{\Pi^{3Y}(m_Z^2) - \Pi^{3Y}(0)}{m_Z^2} \\ &= \frac{1}{4\pi} \left\{ \ln \frac{M_+^2}{M_0^2} + \frac{1}{3} [\bar{F}_0(m_Z^2, M_+, M_+) \right. \\ &\quad \left. - \bar{F}_0(m_Z^2, M_0, M_0)] + 4[\bar{F}_3(m_Z^2, M_+, M_+) \right. \\ &\quad \left. - \bar{F}_3(m_Z^2, M_0, M_0)] \right\},\end{aligned}\tag{3.8}$$

$$\begin{aligned}&\quad - \bar{F}_3(m_Z^2, M_0, M_0)] \Big\}, \\ T_{\text{pol}} &= \frac{4\sqrt{2}G_F}{\alpha} [\Pi^{11}(0) - \Pi^{33}(0)] \\ &= \frac{3\sqrt{2}G_F}{16\pi^2\alpha} \left[M_+^2 + M_0^2 - \frac{2M_+^2 M_0^2}{M_+^2 - M_0^2} \ln \frac{M_+^2}{M_0^2} \right], \\ U_{\text{pol}} &= 16\pi \left[\frac{\Pi^{11}(m_W^2) - \Pi^{11}(0)}{m_W^2} - \frac{\Pi^{33}(m_Z^2) - \Pi^{33}(0)}{m_Z^2} \right] = \\ &\quad - \frac{1}{\pi} \left\{ \frac{2}{3} - \frac{(M_+^2 + M_0^2)}{2m_W^2} [\bar{F}_0(m_W^2, M_+, M_0) \right. \\ &\quad \left. - \bar{F}_0(0, M_+, M_0)] - \frac{1}{4} [\bar{F}_0(m_Z^2, M_+, M_+) \right. \\ &\quad \left. + \bar{F}_0(m_Z^2, M_0, M_0)] + \frac{5}{2} \bar{F}_0(m_W^2, M_+, M_0) \right. \\ &\quad \left. - 3[\bar{F}_3(m_Z^2, M_+, M_+) + \bar{F}_3(m_Z^2, M_0, M_0)] \right. \\ &\quad \left. + \frac{5}{m_W^2} [\bar{F}_4(m_W^2, M_+, M_0) - \bar{F}_4(0, M_+, M_0)] \right. \\ &\quad \left. - 6\bar{F}_3(m_W^2, M_+, M_0) \right\}.\end{aligned}$$

It is known that the bosonic contributions to the S , T , and U parameters defined in terms of conventional self-energies, are gauge dependent and, moreover, divergent unless the restrictive condition [26]

$$\xi_W = c_W^2 \xi_Z + s_W^2 \xi_\gamma$$

is imposed. The parameters become gauge invariant after adding the pinch parts arising from vertex and box diagrams.

The self-energies of electroweak gauge bosons are modified by pinch parts which can be expressed as [26]

$$\begin{aligned}\Pi_{ZZ}(q^2)|_P &= -(q^2 - m_Z^2) [B_0(q^2, M_0, M_0) \\ &\quad + (1 - 2s_W^2)^2 B_0(q^2, M_+, M_+)], \\ \Pi_{ZQ}(q^2)|_P &= -(2q^2 - m_Z^2) (1 - 2s_W^2) \\ &\quad \times B_0(q^2, M_+, M_+),\end{aligned}\tag{3.9}$$

$$\Pi_{QQ}(q^2)|_P = -4q^2 B_0(q^2, M_+, M_+),$$

$$\Pi_{WW}(q^2)|_P = -2(q^2 - m_W^2) B_0(q^2, M_+, M_0),$$

where B_0 is defined by

$$B_0(q^2, M_1, M_2) = \int \frac{d^n k}{i(2\pi)^n} \frac{1}{[M_1^2 - k^2][M_2^2 - (k+q)^2]}$$

$$= \frac{1}{16\pi^2} [\Delta + \ln(M_1 M_2) + \bar{F}_0(q^2, M_1, M_2)].$$

In getting Eq. (3.9) we have used coupling constants of bileptons X , Y with the SM vector bosons: the photon A , weak bosons Z and W . In the notations of Ref. [27] they are given by

$$C_{AYY} = g s_W, \quad C_{AXX} = 0, \quad C_{ZXX} = -\frac{g}{2c_W},$$

$$C_{ZYY} = \frac{g(1-2s_W^2)}{2c_W}, \quad C_{WXY} = \frac{g}{\sqrt{2}}.$$

The above pinch parts give the following corrections to S , T , and U parameters [28]

$$S_{\text{pin}} = \frac{16\pi}{m_Z^2} \text{Re}[\Pi_{ZZ}(m_Z^2) - \Pi_{ZZ}(0) - (1-2s_W^2) \times [\Pi_{ZQ}(m_Z^2) - \Pi_{ZQ}(0)] - s_W^2(1-s_W^2)\Pi_{QQ}(m_W^2)]$$

$$= \frac{1}{\pi} \left[\ln \frac{M_+^2}{M_0^2} + \bar{F}_0(m_Z^2, M_+, M_+) \right],$$

$$T_{\text{pin}} = \frac{4\pi}{s_W^2 c_W^2} \text{Re} \left[c_W^2 \frac{\Pi_{WW}(0)}{m_W^2} - \frac{1}{m_Z^2} \times [\Pi_{ZZ}(0) + 2s_W^2 \Pi_{ZQ}(0)] \right]$$

$$= \frac{1}{4\pi s_W^2} \left[2\bar{F}_0(0, M_+, M_0) + t_W^2 \ln \frac{M_+^2}{M_0^2} \right], \quad (3.10)$$

$$U_{\text{pin}} = 16\pi \text{Re} \left\{ \frac{\Pi_{WW}(m_W^2) - \Pi_{WW}(0)}{m_W^2} + \frac{1}{m_Z^2} [\Pi_{ZZ}(0) - 2s_W^2 [\Pi_{ZQ}(m_Z^2) - \Pi_{ZQ}(0)] - s_W^4 \Pi_{QQ}(m_Z^2)] \right\}$$

$$= \frac{2}{\pi} [s_W^2 \bar{F}_0(m_Z^2, M_+, M_+) - \bar{F}_0(0, M_+, M_0)].$$

The expression S_{pin} , T_{pin} , and U_{pin} in Eq. (3.10) must be added to S_{pol} , T_{pol} , and U_{pol} in Eq. (3.8). Note that due to the term proportional to $\ln(M_+^2/M_0^2)$ the pinch parts can give negative contributions to the S and T parameters. It is well known that the oblique parameter T is positive both for the

case of a heavy left-handed fermion doublet and for the case of a scalar doublet of general hypercharge. On the other hand, the present experimental data seem to favor a negative value for T . So our model is good in that sense.

C. The Z - Z' mixing contribution

The effects of the Z - Z' mixing in a general context has been considered in Ref. [29]. Now, due to the Z - Z' mixing, the observed Z boson mass m_{Z_1} at LEP1 or SLC is shifted from the SM Z boson mass m_Z :

$$\Delta m^2 \equiv m_{Z_1}^2 - m_Z^2 = -\tan^2 \phi \left(M_{Z_2}^2 - \frac{m_W^2}{c_W^2} \right) \leq 0. \quad (3.11)$$

In writing down the last equality of Eq. (3.11), we have employed Eq. (2.8).

The presence of the mass shift affects the T -parameter at tree level [21,22]. The result is [22]

$$T_{ZZ'} = -\frac{\Delta m^2}{\alpha m_{Z_1}^2} = \frac{\tan^2 \phi}{\alpha} \left(\frac{M_{Z_2}^2}{m_{Z_1}^2} - \frac{m_W^2}{c_W^2 m_{Z_1}^2} \right)$$

$$\simeq \frac{\tan^2 \phi}{\alpha} \left(\frac{M_{Z_2}^2}{m_{Z_1}^2} - 1 \right). \quad (3.12)$$

In our model the S and U parameters do not get contributions from the Z - Z' mixing [22,24].

There are a few ways to get constraints on the mixing angle ϕ and the Z^2 mass. For example, a constraint on the Z - Z' mixing can be obtained from the Z -decay data. A bound for the mixing angle is [11] $-0.00018 \leq \phi \leq 0.00285$.

The total values of the S , T , and U parameters in this model are the sum of the bilepton and the Z - Z' contributions

$$S_{\text{RHN}} = \frac{1}{4\pi} \left\{ 5 \ln \frac{M_+^2}{M_0^2} + \frac{1}{3} [13\bar{F}_0(m_Z^2, M_+, M_+) - \bar{F}_0(m_Z^2, M_0, M_0)] + 4[\bar{F}_3(m_Z^2, M_+, M_+) - \bar{F}_3(m_Z^2, M_0, M_0)] \right\}, \quad (3.13)$$

$$T_{\text{RHN}} = \frac{3\sqrt{2}G_F}{16\pi^2\alpha} \left[M_+^2 + M_0^2 - \frac{2M_+^2 M_0^2}{M_+^2 - M_0^2} \ln \frac{M_+^2}{M_0^2} \right]$$

$$+ \frac{1}{4\pi s_W^2} \left[2\bar{F}_0(0, M_+, M_0) + t_W^2 \ln \frac{M_+^2}{M_0^2} \right]$$

$$+ \frac{\tan^2 \phi}{\alpha} \left(\frac{M_{Z'}^2}{m_Z^2} - 1 \right),$$

$$\begin{aligned}
U_{\text{RHN}} = & -\frac{1}{\pi} \left\{ 2[F_0(0, M_+, M_0) - s_W^2 F_0(m_Z^2, M_+, M_+)] \right. \\
& + \frac{2}{3} - \frac{(M_+^2 + M_0^2)}{2m_W^2} [\bar{F}_0(m_W^2, M_+, M_0) \\
& - \bar{F}_0(0, M_+, M_0)] - \frac{1}{4} [\bar{F}_0(m_Z^2, M_+, M_+) \\
& + \bar{F}_0(m_Z^2, M_0, M_0)] + \frac{5}{2} \bar{F}_0(m_W^2, M_+, M_0) \\
& - 3[\bar{F}_3(m_Z^2, M_+, M_+) + \bar{F}_3(m_Z^2, M_0, M_0)] \\
& + \frac{5}{m_W^2} [\bar{F}_4(m_W^2, M_+, M_0) - \bar{F}_4(0, M_+, M_0)] \\
& \left. - 6\bar{F}_3(m_W^2, M_+, M_0) \right\}.
\end{aligned}$$

In Eq. (3.13) we have renamed the physical Z_1 and Z_2 to be usual Z and Z' .

From Eqs. (3.5), (3.6), and (3.7) it is easy to see that in the limit $M_+, M_0, M_{Z'} \rightarrow \infty$ all values S_{RHN} , T_{RHN} , U_{RHN} tend to zero in accord with the decoupling of heavy particles [30]. With the help of Eq. (2.12) we can expand functions in U_{RHN} without any assumption in advance. Most of the effects on precision measurements can be described by the three parameters calculated above.

IV. S , T , U PARAMETERS IN THE MINIMAL 3 3 1 MODEL

Many useful details on the model are given in Ref. [10]. In this model the new charged gauge bosons are doubly charged X^{++} and the singly charged Y^+ bileptons instead of (Y^+, X^0) in the RH neutrino version. The bileptons (X^{++}, Y^+) make an $SU(2)_L$ doublet with hypercharge $Y = \frac{3}{2}$. In Refs. [13,14] the parameters for the considered model are calculated without the Z - Z' mixing contribution. For our aim we note that Eqs. (2.7) and (2.8) are still correct. Therefore as in the above considered model the Z - Z' mixing gives contribution to the T parameter *only*, and the contribution is the same as in the model with right-handed neutrinos

$$\begin{aligned}
T_{\text{min}} = & \frac{3\sqrt{2}G_F}{16\pi^2\alpha} \left[M_{++}^2 + M_+^2 - \frac{2M_{++}^2 + M_+^2}{M_{++}^2 - M_+^2} \ln \frac{M_{++}^2}{M_+^2} \right] \\
& + \frac{1}{4\pi s_W^2} \left[2\bar{F}_0(0, M_{++}, M_+) + 3t_W^2 \ln \frac{M_{++}^2}{M_+^2} \right] \\
& + \frac{\tan^2 \phi}{\alpha} \left(\frac{M_{Z'}^2}{m_Z^2} - 1 \right), \quad (4.1)
\end{aligned}$$

where M_{++}, M_+ stand for $M_{X^{++}}, M_{Y^+}$, respectively. From Eqs. (3.13) and (4.1) we see that the mixing contributions as

expected [31] is positive, while the oblique contributions can be negative in both versions of 3 3 1 models.

The oblique contribution to the S parameter is given in Ref. [14]. However for the U parameter one term was missed in the expression of $U|_P$. The correct expression for this part is

$$\begin{aligned}
U|_P = & \frac{2}{\pi} [2s_W^2 \bar{F}_0(m_Z^2, M_{++}, M_{++}) - s_W^2 \bar{F}_0(m_Z^2, M_+, M_+) \\
& - \bar{F}_0(0, M_{++}, M_{++})]. \quad (4.2)
\end{aligned}$$

From Eq. (4.1) we see that the Z - Z' mixing contribution increases by square of Z' mass. The analysis in Ref. [10] gives $-5 \times 10^{-3} \leq \phi \leq 7 \times 10^{-4}$ from the low-energy experiment. According to the recent analysis [32] the Z' in this model has very large lower limit $M_{Z'} > 14$ TeV. With this mass, the mixing contribution is valuable. With $M_{Z'} = 1$ TeV the mixing contribution is about 4%, that is why it was neglected in the previous analysis [14]. We note that results in this section are correct for another 3 3 1 version—a 3 3 1 model with heavy charged lepton [34]. However, one point should be made here that the condition (2.12) is correct for the mentioned 3 3 1 model with heavy charged lepton, but it is violated in the minimal version.

V. NUMERICAL EVALUATION

For our initial purpose we consider the ρ parameter—one of the most important quantities of the SM, having a leading contribution in terms of the T parameter is very useful to get the new-physics effects (see, for example Refs. [29,31,33]). Defined at the zero point of momentum $Q^2 = 0$ the T parameter which is equivalent to $\Delta\rho$ has some advantage over the U parameter (to deal with F functions there, we have to suggest a prior relationship between bileptons masses and m_Z^2, m_W^2). Neglecting the Z - Z' mixing contribution which is approximately 10% (for $\phi = 10^{-3}, M_{Z'} = 700$ GeV), the S , T parameters can be rewritten in terms of two parameters ϵ and δ as follows:

$$\begin{aligned}
T_{\text{RHN}} = & \frac{1}{4\pi s_W^2 c_W^2} \left[\frac{3}{4} \frac{\epsilon^2}{\delta(M_+)} (2 - 3\epsilon + 2\epsilon^2) + s_W^2 \epsilon - \frac{\epsilon^2}{2} \right. \\
& \left. + O(\epsilon^4) \right], \\
S_{\text{RHN}} = & \frac{1}{4\pi} \left[5\epsilon \left(1 - \frac{\epsilon}{2} + \frac{\epsilon^2}{3} \right) - \frac{2}{3} \delta(M_+) - \frac{\delta^2(M_+)}{15} \right. \\
& + \frac{17}{90} \epsilon \delta(M_+) + \frac{5}{126} \epsilon \delta^2(M_+) \\
& \left. + O[\epsilon^4, \delta^3(M_+)] \right], \quad (5.1)
\end{aligned}$$

where $\epsilon \equiv \epsilon(M_+, M_0) = (M_+^2 - M_0^2)/M_0^2$.

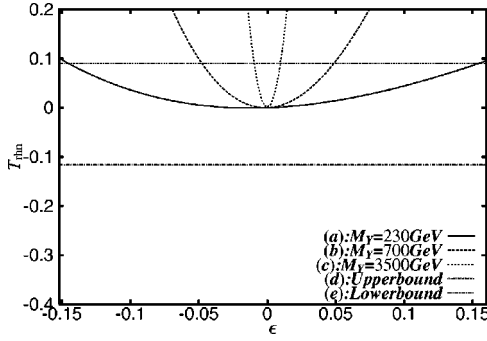


FIG. 2. T_{RHN} as functions of ϵ for three values of M_{Y^+} : (a) $M_{Y^+} = 230$ GeV, (b) $M_{Y^+} = 700$ GeV, and (c) $M_{Y^+} = 3500$ GeV. The horizontal lines (d) and (e) are an upper and a lower limit on the experimental fit subtracted the SM contribution ΔT_{SM} for $m_H = 100$ GeV.

It is to be noted that, due to the mass splitting condition (2.12), for given M_+ , the parameter ϵ is bounded in the interval

$$-\frac{m_W^2}{M_+^2} \leq \epsilon \leq \frac{m_W^2}{M_+^2}, \quad \epsilon < \delta(M_+) = \frac{m_Z^2}{M_+^2}. \quad (5.2)$$

For the heavier M_+ , the interval of definition $\epsilon \in [-m_W^2/M_+^2, m_W^2/M_+^2]$ becomes shorter. With the interval of definition given by Eq. (5.2), the S and T parameters are bounded too. In addition the T_{RHN} is negative in the region $-\epsilon_C \leq \epsilon \leq 0$ where $\epsilon_C \approx (2s_W^2/3)\delta(M_+)$.

In Fig. 2 we plot the T parameter as function of the mass splitting parameter ϵ for the three choices $M_{Y^+} = 230, 700$, and 3500 (GeV), respectively. The horizontal lines are experimental fit [24] after subtracting the SM contributions ΔT_{SM} [35]

$$\Delta T_{\text{SM}} = +(0.130 - 0.003x_H)x_t + 0.003x_t^2 - 0.079x_H - 0.028x_H^2 + 0.0026x_H^3, \quad (5.3)$$

where x_t and x_H are defined by

$$x_t \equiv \frac{m_t - 175 \text{ GeV}}{10 \text{ GeV}}, \quad x_H \equiv \log(m_H/100 \text{ GeV}). \quad (5.4)$$

We choose the standard-model reference point at $m_t = 174$ GeV [24], and $m_H = 100$ GeV. Figure 2 shows that $-0.0095 \leq \epsilon \leq 0.0096$ for $M_{Y^+} = 3500$ GeV, $-0.0475 \leq \epsilon \leq 0.0483$ for $M_{Y^+} = 700$ GeV, and $-0.144 \leq \epsilon \leq 0.154$ for $M_{Y^+} = 230$ GeV. This means that splitting in the bilepton masses is quite narrow about 15% for the $M_+ \sim 200$ GeV, and decreases for the higher M_{Y^+} . This result is approximately consistent with the mass splitting given by the VEV structure (2.13). In Fig. 3 we plot the S parameter as function of the mass splitting parameter ϵ for the two choices $M_{Y^+} = 200$ GeV and 700 GeV, respectively. We see that the S parameter is increasing function of the bilepton mass. However, due to decreasing of the definition interval (5.2), the running interval of the S parameter becomes shorter too, e.g., $-0.018 \leq S \leq 0.05$ for $M_+ = 200$ GeV, while

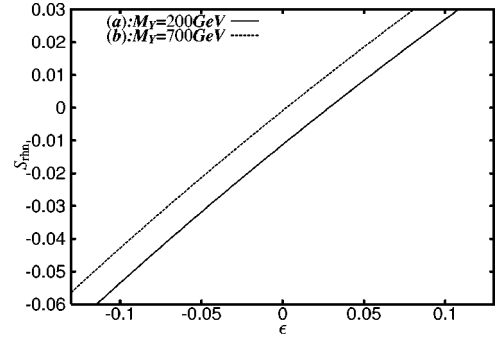


FIG. 3. S_{RHN} as functions of ϵ for two values of M_{Y^+} : (a) $M_{Y^+} = 200$ GeV, (b) $M_{Y^+} = 700$ GeV.

$-0.000082 \leq S \leq 0.0026$ for $M_+ = 1500$ GeV. This means that if experimental data is closed to the SM zero point: $m_H = 100$ GeV, $m_t = 175$ GeV, the bilepton X^0, Y^+ will have large masses. In this case the Z-Z' mixing contribution has to be included. Thus we get a bound for the oblique S parameter $-0.06 \leq S \leq 0.04$.

In Fig. 4 we plot S_{RHN} as function of M_+ for (a) $\epsilon = -0.14$ as its maximum value for M_0 in the range of 230 GeV. As before the horizontal line is a lower bound on the experimental fit subtracting the SM contribution ΔS_{SM} [35]

$$\Delta S_{\text{SM}} = -0.007x_t + 0.091x_H - 0.010x_H^2. \quad (5.5)$$

This figure shows that an allowed region for the mass of the charged bilepton $213 \leq M_{Y^+} \leq 234$ (GeV). It follows an allowed region for the mass of the neutral X^0 : $230 \leq M_{X^0} \leq 251$ (GeV). Note that the result is dependent at the top and Higgs masses.

VI. SUMMARY AND CONCLUSIONS

In this paper we have calculated both the oblique and the mixing contributions to S, T, U parameters. The mixing contribution is negligible if mass of Z' is less than 1 TeV, but it will be valuable for the Z' mass higher than 10 TeV.

We have shown that the oblique contributions to the S and T parameters are bounded, and can be negative. This result is interesting because the present experimental data seem to favor to negative value for T. Since most of the precision measurements can be described by the S, T, and U param-

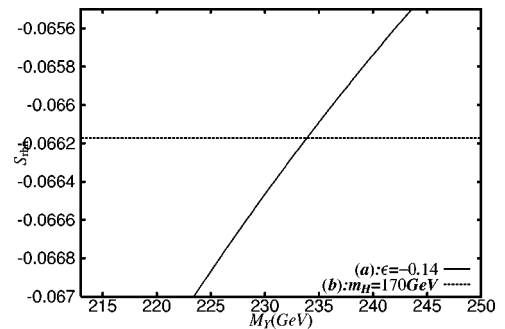


FIG. 4. S_{RHN} as functions of M_{Y^+} for (a): the experimental fit subtracted the SM contribution ΔS_{SM} .

eters the obtained expressions are very important for the future data analysis.

We have mentioned that the bilepton mass splitting by the VEV structure in the 3 3 1 model with RH neutrinos is smaller than those in the minimal version. With this condition, we can get numerical expression for the U parameter without any assumption in advance.

The oblique S and T parameters decrease with higher masses of the bileptons. Thus in this case, the Z - Z' mixing contribution has to be considered.

As a consequence we have found that the bilepton mass splitting is quite narrow about 15% for the singly charged bilepton mass around 200 GeV, and decreases for the higher mass of the bileptons. Therefore in the future studies it is acceptable to put $M_{Y^+} \approx M_{X^0}$.

From the Higgs structure and “wrong” muon decay we have got the first bound on the M_{X^0} . The analysis based on

Eq. (5.1) indicates that the neutral bilepton is heavier, namely, for $213 \leq M_{Y^+} \leq 234$ (GeV), the allowed region for M_{X^0} : $230 \leq M_{X^0} \leq 251$ (GeV).

As mentioned above, the constraints on bilepton masses are dependent upon reference choices of the Higgs mass (even, on the top mass too). Hence discovery of the Higgs particle will give a window to the new particles in the SM extensions. We hope to return to the data analysis in the future.

ACKNOWLEDGMENTS

The authors thank K. Sasaki for very helpful discussions. The work of H.N.L. was supported by the JSPS Grant No. L98509. He thanks Department of Physics, Chuo University for warm hospitality extended to him. This work is partially supported by Grant in Aid of Ministry of Education, Science and Culture, Priority area A, Priority area B (Supersymmetry and Unified Theory), and Basic research C.

APPENDIX

Functions used in this paper are given in Ref. [14], however, we correct a misprint there [in Eq. (A1) below]

$$\begin{aligned} \bar{F}_0(s, M, m) &= \int_0^1 dx \ln[(1-x)M^2 + xm^2 - x(1-x)s] - \ln Mm \\ &= \begin{cases} -\frac{2}{s} \sqrt{(M+m)^2 - s} \sqrt{(M-m)^2 - s} \ln \frac{\sqrt{(M+m)^2 - s} + \sqrt{(M-m)^2 - s}}{2\sqrt{Mm}} \\ \quad + \frac{M^2 - m^2}{s} \ln \frac{M}{m} - 2, & \text{for } s < (M-m)^2, \\ \frac{2}{s} \sqrt{(M+m)^2 - s} \sqrt{s - (M-m)^2} \arctan \sqrt{\frac{s - (M-m)^2}{(M+m)^2 - s}} \\ \quad + \frac{M^2 - m^2}{s} \ln \frac{M}{m} - 2, & \text{for } (M-m)^2 < s < (M+m)^2, \\ \frac{2}{s} \sqrt{s - (M+m)^2} \sqrt{s - (M-m)^2} \left[\ln \frac{\sqrt{s - (M+m)^2} + \sqrt{s - (M-m)^2}}{2\sqrt{Mm}} - i\pi \right] \\ \quad + \frac{M^2 - m^2}{s} \ln \frac{M}{m} - 2, & \text{for } (M+m)^2 < s. \end{cases} \end{aligned} \quad (\text{A1})$$

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