Event by event analysis and entropy of multiparticle systems

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The coincidence method of measuring the entropy of a system, proposed some time ago by Ma, is generalized to include systems out of equilibrium. It is suggested that the method can be adapted to analyze multiparticle states produced in high-energy collisions.

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I. INTRODUCTION

Entropy, being one of the most important characteristics of a system with many degrees of freedom, is — in particular — an important characteristic of multiparticle production processes. In this context it abounds in analyses of dense hadronic matter and in discussions of various models of quark-gluon plasma [1].

Processes in which particles are produced can be considered as so-called *dynamical systems* [2,3] in which — generally — entropy gets produced. Although the application of the mathematical theory of dynamical systems to calculate the entropy in multiparticle production is still out of reach, the existing models suggest that the systems produced in high-energy collisions pass through a stage of (approximate) local statistical equilibrium [4,5].

Recently [6] we proposed to apply the event coincidence method [7] to measure the entropy of a multiparticle system, provided it can be described by a microcanonical ensemble.[1] Since the event-by-event analysis becomes a commonly accepted tool to study the multiparticle phenomena, we feel that it is worthwhile to pursue this problem further. In the present paper we extend the coincidence method to the more realistic case of when the energy of the system in question is not necessarily fixed. We show that the method can be rather effective for investigating *local* properties of the particle spectra. Since the observed particles map the state of the system just before it breaks into freely-moving hadrons (which get registered in the detectors), such a measurement can provide important information on the evolution of the system.[2]

At this point it may be important to stress that to properly estimate the entropy of a multiparticle system one would need information not only on the distribution of momenta but also about positions of particles. In particular, correlations between positions and momenta are very essential. This information cannot be obtained, generally, in a model-independent way. One should thus keep in mind that the entropy we discuss in the present paper only partially reflects the statistical properties of the system: the degrees of freedom related to positions of particles are integrated over. Nevertheless it provides valuable information about the system in question, and can be used to identify its nature. In particular, our method may have a wide range of application for the systems where correlations between momenta and positions of the particles are unimportant.

II. ENTROPY AND THE COINCIDENCE METHOD

In a system at equilibrium with all states having the same probability (microcanonical ensemble), entropy measures the number Γ of states of the system

$$S = \log \Gamma. \tag{1}$$

This formula can be rewritten in terms of the probability p for one of the states of the system to realize. Since all states have equal probabilities we have

$$p = \frac{1}{\Gamma} \tag{2}$$

and thus

$$S = -\log p. \tag{3}$$

Ma observed [7] that the probability p can also be expressed as the probability of ''coincidence,'' i.e., the probability that while sampling the system, one finds two states (configurations) which are identical to each other. Indeed, this probability is given by

$$C_2 = \sum_{\text{all states}} (p^2) = \Gamma p^2 = p \tag{4}$$

so that

$$S = -\log C_2. \tag{5}$$

Now if we measure N configurations and find N_2 coincidences we have (in the limit of large N)

$$C_2 = \frac{N_2}{N(N-1)/2} \tag{6}$$

and thus we obtain a method of estimating p and therefore the entropy S also. The attractive feature of this procedure is

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[1]A direct measurement of the entropy of multiplicity distribution observed in multiparticle production was first reported in [8].

[2]Note that the free movement of the particles from the production point to the detector does not influence this measurement.

that, as seen from Eq. (6), the statistical error drops very fast (like N^{-1}) with an increasing number of the tried configurations.[3]

This method does not work, however, if the energy of the considered system is not precisely fixed (e.g., for a canonical or grand-canonical ensemble) or if the system is not in thermodynamic equilibrium. In such a case the states of the system have, in general, various probabilities of occurrence. Consequently, neither Eqs. (3) nor (4) are valid.

In the present note we argue that even in this general case the coincidence method can nevertheless be used to obtain information on the entropy of the system. To this end it is, however, necessary to measure coincidences of more than two configurations. The argument goes as follows.

For an arbitrary system entropy is defined by the general formula [10]

$$S=-\sum_{\text{all } n} p_n \log p_n, \tag{7}$$

where p_n is the probability of occurrence of the state labeled by n, and the sum runs over all states of the system.

To begin we observe that Eq. (7) can be rewritten as

$$S=-\langle \log p\rangle, \tag{8}$$

where $\langle \cdots \rangle$ denotes the average over all states of the system.

Using now the identity

$$p=\langle p\rangle \frac{p}{\langle p\rangle}=\langle p\rangle\left[1-\left(1-\frac{p}{\langle p\rangle}\right)\right] \tag{9}$$

one can transform Eq. (8) into

$$S=-\log\langle p\rangle+\sum_{m=2}^{\infty}\frac{1}{m}\left\langle\left(1-\frac{p}{\langle p\rangle}\right)^m\right\rangle. \tag{10}$$

In this way we have expressed the entropy by the moments $\langle p^m\rangle$.

Now the point is that these moments have a simple physical interpretation in terms of the coincidence probability. Indeed, let us denote by C_k the probability of coincidence of k configurations. In terms of probabilities p_n it can be expressed as[4]

$$C_k=\sum_{\text{all } n}(p_n)^k=\sum_{\text{all } n}p_n(p_n)^{k-1}=\langle p^{k-1}\rangle. \tag{11}$$

We see that the probability of coincidence of k configurations is given by the $k-1$ moment of p.

We thus conclude from Eq. (10) that the probabilities C_k of coincidences of all orders are, in principle, necessary to determine the entropy of the system.

In terms of C_k's, Eq. (10) reads

$$S=-\log C_2+\sum_{m=2}^{\infty}\frac{1}{m}\sum_{k=0}^{m}(-1)^k\binom{m}{k}\frac{C_{k+1}}{(C_2)^k}. \tag{12}$$

If all states have the same probability of occurrence we trivially obtain $C_{k+1}=(C_2)^k$. Thus all terms in the sum vanish and we fall back to the formula (5).[5]

Of course the series (12) and its approximations may be used for estimation of entropy only if the result is convergent. To this end the consecutive terms must be small enough and thus the parameters $C_{k+1}/(C_2)^k$ cannot be much larger than one.[6] This condition limits seriously the applicability of Eq. (12).

III. APPLICATION OF THE "REPLICA METHOD"

It is useful to rearrange the series (12) using the so-called replica method [11]. To this end, let us consider a system made of M independent replicas of the considered system. The entropy of such a composite system is obviously given by

$$S(M)=MS. \tag{13}$$

On the other hand, since it is made of M independent subsystems the coincidence probabilities are given by

$$C_k(M)=[C_k]^M. \tag{14}$$

Consequently, repeating the argument of the previous section we obtain

$$S(M)=-M\log C_2+\sum_{m=2}^{\infty}\frac{1}{m}\sum_{k=0}^{m}(-1)^k\binom{m}{k}\left(\frac{C_{k+1}}{(C_2)^k}\right)^M. \tag{15}$$

Now, the consistency of Eqs. (13) and (15) requires that the sum on the right-hand side (r.h.s.) of Eq. (15) is proportional to M and thus only the term proportional to M can survive. This term is easy to calculate by observing that

$$\left(\frac{C_{k+1}}{(C_2)^k}\right)^M=1+M\log\left(\frac{C_{k+1}}{(C_2)^k}\right)+\cdots. \tag{16}$$

By substituting this into Eq. (15) we obtain

[3]This holds for N in the region $\sqrt{\Gamma}\ll N\ll\Gamma$, the case of interest in the present context.

[4]This formula can be easily proven by considering the Bernoulli distribution of N independent samplings of the considered system. The error can be estimated with the same technique.

[5]$\sum_{k=0}^{m}(-1)^k\binom{m}{k}=(1-1)^m=0$.

[6]It is not difficult to see that $C_{k+1}/(C_2)^k\geqslant 1$. Indeed, for any positive variable f we have $\langle f^{k-1}(f-\langle f\rangle)^2\rangle\geqslant 0$. It follows that $\langle f^{k+1}\rangle-\langle f\rangle^{k+1}\geqslant 3\langle f\rangle^2(\langle f^{k-1}\rangle-\langle f\rangle^{k-1})$ and one can complete the proof by induction.

$$S(M) = -M \log C_2 + M \sum_{m=2}^{\infty} \frac{1}{m} \sum_{k=0}^{m} (-1)^k \binom{m}{k} \log\left(\frac{C_{k+1}}{[C_2]^k} \right). \tag{17}$$

Using Eq. (13) we thus have

$$S = -\log C_2 + \sum_{m=2}^{\infty} \frac{1}{m} \sum_{k=2}^{m} (-1)^k \binom{m}{k} \log\left(\frac{C_{k+1}}{[C_2]^k} \right), \tag{18}$$

which represents our final formula. It is providing partial resummation of the powers of $C_{k+1}/[C_2]^k$ into logarithms.

IV. RENYI'S ENTROPIES

The formula (18) can be rewritten in terms of the Renyi entropies[7] defined as [12]

$$H_k = -\frac{\log C_k}{k-1}. \tag{19}$$

Using this definition one can easily see that $H_1 = S$. Substituting Eq. (19) into Eq. (18) we obtain, after some algebra,

$$\begin{aligned} S &= H_2 + \sum_{n=1}^{\infty} \sum_{k=0}^{n} (-1)^k \binom{n}{k} H_{k+2} \\ &= \sum_{n=0}^{\infty} \sum_{k=0}^{n} (-1)^k \binom{n}{k} H_{k+2} \\ &= H_2 + (H_2 - H_3) + (H_2 - 3H_3 + H_4) \\ &\quad + (H_2 - 3H_3 + 3H_4 - H_5) + \cdots . \end{aligned} \tag{20}$$

One sees that the first N terms of this series represent the polynomial extrapolation of the function H_k from the points $k = 2,3,4, \ldots ,N+1$ to $k=1$. This observation not only explains the meaning of formulas (18) and (20) but also suggests the way to improve it: one should look for more effective extrapolations. One possibility we have investigated in some detail is to take

$$H_k = a \frac{\log k}{k-1} + a_0 + a_1(k-1) + a_2(k-1)^2 + \cdots . \tag{21}$$

The number of terms is determined by the number of coincidence probabilities one is able to measure. If only C_2 and C_3 are measured we obtain

$$S = H_2 + \frac{1 - \log 2}{\log 2 - (1/2)\log 3} (H_2 - H_3). \tag{22}$$

If three coincidences are measured we have

$$S = H_2 + (H_2 - H_3)(1 + \omega) - \omega(H_3 - H_4), \tag{23}$$

[7]The argument presented in this section was suggested to us by K. Zyczkowski.

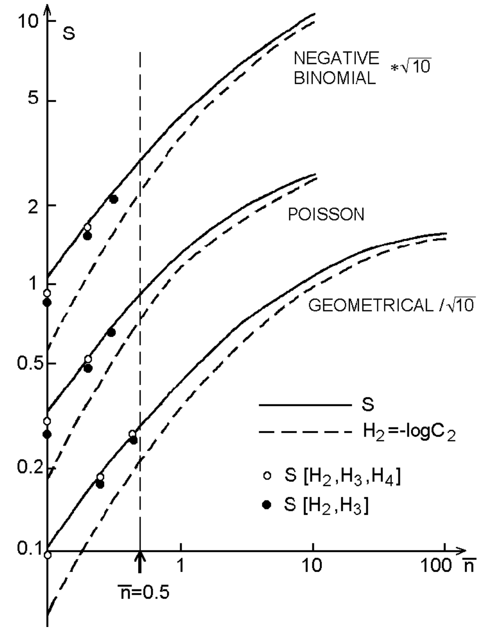

FIG. 1. Estimates of entropy for systems with commonly encountered distributions using the extrapolation given by Eq. (21), plotted versus average multiplicity. Continuous lines: entropy calculated directly from Eq. (7). Dashed lines: entropy calculated from Eq. (5). Open points: Three-term extrapolation from Eq. (23). Full points: Two-term extrapolation from Eq. (22).

where

$$\omega = \frac{1 - 2\log 2 + (1/2)\log 3}{\log(2/3) + (2/3)\log 2}. \tag{24}$$

In Fig. 1 the results of this procedure are shown for three distributions, often encountered in the analysis of multiparticle data: Poisson, negative binomial, and the geometric series. One sees that extrapolation using only two terms is by far sufficient to obtain an accurate value of entropy, provided the average multiplicity is not lower than 1/2. The first term (H_2) is, however, hardly sufficient even for fairly large multiplicities.

For $\overline{n} \to 0$ the extrapolation is rather poor which shows that the method is not well adapted for studies of low multiplicity events.

We have also found that for these three distributions the polynomial extrapolation (20) is less accurate than Eq. (21).

V. ESTIMATE OF ENTROPY THROUGH MEASUREMENTS OF COINCIDENCES

We have suggested recently [6] that the coincidence method of Ma can be used to estimate the entropy of the system of particles produced in a high-energy collision. The idea was to consider the produced events as the randomly chosen configurations of the system. Measurement of the (appropriately defined) probability of coincidence of two events was interpreted, following the formula (5), as a measurement of entropy of the system.[8]

As it is not very likely that the system produced in a high-energy collision can indeed be accurately represented by a microcanonical ensemble at equilibrium[9], however, one may have justified doubts about the accuracy of this method. It is clear from the previous argument that Eqs. (18) and (20) provide a possibility to assess this. Indeed, already measuring the probability of coincidence of three events

$$C_3=\frac{N_3}{N(N-1)(N-2)/6} \tag{25}$$

allows one to estimate the first correction to Eq. (5). As discussed in the previous section, this is often sufficient to obtain an accurate value of the entropy.

VI. DISCRETIZATION

Application of the coincidence method, as described in previous sections, for measurements of entropy in multiparticle production (which is our main objective) requires discretization of the observed multiparticle spectra [6]. The dependence of the results of measurements on discretization can be discussed as follows.

Consider a system consisting of a certain number, say N, of particles produced in a high-energy collision. Let $\Phi(q)dq \equiv \Phi(q_1,\dots,q_N)dq_1\dots dq_N$ be their probability distribution in momentum space. To discretize, we split the distribution into M ($3N$ dimensional) bins of size Δq_m, $m=1,\dots,M$. The probability distribution to find the system in the bin m is

$$w(m,M)=\Phi(q^{(1)}(m),\dots,q^{(N)}(m))\Delta q_m, \tag{26}$$

where $[q^{(1)}(m),\dots,q^{(N)}(m)]$ is the set of N momenta defining the bin m. The coincidence probabilities measured from the distribution (26) are

$$C_k(M)=\sum_{m=1}^{M}(\Delta q_m)^k[\Phi(q^{(i)}(m))]^k. \tag{27}$$

If we now split each bin into λ new bins (and thus multiply the number of bins by factor λ) the probability (26) changes accordingly and we obtain

$$C_k(\lambda M)=\frac{1}{\lambda^{k-1}}\sum_{m=1}^{M}(\Delta q_m)^k \times\sum_{l_m=1}^{\lambda}\frac{1}{\lambda}[\Phi(q^{(1)}(m,l_m),\dots,q^{(N)}(m,l_m))]^k. \tag{28}$$

For nonsingular distribution $\Phi(q_1,\dots,q_N)$ the dependence of the sum on the r.h.s. on λ disappears in the limit $\lambda\to\infty$ and thus using Eq. (18) or Eq. (20) we have

$$S(\lambda M)=\log\lambda+S(M), \tag{29}$$

which summarizes the dependence of the proposed measurement on the resolution used in the procedure of discretization.[10] Note that λ denotes the number of splittings in $3N$ dimensional momentum space. If the splitting procedure is performed by simply splitting the bins in one-dimensional single particle momentum distribution into λ_0 new bins, we have $\lambda=(\lambda_0)^{3N}$ which gives

$$S(\lambda M)=3N\log\lambda_0+S(M). \tag{30}$$

The final question one may ask is how the entropy measured from the distribution (26) is related to the ''true'' entropy[11] of the N particle system described by the distribution function $\Phi(q_1,\dots,q_N)$. To consider this problem we observe that the spacing between the momentum states of a system of N particles is given by the quantum-mechanical relation

$$\delta q=\left(\frac{(2\pi)^3}{v}\right)^N, \tag{31}$$

where v denotes the volume of the system.[12] Denoting the total number of states of the system by Γ the ''true'' entropy is given by

$$\begin{aligned}S(\Gamma)&=-\sum_{i=1}^{\Gamma}p(q^{(1)}(i),\dots,q^{(N)}(i))\\&\quad\times\log[p(q^{(1)}(i),\dots,q^{(N)}(i))]\\&=-\sum_{m=1}^{M}w(q^{(1)}(m),\dots,q^{(N)}(m))\\&\quad\times\log[w(q^{(1)}(m),\dots,q^{(N)}(m))/\Gamma(m)]\\&=S(M)+\sum_{m=1}^{M}w(q^{(1)}(m),\dots,q^{(N)}(m))\log\Gamma(m),\end{aligned} \tag{32}$$

[8] As explained in Sec. I, we are considering only entropy related to the distribution of particle momenta. The volume fluctuations and correlations between the position and momentum of a particle are neglected.

[9] Although this is the case in the Fermi model of multiparticle production [9].

[10] Additional dependence on λ would indicate that the distribution $\Phi(q_1,\dots,q_N)$ is singular (see, e.g., [13]).

[11] We use quotation marks to emphasize that, as explained in Sec. I, the entropy we discuss in this paper is not—in general—the actual entropy of the system since it neglects the positions of particles in configuration space.

[12] The fluctuations of the volume can be—at least in principle—determined if the HBT correlations are measured for each event.

where

$$\Gamma(m)=\frac{\Delta q_m}{\delta q}=\left[\frac{(\Delta_0(M))^3}{(2\pi)^3}v\right]^N \tag{33}$$

is the number of states in the bin m. Here $\Delta_0(M)$ denotes the size of the (one-dimensional) bin in momentum space of one particle.

Equation (32) relates the entropy $S(\Gamma)$ of the considered system to $S(M)$ — the one measured by discretization into M bins. For the simplest case of when all bins used in discretization are equal to each other, Γ_m does not depend on m and the last sum in Eq. (32) can be performed. The result is

$$S(\Gamma)=S(M)+\log(\Gamma(m))=S(M)+3N\log\left(v^{1/3}\frac{\Delta_0(M)}{2\pi}\right). \tag{34}$$

VII. INDEPENDENTLY PRODUCED PARTICLES

To assess the practical possibilities of using the proposed method to the actual multiparticle data, we have estimated the coincidence probabilities for a system of particles produced independently.

Suppose that the produced particles come in a number of species, labeled by f. Then

$$C_k=\prod_f C_k(f), \tag{35}$$

so that it is enough to consider one kind of particle.

We now discretize the system by splitting it into M bins of size Δq. With this procedure, the state of the system is defined by giving the number of particles in each bin. If particles are emitted independently, the probability of a given state is

$$W(n_1,\ldots,n_M)=\prod_{i=1}^{M} P(n_i,\overline{n_i}), \tag{36}$$

where $P(n,\overline{n})$ is the Poisson distribution with average $\overline{n}$ and $\overline{n_i}$ is the average number of particles in a bin labeled i and is given by

$$\overline{n_i}=\int_{q_i-\Delta q/2}^{q_i+\Delta q/2} dq\,\rho(q), \tag{37}$$

where $\rho(q)$ is the single particle momentum distribution: $\int dq\,\rho(q)=\overline{N}$ with N being the total number of particles.

From Eq. (27) we deduce

$$C_k=\sum_{n_1,\ldots,n_M}[W(n_1,\ldots,n_M)]^k=\prod_{i=1}^{M} C_k^{\mathrm{pois}}(\overline{n_i}), \tag{38}$$

where

$$C_k^{\mathrm{pois}}(\overline{n})=\sum_n [P(n,\overline{n})]^k. \tag{39}$$

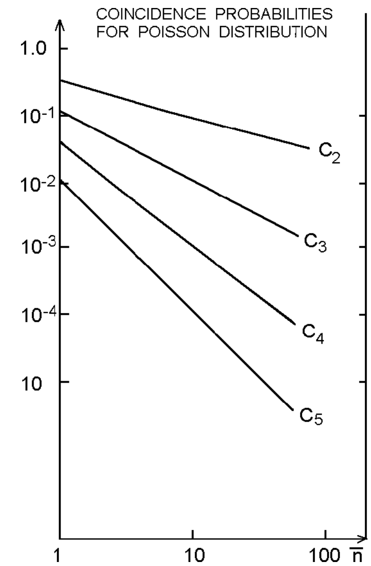

FIG. 2. Coincidence probabilities for Poisson distribution versus average multiplicity.

We have calculated numerically $C_k^{\mathrm{pois}}(\overline{n})$ for $2\leqslant k\leqslant 5$. They are shown in Fig. 2, plotted versus $\overline{n}$. One sees that in the range $1\leqslant\overline{n}\leqslant 50$ they can be well approximated by the formula

$$C_k^{\mathrm{pois}}(\overline{n})\approx\left(\frac{1}{3\sqrt{\overline{n}}}\right)^{k-1}, \tag{40}$$

which shows that they are not prohibitively small even at fairly high multiplicities. We thus conclude that for at least one bin one should be able to measure C_2 and C_3 with a reasonable accuracy even for large systems (i.e., systems containing many particles[13]).

The situation becomes much worse, however, with the increasing number of bins, as easily seen from Eq. (38). For $\overline{N}=100$ and $M=10$ bins, for example, one obtains $C_2\approx 10^{-9.5}$ and $C_3\approx 10^{-19}$. The situation improves somewhat for smaller multiplicities such as $\overline{N}=10$ and $M=10$, where one has $C_2\approx 10^{-5}$ and $C_3\approx 10^{-10}$. As shown in Sec. VI, however, the method does not work if the particle multiplicity in one bin falls below $\overline{n}\sim 1/2$. Therefore it is limited to a study of rather small regions of phase space.

VIII. APPLICATIONS

We are convinced that systematic measurements of the local entropy of multiparticle systems created in high-energy collisions can provide interesting insights into the physics of multiparticle production.

(i) Since the entropy measures the effective number of states of the system, it provides direct information about its internal degrees of freedom. This is well known and actually

[13]For large multiplicities the first term in the asymptotic expansion of C_k is $1/(\sqrt{2k\pi\overline{n}})^{k-1}$.

exploited in numerous models of production of the quark-gluon plasma [1,5,4]. Usually, however, the entropy is simply estimated from the measured number of particles (see, e.g., [4]). This estimate strongly relies on the assumed thermal equilibrium and, moreover, is expected to be valid only in the limit of a very large number of particles. Our method, leading to an independent estimate of the entropy, may thus be used for a verification of the accuracy of the standard approach. For instance, the validity of the relations [4] which express entropy through the thermal energy density and the number of ''wounded nucleons'' [14] can be tested against the entropy estimated from our formulas.

(ii) Most of the models discussing particle production in heavy ion collisions assume local thermal equilibrium of the created system (at least in central collisions). Measurements of entropy allow one to verify this assumption by testing the validity of some thermodynamic identities. One of the most promising ones seems to be [6]

$$\left.\frac{\partial S(E,N,V)}{\partial E}\right|_{N,V}=\frac{1}{T}, \tag{41}$$

where both the left-hand side (l.h.s.) and r.h.s. can be estimated from the data.[14] It would be very interesting therefore to study Eq. (41) in different phase-space regions, and with varying impact parameter of collisions in order to establish the region where the assumption of thermal equilibrium has a chance to be reasonable (let us also note that one may as well envisage an application of this test to ''elementary'' e^+e^- and pp collisions and thus verify the interesting hypothesis of Beccatini [5]).

(iii) If the thermal equilibrium is verified, one may use the identity

$$\left.\frac{\partial S(E,N,V)}{\partial N}\right|_{E,V}=-\frac{\mu}{T} \tag{42}$$

to estimate the chemical potential of the produced particles. This would be another important ingredient in model building.

(iv) The method proposed in the present paper allows one to measure the entropy of systems far from equilibrium. This certainly extends the possibilities of treating multiparticle production phenomena by the methods of statistical physics. For example, the long debated problem as to what extent particle production in heavy ion collisions can be treated as a simple superposition of nucleon-nucleon collisions may be very effectively tested.

[14]This formula is valid under the assumption of a fixed number of particles, N, and a fixed volume, V, where the equilibrium is supposed to be established. The number of particles can be measured but it is difficult to have direct access to the volume. We can conjecture, however, that a selection of a well defined sample of events will come from similar volumes; in fact it is hard to imagine that it could be otherwise. If this is indeed the case, i.e., if the fluctuations of the volume are not too large, it can be measured from the HBT correlations.

(v) It should be noted that many of the properties of the ''true'' entropy of the system are also shared by the Renyi entropies (19) which can be measured directly, without the uncertain extrapolation procedure explained in Sec. IV. Therefore, we feel that they should also be treated seriously and estimated as precisely as possible in theoretical calculations concerning multiparticle production. Actually, in some cases the calculation of Renyi entropies is not more difficult than the calculation of the entropy of the system itself,[15] thus providing a new, powerful tool for the analysis of multiparticle systems.

IX. FINAL COMMENTS

At this point it may be worthwhile to point out that the measurement of event coincidence probabilities represents interesting information about the multiparticle system, independent of its relation to the Shannon entropy. Indeed, it gives valuable information on statistical fluctuations of the system in question and thus may be considered as an alternative approach to the problem of ''erraticity'' [15]. It seems to be a more detailed measure of even-by-event fluctuations than the distribution of the (horizontally averaged) factorial moments [15]. The weak point is that the method seems applicable only to a small part of the available phase space.[16] Some averaging procedure may thus turn out to be necessary also in this case.

It is also worthwhile to emphasize that the event coincidence probabilities are sensitive to an entirely different region of multiparticle spectrum than the widely used factorial moments [13]. Indeed, whereas factorial moments are sensitive mostly to the large multiplicity tail of the spectrum, the coincidence probabilities obtain the largest contributions from the region where the probability distribution is maximal. The two methods thus seem complementary to each other and should best be used in parallel to obtain the maximum of information.

X. CONCLUSIONS

In conclusion, we have proposed a generalization of Ma's coincidence method of entropy determination. It requires measurements of coincidences of 2,3, . . . configurations. The new method can be applied to a more general class of systems. In particular, thermodynamical equilibrium is not necessary.

The method seems well adapted to the analysis of local properties of multiparticle states produced in high-energy collisions. It may thus turn out to be useful for an investiga-

[15]E.g., for a photon gas one finds $H_q=\frac{1}{4}(1+1/q+1/q^2+1/q^3)S$.

[16]An interesting possibility would be to study two (or more) disconnected regions of available phase space, with such a measurement being sensitive to the *long range* correlations in the multiparticle system.

tion of the thermodynamic properties of the dense hadronic matter and/or quark-gluon plasma.

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