Observables in the decays of B to two vector mesons

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In general there are nine observables in the decay of a B meson to two vector mesons defined in terms of polarization correlations of these mesons. Only six of these can be detected via the subsequent decay angular distributions because of parity conservation in those decays. The remaining three require the measurement of the spin polarization of one of the decay products.

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The decay of the B meson into two vector mesons $B\to V_1+V_2$, such as $B\to\rho+\Psi$ or $B\to\rho+K^*$, has been calculated in many models [1–9]. Here we are concerned with observables from a model-independent viewpoint. We limit this discussion to the case of $B^\pm$ decays or B^0 ($\overline{B^0}$) in the absence of B^0-$\overline{B^0}$ mixing effects.

To take advantage of extracting the CP-odd and CP-even or T-odd and T-even components more easily, the angular distribution is often written in the linear polarization (or transversity) basis. Let us define the amplitude of $B\to V_1V_2$ in the rest frame of V_1. According to their polarization combinations, the amplitude can be decomposed into [1]

$$A(B\to V_1V_2)=A_0\epsilon^{*L}_{V_1}\epsilon^{*L}_{V_2}-\frac{A_\parallel}{\sqrt{2}}\overrightarrow{\epsilon^{*T}_{V_1}}\cdot\overrightarrow{\epsilon^{*T}_{V_2}}-i\frac{A_\perp}{\sqrt{2}}\overrightarrow{\epsilon^{*}_{V_1}}\times\overrightarrow{\epsilon^{*}_{V_2}}\cdot\hat{\mathbf{p}}, \quad (1)$$

and similarly for $\overline{B}\to\overline{V_1}\,\overline{V_2}$. In Eq. (1), $\overrightarrow{\epsilon_{V_1}}$ and $\overrightarrow{\epsilon_{V_2}}$ are the unit polarization vectors of V_1 and V_2, respectively. $\hat{\mathbf{p}}$ is the unit vector along the direction of motion of V_2 in the rest frame of V_1, $\epsilon^{*L}_{V_i}\equiv\overrightarrow{\epsilon^{*}_{V_i}}\cdot\hat{\mathbf{p}}$ and $\overrightarrow{\epsilon^{*T}_{V_i}}=\overrightarrow{\epsilon^{*}_{V_i}}-\epsilon^{*L}_{V_i}\hat{\mathbf{p}}$. It is easy to see that $A_\perp$ is odd under the parity transformation because of the appearance of $\overrightarrow{\epsilon^{*}_{V_1}}\times\overrightarrow{\epsilon^{*}_{V_2}}\cdot\hat{\mathbf{p}}$, whereas A_0 and $A_\parallel$ are even.

For B decays, the square of the amplitude A^*A should determine 9 observables proportional to products of the three transversity amplitudes. We can choose these as given by

$$K_1=|A_0|^2,\quad K_4=\mathrm{Re}[A_0^*A_\parallel],\quad L_4=\mathrm{Im}\,[A_0^*A_\parallel],$$
$$K_2=|A_\parallel|^2,\quad K_5=\mathrm{Im}\,[A_0^*A_\perp],\quad L_5=\mathrm{Re}[A_0^*A_\perp],$$
$$K_3=|A_\perp|^2,\quad K_6=\mathrm{Im}[A_\parallel^*A_\perp],\quad L_6=\mathrm{Re}\,[A_\parallel^*A_\perp]. \quad (2)$$

Then

$$A^*A=K_1X_1+K_2X_2+K_3X_3+K_4X_4+L_5X_5+L_6X_6+L_4Y_4+K_5Y_5+K_6Y_6.$$

The observables X_i and Y_i represent polarizations or polarization correlations of the final vector mesons depending on the vector $\hat{\mathbf{p}}$. They are given explicitly in the Appendix.

The polarization state of a spin-1 particle is given [10] in terms of the spin $\mathbf{S}_i$ and a second rank traceless tensor $\mathbf{T}_{ij}$. Sometimes the tensor polarization is referred to as alignment or orientation. The observables can be classified according to their properties with respect to parity P and motion reversal T. By ''motion reversal'' [11] is meant the reversal of all spins and momenta; a nonzero value of a T-odd observable signifies time-reversal violation when there are no final state interactions. We find

	P	T
X_1 to X_4	even	even
Y_5, Y_6	odd	odd
Y_4	even	odd
X_5, X_6	odd	even

The terms which have opposite behavior under P and T are necessarily proportional to the spin polarization $\mathbf{S}_1$ or $\mathbf{S}_2$.

The polarization state of V_i is analyzed via its subsequent decay. If this is a strong or electromagnetic decay into two particles whose angular distribution is measured, then it is impossible to detect the spin polarization $\mathbf{S}_i$ and only $\mathbf{T}_{ij}$ can be detected. This is a consequence of parity conservation since the final decay cannot depend on $\vec{\mathbf{S}}\cdot\vec{\mathbf{k}}$, where $\vec{\mathbf{k}}$ is the relative momentum of the decay products. As a result, one cannot determine L_4, L_5, and L_6 in this way.

In general, the angular distribution of the decay in the transversity basis can be written as

$$\frac{d^3\Gamma}{d\cos\theta_1 d\cos\theta_2 d\phi}=\sum_{i=1}^{6}K_if_i(\theta_1,\theta_2,\phi), \quad (3)$$

where K_i's and L_i's are the amplitude bilinears that contain the dynamics and in the case of B^0 would evolve with time,[1] and $f_i(\theta_1,\theta_2,\phi)$ are the corresponding angular distribution functions which are orthogonal to one another. Here θ_1 (or θ_2) denotes the polar angle of one of the V_1 (or V_2) decay

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[1]The time evolution effects will be addressed in a separate publication [12].

products measured in the rest frame of V_1 (or V_2) relative to the motion of V_1 (or V_2) in the rest frame of the B meson, and ϕ is the angle subtended by the two planes formed by decay products of V_1 and V_2, respectively.

For the case in which the decays of V_1 and V_2 are both into two pseudoscalar mesons, one can immediately translate the tensor correlations into angular distributions as shown in the Appendix. The resulting normalized angular distribution of the decays $B \to V_1(\to P_1 P_1')V_2(\to P_2 P_2')$, where $P_1^{(\prime)}$ and $P_2^{(\prime)}$ denote pseudoscalar mesons, is

$$\frac{1}{\Gamma_0}\frac{d^3\Gamma}{d\cos\theta_1 d\cos\theta_2 d\phi} = \frac{9}{8\pi\Gamma_0}\left\{K_1\cos^2\theta_1\cos^2\theta_2 + \frac{K_2}{2}\sin^2\theta_1\sin^2\theta_2\cos^2\phi + \frac{K_3}{2}\sin^2\theta_1\sin^2\theta_2\sin^2\phi + \frac{K_4}{2\sqrt{2}}\sin 2\theta_1\sin 2\theta_2\cos\phi - \frac{K_5}{2\sqrt{2}}\sin 2\theta_1\sin 2\theta_2\sin\phi - \frac{K_6}{2}\sin^2\theta_1\sin^2\theta_2\sin 2\phi\right\}. \quad (4)$$

Here θ_1 (θ_2) is the angle between the P_1 (P_2) three-momentum vector in the V_1 (V_2) rest frame and the V_1 (V_2) three-momentum vector defined in the B rest frame, and ϕ is the angle between the normals to the planes defined by $P_1 P_1'$ and $P_2 P_2'$, in the B rest frame. An example is the decay $B^- \to K^{*-}\rho^0 \to (K\pi)^-(\pi^+\pi^-)$.

In order to determine L_4, L_5, and L_6 it is necessary to measure the spin polarization of a decay product. For example, if V_2 decays into $\mu^+\mu^-$ and V_1 decays into $P_1 P_1'$ then, as shown in the Appendix, the polarization of either muon is given by

$$\frac{1}{\sqrt{2}}\sin 2\theta_1\sin\theta_2(L_4\sin\phi - L_5\cos\phi) + \frac{L_6}{2}\sin^2\theta_1\cos\theta_2, \quad (5)$$

divided by the sum of the first six terms in Eq. (A11), where θ_1 (θ_2) is, as before, the angle between the P_1 (μ^-) three-momentum vector in the V_1 (V_2) rest frame and the V_1 (V_2) three-momentum vector defined in the B rest frame, and ϕ is the angle between the normals to the planes defined by $P_1 P_1'$ and $\mu^+\mu^-$ in the B rest frame. For the case of L_6 it is only necessary to measure the polarization of one muon independent of observing the decay of V_1. The maximum value of the polarizations is 1 for each of the terms proportional to L_4, L_5, or L_6.

For the case in which V_1 and V_2 are both CP eigenstates, the amplitude given by $A_\perp$ corresponds to a final state with an opposite CP eigenvalue from the other two. This by itself has nothing to do with CP violation, but it leads to the possibility that the coefficients K_5, K_6 involving $A_\perp$ may be good places to look for CP violation.

In particular, in the absence of final state interaction (FSI) it follows from CPT invariance that only ''T-odd'' terms will be odd under CP, so that

$$K_i = \overline{K_i}, \quad \text{for } i = 1,2,3,4;$$

$$K_i = -\overline{K_i}, \quad \text{for } i = 5,6;$$

$$L_5 = \overline{L_5}, \quad L_6 = \overline{L_6}, \quad L_4 = -\overline{L_4}; \quad (6)$$

and the only signals of CP violation are L_4, K_5, and K_6. In fact, in the absence of FSI a nonzero value of L_4 or $K_{5,6}$ by itself is a signal of time reversal violation. On the other hand, in the absence of CP violation (as expected in decays like $B \to \Psi K^*$) a nonzero value of $K_{5,6}$ is a signal of significant final state phases.

In order to have CP violation there must be two contributions with different weak phase factors; we label these as T_η and P_η where $\eta = 0, \|, \perp$. Then each of the three amplitudes entering Eq. (1) has the form

$$A_\eta = e^{i\theta_\eta}(T_\eta + P_\eta e^{i\phi_w} e^{i\delta_\eta}), \quad (7)$$

where θ_η, δ_η are strong phases and ϕ_w is the relative weak phase between the two contributions T_η and P_η, for example, in a picture where the weak amplitude is decomposed into a tree and a penguin contributions. We obtain $\overline{A}_\eta$ by changing ϕ_w to $-\phi_w$. There are in general 12 parameters: T_η, P_η, δ_η, ϕ_w, and two relative phases of the θ_η.

In many cases, ϕ_w is expected to be large, leading to the possibility of large CP violation. Thus for decays such as $B \to K^*\rho$ or $B_s \to \rho\varphi$, $\phi_w = \gamma$ and for $B \to \rho\rho$ or $B_s \to K^*\rho$, $\phi_w = \beta + \gamma$. For the parameters K_5 and K_6 as well as L_4, the CP violation is given by

$$\mathrm{Im}[A_\eta A_{\eta'}^*] - \mathrm{Im}[\overline{A}_\eta \overline{A}_{\eta'}^*] = 2\sin\phi_w[P_\eta T_{\eta'}\cos(\theta_\eta - \theta_{\eta'} + \delta_\eta) - P_{\eta'} T_\eta\cos(\theta_\eta - \theta_{\eta'} - \delta_{\eta'})]. \quad (8)$$

Assuming the strong phases are not very large, the major requirement for a large effect in the above CP asymmetry quantities is that P_η/T_η be quite different from $P_{\eta'}/T_{\eta'}$ for $\eta \neq \eta'$. For the case of K_1 to K_3, the CP violation is given by

$$\frac{|A_\eta|^2 - |\overline{A}_\eta|^2}{|A_\eta|^2 + |\overline{A}_\eta|^2} = \frac{-2T_\eta P_\eta\sin\phi_w\sin\delta_\eta}{T_\eta^2 + P_\eta^2 + 2T_\eta P_\eta\cos\phi_w\cos\delta_\eta} \quad (9)$$

requiring as noted a significant value for $\sin\delta_\eta$. For the parameters K_4, L_5, and L_6, the CP violation is measured by

$$\mathrm{Re}[A_\eta A_{\eta'}^*] - \mathrm{Re}[\overline{A}_\eta \overline{A}_{\eta'}^*] = -2\sin\phi_w[P_\eta T_{\eta'}\sin(\theta_\eta - \theta_{\eta'} + \delta_\eta) - P_{\eta'} T_\eta\sin(\theta_\eta - \theta_{\eta'} - \delta_{\eta'})]. \quad (10)$$

These require a significant value of $\sin(\theta_\eta-\theta_{\eta'})$ or $\sin(\delta_\eta-\delta_{\eta'})$ to have a large effect.

To summarize this paper, we have discussed in a model independent way the observables in the decay of a B meson to two vector mesons and the relations among them. An alternative way of getting the differential angular distribution is provided in the Appendix. We have also explicitly shown how one can determine $L_{4,5,6}$ defined in the text by measuring the polarization of one of the decay products in the final state.

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APPENDIX: CORRELATIONS OF POLARIZATIONS

Analyzing the correlations of polarization vectors appearing in the decay rate provides a way of understanding why only six of the nine amplitude bilinears show up in the differential cross section. The polarization state of a spin 1 particle is described by the density matrix [10] which can be written as a sum of a scalar, vector, and traceless second-rank tensor. With $a=1,2$ for V_1 and V_2 mesons, these are

$$\mathbf{1}_{ij}=\delta_{ij}\,,\ \text{scalar};$$

$$\mathbf{S}_i^a=\frac{1}{2i}\varepsilon_{ijk}\epsilon_j^a\epsilon_k^{a*}\,,\ \text{vector};$$

$$\mathbf{T}_{ij}^a=\frac{1}{2}\left(\epsilon_i^a\epsilon_j^{a*}+\epsilon_i^{a*}\epsilon_j^a-\frac{2}{3}\epsilon\cdot\epsilon^*\delta_{ij}\right),\ \text{tensor}. \tag{A1}$$

Therefore, we have

$$\epsilon_i^a\epsilon_j^{a*}=\mathbf{T}_{ij}^a+i\varepsilon_{ijk}\mathbf{S}_k+\frac{1}{3}\mathbf{1}_{ij}\,, \tag{A2}$$

provided that the polarization vector $\vec{\epsilon^a}$ are normalized to 1.

From Eq. (1), one can get the polarization vector correlations for each of the amplitude bilinears. After simplification, we obtain the following results: For $|A_0|^2$:

$$\begin{aligned}X_1&=\epsilon^{1*L}\epsilon^{1L}\epsilon^{2*L}\epsilon^{2L}\\&=(\hat{\mathbf{p}}\cdot\mathbf{T}^1\cdot\hat{\mathbf{p}})(\hat{\mathbf{p}}\cdot\mathbf{T}^2\cdot\hat{\mathbf{p}})\\&\quad+\frac{1}{3}\hat{\mathbf{p}}\cdot(\mathbf{T}^1+\mathbf{T}^2)\cdot\hat{\mathbf{p}}+\frac{1}{9}.\end{aligned} \tag{A3}$$

For $|A_\parallel|^2$:

$$\begin{aligned}2X_2&=\overrightarrow{\epsilon^{1*T}}\cdot\overrightarrow{\epsilon^{2*T}}\overrightarrow{\epsilon^{1T}}\cdot\overrightarrow{\epsilon^{2T}}\\&=Tr[\mathbf{T}^1\cdot\mathbf{T}^2]+(\hat{\mathbf{p}}\cdot\mathbf{T}^1\cdot\hat{\mathbf{p}})(\hat{\mathbf{p}}\cdot\mathbf{T}^2\cdot\hat{\mathbf{p}})-\frac{1}{3}\hat{\mathbf{p}}\cdot(\mathbf{T}^1+\mathbf{T}^2)\cdot\hat{\mathbf{p}}\\&\quad-2\hat{\mathbf{p}}\cdot\mathbf{T}^1\cdot\mathbf{T}^2\cdot\hat{\mathbf{p}}+\frac{2}{9}-2(\hat{\mathbf{p}}\cdot\mathbf{S}^1)(\hat{\mathbf{p}}\cdot\mathbf{S}^2)+\mathbf{S}^1\cdot\mathbf{S}^2.\end{aligned} \tag{A4}$$

For $|A_\perp|^2$:

$$\begin{aligned}2X_3&=\overrightarrow{\epsilon^{1*}}\times\overrightarrow{\epsilon^{2*}}\cdot\hat{\mathbf{p}}\overrightarrow{\epsilon^1}\times\overrightarrow{\epsilon^2}\cdot\hat{\mathbf{p}}\\&=\varepsilon_{ijk}\varepsilon_{lmn}\mathbf{T}_{il}^1\mathbf{T}_{jm}^2\mathbf{p}_k\mathbf{p}_n\\&\quad-\frac{1}{3}\hat{\mathbf{p}}\cdot(\mathbf{T}^1+\mathbf{T}^2)\cdot\hat{\mathbf{p}}+\frac{2}{9}.\end{aligned} \tag{A5}$$

For $A_0A_\parallel^*$ (apart from an overall minus sign):

$$\begin{aligned}\epsilon^{1*L}\epsilon^{2*L}\overrightarrow{\epsilon^{1T}}\cdot\overrightarrow{\epsilon^{2T}}&=\hat{\mathbf{p}}\cdot\mathbf{T}^1\cdot\mathbf{T}^2\cdot\hat{\mathbf{p}}-(\hat{\mathbf{p}}\cdot\mathbf{T}^1\cdot\hat{\mathbf{p}})(\hat{\mathbf{p}}\cdot\mathbf{T}^2\cdot\hat{\mathbf{p}})\\&\quad-i\hat{\mathbf{p}}\cdot(\mathbf{T}^2\cdot\hat{\mathbf{p}})\times\mathbf{S}^1-i\hat{\mathbf{p}}\cdot(\mathbf{T}^1\cdot\hat{\mathbf{p}})\times\mathbf{S}^2\\&\quad-\mathbf{S}^1\cdot\mathbf{S}^2+(\hat{\mathbf{p}}\cdot\mathbf{S}^1)(\hat{\mathbf{p}}\cdot\mathbf{S}^2).\end{aligned}$$

For $A_0^*A_\parallel$ (apart from an overall minus sign):

$$\begin{aligned}\epsilon^{1L}\epsilon^{2L}\overrightarrow{\epsilon^{1*T}}\cdot\overrightarrow{\epsilon^{2*T}}&=\hat{\mathbf{p}}\cdot\mathbf{T}^1\cdot\mathbf{T}^2\cdot\hat{\mathbf{p}}\\&\quad-(\hat{\mathbf{p}}\cdot\mathbf{T}^1\cdot\hat{\mathbf{p}})(\hat{\mathbf{p}}\cdot\mathbf{T}^2\cdot\hat{\mathbf{p}})+i\hat{\mathbf{p}}\cdot(\mathbf{T}^2\cdot\hat{\mathbf{p}})\times\mathbf{S}^1\\&\quad+i\hat{\mathbf{p}}\cdot(\mathbf{T}^1\cdot\hat{\mathbf{p}})\times\mathbf{S}^2-\mathbf{S}^1\cdot\mathbf{S}^2+(\hat{\mathbf{p}}\cdot\mathbf{S}^1)\\&\quad\times(\hat{\mathbf{p}}\cdot\mathbf{S}^2).\end{aligned}$$

So the net result for $A_0A_\parallel^*$ and $A_0^*A_\parallel$ is

$$\begin{aligned}X_4&=\sqrt{2}[(\hat{\mathbf{p}}\cdot\mathbf{T}^1\cdot\hat{\mathbf{p}})(\hat{\mathbf{p}}\cdot\mathbf{T}^2\cdot\hat{\mathbf{p}})\\&\quad-\hat{\mathbf{p}}\cdot\mathbf{T}^1\cdot\mathbf{T}^2\cdot\hat{\mathbf{p}}-\mathbf{S}^1\cdot\mathbf{S}^2+(\mathbf{S}^1\cdot\hat{\mathbf{p}})(\mathbf{S}^2\cdot\hat{\mathbf{p}})],\\Y_4&=\sqrt{2}[\hat{\mathbf{p}}\cdot(\mathbf{T}^1\cdot\hat{\mathbf{p}})\times\mathbf{S}^2+\hat{\mathbf{p}}\cdot(\mathbf{T}^2\cdot\hat{\mathbf{p}})\times\mathbf{S}^1].\end{aligned} \tag{A6}$$

For $A_0A_\perp^*$ (apart from an i):

$$\begin{aligned}\overrightarrow{\epsilon^1}\times\overrightarrow{\epsilon^2}\cdot\hat{\mathbf{p}}\epsilon^{1*L}\epsilon^{2*L}&=\hat{\mathbf{p}}\cdot(\mathbf{T}^1\cdot\hat{\mathbf{p}})\times(\mathbf{T}^2\cdot\hat{\mathbf{p}})-\hat{\mathbf{p}}\cdot\mathbf{S}^1\times\mathbf{S}^2\\&\quad-i\hat{\mathbf{p}}\cdot\mathbf{T}^2\cdot\mathbf{S}^1\\&\quad+i\hat{\mathbf{p}}\cdot\mathbf{T}^1\cdot\mathbf{S}^2-i(\hat{\mathbf{p}}\cdot\mathbf{T}^1\cdot\hat{\mathbf{p}})(\hat{\mathbf{p}}\cdot\mathbf{S}^2)\\&\quad+i(\hat{\mathbf{p}}\cdot\mathbf{T}^2\cdot\hat{\mathbf{p}})(\hat{\mathbf{p}}\cdot\mathbf{S}^1).\end{aligned}$$

With a similar expression for $A_0^*A_\perp$, we obtain

$$\begin{aligned}X_5&=\sqrt{2}[\hat{\mathbf{p}}\cdot\mathbf{T}^2\cdot\mathbf{S}^1-\hat{\mathbf{p}}\cdot\mathbf{T}^1\cdot\mathbf{S}^2+(\hat{\mathbf{p}}\cdot\mathbf{T}^1\cdot\hat{\mathbf{p}})(\mathbf{S}^2\cdot\hat{\mathbf{p}})-(\hat{\mathbf{p}}\cdot\mathbf{T}^2\cdot\hat{\mathbf{p}})\\&\quad\times(\mathbf{S}^1\cdot\hat{\mathbf{p}})],\\Y_5&=\sqrt{2}[\hat{\mathbf{p}}\cdot(\mathbf{T}^1\cdot\hat{\mathbf{p}})\times(\mathbf{T}^2\cdot\hat{\mathbf{p}})-\hat{\mathbf{p}}\cdot\mathbf{S}^1\times\mathbf{S}^2].\end{aligned} \tag{A7}$$

For $A_\parallel A_\perp^*$ (apart from an i):

$$\begin{aligned}\overrightarrow{\epsilon^1}\times\overrightarrow{\epsilon^2}\cdot\hat{\mathbf{p}}\overrightarrow{\epsilon^{1*T}}\cdot\overrightarrow{\epsilon^{2*T}}&=-\hat{\mathbf{p}}\cdot(\mathbf{T}^1\cdot\hat{\mathbf{p}})\times(\mathbf{T}^2\cdot\hat{\mathbf{p}})\\&\quad+\varepsilon_{ijk}(\mathbf{T}^1\cdot\mathbf{T}^2)_{ij}\mathbf{p}_k+\frac{2}{3}i\hat{\mathbf{p}}\cdot\mathbf{S}^1\\&\quad-\frac{2}{3}i\hat{\mathbf{p}}\cdot\mathbf{S}^2+i(\hat{\mathbf{p}}\cdot\mathbf{T}^1\cdot\hat{\mathbf{p}})(\hat{\mathbf{p}}\cdot\mathbf{S}^2)\\&\quad-i(\hat{\mathbf{p}}\cdot\mathbf{T}^2\cdot\hat{\mathbf{p}})(\hat{\mathbf{p}}\cdot\mathbf{S}^1).\end{aligned}$$

With a similar expression for $A_\parallel^* A_\perp$, we obtain

$$X_6=\frac{2}{3}\hat{\mathbf{p}}\cdot(\mathbf{S}^2-\mathbf{S}^1)+(\hat{\mathbf{p}}\cdot\mathbf{T}^2\cdot\hat{\mathbf{p}})(\mathbf{S}^1\cdot\mathbf{p})-(\hat{\mathbf{p}}\cdot\mathbf{T}^1\cdot\hat{\mathbf{p}})(\mathbf{S}^2\cdot\hat{\mathbf{p}}),$$

$$Y_6=\varepsilon_{ijk}(\mathbf{T}^1\cdot\mathbf{T}^2)_{ij}\mathbf{p}_k-\hat{\mathbf{p}}\cdot(\mathbf{T}^1\cdot\hat{\mathbf{p}})\times(\mathbf{T}^2\cdot\hat{\mathbf{p}}). \tag{A8}$$

Notice that the observables Y_4, X_5, and X_6 are linear in $\mathbf{S}^1$ or $\mathbf{S}^2$. As a result, as discussed in the text, they cannot be detected via the angular distribution of the decays of V_1 and V_2. However, in principle they may be observed in more complicated decays, or in decays such as $B\to K^*(\to PP)J/\Psi(\to l^+l^-)$ by measuring the spin of one of the leptons. In particular X_6 contains $\hat{\mathbf{p}}\cdot\mathbf{S}^2$ and $\hat{\mathbf{p}}\cdot\mathbf{S}^1$ and so could be observed by measuring the polarization of one of the mesons from the decay of V_1 or V_2 without observing the other decay.

The above results can be directly applied to the decays $B\to V_1(\to PP)V_2(\to PP)$ to obtain the angular distribution, Eq. (4), which is uniquely determined by the tensor polarizations of the vector mesons. The angular distributions of $B\to V_1(\to PP)V_2(\to l^+l^-)$ and $B\to V_1(\to P\gamma)V_2(\to P\gamma)$ can be obtained by taking into account that the lepton or photon motions must be perpendicular to the parent particle polarization vector and all possible spins are summed over.

For the case of decay into pseudoscalars $B\to V_1(\to P_1P_1')V_2(\to P_2P_2')$, one can go directly from X_1-X_4, Y_5, and Y_6 to the angular distribution Eq. (4). The polarization vectors of V_2 directly convert to the outgoing relative momentum vectors of the pseudoscalars. Choosing $\hat{\mathbf{p}}=(0,0,1)$, the momentum of P_1, $\vec{k_1}=(\sin\theta_1,0,\cos\theta_1)$, the momentum of P_2, $\vec{k_2}=(\sin\theta_2\cos\phi,\sin\theta_2\sin\phi,\cos\theta_2)$, we have

$$\vec{\epsilon^1}\to(\sin\theta_1,0,\cos\theta_1),\quad \vec{\epsilon^2}\to(\sin\theta_2\cos\phi,\sin\theta_2\sin\phi,\cos\theta_2),$$

$$\mathbf{T}^1\to\begin{pmatrix}\sin^2\theta_1-\frac{1}{3} & 0 & \sin\theta_1\cos\theta_1\\ 0 & -\frac{1}{3} & 0\\ \sin\theta_1\cos\theta_1 & 0 & \cos^2\theta_1-\frac{1}{3}\end{pmatrix},\quad \mathbf{T}^2\to\begin{pmatrix}\sin^2\theta_2\cos^2\phi-\frac{1}{3} & \sin^2\theta_2\sin\phi\cos\phi & \sin\theta_2\cos\theta_2\cos\phi\\ \sin^2\theta_2\sin\phi\cos\phi & \sin^2\theta_2\sin^2\phi-\frac{1}{3} & \sin\theta_2\cos\theta_2\sin\phi\\ \sin\theta_2\cos\theta_2\cos\phi & \sin\theta_2\cos\theta_2\sin\phi & \cos^2\theta_2-\frac{1}{3}\end{pmatrix}. \tag{A9}$$

Putting Eq. (A9) into Eqs. (A3)–(A8) one can immediately get Eq. (4). Notice that terms involving $\mathbf{S}^a$ make no contribution to the result.

For the case of the decay $B\to V_1(\to P_1P_1')V_2(\to l^+l^-)$, suppose we observe that l^- is a right-handed particle and comes out in the direction $\vec{k_2}=(\sin\theta_2\cos\phi,\sin\theta_2\sin\phi,\cos\theta_2)$ with $\hat{\mathbf{p}}=(0,0,1)$ and the momentum of P_1, $\vec{k_1}=(\sin\theta_1,0,\cos\theta_1)$, we have instead

$$\vec{\epsilon^1}\to(\sin\theta_1,0,\cos\theta_1),\quad \vec{\epsilon^2}\to\frac{1}{\sqrt{2}}(\cos\theta_2\cos\phi+i\sin\phi,\cos\theta_2\sin\phi-i\cos\phi,-\sin\theta_2),$$

$$\mathbf{T}^1\to\begin{pmatrix}\sin^2\theta_1-\frac{1}{3} & 0 & \sin\theta_1\cos\theta_1\\ 0 & -\frac{1}{3} & 0\\ \sin\theta_1\cos\theta_1 & 0 & \cos^2\theta_1-\frac{1}{3}\end{pmatrix},\quad \mathbf{S}^2\to\frac{1}{2}\begin{pmatrix}\sin\theta_2\cos\phi\\ \sin\theta_2\sin\phi\\ \cos\theta_2\end{pmatrix},$$

$$\mathbf{T}^2\to\frac{1}{2}\begin{pmatrix}\cos^2\theta_2\cos^2\phi+\sin^2\phi-\frac{2}{3} & -\sin^2\theta_2\sin\phi\cos\phi & -\sin\theta_2\cos\theta_2\cos\phi\\ -\sin^2\theta_2\sin\phi\cos\phi & \cos^2\theta_2\sin^2\phi+\cos^2\phi-\frac{2}{3} & -\sin\theta_2\cos\theta_2\sin\phi\\ -\sin\theta_2\cos\theta_2\cos\phi & -\sin\theta_2\cos\theta_2\sin\phi & \sin^2\theta_2-\frac{2}{3}\end{pmatrix}. \tag{A10}$$

Putting Eq. (A10) into Eqs. (A3)–(A8) one can immediately get the differential angular distribution for the decay $B \to V_1(\to P_1P_2)V_2(\to l^+l^-)$ with a right-handed l^- coming out in the final state:

$$\frac{1}{\Gamma_0}\frac{d^3\Gamma}{d\cos\theta_1 d\cos\theta_2 d\phi}=\frac{9}{16\pi\Gamma_0}\Bigg\{K_1\cos^2\theta_1\sin^2\theta_2+\frac{K_2}{2}(\sin^2\theta_1\cos^2\theta_2\cos^2\phi+\sin^2\theta_1\sin^2\phi)$$
$$+\frac{K_3}{2}(\sin^2\theta_1\cos^2\theta_2\sin^2\phi+\sin^2\theta_1\cos^2\phi)+\frac{K_4}{2\sqrt{2}}\sin2\theta_1\sin2\theta_2\cos\phi$$
$$-\frac{K_5}{2\sqrt{2}}\sin2\theta_1\sin2\theta_2\sin\phi-\frac{K_6}{2}\sin^2\theta_1\sin^2\theta_2\sin2\phi+\frac{L_4}{\sqrt{2}}\sin2\theta_1\sin\theta_2\sin\phi$$
$$-\frac{L_5}{\sqrt{2}}\sin2\theta_1\sin\theta_2\cos\phi+\frac{L_6}{2}\sin^2\theta_1\cos\theta_2\Bigg\}. \tag{A11}$$

Notice that terms involving $\mathbf{S}^1$ do not contribute to the result. To obtain the result for the other possible final state with a left-handed outgoing l^-, one only needs to flip the sign of $\mathbf{S}^2$ and thus the signs of the coefficients of L_4, L_5, and L_6 (namely, the signs of Y_4, X_5, and X_6). The muon polarization is equal to the sum of the terms L_4, L_5, L_6 divided by the sum of the other six terms. For the case of L_6 it is seen that the polarization does not vanish after integrating over θ_1 and ϕ and so the observation can be made without observing the V_1 decay.

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