CP and *T* violation in long baseline experiments with low energy neutrinos from a muon storage ring

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Stimulated by the idea of PRISM, a very high intensity muon ring with a rather low energy, we consider the possibility of observing *CP*-violation effects in neutrino oscillation experiments. More than 10% of *CP*-violation effects can be seen within the experimentally allowed region. The destructive sum of the matter effect and *CP*-violation effect can be avoided with the use of an initial ν_e beam. We finally show that the experiment with (a few) $\times 100$ MeV of neutrino energy and (a few) $\times 100$ km of baseline length, which is considered in this paper, is particularly suitable for the search for *CP* violation in view of the statistical error.

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I. INTRODUCTION

Many experiments and observations have shown evidence for neutrino oscillation. The solar neutrino deficit has long been observed [1–5]. The atmospheric neutrino anomaly has been found [6–9] and recently almost confirmed by SuperKamiokande [10]. There is also another suggestion given by the Liquid Scintillation Neutrino Detector (LSND) [11]. All of them can be understood through neutrino oscillation and hence they indicate that neutrinos are massive and there is mixing in the lepton sector [12].

Since there is mixing in the lepton sector, it is quite natural to imagine that *CP* violation occurs in the lepton sector. Several physicists have considered whether we may see *CP*-violation effects in the lepton sector through long baseline neutrino oscillation experiments. First it has been studied in the context of currently planed experiments [13–18] and recently in the context of a neutrino factory [19–22].

The use of neutrinos from a muon beam has great advantages compared with those from a pion beam. Neutrinos from $\mu^+(\mu^-)$ beam consist of pure ν_e and $\overline{\nu}_\mu$ ($\overline{\nu}_e$ and ν_μ) and will contain no contamination of other kinds of neutrinos. Also their energy distribution will be determined very well. In addition we can test *T* violation in long baseline experiments by using a (anti)electron neutrino [15,16].

Unfortunately those neutrinos have very high energy [23]. The smaller mass scale of the neutrino, determined by the solar neutrino deficit, cannot be seen in most long baseline experiments. Since the *CP*-violation effect arises as three (or more) generation phenomena [24,25], it is difficult to make a *CP*-violation search using neutrinos from such a muon beam.

We are, however, very lucky since we will have a very intense muon source with a rather low energy, PRISM [26]. It will be located at Tokai, Ibaraki Prefecture, about 50 km from KEK. Since the muons will have an energy of less than 1 GeV, we can expect that we will have a very intense neutrino beam with energy less than 500 MeV. It will be very suitable to explore *CP* violation in the lepton sector with neutrino oscillation experiments. With such a low energy beam, we will be able to detect neutrinos experimentally with good energy resolution. Stimulated by the possibility that we will have a low energy neutrino source with very high intensity, we consider here how large a *CP*-violation effect we will see with such a neutrino beam. In this paper we will consider three active neutrinos without any sterile one by attributing the solar neutrino deficit and atmospheric neutrino anomaly to the neutrino oscillation.

II. OSCILLATION PROBABILITIES AND THEIR APPROXIMATED FORMULAS

First we derive approximated formulas [16] of neutrino oscillation [12,25,27,28] to clarify our notation. We assume three generations of neutrinos which have mass eigenvalues $m_i(i=1,2,3)$ and MNS mixing matrix U relating the flavor eigenstates $\nu_\alpha(\alpha=e,\mu,\tau)$ and the mass eigenstates in the vacuum $\nu_i'(i=1,2,3)$ as

$$\nu_\alpha = U_{\alpha i}\nu_i' . \tag{1}$$

We parametrize U [29–31] as

$$U = e^{i\psi\lambda_7}\Gamma e^{i\phi\lambda_5}e^{i\omega\lambda_2} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\psi & s_\psi \\ 0 & -s_\psi & c_\psi \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix} \times \begin{pmatrix} c_\phi & 0 & s_\phi \\ 0 & 1 & 0 \\ -s_\phi & 0 & c_\phi \end{pmatrix}\begin{pmatrix} c_\omega & s_\omega & 0 \\ -s_\omega & c_\omega & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_\phi c_\omega & c_\phi s_\omega & s_\phi \\ -c_\psi s_\omega - s_\psi s_\phi c_\omega e^{i\delta} & c_\psi c_\omega - s_\psi s_\phi s_\omega e^{i\delta} & s_\psi c_\phi e^{i\delta} \\ s_\psi s_\omega - c_\psi s_\phi c_\omega e^{i\delta} & -s_\psi c_\omega - c_\psi s_\phi s_\omega e^{i\delta} & c_\psi c_\phi e^{i\delta} \end{pmatrix}, \tag{2}$$

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where $c_\psi = \cos\psi$, $s_\phi = \sin\phi$, etc.

The evolution equation for the flavor eigenstate vector in the vacuum is

$$i\frac{d\nu}{dx} = \frac{1}{2E} U\,\mathrm{diag}(0, \delta m_{21}^2, \delta m_{31}^2) U^\dagger \nu, \tag{3}$$

where $\delta m_{ij}^2 = m_i^2 - m_j^2$. Similarly the evolution equation in matter is expressed as

$$i\frac{d\nu}{dx} = H\nu, \tag{4}$$

where

$$H \equiv \frac{1}{2E}\tilde{U}\,\mathrm{diag}(\tilde{m}_1^2, \tilde{m}_2^2, \tilde{m}_3^2)\tilde{U}^\dagger, \tag{5}$$

with a unitary mixing matrix $\tilde{U}$ and the effective mass squared $\tilde{m}_i^2$'s ($i=1,2,3$). The matrix $\tilde{U}$ and the masses $\tilde{m}_i$'s are determined by [32–34]

$$\tilde{U}\begin{pmatrix} \tilde{m}_1^2 & & \\ & \tilde{m}_2^2 & \\ & & \tilde{m}_3^2 \end{pmatrix}\tilde{U}^\dagger = U\begin{pmatrix} 0 & & \\ & \delta m_{21}^2 & \\ & & \delta m_{31}^2 \end{pmatrix}U^\dagger + \begin{pmatrix} a & & \\ & 0 & \\ & & 0 \end{pmatrix}. \tag{6}$$

Here

$$a \equiv 2\sqrt{2} G_F n_e E = 7.56\times 10^{-5}\ \mathrm{eV}^2 \times \left(\frac{\rho}{\mathrm{g\,cm^{-3}}}\right)\left(\frac{E}{\mathrm{GeV}}\right), \tag{7}$$

where n_e is the electron density and ρ is the matter density. The solution of Eq. (4) is then

$$\nu(x) = S(x)\nu(0) \tag{8}$$

with

$$S \equiv T\, e^{-i\int_0^x ds H(s)} \tag{9}$$

(T being the symbol for time ordering), giving the oscillation probability for $\nu_\alpha \to \nu_\beta (\alpha, \beta = \mathrm{e}, \mu, \tau)$ at distance L as

$$P(\nu_\alpha \to \nu_\beta; E, L) = |S_{\beta\alpha}(L)|^2. \tag{10}$$

Note that $P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$ is related to $P(\nu_\alpha \to \nu_\beta)$ through $a \to -a$ and $U \to U^*$ (i.e., $\delta \to -\delta$). Similarly, we obtain $P(\bar{\nu}_\beta \to \nu_\alpha)$ from Eq. (10) by replacing $\delta \to -\delta$, $P(\bar{\nu}_\beta \to \nu_\alpha)$ by $a \to -a$.

Attributing both solar neutrino deficit and atmospheric neutrino anomaly to neutrino oscillation, we can assume $a, \delta m_{21}^2 \ll \delta m_{31}^2$. The oscillation probabilities in this case can be considered by perturbation [16]. With the additional conditions

$$\frac{aL}{2E} = 1.93\times 10^{-4} \times \left(\frac{\rho}{\mathrm{g\,cm^{-3}}}\right)\left(\frac{L}{\mathrm{km}}\right) \ll 1 \tag{11}$$

and

$$\frac{\delta m_{21}^2 L}{2E} = 2.53 \frac{(\delta m_{21}^2/\mathrm{eV}^2)(L/\mathrm{km})}{E/\mathrm{GeV}} \ll 1, \tag{12}$$

the oscillation probabilities are calculated, e.g., as

$$\begin{aligned} P(\nu_\mu \to \nu_\mathrm{e}; E, L) = {} & 4\sin^2\frac{\delta m_{31}^2 L}{4E} c_\phi^2 s_\phi^2 s_\psi^2 \\ & \times \left\{ 1 + \frac{a}{\delta m_{31}^2} 2(1 - 2s_\phi^2) \right\} \\ & + 2\frac{\delta m_{31}^2 L}{2E}\sin\frac{\delta m_{31}^2 L}{2E} c_\phi^2 s_\phi s_\psi \\ & \times \left\{ -\frac{a}{\delta m_{31}^2} s_\phi s_\psi (1 - 2s_\phi^2) \right. \\ & \left. + \frac{\delta m_{21}^2}{\delta m_{31}^2} s_\omega (-s_\phi s_\psi s_\omega + c_\delta c_\psi c_\omega) \right\} \\ & - 4\frac{\delta m_{21}^2 L}{2E}\sin^2\frac{\delta m_{31}^2 L}{4E} s_\delta c_\phi^2 s_\phi c_\psi s_\psi c_\omega s_\omega . \end{aligned} \tag{13}$$

As stated, oscillation probabilities such as $P(\bar{\nu}_\mu \to \bar{\nu}_\mathrm{e})$, $P(\nu_\mathrm{e} \to \nu_\mu)$, and $P(\bar{\nu}_\mathrm{e} \to \bar{\nu}_\mu)$ are given from the above formula by some appropriate changes of the sign of a and/or δ.

The first condition (11) of the approximation leads to a constraint for the baseline length of long-baseline experiments as

$$L \ll 1.72\times 10^3\ \mathrm{km}\left(\frac{\rho}{3\ \mathrm{g\,cm^{-3}}}\right). \tag{14}$$

The second condition (12) gives the energy region where we can use the approximation

$$E \gg 76.0\ \mathrm{MeV}\left(\frac{\delta m_{21}^2}{10^{-4}\ \mathrm{eV}^2}\right)\left(\frac{L}{300\ \mathrm{km}}\right). \tag{15}$$

We compare in Fig. 1 the approximated oscillation probabilities [Eq. (13), etc.] with unapproximated ones to show the validity of this approximation. Here we set the baseline length to be 300 km which corresponds to the distance between Tokai and Kamioka. Other parameters are taken from the region allowed by present experiments [37].[1] We see that

[1]Although the Chooz reactor experiment has almost excluded $\sin^2\phi = 0.1$ [38], there still remains a small chance to take this value.

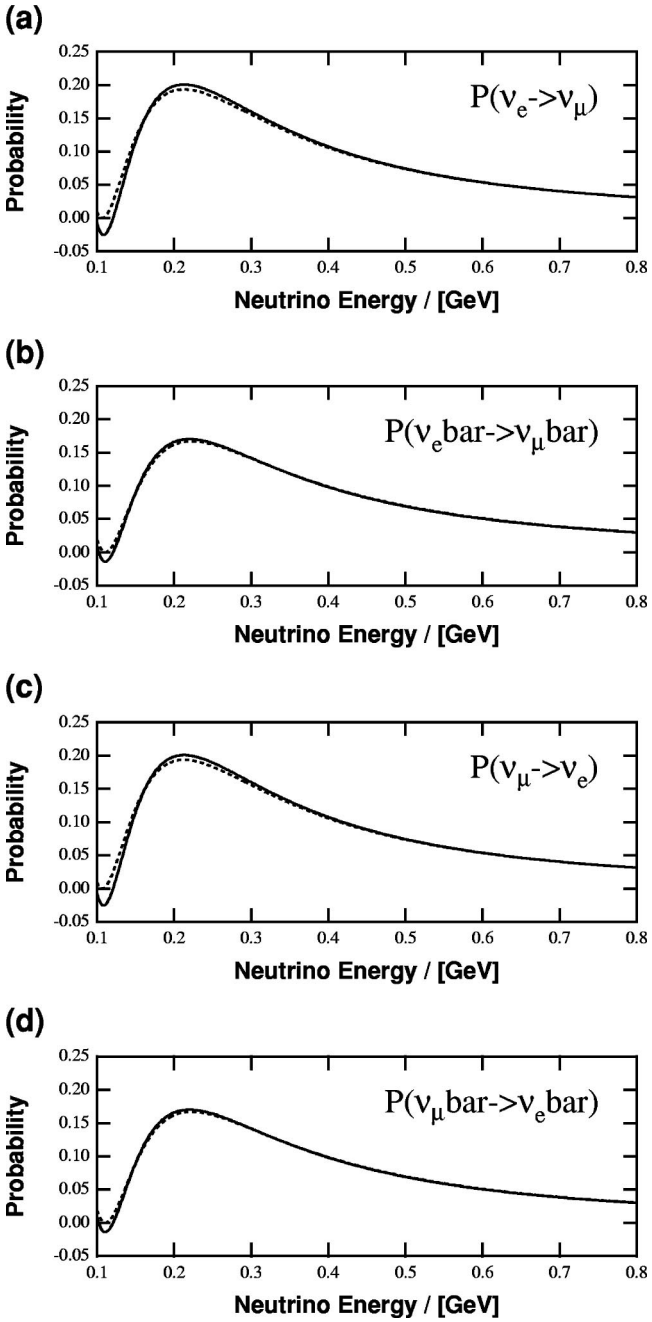

FIG. 1. The approximated oscillation probabilities (solid lines) compared with unapproximated ones (dashed lines). Here the parameters are taken as follows: $\delta m^2_{31}=1.0\times10^{-3}$ eV^2, $\delta m^2_{21}=1.0\times10^{-4}$ eV^2, $\sin^2\psi=1/2$, $\sin^2\omega=1/2$, $\sin^2\phi=0.1$, $\sin\delta=1$; $\rho=2.5$ g/cm^3, and $L=300$ km.

the approximation coincides with full calculation pretty well, and we are safely able to use approximated formulas in the following.

III. *CP* VIOLATION SEARCH IN LONG BASELINE EXPERIMENTS

A. Magnitude of *CP* violation and matter effect

The available neutrinos as an initial beam are ν_μ and $\overline{\nu}_\mu$ in the current long baseline experiments [35,36]. The ''*CP* violation'' gives the nonzero difference of the oscillation probabilities between, e.g., $P(\nu_\mu\to\nu_e)$ and $P(\overline{\nu}_\mu\to\overline{\nu}_e)$ [16]. This gives

$$\begin{aligned}P(\nu_\mu\to\nu_e;L)-P(\overline{\nu}_\mu\to\overline{\nu}_e;L)&=16\frac{a}{\delta m^2_{31}}\sin^2\frac{\delta m^2_{31}L}{4E}c^2_\phi s^2_\phi s^2_\psi(1-2s^2_\phi)\\&\quad-4\frac{aL}{2E}\sin\frac{\delta m^2_{31}L}{2E}c^2_\phi s^2_\phi s^2_\psi(1-2s^2_\phi)\\&\quad-8\frac{\delta m^2_{21}L}{2E}\sin^2\frac{\delta m^2_{31}L}{4E}s_\delta c^2_\phi s_\phi c_\psi s_\psi c_\omega s_\omega.\end{aligned}\tag{16}$$

The difference of these two, however, also includes matter effect, or the fake *CP* violation, proportional to *a*. We must somehow distinguish these two to conclude the existence of *CP* violation as discussed in Ref. [16].

On the other hand, a muon ring enables us to extract ν_e and $\overline{\nu}_e$ beam. It enables direct measurement of pure *CP* violation through ''*T* violation,'' e.g., $P(\nu_\mu\to\nu_e)-P(\nu_e\to\nu_\mu)$ as

$$\begin{aligned}P(\nu_\mu\to\nu_e)-P(\nu_e\to\nu_\mu)&=-8\frac{\delta m^2_{21}L}{2E}\sin^2\frac{\delta m^2_{31}L}{4E}s_\delta c^2_\phi s_\phi c_\psi s_\psi c_\omega s_\omega.\end{aligned}\tag{17}$$

Note that this difference gives pure *CP* violation.

By measuring ''*CPT* violation,'' e.g., the difference between $P(\nu_\mu\to\nu_e)$ and $P(\overline{\nu}_e\to\overline{\nu}_\mu)$, we can check the matter effect

$$\begin{aligned}P(\nu_\mu\to\nu_e;L)-P(\overline{\nu}_e\to\overline{\nu}_\mu;L)&=16\frac{a}{\delta m^2_{31}}\sin^2\frac{\delta m^2_{31}L}{4E}c^2_\phi s^2_\phi s^2_\psi(1-2s^2_\phi)\\&\quad-4\frac{aL}{2E}\sin\frac{\delta m^2_{31}L}{2E}c^2_\phi s^2_\phi s^2_\psi(1-2s^2_\phi).\end{aligned}\tag{18}$$

We present in Fig. 2 the ''*T*-violation'' part (17) and ''*CPT* violation'' part (18) for some parameters allowed by the present experiments [37] with $\sin^2\omega=1/2$, $\sin^2\psi=1/2$, $\sin\delta=1$ fixed. The matter density is also fixed to the constant value $\rho=2.5$ g/cm^3 [39]. The other parameters are taken as $\delta m^2_{31}=3\times10^{-3}$ eV^2 and 1×10^{-3} eV^2, $\delta m^2_{21}=1\times10^{-4}$eV^2 and 3×10^{-5} eV^2.

The ''*T*-violation'' effect is proportional to $\delta m^2_{21}/\delta m^2_{31}$ and, for $\phi\ll1$, also to $\sin\phi$ as seen in Eq. (17) and Fig. 2. Recalling that the energy of neutrino beam is of several hundreds MeV, we see in Fig. 2 that the ''*T*-violation'' effect amounts to at least about 5%, hopefully 10–20 %. This result gives hope that we may detect the pure leptonic *CP* violation directly with neutrino oscillation experiments.

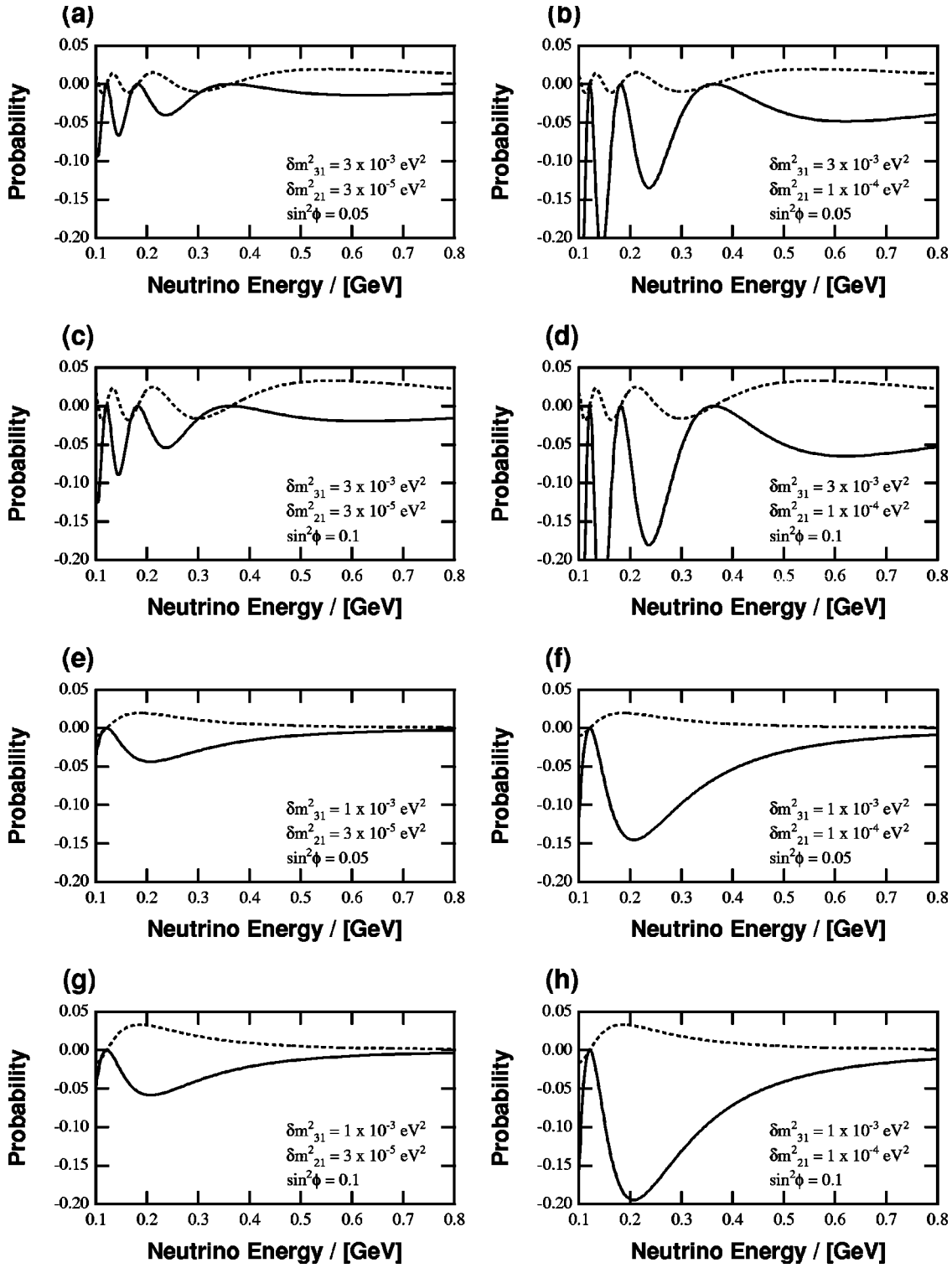

FIG. 2. Graphs of $P(\nu_\mu \to \nu_e) - P(\nu_e \to \nu_\mu)$ (solid lines: pure CP-violation effects) and $P(\nu_\mu \to \nu_e) - P(\bar{\nu}_e \to \bar{\nu}_\mu)$ (dashed lines: matter effects) as functions of neutrino energy. Parameters not shown in the graphs are taken as the same as in Fig. 1; $\sin^2 \omega = 1/2$, $\sin^2 \psi = 1/2$, $\sin \delta = 1$; $\rho = \mathrm{g/cm^3}$ and $L = 300$ km.

The ''T violation'' is, however, less than 10% in the case that δm^2_{21} is as small as 3×10^{-5} eV^2 (see the left four graphs of Fig. 2). In this case matter effect is as large in magnitude as ''T violation'' and has an opposite sign for $\sin\delta > 0$ as seen in Fig. 2. In such a case the sum of the two, Eq. (16), is destructive and has even more smaller magnitude than ''T violation,'' thus the experiments will be more difficult. Thanks to ν_e and $\bar{\nu}_e$ available from low energy muon source, one can measure ''T violation.'' This makes the measurement much easier.

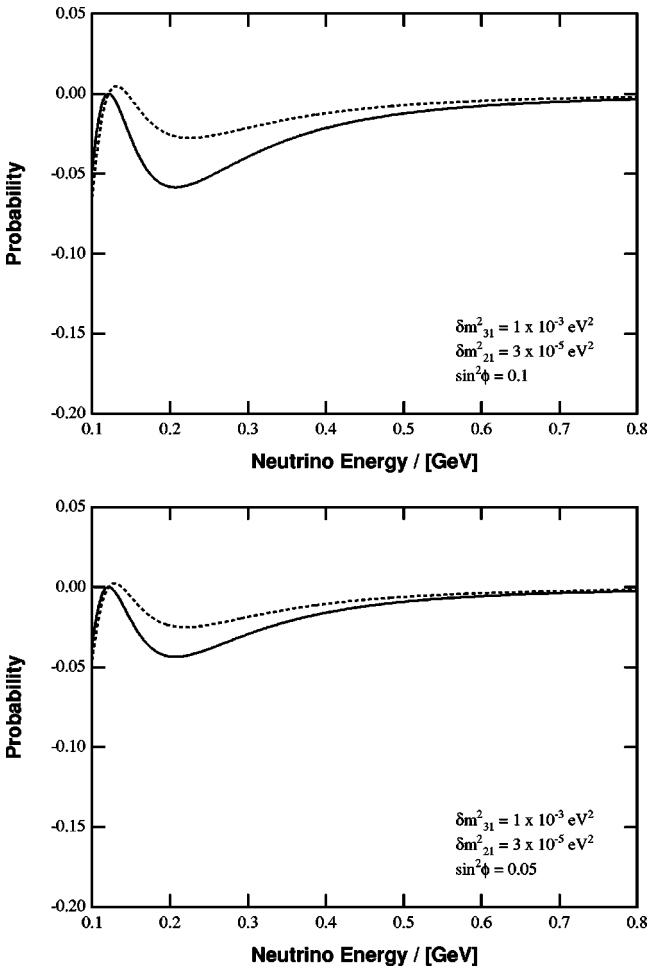

FIG. 3. We compare the magnitudes of ''*T*-violation'' [Eq. (17)] and the ''*CP* violation'' [Eq. (16)] for some parameters. The parameters not shown in the graphs are taken as the same as in Fig. 2.

In Fig. 3 we compare the magnitudes of ''*T* violation'' [Eq. (17)] and the ''*CP* violation'' [Eq. (16)] for some cases. The peak value of ''*T* violation'' is almost twice larger than that of ''*CP* violation.'' We consider that this is a major advantage of the availability of the initial $\nu_e(\overline{\nu_e})$ beam.

B. Estimation of statistical error in *CP*-violation searches

Here we state that the energy range considered here is probably best in view of statistical errors in order to observe the *CP*-violation effect. To this end let us estimate how $\delta P/\Delta P$ scales with E and L, where δP be statistical error of transition probabilities such as $P(\nu_e \to \nu_\mu)$ and $\Delta P = P(\nu_e \to \nu_\mu) - P(\nu_\mu \to \nu_e)$. We denote in this section the transition probabilities $P(\nu_\alpha \to \nu_\beta)(\alpha \neq \beta)$ simply by P. Suppose that n neutrinos out of N detected neutrinos has changed its flavor. With a number of decaying muons fixed, the number of detected neutrinos N are roughly proportional to E^3, and hence $N \sim E^3 L^{-2}$. We estimate δP as

TABLE I. The E-dependence of oscillation envelopes of some quantities with L fixed. Here ''const'' means that the oscillation envelope of the quantity is independent of E. $\delta P/\Delta P$ reaches minimum at the region $E \sim \delta m^2_{31} L$.

E	$\delta m^2_{21} L$		$\delta m^2_{31} L$	
P	''const.''	$1/E$ or ''const.''		$1/E^2$
δP	$1/E^{1.5}$	$1/E^{1.5} \sim 1/E^2$		$1/E^{2.5}$
ΔP	''const''	$1/E$		$1/E^3$
$\delta P/\Delta P$	$1/E^{1.5}$	$1/E^{0.5} \sim 1/E$		$E^{0.5}$
	$\searrow$	$\searrow$	minimum	$\nearrow$

$$\delta P = \delta\left(\frac{n}{N}\right) = \frac{|N\,\delta n| + |n\,\delta N|}{N^2} = \frac{|N\sqrt{NP}| + |NP\sqrt{N}|}{N^2}$$

$$= \frac{\sqrt{P} + P}{\sqrt{N}}, \tag{19}$$

where we used $\delta n = \sqrt{n}$, $\delta N = \sqrt{N}$, and $n = NP$. From Eqs. (13), (17), and (19), we can estimate how $\delta P/\Delta P$ scales for E with L fixed. We summarize the results in Table I. There we see that $\delta P/\Delta P$ reaches minimum at the region $E \sim \delta m^2_{31} L$. Note that this situation is quite different from that for the transition probability P itself.

By a similar consideration one can obtain how $\delta P/\Delta P$ scales for L with E fixed. The result for this case is shown in Table II. We can see there that we should not keep too large an L so that the error $\delta P/\Delta P$ should not get large.

We need a few hundreds MeV of neutrino energy to reach the threshold energy of muon production reaction $N + \nu_\mu \to N + \mu$, where N is the nucleon. We have also seen in Table I that the error comes to minimum at the region $E \sim \delta m^2_{31} L$. Considering these results, we conclude that $E \sim$(a few)$\times$100 MeV and $L \sim$(a few)$\times$100 km, which we have just considered in this paper, is the best configuration to search *CP* violation in view of statistical error.

IV. SUMMARY AND CONCLUSION

We considered how large *CP* or *T* violation effects can be observed making use of low-energy neutrino beam, inspired by PRISM. More than 10%, hopefully 20%, of the pure *CP*-violation effects may be observed within the allowed region of present experiments.

We have also seen that in some cases the pure

TABLE II. The L-dependence of oscillation envelopes of some quantities with E fixed.

L	$\delta m^2_{21}/E$		$\delta m^2_{31}/E$	
P	L^2	L or ''const.''		''const.''
δP	L^3	$L^{1.5} \sim L$		L
ΔP	L^3	L		''const.''
$\delta P/\Delta P$	''const.''	$L^{0.5} \sim$ ''const.''		L
	$\to$	$\nearrow$		$\nearrow$

CP-violation effects are as small as the matter effect but have opposite sign. In such a case the ''CP violation'' gets smaller through the destructive sum of the pure CP-violation effect and matter effect. We pointed out that we can avoid this difficulty by observing ''T-violation'' effect using initial ν_e beam.

We finally discussed that the configuration we have considered here, $E\sim$(a few)$\times$100 MeV and $L\sim$(a few)$\times$100 km is best to search lepton CP violation in terms of statistical error. It is thus worth making an effort to develop leptonic CP violation search using neutrinos from low energy muons.

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