Signatures of the anomalous $Z\gamma$ and ZZ production at lepton and hadron colliders

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The possible form of new physics (NP) interactions affecting the ZZZ, $ZZ\gamma$, and $Z\gamma\gamma$ vertices is critically examined. Their signatures and the possibilities to study them, through ZZ and $Z\gamma$ production, at the CERN e^-e^+ colliders LEP and LC and at the hadronic colliders, the Fermilab Tevatron and CERN LHC, are investigated. Experimental limits obtained or expected on each coupling are collected. A simple theoretical model based on virtual effects due to some heavy fermions is used for acquiring some guidance on the plausible forms of these NP vertices. In such a case specific relations among the various neutral couplings are predicted, which can be experimentally tested and possibly used to constrain the form of the responsible NP structure.

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I. INTRODUCTION

During the last two decades intense activity has taken place about the possible existence of anomalous gauge boson couplings (i.e., nonstandard contributions). The general form of the three-boson couplings was written, in a model-independent way, in terms of a set of seven independent Lorentz and $U(1)_{em}$ invariant operators [1,2]. This general description has been applied to both charged (γWW, ZWW) and neutral ($\gamma\gamma Z$, γZZ, ZZZ) sectors [2].

More recently, anomalous gauge boson couplings were considered in the framework of the effective Lagrangians [3]. Here, the basic assumption is that, beyond the standard model (SM), there exists a new physics (NP) dynamics whose degrees of freedom are so heavy (of mass scale Λ) that they cannot be produced at present or in near future colliders. The only observable effects should then be anomalous interactions of the usual SM particles. Under these conditions, by integrating out these heavy NP states, the observable effects can be described by an effective Lagrangian constructed in terms of operators involving only SM fields [2,4]. As long as Λ is much larger than the actually observable energy range, these operators are dominated by those with the lowest possible dimension. Each operator should be Hermitian, multiplied by a constant coupling, while contributions from higher dimensional operators should be suppressed by powers of s/Λ^2.

The set of anomalous couplings can be classified and restricted using symmetry requirements and constraints on the highest allowed dimensionality. This procedure has been fruitfully applied to various sectors of the SM [5]. Thus, it has allowed a description of anomalous properties of several processes, such as four-fermion, two-fermion–two-boson, three-boson, four-boson interactions, where the fermions are leptons or light or heavy quarks, while the bosons are γ, W, Z, and Higgs.

The charged three-boson sector has been explored in great detail with this method, both theoretically and experimentally [6,7]. The general form with seven types of couplings (four CP conserving and three CP violating for the photon and separately for the Z also) was shown to be reduced to only five independent couplings (three CP conserving and two CP violating) if one restricts to oneself dim=6 $SU(2)\times U(1)$ gauge invariant operators in the linear representation [8], while in the nonlinear representation case (where no light Higgs boson exists) one finds that four independent CP-conserving and three CP violating $SU(2)\times U(1)$ gauge invariant operators contribute to triple gauge couplings, at the level of $d_{chiral}=4$ [9]. Various other assumptions can also reduce the number of independent couplings [10].

Experimental constraints have already been established through W^+W^- production at the CERN e^+e^- collider LEP2 and $W\gamma$, and WZ production at the Fermilab Tevatron [11–13]. Relations between the coupling constants and the effective NP scale Λ have also been established through unitarity relations, which allow one to translate the upper limits on these couplings into lower limits for the effective scale Λ [14]. Using this framework, a comparison of the experimental results already obtained or expected at future colliders in the various processes should allow one to establish interesting constraints on the possible structure of the NP interactions. At least it should show what is the SM sector that NP may affect and what symmetry property it may preserve.

Our first aim in this paper is to explore whether similar information could be obtained in the neutral three-boson sector. Up to now, this sector has received less attention than the charged one. Probably this is because charged boson couplings already received tree level SM contributions, whereas the neutral ones do not, so that they may be considered as purely ''anomalous.'' The situation in it is less simple for several reasons. To the general Lorentz and $U(1)_{em}$ invariance requirements, one should add the constraints due to Bose statistics, as there are always at least two identical particles. This forbids ZZZ, $ZZ\gamma$, or $Z\gamma\gamma$ interactions vertices when all particles are on shell [1]. The appearance of such vertices is only possible if at least one of the gauge bosons involved is off shell. The first discussions about these couplings were given in [15]. The most general allowed form involves only two independent couplings for each of the VZZ vertices ($V=\gamma,\ Z$, one CP conserving and one CP violating) and four independent couplings for each of the $VZ\gamma$ vertices ($V=\gamma,\ Z$, two CP conserving and two CP violating). There is *a priori* no relation between these various couplings. Explicit expressions for these vertices were written in [2] and have then been widely used. However, we noticed that a factor i was omitted in the set of $VZ\gamma$ vertices. This factor i is absolutely necessary in order for the related effective NP Lagrangian to be Hermitian.

As in the charged three-boson sector, this effective Lagrangian may be written in an $SU(2)\times U(1)$ invariant form. The only difference is that, while in the charged sector the NP interactions may be generated already at the level of dim=6 operators,[1] in the neutral sector we need operators of dimension 8 or 10 in order for NP to be generated. So, if we restrict ourselves to dim=6 operators, no NP vertices in the neutral three-boson sector are allowed. Thus, if such interactions exist, it would indicate either that some higher dimensional operators containing neutral three-boson vertices without appreciable admixture from charged ones are somehow enhanced or that the NP scale is rather nearby, so that there is no dimensional ordering on the size of the various operators. But of course, in such a case direct production of the new degrees of freedom may be observable. This fact should also arise when one tries to write unitarity constraints and relate the neutral couplings to the effective NP scale defined as the energy at which the various amplitudes saturate unitarity [14]. To be more precise we take one example of NP structure due to the one loop virtual effects of heavy fermions, and we discuss the corresponding pattern of anomalous couplings that are generated. It is found then that the strength of these couplings may be enhanced compared to what the dimensionality of the related operators would had led us to expect. Moreover, relations among the various couplings are obtained in such models. It will be very interesting to see what constraints the experimental measurements will put on these couplings, i.e., to see how they compare to the above theoretical pattern in the neutral and in the charged sectors.

Thus, our motivation for reconsidering the various ZZ and $Z\gamma$ production processes at LEP2, CERN Linear Collider (LC), Tevatron, and CERN Large Hadron Collider (LHC) is to see how they react to the presence of each of the anomalous couplings. In the next section, Sec. II, we explicitly write the correct neutral three-boson vertices and the effective Lagrangian from which they derive. A toy model for the generation of such couplings is also presented. We then give the corresponding NP contributions to the helicity amplitudes for the $f\bar{f}\to ZZ,\ Z\gamma$ processes. Our conventions are fully defined by the expressions for the SM parts of the amplitudes that we give in Appendix A. The expressions of the observables (cross sections and asymmetries) at the various colliders are given in Appendix B. In Sec. III we give explicit illustrations showing how the observables react to each of the anomalous couplings, in particular the interference patterns for the case of CP-conserving couplings. We emphasize the special role that longitudinal polarization would play at the LC Collider for disentangling photon and Z anomalous couplings. We also devote special attention to the way these anomalous effects would be analyzed at hadron colliders and the respective merits of transverse momentum, invariant mass, and c.m. scattering angle distributions. Finally we summarize our observations and suggestions in Sec. IV.

[1]We assume here the linear scalar sector representation.

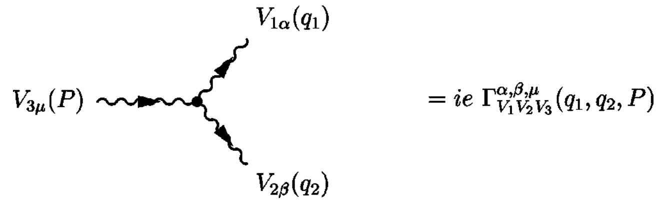

FIG. 1. Feynman rule for the general $V_1V_2V_3$ vertex.

II. DESCRIPTION OF ANOMALOUS NEUTRAL BOSON COUPLINGS

Assuming only Lorentz and $U(1)_{em}$ gauge invariance as well as Bose statistics, the most general form of the $V_1V_2V_3$ vertex function defined in Fig. 1, where V_1,V_2 are on-shell neutral gauge bosons, while ($V_3=Z,\gamma$) is in general off shell but always coupled to a conserved current, has been given in[2] [2]:

$$\Gamma^{\alpha\beta\mu}_{ZZV}(q_1,q_2,P)=\frac{i(P^2-m_V^2)}{m_Z^2}[f_4^V(P^\alpha g^{\mu\beta}+P^\beta g^{\mu\alpha})-f_5^V\epsilon^{\mu\alpha\beta\rho}(q_1-q_2)_\rho], \tag{1}$$

$$\Gamma^{\alpha\beta\mu}_{Z\gamma V}(q_1,q_2,P)=\frac{i(P^2-m_V^2)}{m_Z^2}\left\{h_1^V(q_2^\mu g^{\alpha\beta}-q_2^\alpha g^{\mu\beta})+\frac{h_2^V}{m_Z^2}P^\alpha[(Pq_2)g^{\mu\beta}-q_2^\mu P^\beta]-h_3^V\epsilon^{\mu\alpha\beta\rho}q_{2\rho}-\frac{h_4^V}{m_Z^2}P^\alpha\epsilon^{\mu\beta\rho\sigma}P_\rho q_{2\sigma}\right\}. \tag{2}$$

Compared to [2], we have introduced in Eq. (2) an additional factor i in order for the related effective NP Lagrangian to be Hermitian. Of course, the choice of the sign of this factor is a convention.

The effective Lagrangian generating the vertices (1), (2) is[3]

[2]We define $\epsilon^{0123}=+1$.

[3]Some specific terms of this Lagrangian have been considered in [17].

$$\mathcal{L}_{NP}=\frac{e}{m_Z^2}\Bigg[-[f_4^\gamma(\partial_\mu F^{\mu\beta})+f_4^Z(\partial_\mu Z^{\mu\beta})]Z_\alpha(\partial^\alpha Z_\beta)+[f_5^\gamma(\partial^\sigma F_{\sigma\mu})+f_5^Z(\partial^\sigma Z_{\sigma\mu})]\tilde{Z}^{\mu\beta}Z_\beta-[h_1^\gamma(\partial^\sigma F_{\sigma\mu})+h_1^Z(\partial^\sigma Z_{\sigma\mu})]Z_\beta F^{\mu\beta}$$
$$-[h_3^\gamma(\partial_\sigma F^{\sigma\rho})+h_3^Z(\partial_\sigma Z^{\sigma\rho})]Z^\alpha\tilde{F}_{\rho\alpha}-\left\{\frac{h_2^\gamma}{m_Z^2}[\partial_\alpha\partial_\beta\partial^\rho F_{\rho\mu}]+\frac{h_2^Z}{m_Z^2}[\partial_\alpha\partial_\beta(\Box+m_Z^2)Z_\mu]\right\}Z^\alpha F^{\mu\beta}$$
$$+\left\{\frac{h_4^\gamma}{2m_Z^2}[\Box\partial^\sigma F^{\rho\alpha}]+\frac{h_4^Z}{2m_Z^2}[(\Box+m_Z^2)\partial^\sigma Z^{\rho\alpha}]\right\}Z_\sigma\tilde{F}_{\rho\alpha}\Bigg], \tag{3}$$

where $\tilde{Z}_{\mu\nu}=1/2\epsilon_{\mu\nu\rho\sigma}Z^{\rho\sigma}$ with $Z_{\mu\nu}=\partial_\mu Z_\nu-\partial_\nu Z_\mu$ and similarly for the photon tensor $F_{\mu\nu}$. The couplings $f_4^V,\ h_1^V,\ h_2^V$ violate *CP* invariance, while $f_5^V,\ h_3^V,\ h_4^V$ respect it.

The use of the equations of motion for the photon and Z fields implies that the replacements

$$\partial^\mu F_{\mu\nu}\Rightarrow eQ_f\bar{f}\gamma_\nu f, \tag{4}$$

$$\partial^\mu Z_{\mu\nu}+m_Z^2 Z_\nu\Rightarrow e\bar{f}\left(g_L^Z\gamma_\nu\frac{(1-\gamma_5)}{2}+g_R^Z\gamma_\nu\frac{(1+\gamma_5)}{2}\right)f, \tag{5}$$

$$(\Box+m_Z^2)Z_\nu\Rightarrow e\bar{f}\left(g_L^Z\gamma_\nu\frac{(1-\gamma_5)}{2}+g_R^Z\gamma_\nu\frac{(1+\gamma_5)}{2}\right)f \tag{6}$$

may be done in the first factor of each term in Eq. (3), where f is any fermion with couplings defined in Eq. (A4). Thus, the effective Lagrangian in Eq. (3) is essentially equivalent to a set of contact $f\bar{f}ZZ$ and $f\bar{f}Z\gamma$ interactions.

Of course, the computation of the NP scattering amplitudes for $f\bar{f}\to ZZ$ and $f\bar{f}\to Z\gamma$, either by using these contact interactions or working directly with Eq. (3), gives the same results. They are given below, and should be added to the SM ones, which are due to fermion (f) exchange in the t channel. These SM helicity amplitudes appear in Appendix A and serve to define our notations and conventions.

In $f\bar{f}\to ZZ$, the only nonvanishing NP helicity amplitudes induced by Eq. (1) are those where one Z is transverse ($\tau_1\equiv\tau=\pm1$) and the other longitudinal ($\tau_2=0$). In this case we have

$$F_{\tau 0}^{\lambda}(f\bar{f}\to ZZ;NP)=F_{0,-\tau}^{\lambda *}(f\bar{f}\to ZZ;NP)$$
$$=\frac{e^2\hat{s}^{3/2}\beta}{m_Z^3 2\sqrt{2}}(1+\lambda\tau\cos\vartheta^*)[i(f_4^\gamma Q_f+f_4^Z g_\lambda^Z)-(f_5^\gamma Q_f+f_5^Z g_\lambda^Z)\beta\tau], \tag{7}$$

where the same definitions as in Eq. (A7) are used.

Correspondingly for the NP contribution to $f\bar{f}\to Z\gamma$ [compare Eq. (A12)],

$$F_{\tau_1\tau_2}^{\lambda}(f\bar{f}\to Z\gamma;\ NP)$$
$$=-\frac{e^2(\hat{s}-m_Z^2)\lambda}{4m_Z^2}\sin\vartheta^*[i(h_1^\gamma Q_f+h_1^Z g_\lambda^Z)(1+\tau_1\tau_2)-(\tau_1+\tau_2)(h_3^\gamma Q_f+h_3^Z g_\lambda^Z)]\quad\text{for } \tau_1\tau_2\neq 0, \tag{8}$$

$$F_{0\tau_2}^{\lambda}(f\bar{f}\to Z\gamma;NP)$$
$$=-\frac{e^2\sqrt{\hat{s}}(\hat{s}-m_Z^2)}{m_Z^3 2\sqrt{2}}(1-\lambda\tau_2\cos\vartheta^*)$$
$$\times\left[-i(h_1^\gamma Q_f+h_1^Z g_\lambda^Z)+i(h_2^\gamma Q_f+h_2^Z g_\lambda^Z)\frac{(\hat{s}-m_Z^2)}{2m_Z^2}+\tau_2(h_3^\gamma Q_f+h_3^Z g_\lambda^Z)-\tau_2\frac{(\hat{s}-m_Z^2)}{2m_Z^2}(h_4^\gamma Q_f+h_4^Z g_\lambda^Z)\right], \tag{9}$$

where of course $\tau_2=\pm1$.

Toy model: Heavy fermion contributions at one loop

In order to give at least one illustration of how such anomalous couplings can be generated, we consider the virtual effects of heavy particles at one loop (triangle diagrams with γ and Z external legs), using standard gauge boson couplings. We first observe that heavy scalar particles cannot generate such neutral self-couplings. Heavy fermions can generate f_5^V and h_3^V couplings ($V=\gamma,Z$). No *CP*-violating couplings ($f_4^V,h_{1,2}^V$) and no h_4^V coupling are generated at this level. Higher order effects are needed to get them; see [16] for a detailed discussion.

These results suggest that, indeed, the dominant anomalous couplings may be f_5^V and h_3^V. In fact, at one loop, the results of the computation in [16] for a heavy fermion F interacting with Z and γ as

$$\mathcal{L}=-eQ_F A^\mu\bar{F}\gamma_\mu F-\frac{e}{2s_Wc_W}Z^\mu\bar{F}(\gamma_\mu g_{VF}-\gamma_\mu\gamma_5 g_{AF})F \tag{10}$$

give

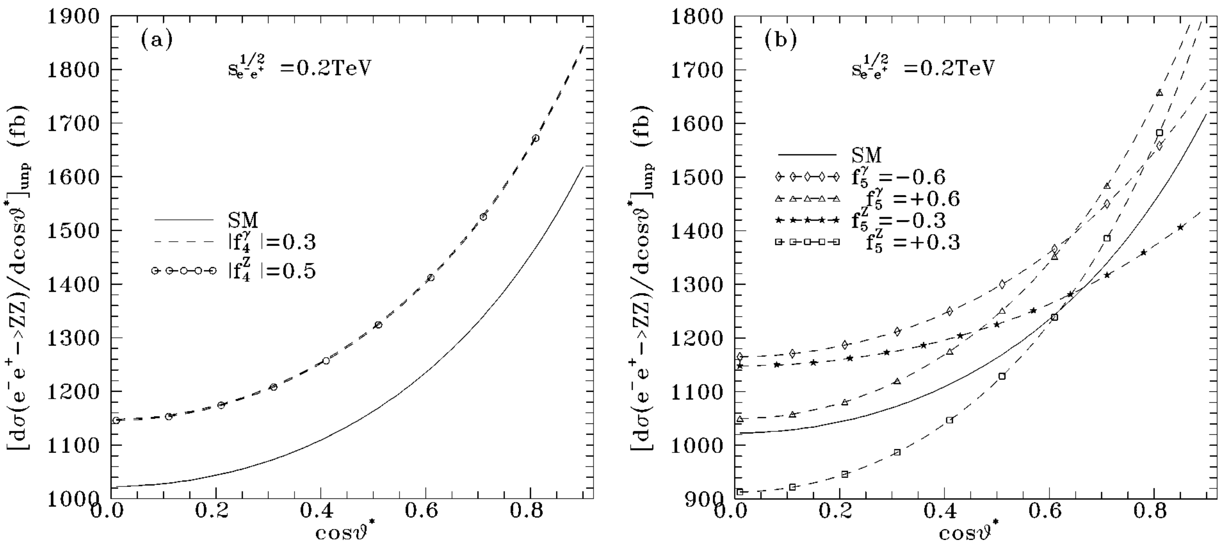

FIG. 2. Standard and anomalous contributions to the unpolarized $e^-e^+\to ZZ$ cross section at LEP.

$$h_3^Z = -f_5^\gamma = -N_f \frac{e^2 Q_F g_{VF} g_{AF}}{96\pi^2 s_W^2 c_W^2}\left(\frac{m_Z^2}{M_F^2}\right), \tag{11}$$

$$h_3^\gamma = -N_F \frac{e^2 Q_F^2 g_{AF}}{48\pi^2 s_W c_W}\left(\frac{m_Z^2}{M_F^2}\right), \tag{12}$$

$$f_5^Z = N_f \frac{e^2 g_{AF}(5g_{VF}^2 + g_{AF}^2)}{960\pi^2 s_W^3 c_W^3}\left(\frac{m_Z^2}{M_F^2}\right), \tag{13}$$

$$h_4^Z = h_4^\gamma = 0, \tag{14}$$

where Q_F is the F electric charge, and g_{VF}, g_{AF} are defined in Eq. (10). N_F is a (color, hypercolor) counting factor which may possibly include enhancement effects due to a strongly interacting sector, while M_F is the F mass.

In general there is no relation to be expected between f_i^V and h_i^V couplings. Note though from Eq. (11) that in the above model the remarkable relation

$$h_3^Z = -f_5^\gamma \tag{15}$$

should hold, which is independent of the fermion couplings. Another striking result is that there are no h_4^Z or h_4^γ couplings in such a model [16]. We also remark that such a model would also generate anapole ZWW and γWW couplings, when the heavy fermion is integrated out at the one-loop level.

Of course, a complete family of exactly degenerate heavy fermions (leptons and quarks with the SM structure) would lead to the vanishing of all the NP couplings in Eqs. (11)–(13). Because in this case the combination of the heavy fermion contributions is the same as in the (mass independent) cancellation in the triangle anomaly. This is the unbroken SU(2)×U(1) situation.

If, instead, one introduces a mass splitting of electroweak size (i.e., $\simeq m_Z^2$) among the multiplets, such as e.g., between the heavy lepton and quark doublets, then the resulting couplings are of the order m_Z^4/M_F^4, which means that they are suppressed by an extra power m_Z^2/M_F^2, as compared to what appears in Eqs. (11)–(13). This case is referred to as a spontaneous broken SU(2)×U(1) situation in [16].

Finally, if a single (or a doublet of a) heavy fermion is much lighter than all the other fermions in the family, then the couplings are as appearing in Eqs. (11)–(13), i.e., just proportional to (m_Z^2/M_F^2). This is obviously the most favorable situation for their observability, and would essentially mean that SU(2)×U(1) is strongly broken in the NP sector.

A final important warning concerning the magnitude of the above couplings must be made. Keeping only standard gauge couplings, the factor $\alpha/4\pi$, which naturally arises in the one loop computations, predicts anomalous couplings of the order of 10^{-3} for M_F in the 100 GeV range. So without a strong enhancement factor there is little hope of observability, except with the very high luminosities expected for the LC collider as we will see in the next section.

III. APPLICATION TO ZZ AND $Z\gamma$ PRODUCTION PROCESSES

In this section we examine how the presence of any of the aforementioned anomalous couplings reflects in ZZ and $Z\gamma$ production at present and future e^+e^- and hadron colliders. The corresponding differential cross sections are given in Appendix B. They are expressed in terms of helicity amplitudes for the basic $f\bar{f}\to ZZ$ and $f\bar{f}\to Z\gamma$ processes.

As expected, the CP-conserving couplings always lead to real amplitudes interfering with the SM ones, so that the various observables are linearly sensitive to these NP terms. On the contrary, the CP-violating couplings always lead to purely imaginary amplitudes that do not interfere with the SM ones.[4] Thus the CP-violating observables depend only

[4]A small interference could only arise for $Z\gamma$ production at energies rather close to the Z pole, where Z-width effects may be non-negligible.

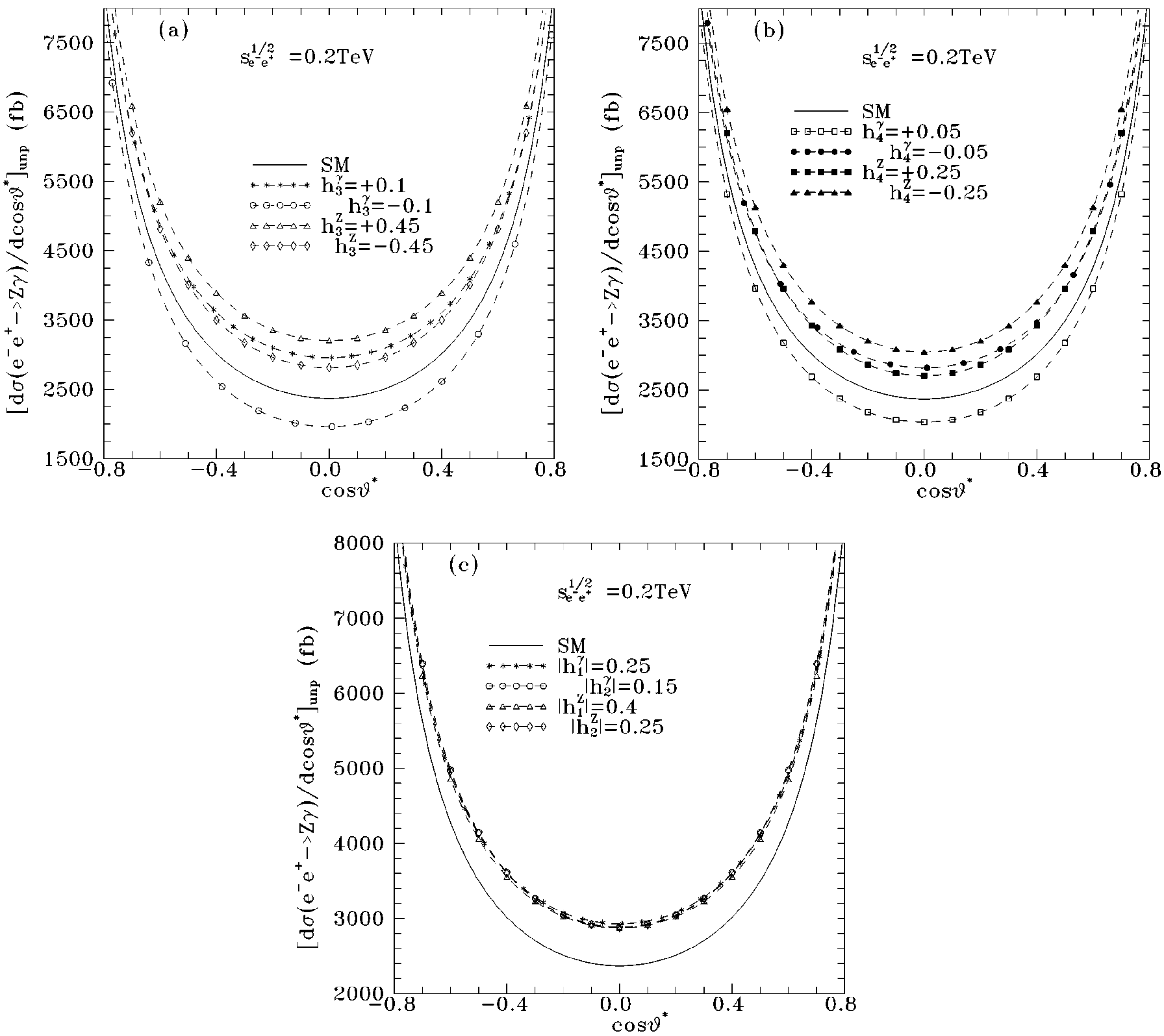

FIG. 3. Standard and anomalous contributions to the unpolarized $e^-e^+\to Z\gamma$ cross sections at LEP.

quadratically on the NP couplings, and their sensitivity limits are accordingly reduced.

Another feature is related to the dimension (dim=6 and dim=8) of the couplings in the Lorentz and $U(1)_{em}$ invariant expression (3). Obviously the dim=8 couplings h_2^V and h_4^V, associated with terms growing with one more power of $\hat{s}$, will be more easily constrained than the dim=6 ones, thus affording a better sensitivity limit.

A. Form factors

Especially at hadron colliders, it has become rather usual to analyze the NP sensitivity limits by multiplying the basic constant anomalous couplings defined in Sec. II by ''form factors'' [19,6]. The reason for this procedure is the following. For a given value of these basic couplings (for example, chosen in order to give a visible effect at an intermediate $\sqrt{\hat{s}}$ energy), the departure from the SM prediction grows rapidly when $\hat{s}$ increases, and may even reach an unreasonable (unitarity-violating) size. In order to cure this behavior, form factors decreasing with $\hat{s}$ with an arbitrary scale (denoted below as Λ_{FF}) are introduced. The form factor usually used is $(1+\hat{s}/\Lambda_{FF}^2)^{-n}$, with $n=3$ for $f_{4,5}^V$, $h_{1,3}^V$ and $n=4$ for $h_{2,4}^V$ [13]. In our illustrations we shall neglect the form factor at LEP2, but for comparison with previous works, we shall keep it for LC where we take $\Lambda_{FF}=1$ TeV, as well as for the Tevatron for which we take $\Lambda_{FF}=0.75$ TeV, and LHC for which $\Lambda_{FF}=3$ TeV is used.

When one analyzes experimental results at a given $\sqrt{\hat{s}}$, it is not of particular importance whether one chooses to use or not to use this procedure, as one can unambiguously translate the limits obtained with form factors to those reached without them. However, at a hadron collider where the limits often arise from an integration over a large range of $\hat{s}$, no simple correspondence is possible.

In fact, the use of form factors is somewhat in contradiction with the basic assumption ($\Lambda \gg \sqrt{\hat{s}}$) that allows one to work with effective Lagrangians, keeping only the lowest dimensions. The additional $\hat{s}$ dependence brought in by the form factor would correspond to the presence of higher dimensional operators with a specific form. Therefore, we would prefer a treatment where no form factors are used, and one instead tries to stay within the basic assumptions, i.e., to keep working within the range $\Lambda \gg \sqrt{\hat{s}}$ and far from the unitarity limit, by considering sufficiently small values for the anomalous couplings for each $\sqrt{\hat{s}}$ domain. We shall come back to this point with some new proposal at the end of this section.

B. Application to LEP2 at 200 GeV

The results for $e^+e^- \to ZZ$ are shown in Fig. 2. As expected, the noninterfering *CP*-violating couplings always produce an increase of the cross section, whereas *CP*-conserving ones produce typical interference patterns with the SM contribution.

The final sensitivity will depend on the integrated luminosity, assumed here to be 150 pb^{-1}, and on the angular cuts and selection of Z decay modes needed for its identification, which should reduce the number of events by roughly a factor of 2. In Fig. 2 we illustrate the additive effects of the *CP*-violating couplings with $|f_4^\gamma| = 0.3$ and $|f_4^Z| = 0.5$, and the interference patterns of the *CP*-conserving ones with $f_5^\gamma = \pm 0.6$ and $f_5^Z = \pm 0.3$. With the expected number of events these values roughly correspond to one standard deviation from SM predictions. This may be compared with recent results obtained at 189 GeV (see, e.g., [20]), in which observability limits were given at the 95% confidence level:

$$-1.9 \leq f_4^Z \leq 1.9, \quad -5.0 \leq f_5^Z \leq 4.5,$$

$$-1.1 \leq f_4^\gamma \leq 1.2, \quad -3.0 \leq f_5^\gamma \leq 2.9. \qquad (16)$$

Note that around 200 GeV we are just above the ZZ threshold where the beta factor [compare Eq. (B2)] strongly affects the cross section. Thus, in this region, the sensitivity to the f_5^V couplings strongly increase with the energy.

In the case of $e^+e^- \to Z\gamma$, the cross section is larger than the ZZ one. It is a factor of 2 larger at large angles, and it has a much larger forward peaking; see Fig. 3. Since the detection of the final photon should be sufficient to characterize the process and all Z decay modes may be used, no reduction factor is probably needed. This should lead to a number of events an order of magnitude larger than for ZZ. Consequently the observability limits should be much better. For the *CP*-violating couplings we then expect one standard deviation effects like $|h_1^\gamma| = 0.1$, $|h_1^Z| = 0.2$, $|h_2^\gamma| = 0.07$, and $|h_2^Z| = 0.12$. Correspondingly, for the *CP*-conserving couplings, we expect asymmetrical one standard deviation effects of the form $h_3^\gamma = \pm 0.02$, $h_3^Z = \pm 0.12$, $h_4^\gamma = \pm 0.015$, and $h_4^Z = \pm 0.09$.

The difference in the sensitivities to the γ and Z couplings can be simply understood as a consequence of the fact that the exchanged photon has a pure vector electron coupling Q_e, whereas the exchanged Z has a weaker (by a factor of $4 s_W c_W$) and essentially purely axial coupling to the electron, so that the interference patterns with the SM amplitude differ in size and in sign for each helicity amplitude; the interplay of linear and quadratic contributions generates further differences.

C. Application to LC at 500 GeV

At energies of 500 GeV and at large angles, the cross section is weaker than at 200 GeV by about a factor of 10. This should be largely compensated for by the expected increase in luminosity [21] (three orders of magnitude for TESLA), which leads to a number of events larger by more than two orders of magnitude. In addition, the NP amplitude increases like s (or even s^2), producing at least an additional order of magnitude in the sensitivity. So, finally, the statistical sensitivity to the above couplings should be increased by *more than two orders of magnitude*.

We present an illustration in Fig. 4 for ZZ and Fig. 5 for $Z\gamma$, by choosing values for the couplings which make the

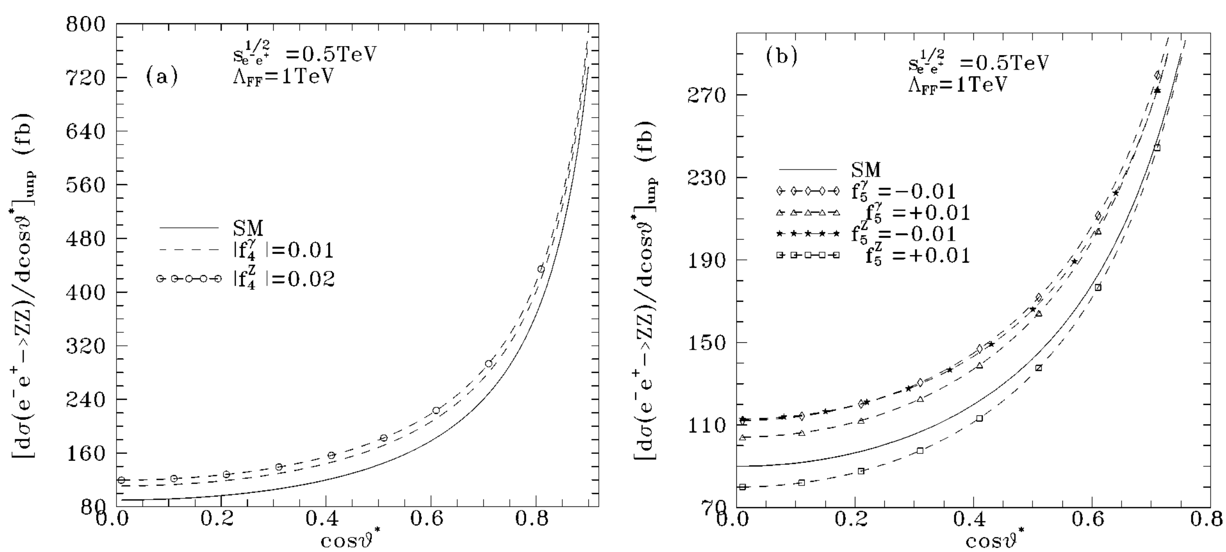

FIG. 4. Standard and anomalous contributions to unpolarized $e^-e^+ \to ZZ$ cross sections at an LC.

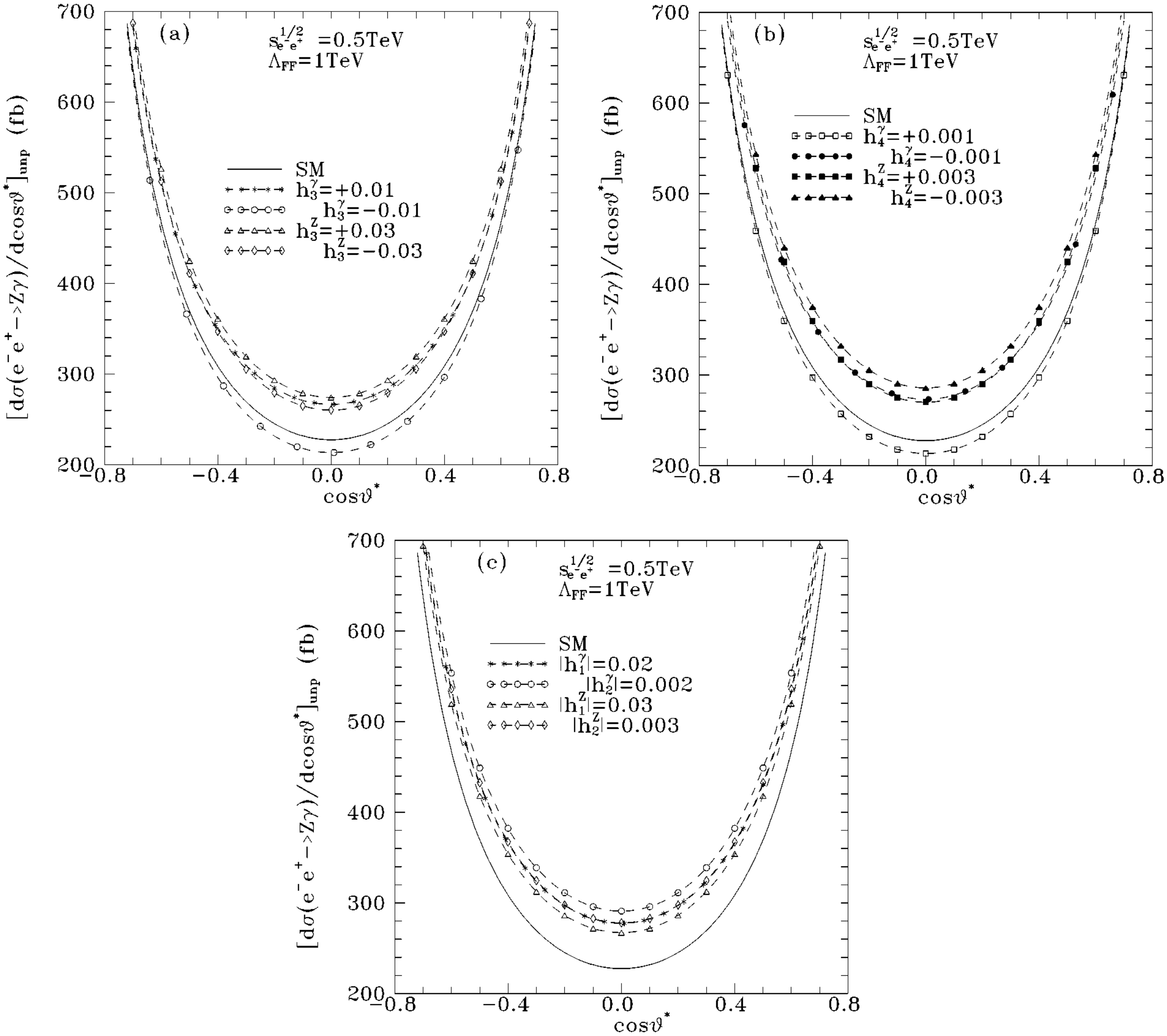

FIG. 5. Standard and anomalous contributions to unpolarized $e^-e^+\to Z\gamma$ cross sections at an LC.

SM and SM+NP curves very visible on the drawing, but of course the observability limits are found to correspond to much lower values. To be more precise, a careful study of the background should be done. One can find some preliminary studies of these effects in Ref. [18]. In the case of $e^+e^-\to ZZ$ there is almost no background for the $q\bar{q}l\bar{l}$ mode, but there is some background in the $q\bar{q}\nu\bar{\nu}$ mode due to the WW channel. Taking them into account, a final (statistical + systematical) accuracy of the order of 1% should be expected, for a conservative integrated luminosity of 100 fb^{-1}. We may even expect a better sensitivity with the higher luminosity of the TESLA design. In any case a 1% accuracy in the cross section at large angles (see Fig. 4), would lead to sensitivity limits for f_4^γ, f_4^Z, f_5^γ, f_5^Z such as 2×10^{-3}, 4×10^{-3}, 3×10^{-3}, 7×10^{-4}, respectively, at the one standard deviation level.

In the case of $e^+e^-\to Z\gamma$ at large angles ($|\cos\theta|<0.8$) no appreciable background is expected [18]. With 100 fb^{-1}, about 50000 events should be selected, leading to an accuracy better than the 0.5% level. The sensitivity (one standard deviation) is now of 3×10^{-3}, 5×10^{-3}, 3×10^{-4}, 4×10^{-4}, 2×10^{-4}, 4×10^{-3}, 4×10^{-5}, 3×10^{-4}, for h_1^γ, h_1^Z, h_2^γ, h_2^Z, h_3^γ, h_3^Z, h_4^γ, h_4^Z, respectively. Indeed, the observability limits should be about two orders of magnitude better than the ones quoted in the LEP2 case.

Another feature of LC is the possibility of having longitudinally polarized $e^\pm$ beams (a polarized e^- beam would be in fact sufficient, like at SLC). We have therefore looked at the effect of the anomalous couplings on the A_{LR} asymmetry whose expression is given in Appendix B. Note, from the expression of the SM amplitudes given in Appendix A, that the SM values of A_{LR} are independent of the scattering angle and energy, taking the values

$$A_{LR}^{SM}(e^-e^+\to ZZ)=\frac{(g_-^Z)^4-(g_+^Z)^4}{(g_-^Z)^4+(g_+^Z)^4}\simeq 0.28, \qquad (17)$$

$$A_{LR}^{SM}(e^+e^-\to Z\gamma)=\frac{(g_-^Z)^2-(g_+^Z)^2}{(g_-^Z)^2+(g_+^Z)^2}\simeq 0.14. \qquad (18)$$

NP departures from these relations arise very differently for

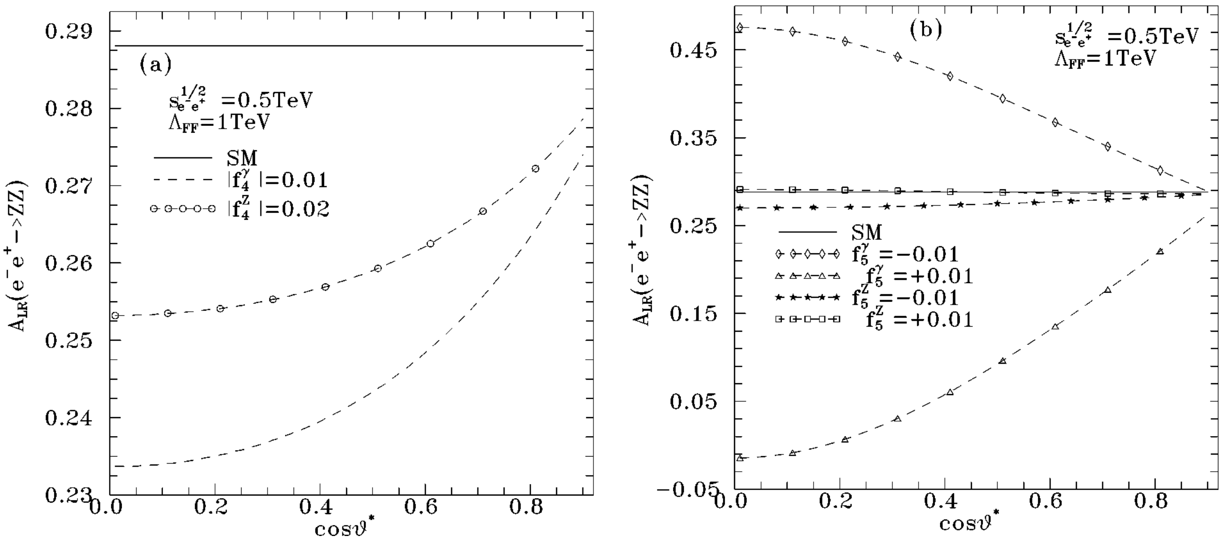

FIG. 6. Standard and anomalous contributions to left-right asymmetries in the $e^-e^+\to ZZ$ cross sections at an LC.

the anomalous photon and Z couplings, especially in the *CP*-conserving case, which interferes with the SM. Thus, one observes a large sensitivity to the sign of certain anomalous *CP*-conserving couplings. See Fig. 6 for *ZZ* and Fig. 7 for $Z\gamma$. It appears, therefore, that measurements of A_{LR} should be very useful for disentangling the anomalous photon and Z couplings.

D. Application to *ZZ* and $Z\gamma$ production at hadron colliders

Finally we have made an illustration for *ZZ* and $Z\gamma$ at the Tevatron (2 TeV) and at LHC (14 TeV). We have chosen to illustrate the transverse momentum (p_T) distribution of one Z (both in the *ZZ* and in the $Z\gamma$ cases), as it is the one which is mostly used in the literature [19]. But we have also checked that the *ZZ* or $Z\gamma$ invariant mass ($\sqrt{\hat{s}}$) distribution shows roughly the same features and gives the same sensitivity to the anomalous couplings. These distributions reflect the fact that *CP*-conserving amplitudes interfere with the SM ones, whereas the *CP*-violating ones always do not, as we can see in Fig. 8 for the Tevatron and Fig. 9 for the LHC.

These interference patterns are somewhat less pronounced than in the illustrations for e^-e^+ collisions. This is due to the fact that the p_T distributions that we are showing are the results of integrations over regions of phase space where the quadratic term is important and partly washes out the interference term. In order to recover the same features as in the e^-e^+ illustrations, one should make severe cuts, selecting a restricted domain in invariant mass and preferably large values of the c.m. scattering angle. In such a domain one can find values of the *CP*-conserving couplings producing a visible effect dominated by the linear (interfering) term. Such a study, which should be carefully done taking into account all the event selection criteria, is beyond the scope of this paper, but we think that it should be tried. For this purpose we have given in Appendix B the expression of the differential cross section with respect to invariant mass and c.m. scattering angle.

As far as the comparison of the sensitivities at hadron colliders and at e^-e^+ colliders is concerned, we would like to come back to the discussion we gave at the begining of this section about the use of form factors, by adding a few (more or less obvious) remarks. This use of form factors is commonly done when analyzing transverse momentum or invariant mass distributions at hadron colliders, in order to take into account the fact that any given nonvanishing value of an anomalous coupling will eventually violate unitarity at sufficiently higher energies. In spite of its apparent necessity, this procedure forbids one to do any clear comparison of observability limits among different colliders because they involve an integration over a large range of invariant mass $\sqrt{\hat{s}}$. We would therefore prefer another procedure that would consist in giving observability limits for the considered basic couplings (without any form factor) in restricted domains (bins) of the subprocess invariant mass. At the same collider, one could then establish a set of different observability limits by taking a set of such domains of invariant masses in which there are enough events to analyze. These observability limits could then be compared among each other and also with observability limits obtained separately at different colliders and different energies.

This is an additional motivation for our suggestion to make analyses in restricted domains of invariant masses, which we already mentioned above.

IV. FINAL DISCUSSION

In this paper we have examined the existing phenomenological description of the anomalous neutral three-boson couplings ($\gamma\gamma Z$, γZZ, ZZZ), and we have reviewed the basic assumptions which allow one to constrain the number and structure of the relavant couplings.

A first observation was that in the set of couplings which were commonly used for studying $Z\gamma$ production, a factor i was missing, making the effective Lagrangian anti-Hermitian. As a result, the interference patterns of the *CP*-

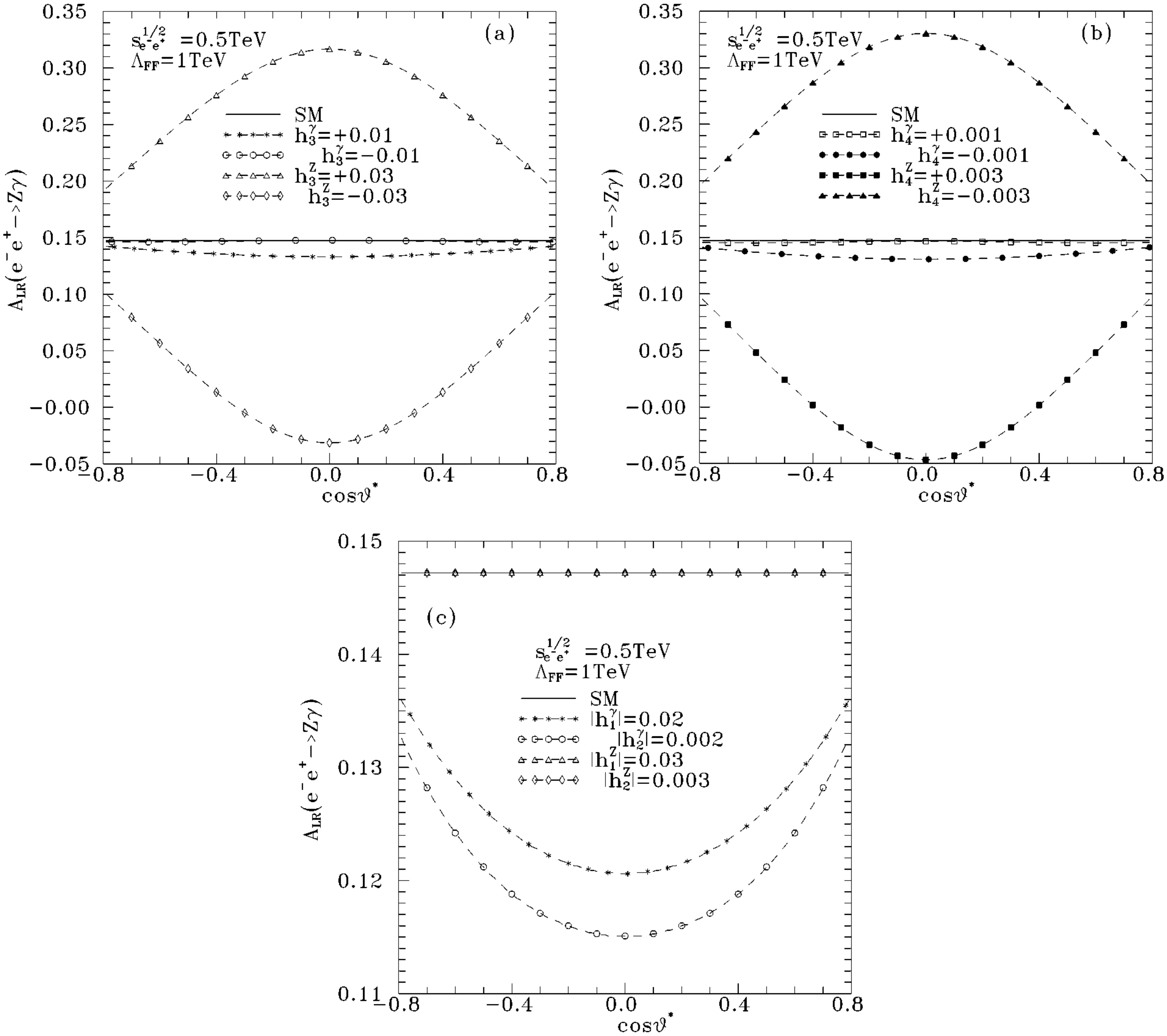

FIG. 7. Standard and anomalous contributions to left-right asymmetries in the $e^-e^+\to Z\gamma$ cross sections at an LC.

conserving and the *CP*-violating NP amplitudes, with the SM ones, were reversed. Nevertheless, this observation does not seem to invalidate most of the presently existing experimental observability limits, since they are so low that they are mainly arising from the quadratic part of the NP contribution. But of course, as the accuracy of the measurements is increasing, it would eventually lead to nonintuitive results.

We have therefore carefully rederived the SM and the anomalous (NP) amplitudes in order to clearly fix all conventions and normalizations. We have illustrated the corresponding effects that the various anomalous couplings induce on ZZ and $Z\gamma$ production at e^-e^+ and hadron colliders. We have made applications for LEP2, for an LC of 500 GeV, and for the Tevatron and the LHC. On the angular distribution, we have shown the interference patterns produced by the *CP*-conserving couplings and the additive contributions given by the *CP*-violating couplings.

In the case of LC, we have emphasized the possible role of longitudinal polarization for disentangling different sets of couplings, as in some cases the interference pattern in the A_{LR} asymmetry is much more pronounced than on the unpolarized cross section.

As far as hadron colliders are concerned, we have suggested to try to make analyses in different restricted domains (bins) of invariant ZZ or $Z\gamma$ masses, with large values of the c.m. scattering angle. In such a case, the inteference patterns should be comparable to the ones observable in e^-e^+ collisions, and much more pronounced than in a (fully integrated) transverse momentum distribution. Of course such an analysis was not possible in the 1.8 TeV Tevatron, but it may be possible, due to the much larger expected statistics, at the upgraded Tevatron and the LHC.

This procedure would also allow one to get rid of the multiplicative form factor introduced in many previous analyses. The comparison of the various observability limits obtained at different e^-e^+ or hadron colliders, each one being defined for a given invariant mass range, would then be straightforward.

We have also mentioned that if we use, e.g., the linear scalar sector representation appropriate for a relatively light

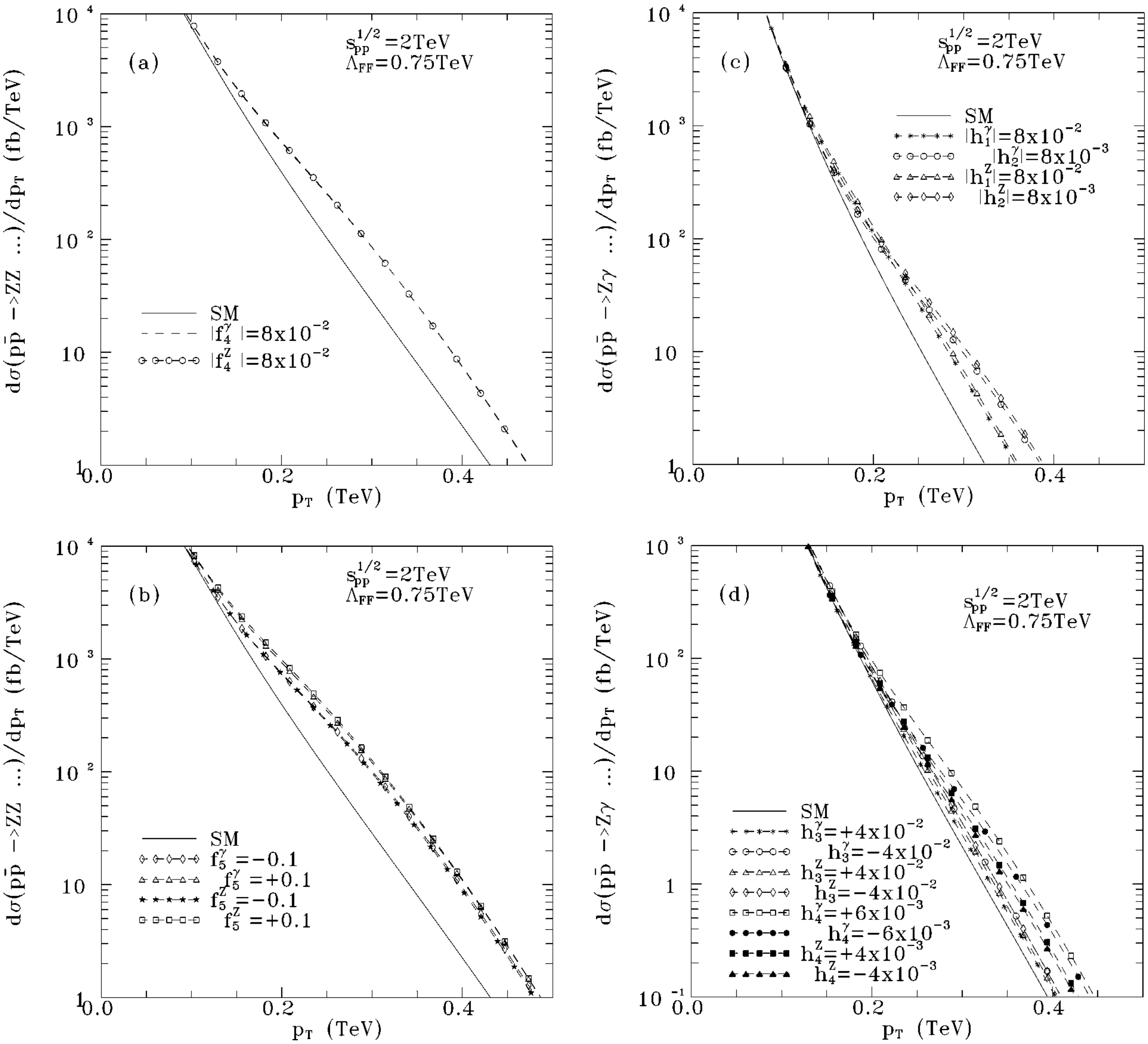

FIG. 8. Standard and anomalous contributions to the $pp\to ZZ$, $Z\gamma$ inclusive cross sections at the Tevatron.

Higgs particle, then at the level of all possible SU(2)×U(1) gauge invariant dim=6 operators, which predict a certain pattern of anomalous γWW and ZWW couplings, no neutral gauge boson couplings are expected. Thus, if such couplings are discovered, it may mean either that for some reasons certain dim=8 or 10 operators are more important than those of dim=6 or that the NP scale is nearby, thus invalidating the dimensional ordering of the SU(2)×U(1) gauge invariant operators.

We have made a specific application, taking as NP effect the one due to the contributions of heavy fermions at one loop. We have discussed two cases, one in which only one set of heavy fermions is lighter than all others and one in which the complete family is nearly degenerate. In both cases one observes that only the f_5^V and h_3^V couplings are generated (together with the anapole ZWW, γWW couplings in the charged sector), the other couplings requiring higher order effects. In addition, we have noticed remarkable relations among these couplings. An important difference between these two aforementioned cases is that, in the first one, the couplings behave like $1/\Lambda_{NP}^2$, whereas in the second they go like $1/\Lambda_{NP}^4$, leading to much poorer bounds on the NP scale. We should also state that, within this type of models, in order to generate observable couplings at present colliders, the NP dynamics must include a strong enhancement effect that would compensate the one loop $\alpha/4\pi$ factor. Otherwise one could expect such virtual effects to be observable only at a very high luminosity LC. In any case this example has shown how experimental constraints established on each coupling could give some indications on the NP properties.

ACKNOWLEDGMENTS

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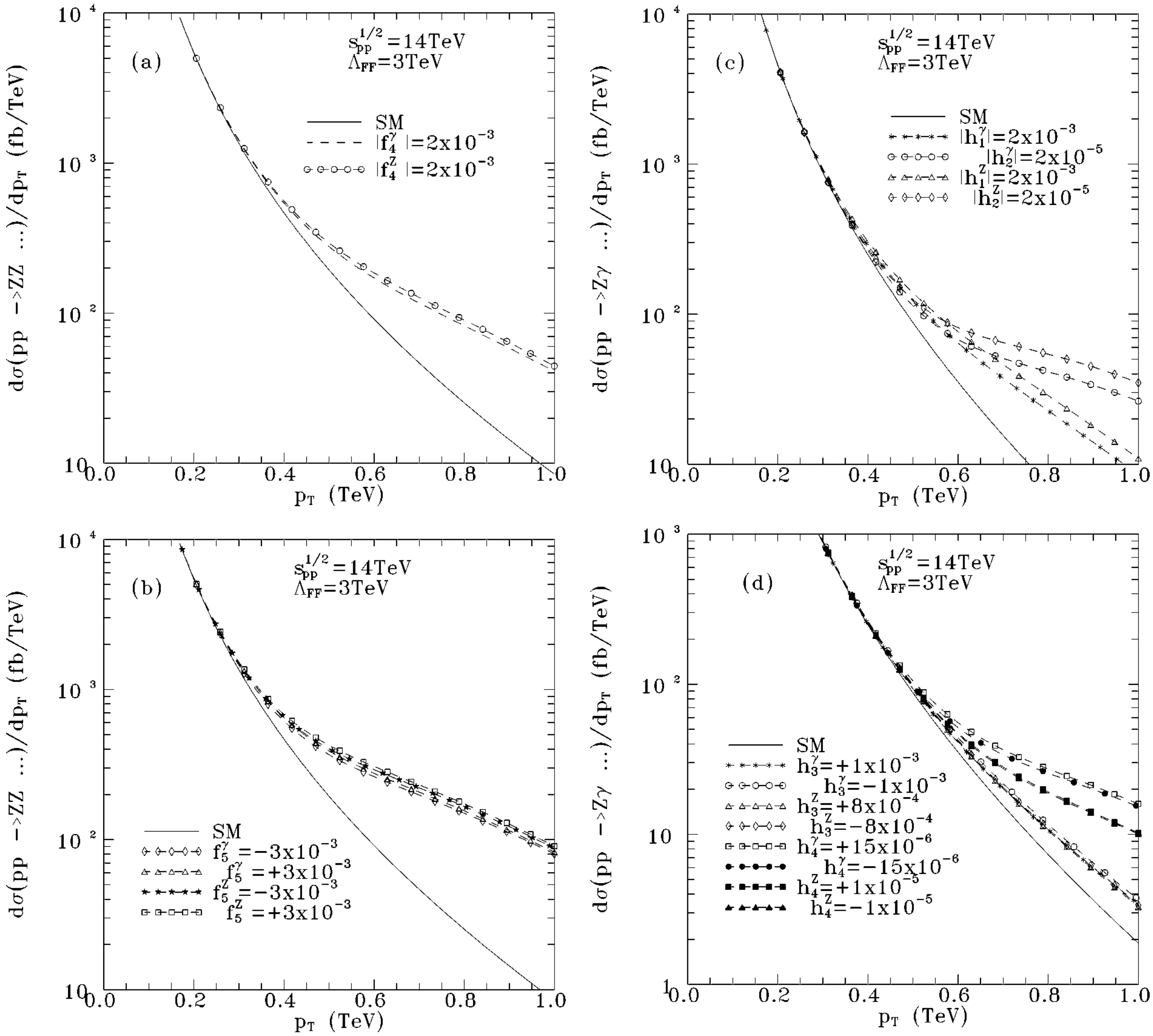

FIG. 9. Standard and anomalous contributions to the $pp \to ZZ$, $Z\gamma$ inclusive cross sections at LHC.

APPENDIX A: THE STANDARD MODEL HELICITY AMPLITUDES FOR $f\bar{f} \to ZZ$ AND $f\bar{f} \to Z\gamma$

The invariant helicity amplitudes for the production processes of the neutral vector bosons V_1, V_2,

$$f(k_1,\lambda_1)\bar{f}(k_2,\lambda_2) \to V_1(q_1,\tau_1)V_2(q_2,\tau_2), \tag{A1}$$

are denoted as[5] $F_{\lambda_1\lambda_2\tau_1\tau_2}$, where the momenta and helicities of the incoming fermions $(f,\bar{f})$ and the outgoing neutral vector bosons are indicated in parentheses in Eq. (A1). Since at collider energies the mass m_f of the incoming fermion can be neglected in all cases,[6] the dependence of the amplitude on the initial helicities is only through the combination $\lambda \equiv \lambda_1 - \lambda_2$, so that the notation $F^{\lambda}_{\tau_1\tau_2}$ will be used below. Thus, there, two possible values for λ are $\lambda = -1(+1)$, corresponding to an $f_{L(R)}$ fermion interacting with an $\overline{f_{R(L)}}$ antifermion, respectively, so that the Z and photon couplings in Eqs. (A3),(A4) may be written as g^Z_λ and g^γ_λ, respectively, as defined by the standard model interaction Lagrangian involving a fermion f of charge Q_f and third isospin component $t^{(3)}_f$,

$$\mathcal{L} = -eV^\mu \bar{f}\left(g^V_L \gamma_\mu \frac{(1-\gamma_5)}{2} + g^V_R \gamma_\mu \frac{(1+\gamma_5)}{2} \right) f, \tag{A2}$$

with

$$g^\gamma_L = g^\gamma_R = Q_f, \tag{A3}$$

$$g^Z_L = \frac{1}{s_W c_W}(t^{(3)}_f - Q_f s^2_W), \quad g^Z_R = \frac{1}{s_W c_W}(-Q_f s^2_W). \tag{A4}$$

For ZZ production, CPT invariance implies, at the tree level,

[5]Its sign is related to the sign of the S matrix through $S_{\lambda_1\lambda_2\tau_1\tau_2} = 1 + i(2\pi)^4 \delta(p_f - p_i) F_{\lambda_1\lambda_2\tau_1\tau_2}$.

[6]Except for the top quark, of course, which is of no relevance here.

$$F^{\lambda}_{\tau_1,\tau_2}(f\bar{f}\to ZZ)=F^{\lambda *}_{-\tau_2,-\tau_1}(f\bar{f}\to ZZ), \quad \text{(A5)}$$

while CP invariance would demand

$$F^{\lambda}_{\tau_1,\tau_2}(f\bar{f}\to ZZ)=F^{\lambda}_{-\tau_2,-\tau_1}(f\bar{f}\to ZZ), \quad \text{(A6)}$$

even at higher orders. Since the standard model amplitudes for $f\bar{f}\to ZZ$ satisfy CP invariance, the tree level standard helicity amplitudes are real and may be written, following the notation of [2], as

$$F^{\lambda}_{\tau_1\tau_2}(f\bar{f}\to ZZ;SM)=-e^2(g^Z_{\lambda})^2\frac{A^{\lambda Z}_{\tau_1\tau_2}}{4\beta^2\sin^2\vartheta^*+\gamma^{-4}}, \quad \text{(A7)}$$

where

$$A^{\lambda Z}_{\tau_1,\tau_2}=2\sin\vartheta^*[2\lambda\cos\vartheta^*(\beta^2-\tau_1\tau_2) -(1+\beta^2)(\tau_2-\tau_1)] \quad \text{for} \quad \tau_1\tau_2\neq 0, \quad \text{(A8)}$$

$$A^{\lambda Z}_{0,\tau}=A^{\lambda Z}_{-\tau,0}=\frac{2\sqrt{2}}{\gamma}(1-\lambda\tau\cos\vartheta^*)[\lambda\tau(1+\beta^2) +2\cos\vartheta^*] \quad \text{for} \quad \tau\neq 0, \quad \text{(A9)}$$

$$A^{\lambda Z}_{00}=-\frac{4\lambda\sin(2\vartheta^*)}{\gamma^2}, \quad \text{(A10)}$$

and ϑ^* is the c.m. Z scattering angle with respect to the f-beam axis, while

$$\beta=\sqrt{1-\frac{4m_Z^2}{\hat{s}}}, \quad \gamma=\frac{1}{\sqrt{1-\beta^2}}. \quad \text{(A11)}$$

Notice that Eqs. (A7)–(A10) define also our conventions on the relative fermion and antifermion phases.

Correspondingly the SM helicity amplitudes for $f\bar{f}\to Z\gamma$ are

$$F^{\lambda}_{\tau_1\tau_2}(f\bar{f}\to Z\gamma;SM)=-e^2g^Z_{\lambda}g^{\gamma}_{\lambda}\ \frac{\hat{s}}{\hat{s}-m_Z^2}\ A^{\lambda\gamma}_{\tau_1\tau_2}, \quad \text{(A12)}$$

where

$$A^{\lambda\gamma}_{\tau_1,\tau_2}=-\frac{1}{\sin\vartheta^*}\left\{\tau_2-\tau_1+\lambda\cos\vartheta^*(-1+\tau_1\tau_2) +\frac{m_Z^2}{\hat{s}}[\tau_1+\tau_2+\lambda(1+\tau_1\tau_2)\cos\vartheta^*]\right\}, \quad \text{(A13)}$$

$$A^{\lambda\gamma}_{0,\tau_2}=2\sqrt{2}\lambda\tau_2\frac{m_Z}{\sqrt{\hat{s}}}. \quad \text{(A14)}$$

APPENDIX B: CROSS SECTION FOR LEP, LC AND THE HADRON COLLIDERS

The full helicity amplitudes for V_1V_2 ($V_1=Z$, $V_2=Z,\gamma$) production through the process Eq. (A1) are obtained by adding the SM contributions in Eqs. (A7), (A12) and the NP ones appearing in Eqs. (7), (8), (9) as

$$F^{\lambda}_{\tau_1\tau_2}(f\bar{f}\to V_1V_2)=F^{\lambda}_{\tau_1\tau_2}(f\bar{f}\to V_1V_2;SM) +F^{\lambda}_{\tau_1\tau_2}(f\bar{f}\to V_1V_2;NP), \quad \text{(B1)}$$

and they are normalized so that the unpolarized differential cross sections are given by

$$\frac{d\hat{\sigma}(f\bar{f}\to ZZ)}{d\cos\vartheta^*}=\frac{\hat{s}}{2}\left(1-\frac{4m_Z^2}{\hat{s}}\right)^{1/2}\frac{d\hat{\sigma}(f\bar{f}\to ZZ)}{d\hat{t}} =\frac{1}{128\pi\hat{s}}\left(1-\frac{4m_Z^2}{\hat{s}}\right)^{1/2} \times\sum_{\tau_1\tau_2}[|F^{\lambda=1}_{\tau_1\tau_2}|^2+|F^{\lambda=-1}_{\tau_1\tau_2}|^2], \quad \text{(B2)}$$

$$\frac{d\hat{\sigma}(f\bar{f}\to Z\gamma)}{d\cos\vartheta^*}=\frac{(\hat{s}-m_Z^2)}{2}\frac{d\hat{\sigma}(f\bar{f}\to Z\gamma)}{d\hat{t}} =\frac{(\hat{s}-m_Z^2)}{128\pi\hat{s}^2}\sum_{\tau_1\tau_2}[|F^{\lambda=1}_{\tau_1\tau_2}|^2+|F^{\lambda=-1}_{\tau_1\tau_2}|^2]. \quad \text{(B3)}$$

Note that in Eq. (B2), the identity of the two final Z is taken into account by imposing the constrain $0\leqslant\cos\vartheta^*\leqslant 1$.

The left-right asymmetry measurable at an LC is defined by

$$A_{LR}(e^-e^+\to V_1V_2) =\left[\frac{d\hat{\sigma}(e^-_Le^+_R\to V_1V_2)}{d\cos\vartheta^*}-\frac{d\hat{\sigma}(e^-_Re^+_L\to V_1V_2)}{d\cos\vartheta^*}\right] \times\left[\frac{d\hat{\sigma}(e^-_Le^+_R\to V_1V_2)}{d\cos\vartheta^*}+\frac{d\hat{\sigma}(e^-_Re^+_L\to V_1V_2)}{d\cos\vartheta^*}\right]^{-1} =\frac{\sum_{\tau_1\tau_2}[|F^{\lambda=1}_{\tau_1\tau_2}|^2-|F^{\lambda=-1}_{\tau_1\tau_2}|^2]}{\sum_{\tau_1\tau_2}[|F^{\lambda=1}_{\tau_1\tau_2}|^2+|F^{\lambda=-1}_{\tau_1\tau_2}|^2]}. \quad \text{(B4)}$$

Finally the p_T distribution at a hadron collider, with c.m. energy $\sqrt{s}$, is determined by

$$\frac{d\sigma(H_1H_2\to V_1V_2+\cdots)}{dp_T^2dy_1dy_2}=\tau\Sigma^{V_1V_2}(x_a,x_b), \quad \text{(B5)}$$

where

$$\Sigma^{ZZ}(x_a,x_b)=\sum_j [q_j(x_a)\overline{q}_j(x_b)+\overline{q}_j(x_a)q_j(x_b)] \times\frac{d\hat{\sigma}(q\overline{q}\to ZZ)}{d\hat{t}}, \tag{B6}$$

$$\Sigma^{Z\gamma}(x_a,x_b)=\sum_j [q_j(x_a)\overline{q}_j(x_b)+\overline{q}_j(x_a)q_j(x_b)] \times\frac{d\hat{\sigma}(q\overline{q}\to Z\gamma)}{d\hat{t}}. \tag{B7}$$

The summation extends over all quark and antiquarks inside the hadrons H_1, H_2. Note that the distributions considered in this paper are symmetrical in $\hat{t}$, $\hat{u}$ interchange. $\tau=\hat{s}/s$, while (x_a,x_b) are fully determined in terms of the rapidities (y_1, y_2) and the (opposite) transverse momenta of the two final gauge bosons:

$$x_a=\tfrac{1}{2}[x_{1T}e^{y_1}+x_{2T}e^{y_2}], \quad x_b=\tfrac{1}{2}[x_{1T}e^{-y_1}+x_{2T}e^{-y_2}], \tag{B8}$$

where $x_{iT}=(2/\sqrt{s})(p_T^2+m_{V_i}^2)^{1/2}$. In the integration over y_1 and y_2 we impose a cut at $|y_i|<2$. We will also discuss the invariant mass and c.m. scattering angle distributions given by

$$\frac{d\sigma(H_1H_2\to V_1V_2+\cdots)}{d\hat{s}\,d\cos\vartheta^*d\overline{y}}=J\tau\Sigma^{V_1V_2}(x_a,x_b), \tag{B9}$$

where $J(Z\gamma)=(\hat{s}-m_Z^2)/2\hat{s}$, $J(ZZ)=\frac{1}{2}\sqrt{1-4m_Z^2/\hat{s}}$. $\overline{y}$ is the boost defined as $\overline{y}=y_1-y_1^*=y_2-y_2^*$, with $thy_1^*=\beta_1^*\cos\vartheta^*$, $thy_2^*=-\beta_2^*\cos\vartheta^*$, and $\beta_1^*=(\hat{s}-m_Z^2)/(\hat{s}+m_Z^2)$, $\beta_2^*=1$ for $Z\gamma$, but $\beta_1^*=\beta_2^*=\sqrt{1-4m_Z^2/\hat{s}}$ for ZZ. With these variables, $x_a=e^{y}\sqrt{\tau}$, $x_b=e^{-y}\sqrt{\tau}$.

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