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Erraticity analysis of multiparticle production

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Event-to-event fluctuations of the spatial patterns of the final states of high-energy collisions, referred to as erraticity, are studied for the data generated by a soft-interaction model called ECOMB. The moments $C_{p,q}$ do not show simple power-law dependences on the bin size. New measures of erraticity are proposed that generalize the bin-size dependence. The method should be applied not only to the soft production data of NA22 and NA27 to check the dynamical content of ECOMB, but also to other collision processes, such as e^+e^- annihilation and heavy-ion collisions.

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I. INTRODUCTION

Progress in the study of multiparticle production has recently been made in two distinct directions among many others. One is in finding measures of event-to-event fluctuations [1] that can probe the production dynamics more deeply than conventional observables, such as the multiplicity distribution and factorial moments [2]. Such measures have been referred to as erraticity [3], which quantifies the erratic nature of the event structure. The other direction is in the construction of a Monte Carlo generator, called ECOMB [4], that simulates soft interactions in hadronic collisions capable of reproducing the intermittency data [5]. ECOMB stands for eikonal color mutation branching, which are the key words of a model that is based on the parton model rather than the string model for low-p_T processes. In this paper we combine the two, using ECOMB to generate events from which we calculate the erraticity measures. The result should be of considerable interest, since, on the one hand, the erraticity analysis of the NA22 data [5] is currently being carried out, and, on the other, it can motivate the investigation and comparison of erraticities in various different collision processes, ranging from e^+e^- annihilation to heavy-ion collisions.

The study of erraticity originated in an attempt to understand possible chaotic behaviors in quark and gluon jets [1], since QCD is intrinsically nonlinear. In the search for a measure of chaos it was realized that the fluctuation of the hadronic final states of a parton jet is the only observable feature of the QCD process that can replace the unpredictable trajectories in classical nonlinear dynamics. A multiparticle final state in momentum space is a spatial pattern. Once a measure is found to quantify the fluctuation of spatial patterns, the usefulness of the method goes far beyond the original purpose of characterizing chaoticity in perturbative QCD processes. Many problems involve spatial patterns; they can range from phase transitions in condensed matter to galactic clustering in astrophysics. Even continuous time series can be transformed by discrete mapping to spatial patterns [6]. Thus the erraticity analysis, which is the study of the fluctuation of spatial patterns, is more general than the determination of chaotic behavior. Indeed, we have applied it to the study of phase transitions in magnetic systems by use of the Ising model [6], as well as to the characterization of heartbeat irregularities in electrocardiogram time series [7].

Multiparticle production at low p_T has always eluded first-principles calculation because of its nonperturbative nature. Various models that simulate the process can generate the average quantities, but fail in getting correctly the fluctuations from the averages [2]. In particular, few models can fit the intermittency data [5]. To our knowledge ECOMB is the only one that can reproduce those data [4] (apart from its predecessor ECCO [8]). Since that model is tuned to fit the data by the adjustment of several parameters, it is necessary to test its predictions on some new features of the production process. Erraticity is such a feature. The fluctuation of final-state patterns presents a severe test of any model.

ECOMB includes many sources of fluctuations in hadronic collisions. In the framework of the eikonal formalism it allows for fluctuations in impact parameter b. For any b there is the fluctuation of the number μ of cut Pomerons. For any μ there is the fluctuation of the number m of partons. A stochastic description of the generation of partons was later given, after the original paper on ECOMB [4], providing an even better fit of the Koba-Nielsen-Olesen (KNO) scaling data on C_q with fewer parameters [9]. For any m numbers of partons the color distribution along the rapidity axis can still fluctuate initially. During the evolution process, the local subprocesses of color mutation, spatial contraction and expansion, branching into neutral subclusters, and hadronization into particles or resonances can all fluctuate. Taken together the model can generate such widely fluctuating events that fitting some average quantity such as $\langle n\rangle$ or dn/dy does not explore the full extent of its characteristics. The dependence of normalized factorial moments F_q on the bin size δ, usually called intermittency, probes deeper, but it is nevertheless a measure that is averaged over all events. Erraticity is a true measure of event-to-event fluctuations.

II. ERRATICITY

There are various ways to characterize a spatial pattern. We shall use the horizontal factorial moments. Given the rapidity distribution of a particular event, we first convert it to a distribution in the cumulative variable X [10,1], in terms of which the average rapidity distribution dn/dX is uniform in X. We then calculate from that distribution for that event the normalized F_q:

$$F_q = \langle n(n-1)\cdots(n-q+1)\rangle / \langle n\rangle^q, \tag{1}$$

where $\langle\cdots\rangle$ signifies (horizontal) average over all bins and n is the multiplicity in a bin. We emphasize that Eq. (1) does not involve any average over events. F_q does not fully describe the structure of an event, since at any fixed q it is insensitive to the rearrangement of the bins. However, it does capture some aspect of the fluctuations from bin to bin and is adequate for our purpose.

Since F_q fluctuates from event to event, one obtains a (vertical) distribution $P(F_q)$ after many events. Let the vertical average of F_q determined from $P(F_q)$ be denoted by $\langle F_q\rangle_v$. Then, in terms of the normalized moments for separate events

$$\Phi_q = F_q / \langle F_q\rangle_v, \tag{2}$$

we can define the vertical pth-order moments of the normalized qth-order factorial (horizontal) moments:

$$C_{p,q} = \langle \Phi_q^p\rangle_v. \tag{3}$$

It should be stressed that $C_{p,q}$ does not involve the $p\times q$ moments of the multiplicity n because for each q there is only one number F_q for each event. It is the fluctuation of F_q from event to event that $C_{p,q}$ measures. The statistical errors of $C_{p,q}$ are, however, more complicated to determine, as we shall explain below.

Erraticity refers to the power-law behavior of $C_{p,q}$ [1,3]:

$$C_{p,q} \propto M^{\psi_q(p)}, \tag{4}$$

where M is the number of bins $1/\delta$ and the length in X space is 1. $\psi_q(p)$ is referred to as the erraticity exponent. If the spatial pattern never changes from event to event, $P(F_q)$ would be a delta function at $\Phi_q=1$ and $C_{p,q}$ would be 1 at all M, p, and q, resulting in $\psi_q(p)=0$. The larger $\psi_q(p)$ is, the more erratic is the fluctuation of the spatial patterns.

Since $\psi_q(p)$ is an increasing function of p with increasing slope, an efficient way to characterize erraticity with one number (for every q) is simply to use the slope at $p=1$, i.e.,

$$\mu_q = \frac{d}{dp}\psi_q(p)\Big|_{p=1}. \tag{5}$$

It is referred to as the entropy index [1]. Experimentally, it is easier to determine first an entropylike quantity Σ_q directly from Φ_q:

$$\Sigma_q = \langle \Phi_q \ln \Phi_q\rangle_v, \tag{6}$$

which follows from Eq. (3) and

$$\Sigma_q = dC_{p,q}/dp\big|_{p=1}, \tag{7}$$

and then to determine μ_q from Σ_q using

$$\mu_q = \frac{\partial \Sigma_q}{\partial \ln M}, \tag{8}$$

provided that $C_{p,q}$ has the scaling behavior (4). In [1] it is found that μ_q is larger for quark jets than for gluon jets, indicating that the branching process of the former is more chaotic or, in other words, the event-to-event fluctuation is more erratic.

If the moments $C_{p,q}$ do not have the exact scaling behavior in M, as in Eq. (4), but have similar nonlinear dependences on M, we can consider a generalized form of scaling:

$$C_{p,q}(M) \propto g(M)^{\tilde{\psi}(p,q)}. \tag{9}$$

If Eq. (9) is approximately valid for a common $g(M)$ for all p and q, it then follows from Eq. (7) that

$$\Sigma_q(M) \propto \tilde{\mu}_q \ln g(M), \tag{10}$$

where

$$\tilde{\mu}_q = \frac{d}{dp}\tilde{\psi}(p,q)\big|_{p=1}. \tag{11}$$

Despite the similarity between Eqs. (5) and (11), $\tilde{\mu}_q$ is distinctly different from μ_q and should not be compared to one another unless $g(M)=M$.

If Eq. (10) is indeed good for a range of q values, then we expect a linear dependence of Σ_q on Σ_2 as M is varied. Let the slope of such a dependence be denoted by ω_q, i.e.,

$$\omega_q = \frac{\partial \Sigma_q}{\partial \Sigma_2}. \tag{12}$$

Then we have

$$\tilde{\mu}_q = \tilde{\mu}_2 \omega_q. \tag{13}$$

A variation of this scheme that makes use of an extra control parameter r in the problem is considered in [6]. It is found there that the entropy indices determined that way are as effective as Lyapunov exponents in characterizing classical nonlinear dynamical systems.

III. SCALING BEHAVIORS

The erraticity analysis described above involves only measurable quantities, so it can be directly applied to the experimental data. The NA22 data at $\sqrt{s}=22$ GeV are ideally suited for this type of analysis, since F_q fluctuates widely from event to event [5]. The nuclear collision data, such as those of NA49, can also be studied, but p_T cuts should be made to reduce the hadron multiplicity to be analyzed, thereby enhancing the erraticity to be quantified.

Here we apply the analysis to hadronic collisions generated by ECOMB. The parameters are tuned to fit $\langle n\rangle$, P_n, dn/dy, and $\langle F_q\rangle_v$ of the NA22 data [5]. Without any further adjustment of the parameters in the model, we calculate $C_{p,q}(M)$, which are therefore our predictions for hadronic

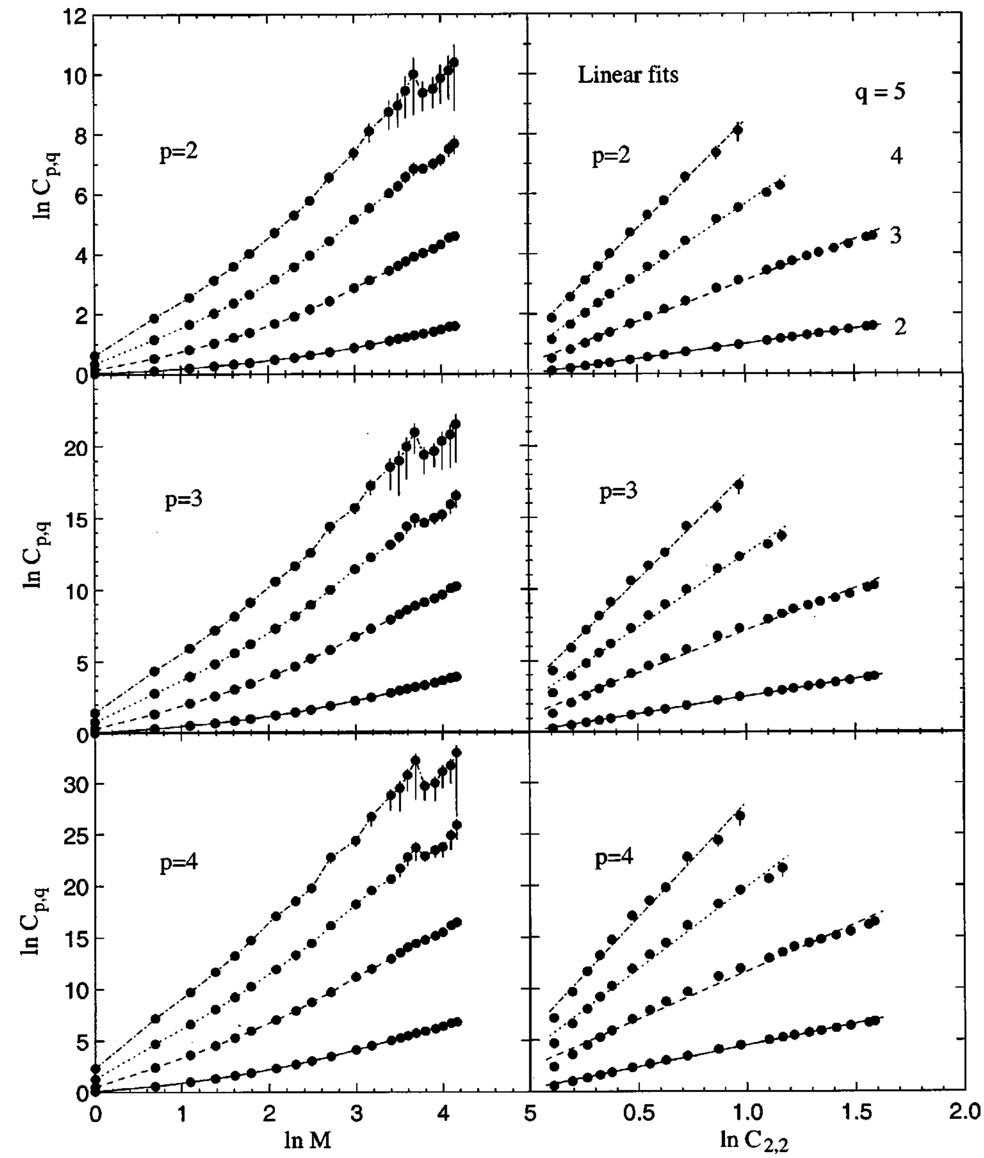

FIG. 1. Log-log plots of $C_{p,q}$ versus M on the left side and versus $C_{2,2}$ on the right side. The lines on the left side are connected between points to guide the eye, while the ones on the right side are linear fits.

collisions at 22 GeV. The results from simulating 10^6 Monte Carlo events are shown on the left side of Fig. 1.

The statistical errors in Fig. 1 are calculated on the assumption that the value of F_q for each event is unique so that the only source of errors arises from the event-to-event fluctuations. Thus the statistical errors are inversely proportional to $\sqrt{N_{\rm ev}}$, where $N_{\rm ev}$ is the total number of events. Unfortunately, that assumption is too strong for events with a total multiplicity not too high, since the bin multiplicities in that case fluctuate greatly from bin to bin. The effect of such fluctuations on F_q is especially severe at high q. It shows up when the bin number M is changed by a small amount, and the average $\langle F_q \rangle_v$ would change by a large amount. That is the phenomenon observed by NA22 [5], where the values of $\langle F_q \rangle_v$, for $q=5$, jump around erratically between neighboring values of M, with the consequence that the overall uncertainty is much greater than the error bars in the individual points. The reason for that phenomenon of fluctuating $\langle F_5 \rangle$ is that not many events in the NA22 data have a bin multiplicity $n \geqslant 5$, when the bin size is small and the event multiplicity is only between 10 and 15. Statistical errors can be unambiguously determined only when the underlying distribution is smooth. However, when most of the events give $F_5=0$ and only a few yield $F_5 \neq 0$, no meaningful statistical error in $\langle F_5 \rangle$ can be given. If the problem is severe for $\langle F_5 \rangle$, then it is much worse for $\langle F_5^p \rangle$ for $p>1$.

In our simulation using ECOMB the result in Fig. 1 shows a kink at $\ln M=3.8$ for $q=4$ and 5. That is due to the bin-to-bin fluctuations mentioned above. Such fluctuations do not affect the $q=2$ and 3 cases, since F_q does not require bin multiplicities to be large when q is small. The statistical errors calculated for the $q=4$ and 5 cases are done using the standard method, which is probably inadequate when the underlying distribution is not smooth, but they give a rough idea. The kinks indicate that the overall errors are larger. The lines on the left side of Fig. 1 are drawn by connecting the

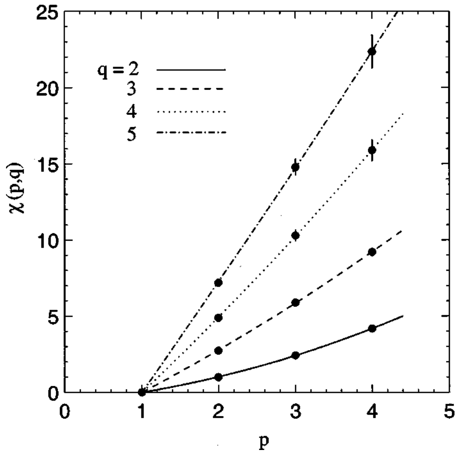

FIG. 2. The slopes of the linear fits on the right side of Fig. 1 are plotted against p for various values of q. The lines are fits by a quadratic formula.

points just to guide the eye.

From the points shown, it is clear that the dependences of $C_{p,q}$ on M in the log-log plots are not very linear. Thus the power-law behavior in Eq. (4) is not well satisfied. Since the general behaviors of $C_{p,q}$ are rather similar in shape, we can regard $C_{2,2}$ as the reference that carries the typical dependence on M and examine $C_{p,q}$ vs $C_{2,2}$ when M is varied as an implicit variable. The results are shown on the right side of Fig. 1. The straight lines are linear fits of the points shown and lend support to the scaling behavior

$$C_{p,q} \propto C_{2,2}^{\chi(p,q)}. \tag{14}$$

The slopes of the fits are $\chi(p,q)$, which are shown in Fig. 2. One may regard $\chi(p,q)$ as a representation of the erraticity properties of the particle production data, when there is no strict scaling law as in Eq. (4).

The behavior of $\chi(p,q)$ exhibited in Fig. 2 can be described analytically, if we fit the points by a quadratic formula for each q. The result is shown by the lines in Fig. 2. Evidently, the fits are very good. Since, as mentioned above, the errors in the points for $q=4$ and 5 are hard to determine precisely, it is prudent not to pursue the quantitative implications of those points in Fig. 2, even though they admit smooth curves. In the following we shall put emphasis only on the $q=2$ and 3 data points. The properties of the smooth behaviors can be further summarized by their derivatives at $p=1$:

$$\chi_q' \equiv \frac{d}{dp}\chi(p,q)\big|_{p=1}. \tag{15}$$

The values of χ_q' for $q=2$ and 3 are

$$\chi_2' = 0.83 \pm 0.01, \quad \chi_3' = 2.64 \pm 0.04. \tag{16}$$

We suggest that these values of χ_q' be used to compare with the experimental data.

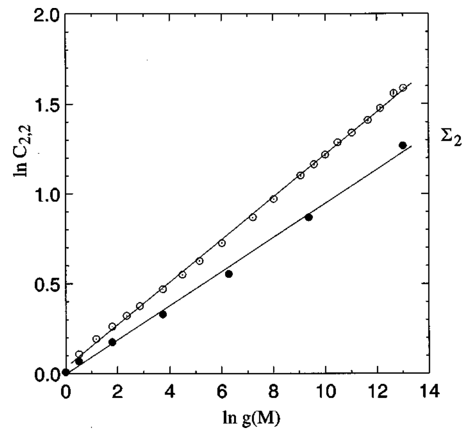

FIG. 3. The open circles are for $C_{2,2}$ and the solid points are for Σ_2. The lines are linear fits, whose slopes are $\tilde{\psi}(2,2)$ and $\tilde{\mu}_2$, respectively.

Although $C_{p,q}(M)$ do not satisfy Eq. (4), we can consider the more general form (9). If the same function $g(M)$ is good enough in Eq. (9) for all p and q, then it follows from Eq. (14) that

$$\chi(p,q) = \tilde{\psi}(p,q)/\tilde{\psi}(2,2). \tag{17}$$

Using Eq. (11), we then have

$$\tilde{\mu}_q = \tilde{\psi}(2,2)\chi_q'. \tag{18}$$

It should be noted that, whereas χ_q' follows only from the scaling property of Eq. (14), the determination of $\overline{\psi}(2,2)$, and therefore $\tilde{\mu}_q$, requires knowledge of $g(M)$ in Eq. (9).

To determine $g(M)$, we write it in the form

$$\ln g(M) = (\ln M)^a. \tag{19}$$

By varying a, we can find a good linear behavior of $\ln C_{2,2}$ vs $\ln g(M)$, as shown by the open circles in Fig. 3 for $a=1.8$. (The solid points should be ignored for now.) The corresponding value of $\tilde{\psi}(2,2)$ determined by the slope of the straight line fit is

$$\tilde{\psi}(2,2) = 0.119 \pm 0.001. \tag{20}$$

Using that in Eq. (18) in conjunction with Eq. (16) yields

$$\tilde{\mu}_2 = 0.099 \pm 0.002, \quad \tilde{\mu}_3 = 0.314 \pm 0.007. \tag{21}$$

These values will be compared below with those calculated by an alternative method.

We remark that in checking the validity of Eq. (9) for values of p and q other than 2, one can improve the linearity of the points for each p and q by slight adjustments of the value of a. If there is a range of possible $g(M)$ that depends on p and q to yield the best fits, however small the variations in a may be, the scheme defeats the point of defining a universal $\tilde{\psi}(p,q)$. We thus propose that the emphasis of the erraticity analysis should be placed on Eq. (14), which is

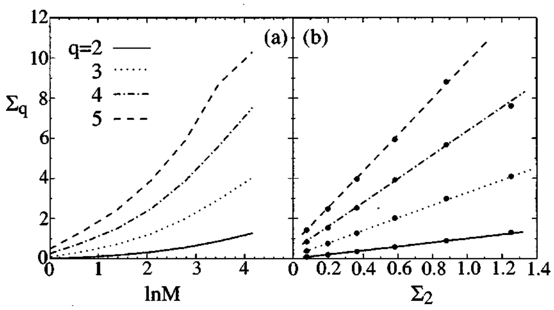

FIG. 4. (a) Σ_q vs $\ln M$ for various q; (b) Σ_q vs Σ_2 with the lines being linear fits.

independent of $g(M)$, and that Eq. (9) is examined only for $p=2$, $q=2$ so that Eq. (18) can be evaluated.

Since $\tilde{\mu}_q$ is distinct from μ_q, we cannot compare our result on $\tilde{\mu}_q$ with the theoretical values of μ_q found for quark and gluon jets [1] or with the experimental values of μ_q determined from pp collisions at 400 GeV/c (NA27) [11].

The values of $\tilde{\mu}_q$ can also be determined independently by use of $\Sigma_q(M)$. From the definition in Eq. (6) we have calculated Σ_q as functions of $\ln M$, as shown in Fig. 4(a). Not surprisingly, the dependences are not linear. However, when Σ_q is plotted against Σ_2 in Fig. 4(b), they all fall into straight lines. The slopes, which give ω_q defined in Eq. (12), are $\omega_2=1.0$ by definition and

$$\omega_3=3.24\pm0.12 \tag{22}$$

by fitting. We resist the inclination to give the values of ω_4 and ω_5, since we have chosen to relinquish the quantitative study of the higher-q cases. If we examine Eq. (10) for $q=2$ only and plot Σ_2 vs $\ln g(M)$ with $a=1.8$, shown by the solid points in Fig. 3, we obtain a linear behavior with a slope

$$\tilde{\mu}_2=0.095\pm0.004. \tag{23}$$

This value is consistent with that in Eq. (21). Of the two methods of determining $\tilde{\mu}_2$, this latter approach is more reliable, since the derivative in p at $p=1$ is done analytically in the definition of Σ_q in Eq. (7), whereas in the former approach the differentiation is done in Eq. (15) using the fitted curve in Fig. 2. Substituting Eq. (23) into Eq. (13), we can determine the value of $\tilde{\mu}_3$ from the value of ω_3 in Eq. (22). The result is

$$\tilde{\mu}_3=0.308\pm0.024. \tag{24}$$

The two values of $\tilde{\mu}_3$ in Eqs. (21) and (24) agree within errors.

IV. CONCLUSION

In conclusion, we recapitulate the two essential points of this paper. One is the prediction of ECOMB on the nature of fluctuations of the factorial moments F_q from event to event. The other is the proposed method of summarizing the scaling behaviors of $C_{p,q}$ that do not have strict power-law dependences on the bin size. The two aspects of this paper converge on the new erraticity measures $\chi(p,q)$, χ_q', ω_q, and $\tilde{\mu}_q$.

The proposed measures of erraticity are, of course, more general than the application made here to soft production. Event-to-event fluctuation has recently become an important theme in collisions of all varieties: e^+e^- annihilation, leptoproduction, hadronic collisions at very high energies where hard subprocesses are important, and heavy-ion collisions. What was lacking previously is an efficient measure of such fluctuations. The erraticity measures proposed in [1,3], now generalized to $\chi(p,q)$, χ_q', ω_q, and $\tilde{\mu}_q$, are well suited for that purpose. They may be redundant, if strict scaling in M is good enough to give the erraticity indices $\psi(p,q)$. The method of treating the less-strict scaling properties proposed here may well be more generally applicable to a wide range of collision processes amenable to erraticity study.

V. SOME FINAL REMARKS

After the completion of this work, two groups of investigators have followed the suggestions made in this paper and obtained results that are worth commenting on. One is by a Hefei group [12], who showed that the erraticity analysis of the NA27 data yields results that are ''similar'' to ours shown in this paper. A more critical paper is by a group in Wuhan [13], who investigated the effect of statistical fluctuations. In the framework of a simple model what Fu *et al.* have found is that when the event multiplicity is low, the horizontal factorial moments are dominated by the statistical fluctuations and are therefore ineffective in quantifying the event structure. In that case the moments F_q are not suitable as measures of dynamical fluctuations for erraticity analysis.

The origin of the problem can easily be seen by the following argument. If the event multiplicity N is low and the number of bins M is high, then the average bin multiplicity in an event, N/M, is $\ll 1$. Thus only by a large fluctuation can a bin have $n \geqslant q$ so as to contribute to F_q. Since Eq. (1) does not depend on the locations of the few bins that contribute, F_q does not describe the spatial pattern of an event very well. This problem does not arise if N/M is large, as shown quantitatively in [13].

The problem with low-multiplicity events must be treated in a very different way that is almost orthogonal to the use of the factorial moments. Such a method has already been found [14]; however, since it is totally outside the scope of this paper, it will not be presented here. The method discussed in this paper is, nevertheless, effective in analyzing the dynamical fluctuations in high-multiplicity events, whether in hadronic collisions at very high energies or nuclear collisions at lower energies.

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