

## Mechanism for a next-to-lowest lying scalar meson nonet

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Recent work suggests the existence of a nonconventional lowest-lying scalar nonet containing the  $a_0(980)$ . Then the  $a_0(1450)$  and also the  $K_0^*(1430)$  are likely candidates to belong to a conventional  $p$ -wave  $q\bar{q}$  nonet. However, a comparison of their properties with those expected on this basis reveals a number of puzzling features. It is pointed out that these puzzles can be resolved in a natural and robust way by assuming a ‘‘bare’’ conventional  $p$ -wave scalar  $q\bar{q}$  nonet to mix with a lighter four-quark  $qq\bar{q}\bar{q}$  scalar nonet to form new ‘‘physical’’ states. The essential mechanism is driven by the fact that the isospinor is lighter than the isovector in the unmixed  $qq\bar{q}\bar{q}$  multiplet.

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### I. INTRODUCTION

The identification and interpretation of the low-lying scalar mesons are questions of great current interest. A variety of approaches and models have been explored [1–22]. In the effective chiral Lagrangian approach from which this paper is motivated, a light isoscalar  $\sigma(560)$  in addition to the known light isoscalar  $f_0(980)$  are needed [1] to produce a  $\pi\pi$  scattering amplitude which agrees with experiment. Similarly, a light strange  $\kappa(900)$  state is needed [2] to understand the experimental  $\pi K$  amplitude. These three particles were postulated [3] to form a nonet, taken together with the known isovector  $a_0(980)$ . Consistency of this picture with the properties of the  $a_0(980)$  as seen in  $\eta' \rightarrow \eta\pi\pi$  decay [4] and as required in  $\pi\eta$  scattering [5] was checked. The pattern of masses, coupling constants, and especially the isoscalar mixing angle was observed [3] to be much closer to the one expected from a four-quark ( $qq\bar{q}\bar{q}$ ) picture rather than from the conventional two-quark ( $q\bar{q}$ ) picture for this scalar nonet. The four-quark picture was proposed by Jaffe [23] in the framework of the MIT bag model. Very recent experiments [24] on the radiative decays  $\phi \rightarrow \pi\eta\gamma$  and  $\phi \rightarrow \pi\pi\gamma$  have been interpreted [25,26] as evidence in favor of the four-quark picture of the low-lying scalars  $a_0(980)$  and  $f_0(980)$ .

Now if one adopts the above picture or, as a matter of fact, any other picture in which an unconventional non- $q\bar{q}$  nonet made of the  $\sigma(560)$ ,  $\kappa(900)$ ,  $a_0(980)$ , and  $f_0(980)$  exists, there is an interesting puzzle concerning the conventional  $q\bar{q}$  scalar nonet. Such a nonet has an interpretation in the constituent quark model as a  $p$ -wave excitation and should, therefore, share many characteristics of the other  $p$ -wave states (the tensor nonet and two axial vector nonets with different charge conjugation properties). To see the puzzling features let us focus attention on the experimental scalar candidates with nontrivial isospin quantum numbers in the greater than 1 GeV energy range. These are the isovector  $a_0(1450)$  and the strange isospinor  $K_0^*(1430)$ . According to

the Particle Data Group survey [27] (see Table 13.2 on p. 110), they are the likely candidates for a  $q\bar{q}$  scalar nonet. Then one has the following unusual features.

(i) The mass of the  $a_0(1450)$  (presumably  $a_0^+ \sim u\bar{d}$ ) is listed as  $1474 \pm 19$  MeV, about 50 MeV heavier than the strange  $K_0^*(1430)$  (presumably  $K_0^{*+} \sim us$ ), which has a listed mass of  $1429 \pm 6$  MeV. Our normal expectation is that the replacement of the  $\bar{d}$  quark in a  $ud$  composite by an  $\bar{s}$  quark should make the resulting state heavier rather than lighter.

(ii) Upon comparison with the corresponding members of the  $p$ -wave  $J^{PC} = 2^{++}$  nonet, we see that the  $q\bar{q}$  scalar meson candidates are not lighter; specifically  $m[a_0(1474 \pm 19)] > m[a_2(1318.1 \pm 0.7)]$  and  $m[K_0^*(1429 \pm 6)] \approx m[K_2^*(1432.3 \pm 1.3)]$ . Usually it is expected in the constituent quark model that  $\mathbf{L} \cdot \mathbf{S}$  forces should make the spin-0 particle lighter than the corresponding spin-2 particle. This is experimentally evident in the (perhaps too simple)  $c\bar{c}$  system where  $m[\chi_{c2}(1P)] = 3556.17 \pm 0.13$  MeV and  $m[\chi_{c0}(1P)] = 3415.1 \pm 0.1$  MeV.

(iii) If  $a_0(1450)$  and  $K_0^*(1430)$  belong to a conventional nonet their decay widths into pseudoscalars should be related. Now, only decay modes into two pseudoscalars have been observed for these particles:  $K_0^*(1430) \rightarrow \pi K$  and  $a_0(1450) \rightarrow \pi\eta$ ,  $K\bar{K}$  and  $\pi\eta'$ . As we will see later, SU(3) symmetry predicts

$$\Gamma[a_0(1450)] = 1.51\Gamma[K_0^*(1430)], \quad (1.1)$$

assuming that the total widths are saturated by the decay modes mentioned. On the other hand, the experimental result is

$$\Gamma[a_0(1450)] = (0.92 \pm 0.12)\Gamma[K_0^*(1430)], \quad (1.2)$$

which clearly differs from the SU(3) prediction.

In this note we will show that there exists a model which naturally provides a solution to these three problems. This model simply consists of allowing the  $q\bar{q}$  nonet to mix with a lighter  $qq\bar{q}\bar{q}$  nonet. Notice that the isovector in the lighter nonet has a structure  $u\bar{d}s\bar{s}$ , with two strange quarks. On the other hand, the isospinor in the lighter nonet has a structure  $u\bar{s}d\bar{d}$ , with only one strange quark. Thus, before mixing the

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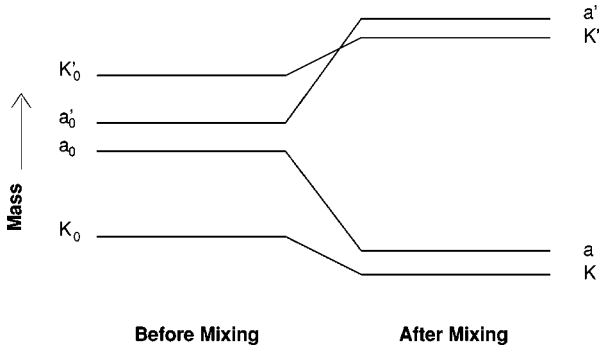


FIG. 1. Mixing of two nonets.  $a'$ ,  $K'$ ,  $a$ , and  $K$  stand, respectively, for the “physical” states  $a_0(1450)$ ,  $K_0^*(1430)$ ,  $a_0(980)$ , and  $\kappa(900)$ .  $K_0$  and  $a_0$  are the unmixed isospinor and isovector  $qq\bar{q}\bar{q}$  states, while  $K'_0$  and  $a'_0$  are the corresponding unmixed  $q\bar{q}$  states.

lighter nonet will have the isovector heavier than the strange isospinor. This situation is illustrated in Fig. 1, where the notation is explained. Details will be given later, but we can easily see how the scheme works. The two isovectors mix with each other as do the two isospinors. Since the mixing of the two levels repels them, this explains point (ii), which is why the  $q\bar{q}$  scalars appear heavier than expected. Similarly the  $qq\bar{q}\bar{q}$  scalars are pushed down in mass. Point (i), the level crossing of the  $q\bar{q}$  isovector and isospinor, can be simply understood in the perturbation theory approximation: since the  $a_0 - a'_0$  splitting is smaller than the  $K_0 - K'_0$  splitting the “energy denominator” for the isovector mixing will be smaller than the one for the isospinor mixing. Hence the isovectors will be more strongly repelled. We must assume that the  $a_0 - K_0$  splitting is large enough so that there is no level crossing for the lower mass scalars. Finally, point (iii), the difference in coupling constants of the  $K_0^*(1430)$  and the  $a_0(1450)$ , can be readily understood from the greater “contamination” of the  $a_0(1450)$  wave function with the four-quark isovector state.

In our present work we do not discuss in detail the isoscalars of the scalar  $q\bar{q}$  nonet. The reason is that the experimental situation is rather fluid at the moment, with many candidates. These include the  $f_0(1370)$  (which may actually correspond to two different states), the  $f_0(1500)$  (which may be a glueball state) and the  $f_J(1710)$ . The present scheme suggests a fivefold mixing between the  $\sigma(560)$ , the  $f_0(980)$ , two heavier  $q\bar{q}$  isoscalar scalars, and a glueball.

## II. MIXING FORMALISM AND MASS SPECTRA

Our interest is in investigating the mass spectra and later the decay properties of scalar mesons which are a mixture of “conventional”  $q\bar{q}$   $p$ -wave states and  $qq\bar{q}\bar{q}$  states.

In the quark model the usual  $q\bar{q}$  type scalars are grouped into a nonet, say  $N'$ , with

$$N'^b_a \sim q_a \bar{q}^b, \quad (2.1)$$

where  $a$  and  $b$  are flavor indices and  $q_1, q_2, q_3 = u, d, s$ . So, for example,  $N_3^{\prime 3}$  contains one strange quark and one anti-

strange quark,  $N_3^{\prime 1}$  and  $N_3^{\prime 2}$  contain one strange quark and one light antiquark, while  $N_1^{\prime 1}$  and  $N_2^{\prime 2}$  have no strange quarks. One can also construct “multiquark” hadrons, an idea originally discussed by Jaffe [23]. In this paper we focus on a scalar flavor SU(3) nonet  $N$  of color SU(3) singlet states with quark composition  $qq\bar{q}\bar{q}$ . Within the context of the MIT bag model, Jaffe showed moreover that the lightest such scalar nonet  $N$  should have a mass less than or in the vicinity of 1 GeV due to the strong binding energy of the  $qq\bar{q}\bar{q}$  configuration arising from hyperfine interactions between the quarks. The four-quark scalar nonet  $N$ , which transforms in an identical manner to  $N'$  under flavor SU(3), can naturally be decomposed (this discussion is a summary of Sec. II of [3]) in terms of “dual” flavor quarks (actually diquarks):

$$N^b_a \sim T_a \bar{T}^b, \quad (2.2)$$

where

$$T_a = \epsilon_{abc} \bar{q}^b \bar{q}^c, \quad \bar{T}^a = \epsilon^{abc} q_b q_c. \quad (2.3)$$

So  $N_3^3$  contains no strange quarks,  $N_1^3$  and  $N_2^3$  contain one strange antiquark each, while  $N_1^1$ ,  $N_2^2$ , and  $N_1^2$  contain two strange constituents each. As explained in the Introduction we are not including the experimentally ambiguous isoscalars in our present discussion and so the pure  $q\bar{q}$  states in  $N'$  of interest are the isovector and isospinor; their charged components are (using the notation of Fig. 1):

$$a_0^+ \sim u\bar{d}, \quad K_0^+ \sim u\bar{s}, \quad (2.4)$$

and the corresponding members of the  $qq\bar{q}\bar{q}$  nonet  $N$  are

$$a_0^+ \sim us\bar{d}\bar{s}, \quad K_0^+ \sim ud\bar{s}\bar{d}. \quad (2.5)$$

By simply considering the strange quark content of these states, and also bearing in mind that the  $q\bar{q}$  nonet  $N'$  presumably lies in the same mass range as the  $p$ -wave axial and tensor meson nonets whereas the bag-model indication is that the  $qq\bar{q}\bar{q}$  nonet  $N$  should be less than about 1 GeV, we expect an ordering of the masses of these states

$$m_{K_0} < m_{a_0} \leq m_{a_0'} < m_{K_0'}, \quad (2.6)$$

as illustrated in Fig. 1. Suppose initially that the scalar meson nonet  $N'$  is “ideally mixed” according to the classic idea of Okubo [28], applied originally to the lightest vector mesons. “Ideal mixing” within the nonet may be defined by the following mass terms of an effective Lagrangian density for the  $q\bar{q}$  scalars:

$$\mathcal{L}'_{\text{mass}} = -a' \text{Tr}(N'N') - b' \text{Tr}(N'N'\mathcal{M}). \quad (2.7)$$

In fact as discussed in [3] we may define a generalized ideal mixing model for the  $qq\bar{q}\bar{q}$  nonet  $N$  by the mass terms:

$$\mathcal{L}_{\text{mass}} = -a \text{Tr}(NN) - b \text{Tr}(NN\mathcal{M}). \quad (2.8)$$

Here  $\mathcal{M}$  is the “spurion matrix” [ $\mathcal{M} = \text{diag}(1, 1, x)$ ] where  $x$  is the ratio of strange to nonstrange quark masses in the usual

interpretation]. It is worth remarking that although Eqs. (2.7) and (2.8) are similar in appearance, the difference at the quark level between  $N$  and  $N'$  manifests itself through opposite signs of  $b$  and  $b'$ . This can be seen by noting that<sup>1</sup>

$$\begin{aligned} m_{K_0}^2 - m_{a_0}^2 &= (x-1)b, \\ m_{K'_0}^2 - m_{a'_0}^2 &= (x-1)b', \end{aligned} \quad (2.9)$$

where the numerical value of  $x$  is around 20.5 [29]. Here  $b'$  is taken positive, while  $b$  is taken negative; this agrees with counting the number of constituent strange quarks.

To see whether a mixing between the nonets  $N'$  and  $N$  can give states whose properties reproduce those of the experimental scalar isovector and isospinor candidates above 1 GeV, we consider the simplest invariant term which will induce mixing between  $N$  and  $N'$ , namely

$$\mathcal{L}'_{\text{mass}} = -\gamma \text{Tr}(NN'). \quad (2.10)$$

For the isovectors and isospinors we have the  $2 \times 2$  mixing matrices

$$M_a^2 = \begin{bmatrix} m_{a_0}^2 & \gamma \\ \gamma & m_{a'_0}^2 \end{bmatrix} \quad \text{and} \quad M_K^2 = \begin{bmatrix} m_{K_0}^2 & \gamma \\ \gamma & m_{K'_0}^2 \end{bmatrix}. \quad (2.11)$$

For orientation, we first consider the mixing from the point of view of simple perturbation theory applied to two two-state systems. At second order in perturbation theory we see that the shifts in the square masses for the  $a_0$ - $a'_0$  and  $K_0$ - $K'_0$  systems have magnitudes

$$\Delta_a = \frac{\gamma^2}{m_{a'_0}^2 - m_{a_0}^2} \quad \text{and} \quad \Delta_K = \frac{\gamma^2}{m_{K'_0}^2 - m_{K_0}^2}, \quad (2.12)$$

respectively. Clearly the ordering of the masses in Eq. (2.6) implies that  $\Delta_a > \Delta_K$  and so if the gap between  $m_{a_0}$  and  $m_{K_0}$  is sufficiently large relative to the  $a'_0$ - $K'_0$  mass difference we will naturally be able to obtain the level-crossing behavior of Fig. 1.

Next we proceed to an exact treatment. Invariance of the trace of  $M_a^2$  upon diagonalization implies that

$$m_{a_0}^2 + m_{a'_0}^2 = m_a^2 + m_{a'}^2, \quad (2.13)$$

with an analogous equation holding for the isospinors. Using this condition and the eigenvalue equation

<sup>1</sup>Squared masses are being used since we are working in an effective Lagrangian framework. Consequently the relativistic invariance of our treatment is manifest. A similar mechanism would be expected to hold in a nonrelativistic treatment where the masses are not squared.

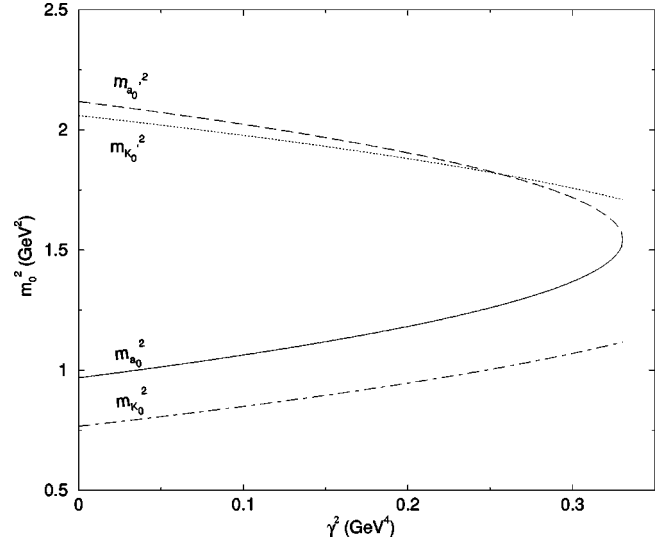


FIG. 2. Evolution, as a function of  $\gamma^2$ , of the bare masses needed to produce the physical ones. Of course, the bare and physical masses coincide for  $\gamma^2=0$ . This picture corresponds to the case  $m_{K'}^2 - m_K^2 > m_{a'}^2 - m_a^2$ , which holds for the central “experimental” values. Here the plot is shown for the physical masses chosen at the end of Sec. II. The dot-dashed curve is  $m_{K_0}^2$ , the solid curve is  $m_a^2$ , the dotted curve is  $m_{K'}^2$ , and the dashed curve is  $m_{a'}^2$ .

$$(m_{a_0}^2 - m_a^2)(m_{a'_0}^2 - m_a^2) - \gamma^2 = 0, \quad (2.14)$$

we solve for the masses of the original unmixed states to obtain

$$m_{a_0/a'_0}^2 = \frac{1}{2} [m_a^2 + m_{a'}^2 \mp \sqrt{(m_a^2 - m_{a'}^2)^2 - 4\gamma^2}]. \quad (2.15)$$

Analogous equations follow from the diagonalization of  $M_K^2$ . These equations may be read as giving for each value  $\gamma^2$  the corresponding masses of the unmixed states which will, upon inclusion of  $\mathcal{L}'_{\text{mass}}$ , lead to the experimentally known physical masses. Reality of the masses implies that  $4\gamma^2 \leq \text{Min}\{(m_a^2 - m_{a'}^2)^2, (m_{K'}^2 - m_K^2)^2\}$ .

We are interested in a scenario where the ordering of the unmixed masses is as in Eq. (2.6). We find that this can happen provided that  $m_{a'}^2 - m_a^2 < m_{K'}^2 - m_K^2$  (which holds for most of the experimentally allowed range of masses) because in this case the behavior of the bare masses is as shown in Fig. 2. Of course the bare and physical masses must coincide for  $\gamma=0$ . We define  $\gamma_{\text{max}}^2 = 1/4(m_{a'}^2 - m_a^2)^2$ ; the value of  $\gamma_{\text{max}}^2$  depends (since  $m_a$  is very accurately known) on the exact value of  $m_{a'}$ . For  $\gamma = \gamma_{\text{max}}$  the  $I=1$  states are maximally mixed and the unmixed states are degenerate with square masses equal to  $1/2(m_a^2 + m_{a'}^2)$ . We see from Fig. 2 that the choice  $\gamma = \gamma_{\text{max}}$  is expected to result in the largest splitting of the bare  $q\bar{q}$  masses  $m_{K'_0}^2 - m_{a'_0}^2$ .

A detailed numerical search shows that the largest value of this splitting is in fact obtained for  $\gamma^2 = \gamma_{\text{max}}^2 = 0.33 \text{ GeV}^4$  and with the choice of physical masses (within the allowed “experimental” range)

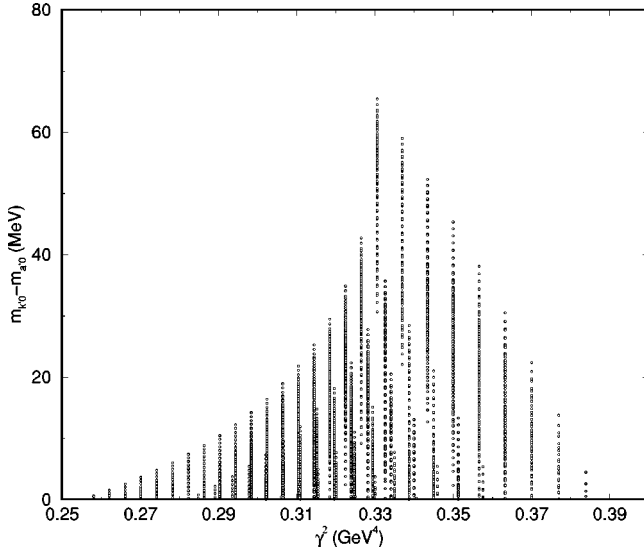


FIG. 3. Scan showing the values of the bare mass splitting  $m_{K'_0} - m_{a'_0}$  resulting from different experimentally allowed masses of  $a$ ,  $a'$ ,  $K$ ,  $K'$ , and of  $\gamma^2$ . The mechanism gives the correct ordering for the approximate range  $0.26 < \gamma^2 < 0.38 \text{ GeV}^4$ .

$$\begin{aligned} m_a &= 0.9835 \text{ GeV}, & m_{a'} &= 1.455 \text{ GeV}, \\ m_K &= 0.8750 \text{ GeV}, & m_{K'} &= 1.435 \text{ GeV}. \end{aligned} \quad (2.16)$$

Figure 3 shows all the allowed points and their corresponding mass splittings. Notice that  $m_K$  is obtained from the analysis of  $\pi K$  scattering given in [2]. This then yields the following masses for the unmixed states:

$$\begin{aligned} m_{a_0} &= m_{a'_0} = 1.24 \text{ GeV}, \\ m_{K_0} &= 1.06 \text{ GeV}, & m_{K'_0} &= 1.31 \text{ GeV}. \end{aligned} \quad (2.17)$$

We see that  $m_{K'_0} - m_{a'_0} \approx 65 \text{ MeV}$  which is comparable with the analogous splitting of the tensor and axial families of order 100 MeV. We also notice that in addition to satisfying the ordering in Eq. (2.6) [which can be an explanation for puzzle (i)], we can understand puzzle (ii) in this picture since the unmixed  $q\bar{q}$  scalar states are lighter than the analogous tensors. Specifically, we have that  $m[a'_0] < m[a_2(1318.1)]$  and  $m[K'_0] < m[K_2^*(1432.3 \pm 1.3)]$ .

### III. DECAY WIDTHS

In this section we address the third puzzle presented in the introduction. The total widths of the  $a_0(1450)$  and the  $K_0^*(1430)$  are listed in the Review of Particle Physics [27] as

$$\begin{aligned} \Gamma^{\text{tot}}[K_0^*(1430)] &= 287 \pm 23 \text{ MeV} \quad \text{and} \\ \Gamma^{\text{tot}}[a_0(1450)] &= 265 \pm 13 \text{ MeV}, \end{aligned} \quad (3.1)$$

which implies the ratio in Eq. (1.2). The only listed decay mode of the  $K_0^*(1430)$  is  $\pi K$  with a branching fraction of  $(93 \pm 10) \%$  which is close to 100%. On the other hand, for

the  $a_0(1450)$ , the experimental knowledge of the exclusive decay modes is less certain; the  $\pi\eta$ ,  $K\bar{K}$  and  $\pi\eta'$  modes are listed as ‘‘seen’’ without stating any branching fractions. In the detailed listings the following ratios are presented:

$$\begin{aligned} \frac{\Gamma[a_0(1450) \rightarrow K\bar{K}]}{\Gamma[a_0(1450) \rightarrow \pi\eta]} &= 0.88 \pm 0.23 \quad \text{and} \\ \frac{\Gamma[a_0(1450) \rightarrow \pi\eta']}{\Gamma[a_0(1450) \rightarrow \pi\eta]} &= 0.35 \pm 0.16. \end{aligned} \quad (3.2)$$

In this section we also denote the physical state  $a' = a_0(1450)$  by  $a_0^*$  and the physical state  $K' = K_0^*(1430)$  by  $K_0^*$ . Despite the uncertainty, for the purpose of our analysis we shall assume that the  $\pi\eta$ ,  $K\bar{K}$  and  $\pi\eta'$  modes saturate the  $a_0(1450)$  decays and that their ratios expressed above hold as stated.

Using isotopic spin invariance the scalar-pseudoscalar-pseudoscalar trilinear interaction terms relevant for these decay channels can be written as<sup>2</sup>

$$\begin{aligned} -\mathcal{L} &= \frac{\gamma_{a_0^* K K}}{\sqrt{2}} \partial_\mu \bar{K} \boldsymbol{\tau} \cdot \mathbf{a}_0^* \partial_\mu K + \gamma_{a_0^* \pi \eta} \mathbf{a}_0^* \cdot \partial_\mu \boldsymbol{\pi} \partial_\mu \eta \\ &+ \gamma_{a_0^* \pi \eta'} \mathbf{a}_0^* \cdot \partial_\mu \boldsymbol{\pi} \partial_\mu \eta' \\ &+ \frac{\gamma_{K_0^* K \pi}}{\sqrt{2}} (\partial_\mu \bar{K} \boldsymbol{\tau} \cdot \partial_\mu \boldsymbol{\pi} K_0^* + \text{H.c.}). \end{aligned} \quad (3.3)$$

Hence, the perturbative decay width of the  $K_0^*(1430)$  is

$$\Gamma(K_0^* \rightarrow \pi K) = \frac{3}{2} \frac{\gamma_{K_0^* K \pi}}{32\pi} \frac{q}{m_{K_0^*}^2} (m_{K_0^*}^2 - m_\pi^2 - m_K^2)^2, \quad (3.4)$$

where  $q$  is the momentum of the decay products in the  $k_0^*$  rest frame. Analogous expressions follow for the  $a_0(1450)$  partial widths. Thus we have that

$$\begin{aligned} \Gamma(a_0^* \rightarrow \pi\eta) &= 0.0099 \gamma_{a_0^* \pi \eta}^2, \\ \Gamma(a_0^* \rightarrow \pi\eta') &= 0.0028 \gamma_{a_0^* \pi \eta'}^2, \\ \Gamma(a_0^* \rightarrow K\bar{K}) &= 0.0070 \gamma_{a_0^* K \bar{K}}^2, \\ \Gamma(K_0^* \rightarrow \pi K) &= 0.0143 \gamma_{K_0^* \pi K}^2. \end{aligned} \quad (3.5)$$

Let us initially suppose that the  $a_0(1450)$  and  $K_0^*(1430)$  are members of a hypothetical unmixed scalar  $q\bar{q}$  nonet  $N'$ , i.e.,  $\gamma = 0$ . Then their decays into two pseudoscalars are presumably described by the interaction

<sup>2</sup>Derivative coupling is being used because we want our Lagrangian to be a piece of a chiral invariant object. See Appendix B of [3].



$$\mathcal{L}_{N'\phi\phi} = 2A' \text{Tr}(N' \partial_\mu \phi \partial_\mu \phi), \quad (3.6)$$

where  $\phi_a^b$  is the matrix of pseudoscalar fields. This pure  $q\bar{q}$  coupling term, when expanded into individual fields and compared with Eq. (3.3) above (where the coupling constant conventions are defined), leads to the identifications:

$$\begin{aligned} \gamma_{a_0^* \pi \eta} &= -2\sqrt{2} \cos \theta_p A', & \gamma_{a_0^* \pi \eta'} &= -2\sqrt{2} \sin \theta_p A', \\ \gamma_{a_0^* K \bar{K}} &= \gamma_{K_0^* \pi K} = -2A'. \end{aligned} \quad (3.7)$$

where  $\theta_p$  is the pseudoscalar mixing angle, which we take to be  $37^\circ$  [29]. Now if we substitute into Eq. (3.5) we find the  $q\bar{q}$  SU(3) predictions for the ratios of the total widths:

$$\frac{\Gamma^{\text{tot}}(a_0^*)}{\Gamma(K_0^* \rightarrow \pi K)} = 1.51, \quad (3.8)$$

and for the partial  $a_0(1450)$  widths:

$$\frac{\Gamma(a_0^* \rightarrow K \bar{K})}{\Gamma(a_0^* \rightarrow \pi \eta)} = 0.55, \quad \frac{\Gamma(a_0^* \rightarrow \pi \eta')}{\Gamma(a_0^* \rightarrow \pi \eta)} = 0.16. \quad (3.9)$$

We see that while Eqs. (3.9) are just a little below the experimentally allowed ratios (3.2), the ratio (3.8) is not consistent with the experimental ratio which follows from Eq. (3.1). Thus considering the  $a_0(1450)$  and  $K_0^*(1430)$  as members of a pure  $q\bar{q}$  SU(3) nonet does not give good agreement with experiment.

Next we study the predictions for the decay widths of the  $a_0(1450)$  and the  $K_0^*(1430)$  in the mixing picture of Sec. II. In [3] we discussed the general SU(3) flavor invariant coupling of members of a scalar nonet to two pseudoscalars. For the case of the  $q\bar{q}$  scalar nonet  $N'$  the most standard form is as taken in Eq. (3.6) above. However, for the four-quark nonet,  $N$ , a more natural structure which to a first approximation reproduces the scalar decay pattern is

$$\mathcal{L}_{N\phi\phi} = A \epsilon^{abc} \epsilon_{def} N_a^d \partial_\mu \phi_b^e \partial_\mu \phi_c^f. \quad (3.10)$$

For  $qq\bar{q}\bar{q}$  mesons it seems reasonable that the dominant decays will simply be ones that involve a ‘‘falling apart’’ [23], or rearrangement, of the four quarks into two  $q\bar{q}$  mesons. So, for example, since  $N_3^3$  contains no strange quarks one would expect its decay into  $K\bar{K}$  to be suppressed. Indeed the Lagrangian (3.10) predicts zero coupling of  $N_3^3$  into  $K\bar{K}$ .

Upon diagonalization of Eq. (2.11) the physical isospinors are  $K = \kappa(900)$  and  $K' = K_0^*(1430)$  and we take the mixing convention:

$$\begin{pmatrix} \kappa(900) \\ K_0^*(1430) \end{pmatrix} = \begin{pmatrix} \cos \psi_K & -\sin \psi_K \\ \sin \psi_K & \cos \psi_K \end{pmatrix} \begin{pmatrix} K_0 \\ K_0' \end{pmatrix}. \quad (3.11)$$

Likewise the isovector mass eigenstates are  $a = a_0(980)$  and  $a' = a_0(1450)$  given by

$$\begin{pmatrix} a_0(980) \\ a_0(1450) \end{pmatrix} = \begin{pmatrix} \cos \psi_a & -\sin \psi_a \\ \sin \psi_a & \cos \psi_a \end{pmatrix} \begin{pmatrix} a_0 \\ a_0' \end{pmatrix}, \quad (3.12)$$

where the mixing angles are obtained as

$$\tan(2\psi_K) = \frac{2\gamma}{m_{K_0'}^2 - m_{K_0}^2} \quad \text{and} \quad \tan(2\psi_a) = \frac{2\gamma}{m_{a_0'}^2 - m_{a_0}^2}. \quad (3.13)$$

Now if we take the total trilinear interaction Lagrangian density to be the sum of Eqs. (3.6) and (3.10) and expand the relevant unmixed isovector and isospinor members of  $N$  and  $N'$  in terms of the physical fields using the mixing convention above, we find that [see Eq. (3.3) and compare with the unmixed case (3.7)]

$$\begin{aligned} \gamma_{a_0^* \pi \eta} &= -2(\sin \psi_a \sin \theta_p A + \sqrt{2} \cos \theta_p \cos \psi_a A'), \\ \gamma_{a_0^* \pi \eta'} &= 2(\sin \psi_a \cos \theta_p A - \sqrt{2} \sin \theta_p \cos \psi_a A'), \\ \gamma_{a_0^* K \bar{K}} &= -2(\sin \psi_a A + \cos \psi_a A'), \\ \gamma_{K_0^* \pi K} &= -2(\sin \psi_K A + \cos \psi_K A'). \end{aligned} \quad (3.14)$$

Again we calculate the widths using Eqs. (3.5) and compare their ratios with experiment. It turns out to be helpful to begin by analyzing these ratios in different regions.

First we consider the limit where  $|A'/A|$  is large. In this region,

$$\frac{\Gamma(K_0^* \rightarrow \pi K)}{\Gamma(a_0^* \rightarrow \pi \eta)} \sim \left[ 1.444 \frac{1}{2 \cos^2 \theta_p} \right] \left[ \frac{\cos \psi_K}{\cos \psi_a} \right]^2. \quad (3.15)$$

The term in the first bracket is what we obtained above from the couplings in Eq. (3.7) and so gives the prediction (3.8) which is smaller than experiment. Recalling Eq. (3.13) we see that for mixing angles  $\psi_a, \psi_K \in [0, \pi/4]$  (which is all that is needed in this limit since the relative sign introduced by considering  $\psi_a, \psi_K \in [-\pi/4, 0]$ , and so for the opposite sign of  $\gamma$  may be absorbed in a change of the relative sign of  $A$  and  $A'$ ), we will always have that  $\psi_a > \psi_K$  since we are considering the scheme where the ordering of the masses is as in Eq. (2.6). Hence, the ratio (3.15) always *increases* relative to the  $q\bar{q}$  prediction in this limit and so the ratio of the total widths (3.8) will *decrease*, as required. This behavior is independent of the choice of experimental masses as long as they satisfy  $m_{K'}^2 - m_K^2 > m_{a'}^2 - m_a^2$ , which is true whenever the mechanism works in order to produce the correct level-crossing behavior for the masses. For example, for the illustrative input masses considered at the end of Sec. II we have that  $\psi_a = \pi/4$  and  $\psi_K \approx 31^\circ$  which implies that

$$\frac{\Gamma(a_0^* \rightarrow \pi \eta)}{\Gamma(K_0^* \rightarrow \pi K)} \approx 0.606 \quad (3.16)$$

giving that

$$\frac{\Gamma^{\text{tot}}(a_0^*)}{\Gamma(K_0^* \rightarrow \pi K)} \approx 1.036. \quad (3.17)$$

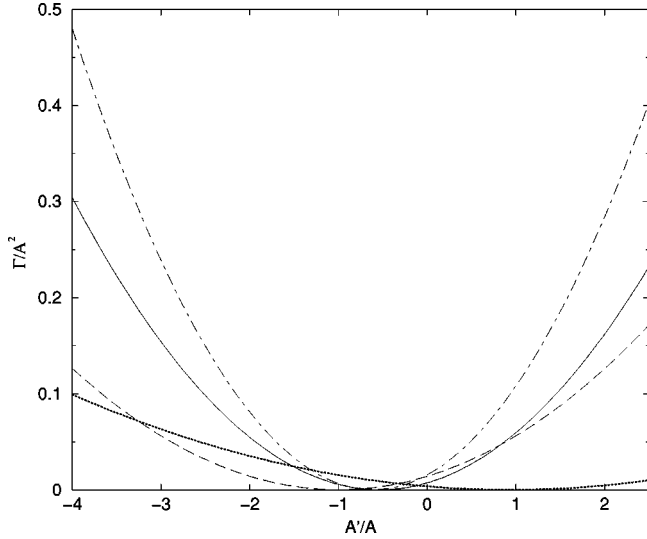


FIG. 4. Plot of  $\Gamma/A^2$  against  $A'/A$ . The dot-dashed curve is  $\Gamma[K_0^*(1430) \rightarrow \pi K]$ , the solid curve is  $\Gamma[a_0(1450) \rightarrow \pi \eta]$ , the dashed curve is  $\Gamma[a_0(1450) \rightarrow K \bar{K}]$  and the dotted curve is  $\Gamma[a_0(1450) \rightarrow \pi \eta']$ .

Within the errors quoted in [27] this agrees with the experimental result (3.1) and is much closer than the  $q\bar{q}$  prediction of Eq. (3.8). Furthermore, from Eq. (3.14) for large  $|A'/A|$  we have the same prediction as in Eq. (3.9).

Next we suppose conversely that  $|A/A'|$  is large. In this region,

$$\frac{\Gamma(K_0^* \rightarrow \pi K)}{\Gamma(a_0^* \rightarrow \pi \eta)} \sim \left[ 1.444 \frac{1}{\cos^2 \theta_p} \right] \left[ \frac{\sin \psi_K}{\sin \psi_a} \right]^2 \approx 2.115, \quad (3.18)$$

for  $\psi_K = 31^\circ$ . The ratios of the  $a_0(1450)$  widths now become

$$\frac{\Gamma(a_0^* \rightarrow K \bar{K})}{\Gamma(a_0^* \rightarrow \pi \eta)} \sim 0.7071 \frac{1}{\sin^2 \theta_p} = 1.95, \quad (3.19)$$

$$\frac{\Gamma(a_0^* \rightarrow \pi \eta')}{\Gamma(a_0^* \rightarrow \pi \eta)} = 0.2828 \cot^2 \theta_p = 0.49.$$

In this limit, where it is the  $qq\bar{q}\bar{q}$  decay modes of the  $a_0(1450)$  and the  $K_0^*(1430)$  that dominate, we see that in particular the first ratio in Eq. (3.19) is well outside the experimentally allowed range.

For  $A' \sim A$  a graphical analysis is helpful since in this region the ratios of the widths blow up. In Fig. 4 we plot the widths themselves (up to an overall normalization of  $1/A^2$ ). It is seen that, for the central values of Eqs. (3.1) and (3.2) the correct width order  $\Gamma(K_0^* \rightarrow \pi K) > \Gamma(a_0^* \rightarrow \pi \eta) > \Gamma(a_0^* \rightarrow K \bar{K}) > \Gamma(a_0^* \rightarrow \pi \eta')$  is obtained for the ‘‘asymptotic’’ regions  $A'/A > 1.2$  and  $A'/A < -3.2$ . Inside, where  $-3.2 < A'/A < 1.2$ , the correct width order cannot be obtained for the central values.

In summary, the above analysis shows that for large  $|A'/A|$  the mechanism significantly improves [see, for example, Eq. (3.17)] the ratio  $\Gamma^{\text{tot}}(a_0^*)/\Gamma(K_0^* \rightarrow \pi K)$  compared

with the prediction based on a pure  $q\bar{q}$  description of the  $a_0(1450)$  and the  $K_0^*(1430)$ . Outside of this asymptotic region more detailed analysis is needed and requires additional experimental guidance.

#### IV. NUMERICAL ANALYSIS OF THE DECAY WIDTHS

In this section we give a more detailed numerical analysis of the decay widths. We will take into account the experimental uncertainties for comparison with theory. Furthermore we will include a more general form of decay interaction. Finally, the decay widths of the lighter scalars  $K = \kappa(900)$  and  $a = a_0(980)$  will also be discussed. The input masses will be kept the same as in Eq. (2.16), and we will continue to use  $\gamma^2 = \gamma_{\text{max}}^2 = 0.33 \text{ GeV}^4$ . The general interaction Lagrangian describing the decay widths has the form (see [3])

$$\begin{aligned} \mathcal{L}_{\text{int}} = & A \epsilon^{abc} \epsilon_{def} N_a^d \partial_\mu \phi_b^3 \partial_\mu \phi_c^f + C \text{Tr}(N \partial_\mu \phi) \text{Tr}(\partial_\mu \phi) \\ & + A' \epsilon^{abc} \epsilon_{def} N_a^d \partial_\mu \phi_b^e \partial_\mu \phi_c^f + C' \text{Tr}(N' \partial_\mu \phi) \text{Tr}(\partial_\mu \phi) \\ & + \dots, \end{aligned} \quad (4.1)$$

where the three dots stand for terms which do not contribute to isovector or isospinor decays.

We first consider the limit  $C=0$  and  $C'=2A'$ , as in Sec. III. In this limit, the above Lagrangian simplifies<sup>3</sup> to

$$\mathcal{L}_{\text{int}} = A \epsilon^{abc} \epsilon_{def} N_a^d \partial_\mu \phi_b^e \partial_\mu \phi_c^f + 2A' \text{Tr}(N' \partial_\mu \phi \partial_\mu \phi) + \dots \quad (4.2)$$

We scan the  $AA'$  parameter space numerically and search for regions consistent with the available experimental data on the decay widths of these scalars. We start with the  $a_0(1450)$  decay widths as they impose the strongest restrictions on the parameter space. First, we find that the experimental estimate  $\Gamma^{\text{tot}}[a_0(1450)] = 265 \pm 13 \text{ MeV}$  restricts  $A$  and  $A'$  to the perimeter of the ellipse shown in Fig. 5—the thickness of the perimeter is related to the 13 MeV uncertainty of the decay width. We then search for regions that are consistent with the current experimental estimates on the ratios  $\Gamma[a_0(1450) \rightarrow K \bar{K}]/\Gamma[a_0(1450) \rightarrow \pi \eta] = 0.88 \pm 0.23$ , and  $\Gamma[a_0(1450) \rightarrow \pi \eta']/\Gamma[a_0(1450) \rightarrow \pi \eta] = 0.35 \pm 0.16$ . Regions consistent with the first and second ratios are, respectively, shown by dark and light shading. The vertical axis corresponds to the conventional interaction term for  $q\bar{q}$  nonets, whereas the horizontal axis represents a natural interaction for  $qq\bar{q}\bar{q}$  nonets, as previously discussed. We see in Fig. 5 that within our model we cannot exactly describe the current experimental data on the partial decay widths of  $a_0(1450)$ . Obviously a natural four-quark interaction is far from the allowed regions, while a natural two-quark interaction seems to be a

<sup>3</sup>It is helpful to use the identity:

$$\begin{aligned} A' \epsilon^{abc} \epsilon_{def} N_a^d \partial_\mu \phi_b^e \partial_\mu \phi_c^f = & 2A' \text{Tr}(N' \partial_\mu \phi \partial_\mu \phi) \\ & - A' \text{Tr}(N') \text{Tr}(\partial_\mu \phi \partial_\mu \phi) \\ & - 2A' \text{Tr}(N' \partial_\mu \phi) \text{Tr}(\partial_\mu \phi) \\ & + A' \text{Tr}(N') \text{Tr}(\partial_\mu \phi) \text{Tr}(\partial_\mu \phi). \end{aligned}$$

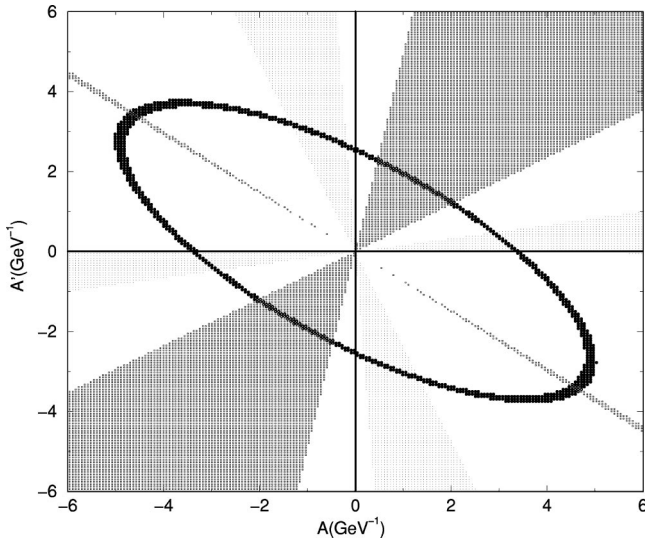


FIG. 5. Regions in the  $AA'$  parameter space [see Eq. (4.2)] consistent with the currently available experimental estimates on the decay widths of  $a_0(1450)$ . Points on the ellipse are consistent with the total decay width of  $a_0(1450)$ . Dark and light gray regions, respectively, represent points consistent with the experimental ratio  $\Gamma[a_0(1450) \rightarrow K\bar{K}]/\Gamma[a_0(1450) \rightarrow \pi\eta] = 0.88 \pm 0.23$ , and  $\Gamma[a_0(1450) \rightarrow \pi\eta']/\Gamma[a_0(1450) \rightarrow \pi\eta] = 0.35 \pm 0.16$ .

favorable scenario for the description of the available experimental data. Also small distortions from the natural two-quark interaction, although slightly improving the situation, do not exactly describe the data. This is described more quantitatively in Table I. We have fitted the prediction of our model for the total decay width as well as the decay ratios, to the above experimental estimates and searched for the best values of  $A$  and  $A'$ . The natural two-quark interaction (column one) is compared with the more general case that natural four-quark interactions are also allowed (column two). Although the  $\chi^2$  of the fit gets slightly reduced, effectively

the best point remains around the natural two-quark interaction.

In order to see whether we could get a better description of  $a_0(1450)$ , we have also extended our investigations to the more general case where  $C \neq 0$  and  $C' \neq 2A'$  i.e., working with the general Lagrangian (4.1). The result is given in column three of Table I, indicating that even with the introduction of more general interaction terms, the current experimental data is still not exactly described. We also notice that in this case  $C' - 2A' \approx 0$  and that  $|2A'|$  dominates  $A$  and  $C$ . Thus the best fit in this case also is similar to column I, although the fit is slightly improved. Therefore, the simplified model Eq. (4.2) already provides a reasonable picture for understanding the nature of the  $a_0(1450)$ . We should emphasize, however, that the available experimental estimates of the decay channels of  $a_0(1450)$  are not very accurately known. More accurate experimental data on  $a_0(1450)$  would be useful for our purposes.

Next, we include the  $K_0^*(1430)$  in the picture. We take experimental values [27]  $\Gamma^{\text{tot}} = 287 \pm 10 \pm 21$ , and  $\Gamma_{\pi K}/\Gamma^{\text{tot}} = 93 \pm 10\%$ , and search for regions that give  $\Gamma[K_0^*(1430) \rightarrow \pi K] \approx 267 \pm 50$  MeV. These are shown in Fig. 6 with two parallel strips in the northwest to southeast direction. We see in the figure that within our model (4.2) there are overlaps of regions in parameter space  $AA'$  that explain most of the decay properties of both  $a_0(1450)$  and  $K_0^*(1430)$ .

Now that we can understand the decay widths of the heavier scalars, we explore the possibility of explaining the decay widths of the light scalars within the same theoretical setup. We proceed by further exploring the parameter space  $AA'$  in the limit  $C = 0$  and  $C' = 2A'$ , for regions that explain decay properties of the lighter physical nonet members  $a_0(980)$  and  $\kappa(900)$ . We search for regions consistent with  $\Gamma[a_0(980) \rightarrow \pi\eta] \approx 65 \pm 5$  MeV in agreement with experimental measurement in [30] as well as the theoretical estimate in [4]. We also search for regions consistent with<sup>4</sup>

TABLE I. Best numerical values for the free parameters in the scalar pseudoscalar pseudoscalar interaction Lagrangian, found by fitting the prediction of our model for the total decay width and ratio of the partial decay widths of  $a_0(1450)$  to the experimental data. The first column corresponds to an interaction natural for  $q\bar{q}$ , while in the second column interaction terms natural for  $qq\bar{q}\bar{q}$  are also included. In the third column the more general interaction Eq. (4.1) is considered.

Fitted parameters	Eq. (3.6)	Eq. (4.2)	Eq. (4.1)
$A(\text{GeV}^{-1})$	0	$0.10 \pm 0.12$	$1.03 \pm 0.12$
$A'(\text{GeV}^{-1})$	$-2.55 \pm 0.06$	$-2.60 \pm 0.06$	$-3.53 \pm 0.12$
$C(\text{GeV}^{-1})$	0	0	$1.36 \pm 0.27$
$C'(\text{GeV}^{-1})$	$2A'$	$2A'$	$-6.56 \pm 0.27$
	Predicted decay widths		
$\Gamma^{\text{tot}}[a_0(1450)](\text{MeV})$	265	265	265
$\Gamma[a_0(1450) \rightarrow K\bar{K}]/\Gamma[a_0(1450) \rightarrow \pi\eta]$	0.55	0.53	0.53
$\Gamma[a_0(1450) \rightarrow \pi\eta']/\Gamma[a_0(1450) \rightarrow \pi\eta]$	0.16	0.18	0.18
$\chi^2$	1.161	1.157	1.157

<sup>4</sup>This is a width corresponding to the numerator, rather than denominator of a partial wave amplitude as explained in [2].



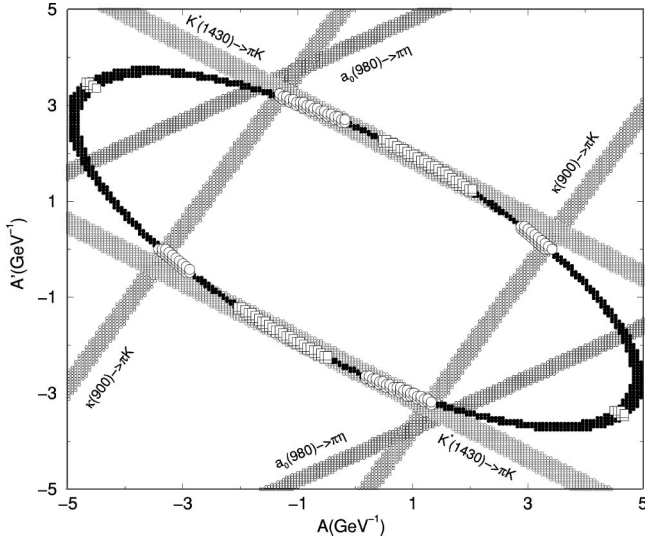


FIG. 6. Regions in the  $AA'$  parameter space [see Eq. (4.2)] consistent with the current experimental and theoretical estimates on the decay widths of  $a_0(1450)$ ,  $K_0^*(1430)$ ,  $a_0(980)$ , and  $\kappa(900)$ . Points on the ellipse are consistent with the total decay width of  $a_0(1450)$ . Squares and circles, respectively, represent points consistent with the experimental ratio  $\Gamma[a_0(1450) \rightarrow K\bar{K}]/\Gamma[a_0(1450) \rightarrow \pi\eta] = 0.88 \pm 0.23$ , and  $\Gamma[a_0(1450) \rightarrow \pi\eta']/\Gamma[a_0(1450) \rightarrow \pi\eta] = 0.35 \pm 0.16$ .

$\Gamma'[\kappa(900) \rightarrow \pi K] \approx 40 \pm 5$  MeV in agreement with theoretical estimates of the properties of  $\kappa(900)$  given in [3]. The result is also shown in Fig. 6, indicating that there are regions in the parameter space of our model ( $A \approx \pm 1$  and  $A' \approx \mp 3$ ) that are approximately consistent with the decay properties of the light scalars in addition to those of the heavy scalars.

We have given in Table II our best fits for  $A$ ,  $A'$ ,  $C$ , and  $C'$ , resulting from comparing our theoretical prediction to the experimental data. We have also displayed in the same table the predicted decay widths. In the limit  $C=0$  and  $C'=2A'$  (column one), the resulting decay widths have the right order of magnitude, although some of them are not within the ranges allowed by experiment. The fit gives  $A'/A = -2.4$  and so, as expected from the discussion of Fig. 4 the widths  $\Gamma[a_0(1450) \rightarrow K\bar{K}]$  and  $\Gamma[a_0(1450) \rightarrow \pi\eta]$  have the wrong order. Outside this limit (column two), we get a better agreement with experiment (as  $\chi^2$  of fit also indicates), and except for the ratio  $\Gamma[a_0(1450) \rightarrow K\bar{K}]/\Gamma[a_0(1450) \rightarrow \pi\eta]$ , all other decay widths are within their experimentally allowed ranges. We notice that in the general case (column two),  $C' \approx 2A'$ , which means that the decay interaction for nonet  $N'$  remains close to that natural for  $q\bar{q}$ . We also notice that in this general case,  $C \neq 0$ , which is expected from our previous results on decays of the low-lying light scalars [3].

## V. DISCUSSION

(i) We studied the properties of the  $a_0(1450)$  and  $K_0^*(1430)$  scalar mesons (which are usually considered to

belong to a conventional  $p$ -wave  $q\bar{q}$  nonet in the quark model) in a framework where a lighter scalar nonet (of  $qq\bar{q}\bar{q}$  type) was also present. It was found that certain puzzling features of these two particles could be naturally explained if the  $q\bar{q}$  and  $qq\bar{q}\bar{q}$  nonets mix with each other to form new physical states. The essential mechanism is driven simply by the fact that the isospinor is lighter than the isovector in the unmixed  $qq\bar{q}\bar{q}$  multiplet.

(ii) Although we carried out the analysis in a  $qq\bar{q}\bar{q}$  picture for the unmixed light scalar nonet, it seems reasonable that it could also be done for other models of the light scalars (like the unitarized quark model [6,14], or molecular models [31]) in which they have somewhat different four-quark interpretations. Indeed it seems likely that the mixing of  $q\bar{q}$  states with  $qq\bar{q}\bar{q}$  states already has a lot of similarity to the mixing with two mesons states induced by unitarization in those schemes. For example, the internal  $qq\bar{q}\bar{q}$  wave function can be rewritten as a linear combination of color singlet  $q\bar{q} \times$  color singlet  $q\bar{q}$  and other pieces (see, for example, Table VI of [3]).

(iii) We did not discuss the heavier isoscalar particles in this paper, mainly because the experimental situation is still rather ambiguous. Clearly, this is an interesting future project. Nevertheless, it seems interesting to discuss some aspects of this question. The simplest way to proceed is to assume the validity of the dual ideal mixing model (2.8) for the lighter scalar nonet and the ordinary ideal mixing model (2.7) for the heavier scalar nonet. Actually in our earlier treatment [see Eq. (2.10) of [3]] it was necessary to include additional terms labeled by parameters  $c$  and  $d$  to fully describe the isoscalar masses and mixings of the light scalar nonet. These terms do not contribute to the isovector and isospinor particle masses and so are irrelevant for our preceding discussion. But when the isoscalar particles are examined they can definitely contribute as can two additional terms labeled by  $c'$  and  $d'$  for the heavier nonet. If we set  $c=d=c'=d'=0$  we would predict in the present model the isoscalars  $\sigma$ ,  $\sigma'$ ,  $f$ ,  $f'$  defined by

$$\begin{pmatrix} \sigma \\ \sigma' \end{pmatrix} = \begin{pmatrix} \cos \psi_\sigma & -\sin \psi_\sigma \\ \sin \psi_\sigma & \cos \psi_\sigma \end{pmatrix} \begin{pmatrix} N_3^3 \\ N_3'^3 \end{pmatrix}, \quad (5.1)$$

$$\begin{pmatrix} f \\ f' \end{pmatrix} = \begin{pmatrix} \cos \psi_f & -\sin \psi_f \\ \sin \psi_f & \cos \psi_f \end{pmatrix} \begin{pmatrix} (N_1^1 + N_2^2)/\sqrt{2} \\ (N_1'^1 + N_2'^2)/\sqrt{2} \end{pmatrix}, \quad (5.2)$$

with  $\psi_\sigma = 22.1^\circ$ ,  $\psi_f = 45^\circ$  and masses  $m_\sigma = 0.69$  GeV,  $m_f = 0.98$  GeV,  $m_{\sigma'} = 1.45$  GeV, and  $m_{f'} = 1.46$  GeV. With our previous interpretation this makes  $\sigma$  mostly of  $udud$  type,  $\sigma'$  mostly of  $ss$  type, while  $f$  and  $f'$  would be equal mixtures of  $(u\bar{u} + d\bar{d})/\sqrt{2}$  and  $s\bar{s}(u\bar{u} + d\bar{d})/\sqrt{2}$ . Amusingly, these masses, which were obtained by using the ‘‘external’’ mixing term (2.10); have reasonable orders of magnitude. However, the predicted masses do not seem to be accurate in detail. Thus we should include the  $c$ ,  $d$ ,  $c'$ ,  $d'$  terms and compare with the experimental heavier isoscalar candidate masses (which are apparently not definitively established). This would lead us to a  $4 \times 4$  mixing matrix ( $5 \times 5$  if a scalar glueball state is included) instead of the two  $2 \times 2$  mixing matrices above.



TABLE II. Best numerical values for the free parameters in the scalar-pseudoscalar-pseudoscalar interaction Lagrangian, found by fitting the prediction of our model for both the low lying and next-highest scalars to the experimental data. The first and second columns correspond, respectively, to the limit ( $C=0$  and  $C'=2A'$ ) and to the general case outside this limit.

Fitted parameters	Eq. (4.2)	Eq. (4.1)
$A(\text{GeV}^{-1})$	$1.40 \pm 0.12$	$1.19 \pm 0.16$
$A'(\text{GeV}^{-1})$	$-3.26 \pm 0.07$	$-3.37 \pm 0.16$
$C(\text{GeV}^{-1})$	0	$1.05 \pm 0.49$
$C'(\text{GeV}^{-1})$	$2A'$	$-6.87 \pm 0.50$
Predicted decay widths		
$\Gamma^{\text{tot}}[a_0(1450)](\text{MeV})$	274	263
$\Gamma[a_0(1450) \rightarrow K\bar{K}]/\Gamma[a_0(1450) \rightarrow \pi\eta]$	0.30	0.42
$\Gamma[a_0(1450) \rightarrow \pi\eta']/\Gamma[a_0(1450) \rightarrow \pi\eta]$	0.52	0.32
$\Gamma[K^*(1430) \rightarrow \pi K](\text{MeV})$	245	298
$\Gamma[a_0(980) \rightarrow \pi\eta](\text{MeV})$	57	65
$\Gamma[\kappa(900) \rightarrow \pi K](\text{MeV})$	45	41
$\Gamma^{\text{tot}}[a_0(1450)]/\Gamma[K^*(1430) \rightarrow \pi K]$ (Expected value: $0.99 \pm 0.24$ )	1.12	0.88
$\chi^2$	1.864	0.757

For the estimation of the decay widths of the isoscalar particles the simplest procedure would be just to use the  $A$  and  $A'$  terms in Eq. (4.2). We have found that this does not give an especially good description of the perturbative widths. Again the full situation is more complicated as one may see from Eq. (3.8) of [3]. Previously we required the four terms defined there proportional to  $A$ ,  $B$ ,  $C$ , and  $D$  for light scalars. Now the parameters  $B$ ,  $D$ ,  $B'$ , and  $D'$  do not contribute to the previously discussed isovector and isospinor particle decays but will contribute to the decays of the isoscalars. Altogether eight additional parameters may be involved in the treatment of the isoscalars.

(iv) In our treatment we used the simplest mixing term (2.10) and obtained fairly good agreement with experiment. The model can easily be generalized to include different mixing terms in the effective Lagrangian; for example,

$$\begin{aligned} & \text{Tr}[\mathcal{M}(NN' + N'N)], \quad \text{Tr}(N)\text{Tr}(N'), \\ & \text{Tr}(\mathcal{M}N)\text{Tr}(N'), \quad \text{Tr}(\mathcal{M}N')\text{Tr}(N). \end{aligned} \quad (5.3)$$

(iv) Although our focus in this paper was on the heavier scalars, the model of course describes the lighter ones too. If we want to describe only the lighter scalars, as in [3,4], we can imagine “integrating out” the heavier scalars. In the simplest approximation, based on neglecting the symmetry-breaking terms in Eqs. (2.7) and (2.8), we would just replace

$$N' \rightarrow -\frac{\gamma}{2a'}N. \quad (5.4)$$

To check the consistency of this with our previous work we might ask how much the decay coefficient of the light scalars [ $A$  in Eq. (3.10)] gets modified due to this replacement. Using Eq. (3.6) and the identity in footnote 3 then gives

$$A \rightarrow A - \frac{\gamma}{2a'}A'. \quad (5.5)$$

Using  $A = 1.2 \text{ GeV}^{-1}$  and  $A' = -3.4 \text{ GeV}^{-1}$  from column 2 of Table II, together with  $\gamma^2 = 0.33 \text{ GeV}^4$  and  $a' = 0.76 \text{ GeV}^2$  from Eq. (2.8) shows that  $A = 1.2 \text{ GeV}^{-1}$  in the present paper is to be replaced by  $A = 2.5 \text{ GeV}^{-1}$  in a model where the heavy scalars have been eliminated. This is in rough agreement with  $A = 2.9 \text{ GeV}^{-1}$  found in Table III of [3].

## ACKNOWLEDGMENTS

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- [1] F. Sannino and J. Schechter, Phys. Rev. D **52**, 96 (1995); M. Harada, F. Sannino, and J. Schechter, *ibid.* **54**, 1991 (1996); Phys. Rev. Lett. **78**, 1603 (1997).  
 [2] D. Black, A. H. Fariborz, F. Sannino, and J. Schechter, Phys. Rev. D **58**, 054012 (1998).  
 [3] D. Black, A. H. Fariborz, F. Sannino, and J. Schechter, Phys.

Rev. D **59**, 074026 (1999).

- [4] A. H. Fariborz and J. Schechter, Phys. Rev. D **60**, 034002 (1999).  
 [5] D. Black *et al.*, hep-ph/9910351, Phys. Rev. D (to be published).  
 [6] See, N. A. Törnqvist, Z. Phys. C **68**, 647 (1995), and references therein. In addition, see, N. A. Törnqvist and M. Roos,

- Phys. Rev. Lett. **76**, 1575 (1996); N. A. Törnqvist, hep-ph/9711483; Phys. Lett. B **426**, 105 (1998).
- [7] S. Ishida, M. Y. Ishida, H. Takahashi, T. Ishida, K. Takamatsu, and T. Tsuru, Prog. Theor. Phys. **95**, 745 (1996); S. Ishida, M. Y. Ishida, T. Ishida, K. Takamatsu, and T. Tsuru, *ibid.* **98**, 621 (1997); see, also, M. Ishida and S. Ishida, Talk given at 7th International Conference on Hadron Spectroscopy (Hadron 97), Upton, NY, 25–30 Aug. 1997, hep-ph/9712231.
- [8] D. Morgan and M. Pennington, Phys. Rev. D **48**, 1185 (1993).
- [9] G. Janssen, B. C. Pearce, K. Holinde, and J. Speth, Phys. Rev. D **52**, 2690 (1995).
- [10] A. A. Bolokhov, A. N. Manashov, M. V. Polyakov, and V. V. Vereshagin, Phys. Rev. D **48**, 3090 (1993); see, also, V. A. Andrianov and A. N. Manashov, Mod. Phys. Lett. A **8**, 2199 (1993). Extension of this stringlike approach to the  $\pi K$  case has been made in V. V. Vereshagin, Phys. Rev. D **55**, 5349 (1997), and very recently in A. V. Vereshagin and V. V. Vereshagin, *ibid.* **59**, 016002 (1999), which is consistent with a light  $\kappa$  state.
- [11] N. N. Achasov and G. N. Shestakov, Phys. Rev. D **49**, 5779 (1994).
- [12] R. Kamínski, L. Leśniak, and J. P. Maillet, Phys. Rev. D **50**, 3145 (1994).
- [13] M. Svec, Phys. Rev. D **53**, 2343 (1996).
- [14] E. van Beveren, T. A. Rijken, K. Metzger, C. Dullemond, G. Rupp, and J. E. Ribeiro, Z. Phys. C **30**, 615 (1986); E. van Beveren and G. Rupp, hep-ph/9806246, p. 248. See, also, J. J. de Swart, P. M. M. Maessen, and T. A. Rijken, U.S./Japan Seminar on the YN Interaction, Maui, 1993 [Nijmegen report THEF-NYM 9403].
- [15] R. Delbourgo and M. D. Scadron, Mod. Phys. Lett. A **10**, 251 (1995); see, also, D. Atkinson, M. Harada, and A. I. Sanda, Phys. Rev. D **46**, 3884 (1992).
- [16] J. A. Oller, E. Oset, and J. R. Pelaez, Phys. Rev. Lett. **80**, 3452 (1998); see, also, K. Igi and K. Hikasa, Phys. Rev. D **59**, 034005 (1999).
- [17] A. V. Anisovich and A. V. Sarantsev, Phys. Lett. B **413**, 137 (1997).
- [18] V. Elias, A. H. Fariborz, Fang Shi, and T. G. Steele, Nucl. Phys. A **633**, 279 (1998).
- [19] V. Dmitrasinović, Phys. Rev. C **53**, 1383 (1996).
- [20] P. Minkowski and W. Ochs, Eur. Phys. J. C **9**, 283 (1999).
- [21] S. Godfrey and J. Napolitano, hep-ph/9811410.
- [22] L. Burakovsky and T. Goldman, Phys. Rev. D **57**, 2879 (1998).
- [23] R. L. Jaffe, Phys. Rev. D **15**, 267 (1977).
- [24] M. N. Achasov *et al.*, Phys. Lett. B **440**, 442 (1998) and very recently R. R. Akhmetshin *et al.*, Report Budker INP 99-11, 1999 and also E. P. Solodov, Talk at the International Workshop on  $e^+e^-$  collisions from  $\phi$  to  $J/\psi$ , Budker Institute of Nuclear Physics, Novosibirsk, Russia, March 1–5, 1999.
- [25] N. N. Achasov, Phys. Usp. **41**, 1149 (1998); see also, N. N. Achasov and V. V. Gubin, Phys. Rev. D **56**, 4084 (1997); N. N. Achasov and V. N. Ivanchenko, Nucl. Phys. B **315**, 465 (1989).
- [26] See, also, F. E. Close, N. Isgur, and S. Kumano, Nucl. Phys. B **389**, 513 (1993).
- [27] Particle Data Group, C. Case *et al.*, Eur. Phys. J. C **3**, 1 (1999).
- [28] S. Okubo, Phys. Lett. **5**, 165 (1963).
- [29] See, for example, M. Harada and J. Schechter, Phys. Rev. D **54**, 3394 (1996).
- [30] S. Teige *et al.*, Phys. Rev. D **59**, 012001 (1999).
- [31] J. Weinstein and N. Isgur, Phys. Rev. Lett. **48**, 659 (1982); Phys. Rev. D **27**, 588 (1983).