Logarithmic expansion of electroweak corrections to four-fermion processes in the TeV region

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Starting from a theoretical representation of the electroweak component of four-fermion neutral current processes that uses as theoretical input the experimental measurements at the Z peak, we consider the asymptotic high energy behavior in the standard model at one loop of those gauge-invariant combinations of self-energies, vertices and boxes that contribute all the different observables. We find that the logarithmic contribution due to the renormalization group running of the various couplings is numerically overwhelmed by single and double logarithmic terms of purely electroweak (Sudakov-type) origin, whose separate relative effects grow with energy, reaching the 10% size at about 1 TeV. We then propose a simple ''effective'' parametrization that aims at describing the various observables in the TeV region, and discuss its validity both beyond and below 1 TeV, in particular in the expected energy range of future linear electron-positron (LC) and muon-muon colliders.

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I. INTRODUCTION

The construction of future lepton-antilepton (l^+l^-) colliders in energy ranges varying from a few hundreds of GeV (LC) to a few TeV (muon collider) is being thoroughly investigated at the moment [1,2]. One of the key points of all existing proposals is the availability of extremely high luminosities. These would lead to an experimental accuracy for standard four fermion processes (i.e. $l^+l^- \to f\bar{f}$) comparable with that obtained at the Z peak, thus allowing high precision tests of electroweak models at one loop to be performed in the same spirit.

On the theoretical side, the possibility that such extremely precise measurements are performed requires imperatively the existence of suitable computational programs that are able to provide numerical predictions of comparable accuracy, both for the standard model (SM) case and, possibly, for electroweak models of different origin. For CERN e^+e^- collider LEP2 physics such programs already exist at the one loop level for the SM [3] and, at least in principle, their extrapolation to higher energies should be conceivable. There are, though, in our opinion, at least two points that deserve special attention in this respect.

The first one is the importance of understanding the role of the several terms that contribute the various observables when the energy becomes very large. At the Z peak, the most spectacular one loop effects were provided by fermion contributions to gauge boson self-energies (the Higgs effect is notoriously [4] screened), with the exception of the significant contribution due to the celebrated $Zb\bar{b}$ vertex [5]. When the energy increases and moves towards the 1 TeV region, this fermion dominance is apparently weakened, and bosonic effects of vertex and particularly of box type appear to rise, as first stressed in a previous paper [6]. To confirm this rise and to understand in a simple way its physical origin would be, in our opinion, an important achievement both within the standard model framework and beyond it.

The second point that might be worthwhile to examine is that of the reliability of a perturbative description at the one loop level when the energy becomes very large, say beyond the 1 TeV orientative value. In particular, a problem that might arise is that of having to perform a resummation that takes into account higher order leading effects, as in the case of the running of α_{QED}. In fact, it is well known that this problem is present in QED and QCD diagrams when Sudakov logarithms of the type $\ln q^2/\mu^2$, $\ln^2 q^2/\mu^2$ (where $\sqrt{q^2}$ is the available energy and μ is the mass scale of the diagram) appear in vertices and boxes [7]. In the standard model, logarithms of a similar type (usually called ''of Sudakov type'') are also known to appear [8,9]. In Ref. [9] the leading asymptotic behavior growing like $\ln^2 q^2/M_{W,Z}^2$ was calculated for the relevant observables. The physical origin of these double logarithms was also discussed. In this paper we also calculate the subleading terms, growing like a single logarithm $\ln q^2/M_{W,Z}^2$, where $M_{W,Z}$ are the physical gauge boson masses. If the relative contribution coming from these diagrams became large, typically beyond a ''reasonable'' percentage amount (which could also be quantitatively fixed by knowledge of the requested theoretical accuracy), the necessity for a resummation would become stringent (and, to our knowledge, this study has not yet been performed).

The aim of this paper is precisely that of discussing the two previous points. In particular, we shall perform in Sec. II an explicit calculation of the coefficients of the three types of logarithms that dominate the large energy behavior of the different one loop gauge-invariant combinations that make up all observables of the process. These come from the renormalization group (RG) running of the gauge couplings and from the typically electroweak Sudakov-type effects. The first ones generate linear logarithms, the second ones linear and quadratic logarithms. As we shall show, the numerical size of the various Sudakov logarithms largely overwhelms that of the RG ones. Technically speaking, the main contribution turns out to be provided by diagrams with W

bosons (those with a Z boson are relatively less important), more precisely by special combinations of the non-universal components of vertices and boxes (''oblique'' self-energies are only contributing the RG component). Thus, at very high energy, the relevance of ''non-oblique'' contributions appears essential, in opposition to the situation that is met at the Z peak.

Having computed the various gauge-invariant combinations, we shall then evaluate in Sec. III the asymptotic expression of all the possible observables (i.e. cross sections and asymmetries). Here the reliability of the one loop expansion appears to depend critically on the c.m. energy of the process. Typically, we find that, beyond a certain value that depends on the chosen observable, the *separate* gauge-invariant relative effects begin to cross the dangerous 10% value (orientatively chosen as a realistic limit). This makes the validity of the approximation rather doubtful if the requested precision must be below the 1% level, and indicates the need for a proper resummation already in the TeV range.

As a by-product of our calculation, we shall propose in Sec. IV a simple ''effective'' parametrization of all observables, aiming to provide a satisfactory approximation of the one loop component in the energy range between 1 TeV and a few TeV. We shall also compare this expression with the complete SM calculations in the region below 1 TeV and discuss its possible practical interest and its limitations in this not rigorously asymptotic regime, in the particular theoretical framework provided by the SM. A final discussion given in Sec. V will conclude the paper, and Appendixes A and B will contain the analytic expressions of the most relevant one loop diagrams (self-energies, vertices and boxes) and the way they contribute to the various observables.

II. ASYMPTOTIC EXPRESSIONS OF THE GAUGE-INVARIANT RELEVANT COMBINATIONS AT ONE LOOP

A. Preliminary discussion

The starting point of our paper, which has been already illustrated in several previous references [6,10], is the choice of a theoretical representation of four fermion neutral current processes, strictly valid at one loop, that uses as theoretical input in addition to the conventionally defined electric charge, the extremely high precision measurements performed on top of the Z resonance at LEP1 or the SLAC Linear Collider (SLC). The consequences of this attitude, in which the usual parameter G_μ is replaced by Z peak observables (this does not introduce any appreciable theoretical error [10]), are that all the physical information for the process $l^+l^- \to f\bar{f}$ (where f is either a charged lepton, in which case $f \equiv l$, or a u,d,s,c,b quark,[1] and *for the moment* all external fermions are supposed to be massless) is provided by four different functions of q^2 and θ (the squared c.m. energy and scattering angle). The first one is the ''photonic form factor'' $\Delta_{\alpha,lf}(q^2,\theta)$ which is subtracted at $q^2=0$, whereas the three other ones, $R_{lf}(q^2,\theta)$, $V_{lf}^{\gamma Z}(q^2,\theta)$, $V_{lf}^{Z\gamma}(q^2,\theta)$ corresponding to the one loop ZZ, γZ and $Z\gamma$ transitions, are subtracted at $q^2=M_Z^2$. All four functions are separately gauge independent, as discussed in Ref. [10] on the basis of previous observations made by Degrassi and Sirlin [11,12]. These functions receive two types of one loop contributions: universal (i.e. lf-independent) ones, coming from fermionic and bosonic self-energies as well as from a part of some vertex diagrams, and non-universal ones, coming from the remaining parts of the vertex diagrams and from the box diagrams. As we shall see in the next subsection, some universal contributions correspond to the running of α, g_Z^2 and s_W^2. The other contributions lead asymptotically to the ''Sudakov-type'' terms that we shall explicitly review in Appendix A. The way these four functions enter the various $e^+e^- \to f\bar{f}$ observables will be shown in Sec. III and in Appendix B.

B. RG contributions to the gauge-invariant combinations

Following our previous discussion, we shall first consider the contributions to $\tilde{\Delta}_\alpha$, R, V from the class of diagrams that are supposed to reproduce the canonical RG asymptotic behavior. In the R_ξ gauge in which we are working there will be no contributions from the (finite) boxes to this sector. A simple request will select the possible contributions from vertices. In fact, the various combinations that we consider are all, as we stressed several times, gauge independent. But the contribution from self-energies is not such, owing to the set of bosonic ''bubbles'' that must be considered. Therefore extra terms from vertices must be properly added. Since self-energy contributions are of universal type, the same property must obtain for the selected vertices. In practice, this limits the choice to vertices where two W's are involved. Note that since we shall be working in the $\xi=1$ 't Hooft gauge, diagrams with would-be Goldstone bosons must not be neglected in all self-energies (in fact, also, those with ghosts must be considered). In the *universal* component of the vertices, the would-be contribution vanishes (in practice, it must only be retained in the not universal component of vertices with final $b\bar{b}$ pairs).

Our previous statement can be reformulated exactly using a previous definition that can be found in Ref. [12]. In fact, it would be easy to show that the amount of WW vertices that must be added to the various self-energies is fully provided by the so-called ''pinch'' component [13], and for more details we defer to Ref. [13]. Otherwise stated, the combinations of self-energies and ''pinch'' vertices that make up the RG behavior in $\tilde{\Delta}_\alpha$, R, V correspond rigorously to what was called ''gauge-invariant self-energies'' by Degrassi and Sirlin [11,12].

In terms of Feynman diagrams, we must then consider the set represented in Figs. 1 and 2. The full expression of the various self-energies and vertices of Figs. 1 and 2 can be easily computed. One finds it in the Appendix of Ref. [6],

[1]Note that our theoretical formalism cannot be applied in its present formulation to the production of $t\bar{t}$ pairs for the simple reason that the necessary LEP1-SLC information does not exist ($t\bar{t}$ cannot be produced). The study of this process requires modifications that are being studied.

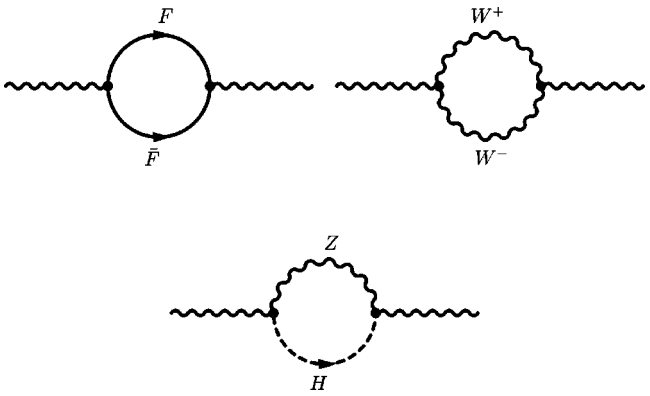

FIG. 1. Self-energy diagrams for γ,Z gauge bosons. It must be understood that a W or Z running inside the loop is accompanied by its corresponding Goldstone boson and ghost states.

and we shall not rewrite it here since several other new lengthy formulas will have to be written. From those expressions one derives in a straightforward way the *universal* (i.e., θ, lf-independent) asymptotic behavior of $\tilde{\Delta}_\alpha$, R, V, which read, for $q^2 \gg \mu^2$,

$$\tilde{\Delta}_\alpha^{(RG)}(q^2,\theta) \to \frac{\alpha(\mu^2)}{12\pi}\left[\frac{32}{3}N-21\right]\ln\left(\frac{q^2}{\mu^2}\right) \tag{1}$$

$$R^{(RG)}(q^2,\theta) \to -\frac{\alpha(\mu^2)}{4\pi s^2 c^2}\left[\frac{20-40c^2+32c^4}{9}N + \frac{1-2c^2-42c^4}{6}\right]\ln\left(\frac{q^2}{\mu^2}\right) \tag{2}$$

$$\frac{c}{s}V_{\gamma Z}^{(RG)}(q^2,\theta) = \frac{c}{s}V_{Z\gamma}^{(RG)}(q^2,\theta) \to \frac{\alpha(\mu^2)}{3\pi s^2}\left[\frac{10-16c^2}{6}N+\frac{1+42c^2}{8}\right] \times \ln\left(\frac{q^2}{\mu^2}\right). \tag{3}$$

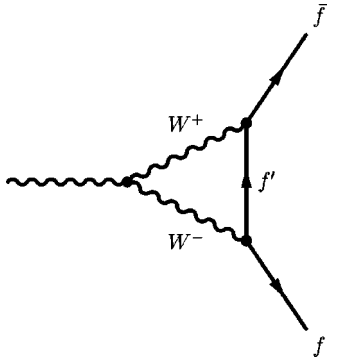

FIG. 2. WW triangle contribution to the γ-$f\bar{f}$, Z-$f\bar{f}$ vertex. Here also the contribution of the corresponding Goldstone bosons is to be added.

In the previous equations, N is the number of fermion families, and μ is a ''reference'' scale that will be chosen following practical arguments. In our special case, where a major part of the theoretical input is fixed at the Z mass, it seems rather natural to fix, correspondingly, the value $\mu = M_Z$. This sets the scale at which the asymptotic expression should become valid, $q^2 \gg M_Z^2$. Note that, in order to follow consistently this choice, we should replace the theoretical input $\alpha(0)$ with $\alpha(M_Z^2)$, even in the photonic component. This does not introduce any substantial theoretical error, given the available accuracy of the (theoretical) determination of $\alpha(M_Z^2)$ [14]. Finally, choosing $\mu = M_Z$ suggests also to identify the parameters s^2, c^2 of Eqs.(1)–(3) with the experimentally measured quantities $s_l^2(M_Z^2)$ and $c_l^2(M_Z^2)$ for which we shall take, following the common attitude, the LEP1 and SLAC Large Detector (SLD) average [15]:

$$s_l^2(M_Z^2) = 0.23157(18). \tag{4}$$

To verify that Eqs. (1)–(3) do indeed reproduce the running of the various standard model couplings is now straightforward. The relevant expressions can be found in previous references, e.g., [16]. In order to make this discussion reasonably self-consistent we write the two following formulas:

$$\alpha^{(RG)}(q^2) = \alpha^{(RG)}(\mu^2)\Big/\left\{1-\left[\frac{\alpha^{(RG)}(\mu^2)}{12\pi}\left(\frac{32}{3}N-21\right)\right]\ln\frac{q^2}{\mu^2}\right\} \tag{5}$$

$$g^{2(RG)}(q^2) = g^2(\mu^2)\Big/\left\{1+\left[\frac{g^2(\mu^2)}{96\pi^2}(43-8N)\right]\ln\frac{q^2}{\mu^2}\right\} \tag{6}$$

where μ is the arbitrary reference scale. To derive the corresponding expressions for g_Z^2, s_l^2 is immediate using the corresponding definitions ($s_l^2 \equiv e^2/g^2$, $g_Z^2 \equiv g^2/[1-(e^2/g^2)]$).

One can thus realize that the RG running of $R(q^2,\theta)$ is that of g_Z^2 and that of $(c/s)V_{\gamma Z}^{(RG)}(q^2,\theta)$ is exactly that of $\sin^2\theta_W$ as implied by the definitions of Refs. [10,11]: i.e.,

$$g_{Vl}^{(1)}(q^2,\theta) = I_{3L} - 2Q_l s_l^2(q^2,\theta) \tag{7}$$

with

$$s_l^2(q^2,\theta) = \sin^2\theta_{W,0} + sc\tilde{F}_{\gamma Z,lf}(q^2,\theta) = s_l^2(M_Z^2)\left[1+\frac{c}{s}V_{lf}^{\gamma Z}(q^2,\theta)\right]. \tag{8}$$

We have therefore derived the universal RG component of the gauge invariant functions $\tilde{\Delta}_\alpha$, R, V that leads to an asymptotic (linear) logarithmic behavior. From a formal point of view, the existence of these components is due to the ultraviolet *divergence* of the generating Feynman diagrams (this is, of course, cancelled in the physical subtracted combinations). In fact, the coefficients of the logarithmic terms $\ln q^2$ are exactly the same (with opposite sign) as those

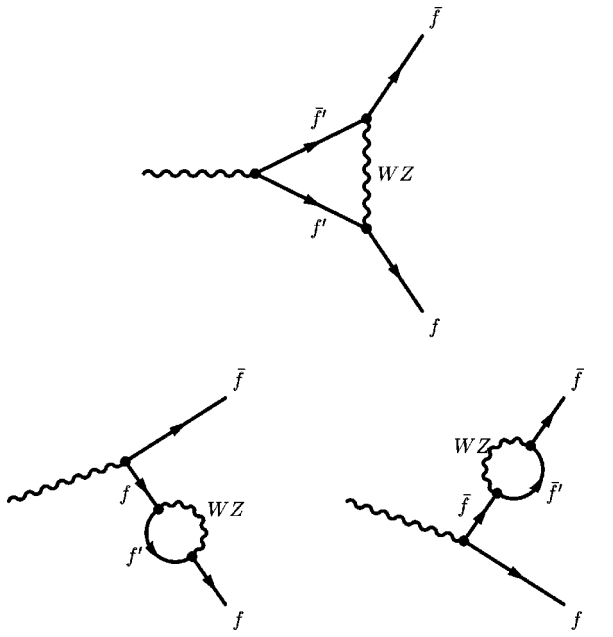

FIG. 3. Single W or Z exchange contribution to the γ-$f\bar{f}$, Z-$f\bar{f}$ vertex. Here also the contribution of the corresponding Goldstone bosons is to be added.

of the divergent terms $\propto 1/(d-4)$ in the various diagrams. When we turn to the task of evaluating other possible asymptotic logarithmic contributions, we expect that these will actually be produced by ultraviolet *finite* quantities, more precisely by non-universal vertices and boxes, as will be discussed in the following section, Sec. II C.

C. Sudakov-type contributions to the gauge-invariant combinations

From the previous discussion we have learned that the set of one loop contributions that have not been considered yet must be ultraviolet finite. This simple statement already enables us to write the list of the remaining quantities, which with our choice of $\xi=1$ gauge must be either vertices or boxes. More precisely, we shall find here (a) the ''non-pinch'' component of the vertices with two W's where the pure non-RG behavior survives, Fig. 2; (b) vertices with one

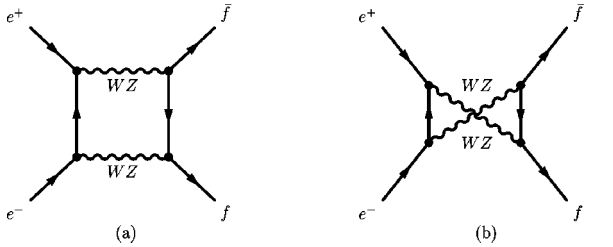

FIG. 4. WW and ZZ box contributions to $e^+e^-\to f\bar{f}$. In the WW case diagram (a) contributes for $I_{3f}=-\frac{1}{2}$, whereas diagram (b) contributes for $I_{3f}=+\frac{1}{2}$. In the ZZ case both diagrams contribute.

W or Z exchange, Fig. 3; and (c) boxes, Fig. 4. Before performing the explicit calculation of the asymptotic behavior of the diagrams represented in Figs. 2–4, a brief preliminary discussion is now suited.

Infrared (IR) divergences arise in perturbative calculations from regions of integration over the momentum k where k is small compared to the typical scales of the process. This is a well-known fact in QED for instance, where the problem of an unphysical divergence is solved by giving the photon a fictitious mass which acts as a cutoff for the IR divergent integral. When real (bremsstrahlung) and virtual contributions are summed, the dependence on this mass cancels and the final result is finite. The (double) logarithms coming from these contributions are large and, growing with the scale, can spoil perturbation theory and need to be resummed. They are usually called Sudakov double logarithms [7]. In the case of electroweak corrections, similar logarithms arise when the typical scale of the process considered is much larger than the mass of the particles running in the loops, typically the $W(Z)$ mass [8]. The expansion parameter that results is then $(\alpha/4\pi s^2)\ln^2(q^2/M_W^2)$, which is already 10% for energies $\sqrt{q^2}$ of the order of 1 TeV. This kind of correction becomes therefore particularly relevant for next generation of linear colliders (LC's [2]). In the case of corrections coming from loops with $W(Z)$'s, there is no equivalent of ''bremsstrahlung'' like in QED or QCD: the $W(Z)$, unlike the photon and the gluon, has a definite non-zero mass and is experimentally detected like a separate particle. In this way the full dependence on the $W(Z)$ mass is retained in the corrections.

In conclusion, for the process that we consider here, i.e. $l^+l^-\to f\bar{f}$ in the limit of massless external fermions at the one loop level, we expect three kinds of contributions to become ''large'' in the asymptotic $q^2\gg M_{W,Z}^2$ region:

(i) Single logarithms $[\ln(s/\mu^2)]$ coming from UV divergences which can be reabsorbed by running of bare parameters.

(ii) Single logarithms $[\ln(s/m_{Z,W}^2)]$ coming from the analogue of QED collinear divergences.

(iii) Double logarithms $[\ln^2(s/m_{Z,W}^2)]$ coming from the analogue of QED divergences that are of IR and collinear origin.

The double logarithmic contributions come from vertex corrections in which one gauge boson is exchanged and from (direct and crossed) boxes with two Z's or two W's. The single collinear logarithms come also from vertex and box diagrams. The single UV logarithms affect self-energies and the vertex.

Let us consider $\gamma(Z)f\bar{f}$ vertices first. Using the definition

$$\Gamma^{\gamma(Z)}_{\mu,f}\equiv\bar{v}_f(p_1)\gamma_\mu(\mathcal{V}^{\gamma(Z)}_{fL}P_L+\mathcal{V}^{\gamma(Z)}_{fR}P_R)u_f(p_2)$$
$$\text{with}\quad P_{L,R}=\frac{1\mp\gamma_5}{2} \tag{9}$$

we make an asymptotic expansion of the various vertices.

Subtracting the ''RG divergent component'' of the vertices that has been discussed previously, we obtain

$$\mathcal{V}^{\gamma}_{fL}=igsQ_f-\frac{1}{16\pi^2}\frac{g^2}{c^2}g^2_{fL}F[m_Z] \tag{10}$$

$$-\frac{1}{16\pi^2}\frac{g^2}{2}\frac{Q_{f'}}{Q_f}F[m_W]$$

$$-\frac{1}{16\pi^2}g^2\frac{T_{3f}}{Q_f}G[m_W] \tag{11}$$

$$\mathcal{V}^{\gamma}_{fR}=igsQ_f\left(-\frac{1}{16\pi^2}\frac{g^2}{c^2}g^2_{fR}F[m_Z]\right) \tag{12}$$

and

$$\mathcal{V}^{Z}_{fL}=i\frac{g}{c}g_{fL}-\frac{1}{16\pi^2}\frac{g^2}{c^2}g^2_{fL}F[m_Z] \tag{13}$$

$$-\frac{1}{16\pi^2}\frac{g^2}{2}\frac{g_{f'L}}{g_{fL}}F[m_W]$$

$$-\frac{1}{16\pi^2}g^2\frac{T_{3f}c^2}{g_{fL}}G[m_W] \tag{14}$$

$$\mathcal{V}^{Z}_{fR}=\frac{g}{c}g_{fR}\left(-\frac{1}{16\pi^2}\frac{g^2}{c^2}g^2_{fR}F[m_Z]\right) \tag{15}$$

where

$$F[m]\equiv-4\ln\frac{q^2}{m^2}+\ln^2\frac{q^2}{m^2},\quad G[m]\equiv-4\ln\frac{q^2}{m^2}. \tag{16}$$

Here f is the external fermion and f' its isospin partner. Moreover, $g_{f(f')R}=-Q_{f(f')}s^2$, $g_{f(f')L}=T_3^{f(f')}-Q_{f(f')}s^2$ and $Q_f-Q_{f'}=2T_{3f}$, $T_{3f}=-T_{3f'}$.

For the boxes, defining

$$\bar{v}_l(p_1)\gamma_\mu P_{L,R}u_l(p_2)\bar{u}_f(p_3)\gamma_\mu P_{L,R}v_f(p_4)\equiv\tilde{P}_{L,R}\otimes\tilde{P}_{L,R} \tag{17}$$

we have computed the corrections from direct and crossed box diagrams as a sum of projected amplitudes on the left-right chiral basis:

$$A^{Box}_{LL,lf}\tilde{P}_L\otimes\tilde{P}_L+A^{Box}_{LR,lf}\tilde{P}_L\otimes\tilde{P}_R+A^{Box}_{RL,lf}\tilde{P}_R\otimes\tilde{P}_L$$
$$+A^{Box}_{RR,lf}\tilde{P}_R\otimes\tilde{P}_R$$

(projecting on the ''photon,'' ''Z'' Lorentz structures is then straightforward).

For the various components we find the following asymptotic expansions:

$$A^{Box}_{LL,lf}=\frac{\alpha e^2}{4\pi s^4}D^W_f+\frac{\alpha e^2}{\pi s^4c^4}[g^2_{lL}g^2_{fL}]D^Z \tag{18}$$

$$A^{Box}_{LR,lf}=\frac{\alpha e^2}{\pi s^4c^4}[g^2_{lL}g^2_{fR}]D^Z \tag{19}$$

$$A^{Box}_{RL,lf}=\frac{\alpha e^2}{\pi s^4c^4}[g^2_{lR}g^2_{fL}]D^Z \tag{20}$$

$$A^{Box}_{RR,lf}=\frac{\alpha e^2}{\pi s^4c^4}[g^2_{lR}g^2_{fR}]D^Z \tag{21}$$

where the functions that appear above are

$$D^W_{\mu,d}=-\frac{1}{2q^2}\ln^2\frac{q^2}{M^2_W}-\frac{1}{q^2}\ln\frac{1-\cos\theta}{2}\ln\frac{q^2}{M^2_W}$$

$$D^W_u=\frac{1}{2q^2}\ln^2\frac{q^2}{M^2_W}+\frac{1}{q^2}\ln\frac{1+\cos\theta}{2}\ln\frac{q^2}{M^2_W} \tag{22}$$

$$D^Z=\frac{1}{q^2}\ln\frac{1+\cos\theta}{1-\cos\theta}\ln\frac{q^2}{M^2_Z}. \tag{23}$$

Equations (10)–(23) are the main results of this paper. They contain in a compact form the leading asymptotic Sudakov-type (double and single logarithmic) contributions. Using the procedure given in [10] it is now straightforward to compute the corresponding contributions to the four gauge-invariant combinations that also depend on the chosen final fermion. These are written in the complete form that is given in Appendix A. For the practical purposes of our paper we shall now write the numerical expressions of $\tilde{\Delta}_\alpha$, R, V that are obtained by summing the RG contributions of Eqs. (1)–(3) to those given in Appendix A. Taking the value of s^2 given in Eq. (4) leads to the following result:

$$\tilde{\Delta}_{\alpha,l\mu}(q^2,\theta)\to\frac{\alpha(\mu^2)}{4\pi}\Bigg\{(-7.00+10.67)\ln\frac{q^2}{\mu^2}+6.00\ln^2\frac{q^2}{M_W^2}+2.10\ln\frac{q^2}{M_Z^2}-0.70\ln^2\frac{q^2}{M_Z^2}$$
$$+\left[-2.00\ln^2\frac{q^2}{M_W^2}-4.00\ln\frac{1-\cos\theta}{2}\ln\frac{q^2}{M_W^2}+0.49\ln\frac{1+\cos\theta}{1+\cos\theta}\ln\frac{q^2}{M_Z^2}\right]\Bigg\} \tag{24}$$

$$R_{l\mu}(q^2,\theta)\to\frac{\alpha(\mu^2)}{4\pi}\Bigg\{(23.73-15.28)\ln\frac{q^2}{\mu^2}-6.96\ln\frac{q^2}{M_W^2}-4.32\ln^2\frac{q^2}{M_W^2}-2.13\ln\frac{q^2}{M_Z^2}+0.71\ln^2\frac{q^2}{M_Z^2}$$
$$+\left[6.64\ln^2\frac{q^2}{M_W^2}+13.27\ln\frac{1-\cos\theta}{2}\ln\frac{q^2}{M_W^2}-0.03\ln\frac{1+\cos\theta}{1+\cos\theta}\ln\frac{q^2}{M_Z^2}\right]\Bigg\} \tag{25}$$

$$V_{\gamma Z,l\mu}(q^2,\theta)=V_{Z\gamma,l\mu}(q^2,\theta)\to\frac{\alpha(\mu^2)}{4\pi}\Bigg\{(13.15-3.63)\ln\frac{q^2}{\mu^2}-7.37\ln\frac{q^2}{M_W^2}-1.19\ln^2\frac{q^2}{M_W^2}-0.35\ln\frac{q^2}{M_Z^2}$$
$$+0.12\ln^2\frac{q^2}{M_Z^2}+\left[3.64\ln^2\frac{q^2}{M_W^2}+7.29\ln\frac{1-\cos\theta}{2}\ln\frac{q^2}{M_W^2}-0.12\ln\frac{1+\cos\theta}{1+\cos\theta}\ln\frac{q^2}{M_Z^2}\right]\Bigg\} \tag{26}$$

$$\tilde{\Delta}_{\alpha,lu}(q^2,\theta)\to\frac{\alpha(\mu^2)}{4\pi}\Bigg\{(-7.00+10.67)\ln\frac{q^2}{\mu^2}+5.00\ln\frac{q^2}{M_W^2}+0.33\ln^2\frac{q^2}{M_W^2}+1.95\ln\frac{q^2}{M_Z^2}-0.65\ln^2\frac{q^2}{M_Z^2}$$
$$+\left[-2.00\ln^2\frac{q^2}{M_W^2}-4.00\ln\frac{1+\cos\theta}{2}\ln\frac{q^2}{M_W^2}-0.63\ln\frac{1+\cos\theta}{1+\cos\theta}\ln\frac{q^2}{M_Z^2}\right]\Bigg\} \tag{27}$$

$$R_{lu}(q^2,\theta)\to\frac{\alpha(\mu^2)}{4\pi}\Bigg\{(23.73-15.28)\ln\frac{q^2}{\mu^2}-7.96\ln\frac{q^2}{M_W^2}-3.99\ln^2\frac{q^2}{M_W^2}-2.59\ln\frac{q^2}{M_Z^2}+0.86\ln^2\frac{q^2}{M_Z^2}$$
$$+\left[6.64\ln^2\frac{q^2}{M_W^2}+13.27\ln\frac{1+\cos\theta}{2}\ln\frac{q^2}{M_W^2}+0.16\ln\frac{1+\cos\theta}{1+\cos\theta}\ln\frac{q^2}{M_Z^2}\right]\Bigg\} \tag{28}$$

$$V_{\gamma Z,lu}(q^2,\theta)=V_{Z\gamma,l\mu}(q^2,\theta)\to\frac{\alpha(\mu^2)}{4\pi}\Bigg\{(13.15-3.63)\ln\frac{q^2}{\mu^2}-5.55\ln\frac{q^2}{M_W^2}-1.79\ln^2\frac{q^2}{M_W^2}-1.00\ln\frac{q^2}{M_Z^2}$$
$$+0.33\ln^2\frac{q^2}{M_Z^2}+\left[3.64\ln^2\frac{q^2}{M_W^2}+7.29\ln\frac{1+\cos\theta}{2}\ln\frac{q^2}{M_W^2}+0.63\ln\frac{1+\cos\theta}{1+\cos\theta}\ln\frac{q^2}{M_Z^2}\right]\Bigg\} \tag{29}$$

$$V_{Z\gamma,lu}(q^2,\theta)\to\frac{\alpha(\mu^2)}{4\pi}\Bigg\{(13.15-3.63)\ln\frac{q^2}{\mu^2}-7.92\ln\frac{q^2}{M_W^2}-1.00\ln^2\frac{q^2}{M_W^2}-0.87\ln\frac{q^2}{M_Z^2}+0.29\ln^2\frac{q^2}{M_Z^2}$$
$$+\left[3.64\ln^2\frac{q^2}{M_W^2}+7.29\ln\frac{1+\cos\theta}{2}\ln\frac{q^2}{M_W^2}+0.16\ln\frac{1+\cos\theta}{1+\cos\theta}\ln\frac{q^2}{M_Z^2}\right]\Bigg\} \tag{30}$$

$$\tilde{\Delta}_{\alpha,ld}(q^2,\theta)\to\frac{\alpha(\mu^2)}{4\pi}\Bigg\{(-7.00+10.67)\ln\frac{q^2}{\mu^2}+4.00\ln\frac{q^2}{M_W^2}+0.67\ln^2\frac{q^2}{M_W^2}+1.60\ln\frac{q^2}{M_Z^2}-0.53\ln^2\frac{q^2}{M_Z^2}$$
$$+\left[-2.00\ln^2\frac{q^2}{M_W^2}-4.00\ln\frac{1-\cos\theta}{2}\ln\frac{q^2}{M_W^2}+0.77\ln\frac{1+\cos\theta}{1+\cos\theta}\ln\frac{q^2}{M_Z^2}\right]\Bigg\} \tag{31}$$

$$R_{ld}(q^2,\theta)\to\frac{\alpha(\mu^2)}{4\pi}\Bigg\{(23.73-15.28)\ln\frac{q^2}{\mu^2}-8.96\ln\frac{q^2}{M_W^2}-3.65\ln^2\frac{q^2}{M_W^2}-3.64\ln\frac{q^2}{M_Z^2}+1.21\ln^2\frac{q^2}{M_Z^2}$$
$$+\left[6.64\ln^2\frac{q^2}{M_W^2}+13.27\ln\frac{1-\cos\theta}{2}\ln\frac{q^2}{M_W^2}-0.29\ln\frac{1+\cos\theta}{1+\cos\theta}\ln\frac{q^2}{M_Z^2}\right]\Bigg\} \tag{32}$$

$$V_{\gamma Z,ld}(q^2,\theta)\to\frac{\alpha(\mu^2)}{4\pi}\left\{(13.15-3.63)\ln\frac{q^2}{\mu^2}-3.73\ln\frac{q^2}{M_W^2}-2.40\ln\frac{q^2}{M_W^2}-0.91\ln\frac{q^2}{M_Z^2}+0.30\ln^2\frac{q^2}{M_Z^2}\right.$$
$$\left.+\left[3.64\ln\frac{q^2}{M_W^2}+7.29\ln\frac{1-\cos\theta}{2}\ln\frac{q^2}{M_W^2}-1.15\ln\frac{1+\cos\theta}{1+\cos\theta}\ln\frac{q^2}{M_Z^2}\right]\right\} \tag{33}$$

$$V_{Z\gamma,ld}(q^2,\theta)\to\frac{\alpha(\mu^2)}{4\pi}\left\{(13.15-3.63)\ln\frac{q^2}{\mu^2}-8.47\ln\frac{q^2}{M_W^2}-0.82\ln^2\frac{q^2}{M_W^2}-1.62\ln\frac{q^2}{M_Z^2}+0.54\ln^2\frac{q^2}{M_Z^2}\right.$$
$$\left.+\left[3.64\ln^2\frac{q^2}{M_W^2}+7.29\ln\frac{1-\cos\theta}{2}\ln\frac{q^2}{M_W^2}-0.19\ln\frac{1+\cos\theta}{1+\cos\theta}\ln\frac{q^2}{M_Z^2}\right]\right\}. \tag{34}$$

We have left three scale parameters μ, M_W, M_Z as evidence of the different origin [RG equations (RGEs), W diagrams, Z diagrams] of the various contributions. In the first two lines of each expression there are the contributions coming from bubbles and vertices. In the RGE part of them, the contributions of fermionic and bosonic degrees of freedom are left separated. In practice for numerical estimates we set $\mu = M_Z = 91.187$ GeV. Finally, the contributions from the WW and ZZ box diagrams are systematically grouped in the last three terms inside the square brackets.

Equations (24)–(34) allow us to derive the leading asymptotic behavior of all relevant observables (i.e. cross sections and asymmetries) of the considered four-fermion process. This will be done in the following section.

III. ASYMPTOTIC EXPRESSION OF THE PHYSICAL OBSERVABLES

Starting from Eqs. (24)–(34) it is now straightforward to derive the asymptotic behavior of any given observable. In order to proceed in a systematic way, we shall start from the very general formula

$$\frac{d\sigma_{lf}}{d\cos\theta}=\frac{4\pi}{3}\mathcal{N}_f q^2\left\{\frac{3}{8}(1+\cos^2\theta)[(1-PP')U_{11}+(P'-P)U_{21}]+\frac{3}{4}\cos\theta[(1-PP')U_{12}+(P'-P)U_{22}]\right\} \tag{35}$$

where the quantities U_{ij} are defined in terms of $\tilde{\Delta}^{(lf)}$, $R^{(lf)}$, $V_{\gamma Z}^{(lf)}$, $V_{Z\gamma}^{(lf)}$ in Appendix B, P, P' are the (conventionally defined) *longitudinal* polarization degree of the initial lepton and antilepton, and $\mathcal{N}_f$ is the color factor for the $f\bar{f}$ channel which includes the appropriate QCD corrections to the input.

From the previous equation, and from Eqs. (24)–(34), we can now derive the asymptotic expansion of the relevant observables. We shall first consider the case of unpolarized beams and concentrate our attention on a set of ''typical'' observables that one expects to measure at future colliders. These will be the cross section for muon (and/or tau) production σ_μ, the related forward-backward asymmetry $A_{FB,\mu}$ and the cross section for ''light'' u,d,s,c,b production σ_5.

As we anticipated in the Introduction, we shall treat quark production in the massless quark limit. This will introduce a certain approximation in the treatment of the cross section for b production σ_b where the effects of the non-vanishing top quark mass in the asymptotic regime must be carefully estimated. Since the contribution of σ_b to σ_5 is relatively small, we shall not treat this extra effect in this paper. We shall rather postpone a complete rigorous discussion of m_t effects to a forthcoming article.

Collecting all our numerical formulas leads to the final asymptotic expressions of the previous observables, which read ($N=3$ is the number of fermion families and $\mu^2=M_Z^2 \approx M_W^2$)

$$\sigma_\mu=\sigma_\mu^B\left[1+\frac{\alpha}{4\pi}\left\{(7.72N-20.58)\ln\frac{q^2}{\mu^2}+35.27\ln\frac{q^2}{M_W^2}-4.59\ln^2\frac{q^2}{M_W^2}+4.79\ln\frac{q^2}{M_Z^2}-1.43\ln^2\frac{q^2}{M_Z^2}+\cdots\right\}\right] \tag{36}$$

$$A_{FB,\mu}=A_{FB,\mu}^B+\frac{\alpha}{4\pi}\left\{(0.54N-5.90)\ln\frac{q^2}{\mu^2}+10.19\ln\frac{q^2}{M_W^2}-0.08\ln^2\frac{q^2}{M_W^2}+1.25\ln\frac{q^2}{M_Z^2}-0.004\ln^2\frac{q^2}{M_Z^2}+\cdots\right\} \tag{37}$$

$$\sigma_5=\sigma_5^B\left[1+\frac{\alpha}{4\pi}\left\{(9.88N-42.66)\ln\frac{q^2}{\mu^2}+46.58\ln\frac{q^2}{M_W^2}-6.30\ln^2\frac{q^2}{M_W^2}+7.25\ln\frac{q^2}{M_Z^2}-2.03\ln^2\frac{q^2}{M_Z^2}+\cdots\right\}\right]. \tag{38}$$

The precise definition of the ''Born'' (B) quantities that appear in these equations will be given in the forthcoming section. The ellipses that appear in the brackets refer to the ''non-leading'' terms. These could either be constants or $O(1/q^2)$ components whose asymptotic effect vanishes. At

non-asymptotic energies, such terms (in particular the constants) should be discussed; we shall return to this point in the following section.

Next, we consider the simplest case of full lepton longitudinal polarization and treat one simple observable: the longitudinal polarization asymmetry for final lepton production, $A_{LR,\mu}$. Using the same procedure leads to the following expressions:

$$A_{LR,\mu}=A^B_{LR,\mu}+\frac{\alpha}{4\pi}\left\{(1.82N-19.79)\ln\frac{q^2}{\mu^2}+30.76\ln\frac{q^2}{M_W^2}-3.52\ln^2\frac{q^2}{M_W^2}+0.78\ln\frac{q^2}{M_Z^2}-0.17\ln^2\frac{q^2}{M_Z^2}+\cdots\right\}, \tag{39}$$

where the ''Born'' asymmetry will be defined later on.

Equations (36)–(39) are the physical predictions of this paper. Looking at their expressions one immediately notices the following main features:

(a) The coefficient of the RG linear logarithm is much smaller than that of the Sudakov one. A naive expectation of an asymptotic behavior essentially reproduced by the RG logarithms would be therefore, at this stage, completely wrong. In other words, the high energy behavior of SM observables is only partially reabsorbed by the running of the coupling constants.

(b) The role of the Sudakov squared logarithm is numerically relevant in almost all the considered observables, with the exception of the muon asymmetry where it almost vanishes.

(c) Both the linear and the quadratic logarithmic terms are, separately taken, relatively ''large.'' However, at the considered one loop level, they have opposite sign and the overall contribution is smaller. This raises a few important questions that we shall try now to investigate.

The first question is that of whether the various logarithmic terms have separately a physical meaning. In our opinion, at the considered one loop level, this must be evidently the case. In fact, since the overall expansion is necessarily gauge independent, the different powers of the logarithm must satisfy the same request (note that this applies, in general, to the *sum* of the separate contributions from vertices and boxes). *A priori*, one expects that a gauge-independent quantity might (should) have a physical meaning. For the RG component, this is connected with the running of the couplings. For the remaining terms, their origin is related, as we have seen previously, to the fact that a final state W is supposed to be experimentally detectable.

The second question is that of the reliability of the one loop perturturbative electroweak corrections to the considered observables. This is fixed by the aimed theoretical accuracy, which in turn is dictated by the expected experimental precision. For future l^+l^- colliders we shall stick to a conservative expectation of about 1 % experimental (relative) accuracy. In this spirit, we shall assume a ''reliability barrier'' for one loop effects at the relative 10% level; of course, these numbers can be easily changed without modifying the philosophy of the approach. Having fixed the accuracy level, we can proceed following two different criteria. The first one is a global one: if the total $O(\alpha(\mu)/\pi)$ correction remains below the 10% threshold, we can consider the perturbative expansion to be under control and the one loop approximation to be a satisfactory one. The second point of view, the most correct in our opinion, is that in which one requires that *all* the different logarithmic effects individually satisfy the previous 10% criterion. Starting from these requirements, we have therefore examined how the relative size of the various components in the considered observables varies with energy, in an energy range where our asymptotic expansion might be reasonably accurate. Naively we would expect that this corresponds orientatively to energies of about 1 TeV and beyond. In fact, we have verified by an exact one loop numerical calculation of the various observables, where all the contributions from the various diagrams are retained without approximation, that the rigorous expressions are reproduced by our asymptotic expansions within a few percent at most, in the energy range between 1 TeV and 10 TeV (larger energy values seem to us not realistic). We can therefore reasonably conclude that, in this energy range, Eqs. (36)–(38) contain the bulk of the one loop electroweak corrections, and can be used for a meaningful discussion of the size of the various effects.

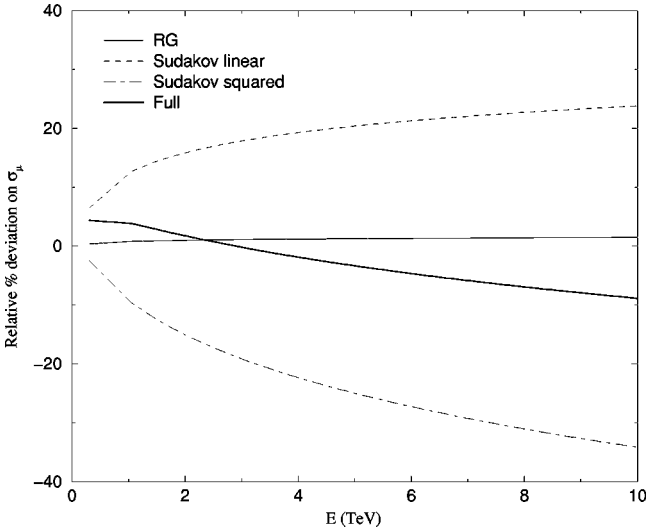

FIG. 5. Logarithmic contributions to the asymptotic cross section $\sigma(e^+e^-\to\mu^+\mu^-)$ from Eq. (36).

The results of our analysis are shown in Figs. 5–8. As one sees from inspection of these figures, the situation is quite different for the two criteria and for the various observables. To be more precise, we list the various cases separately:

(a) σ_μ. Here the global relative effect remains below the 10% limit in the full range 1 TeV$<\sqrt{q^2}<$10 TeV. However, the individual Sudakov components both cross the ''safety limit'' practically in the full range.

(b) σ_5. Here the global relative effect rises beyond the ''safety limit'' at energies larger than 4 TeV. The separate Sudakov contributions are over the limit in the full considered range.

(c) $A_{FB,\mu}$. Here the global relative effects remain always below the limit. The linear Sudakov crosses the limit at about

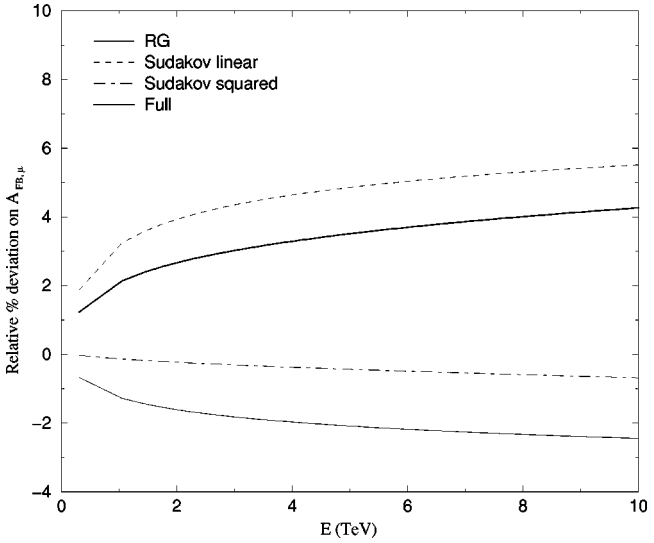

FIG. 6. Logarithmic contributions to the asymptotic asymmetry $A_{FB,\mu}$ from Eq. (37).

3 TeV (the quadratic one is almost absent in this case).

(d) $A_{LR,\mu}$. In principle, both global and individual relative effects are systematically well beyond the 10% limit in the full range. A word of caution is, though, suited since for this special observable the Born approximation is particularly small (≈ 0.07). Moreover, the most important one loop effects are produced by W diagrams, which only contribute to the left-handed cross section and generate therefore particularly large effects in this observable.

The conclusion of this preliminary investigation is that, in the considered energy range beyond 1 TeV, the validity of the one loop SM electroweak expansion is debatable. The necessity of a two loop calculation, leading if possible to a resummation af Sudakov effects, appears to us strongly motivated.

Having examined the situation that occurs in the region beyond 1 TeV, we want now to turn our attention to the

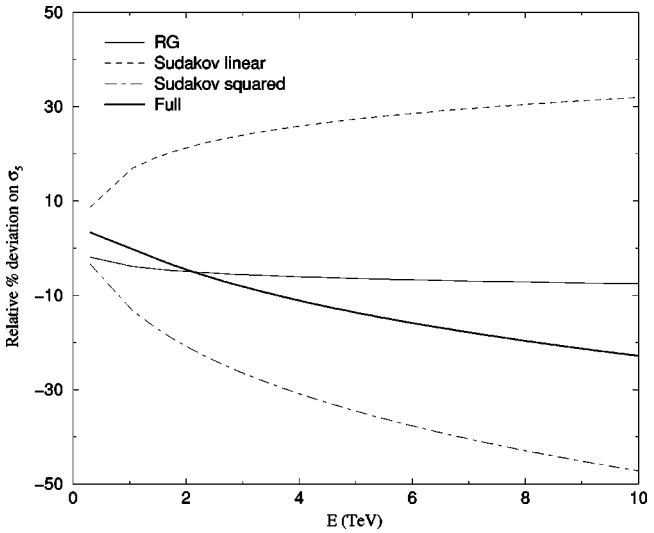

FIG. 7. Logarithmic contributions to the asymptotic cross section $\sigma(e^+e^- \to \text{hadrons})$ from Eq. (38).

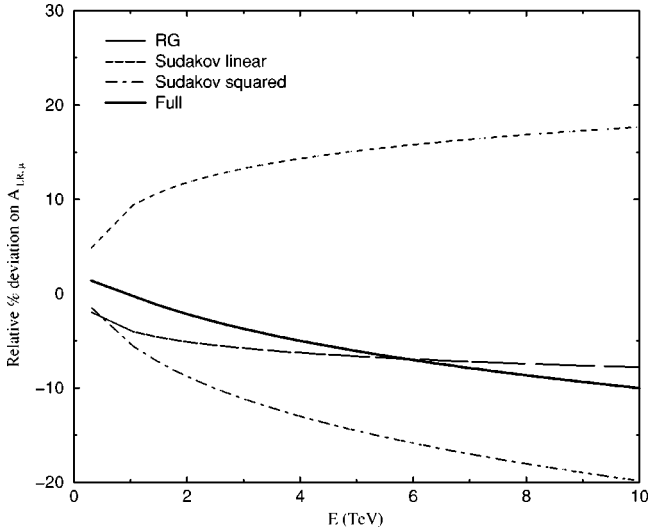

FIG. 8. Logarithmic contributions to the asymptotic asymmetry $A_{LR,\mu}$ from Eq. (39).

energy region below. Here an asymptotic expansion is certainly less justified. We shall discuss the emerging picture in the next section.

IV. SIMPLE FORMULA FOR CROSS SECTIONS AND ASYMMETRIES BELOW 1 TeV

In the previous sections we have derived a one loop asymptotic expansion for various observables and we have explicitly shown in Sec. III that in the energy range beyond 1 TeV, where the expansion is able to describe well the one loop corrections, the reliability of the one loop approximation is not evident. These conclusions would be particularly relevant for a possible future muon collider, operating in an energy region of a few TeV. On the other hand, the simplicity and the well-established physical interpretation of the various terms of the expansion pushes us to investigate the possibility of using them in the energy range *below* 1 TeV, where, in principle, the validity of an asymptotic expansion is not guaranteed. In particular, we are interested in simple formulas that describe cross sections and asymmetries e.g. in the energy range 300 GeV$<\sqrt{q^2}<$1 TeV, where the future linear colliders (LC's) will be operating at an energy that we assume to be of 500 GeV. For what concerns the experiment accuracy, we shall stick to the previous conservative assumption of a 1% precision, which is rather pessimistic compared to the latest expectations [1].

Our empirical procedure has been the following. We have first plotted the *exact* SM predictions for σ_μ and $A_{FB,\mu}$ in the considered energy region (a full discussion of σ_5, which requires a rigorous calculation of the top quark effects, will be given in the annunciated following paper). The calculation has been made using the program TOPAZ0 [3].[2] We have then compared the TOPAZ0 one loop results with those of the

[2]We thank G. Passarino for kindly providing us with the numerical data.

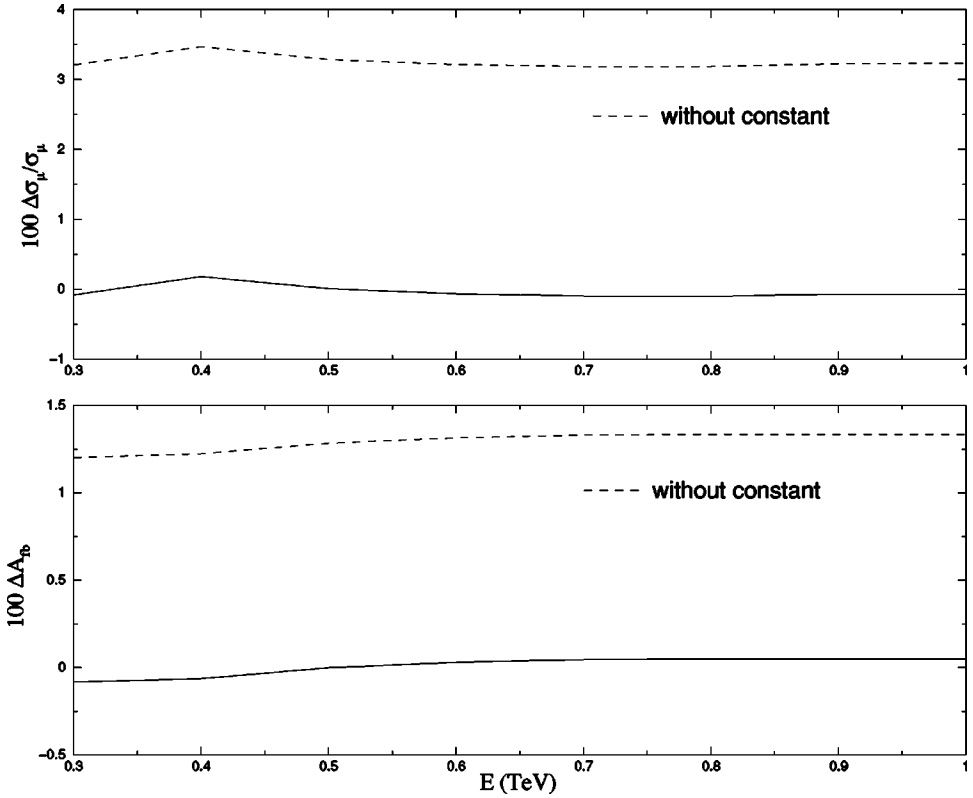

FIG. 9. Comparison between the asymptotic expressions for σ_μ and $A_{FB,\mu}$, Eqs. (36), (37), and the exact one loop calculation by TOPAZ0.

simple asymptotic expansion, Eqs. (36)–(38). *A priori*, we expect that ''some'' modifications of our formulas are unavoidably requested.

The results of our first comparison are shown in Fig. 9. As one sees, the difference between the full calculation and the logarithmic approximation remains impressively practically constant in the whole considered energy range. This (in our opinion) remarkable fact allows us to conclude that the simplest modification of the logarithmic expansion obtained by adding a constant term seems sufficient to obtain a very accurate agreement.

For the two observables we therefore write

$$\sigma_\mu = \sigma_\mu^B \left(1 + \frac{\alpha}{4\pi}(c_\mu + \text{logarithms}) \right) \tag{40}$$

$$A_{FB} = A_{FB}^B + \frac{\alpha}{4\pi}(c_{FB} + \text{logarithms}) \tag{41}$$

where ''logarithms'' stands for the logarithmic terms of Eqs. (36)–(38) and where, in agreement with our overall philosophy, σ_μ^B and $A_{FB,\mu}^B$ are the Born level expressions in which we have set $\alpha = \alpha(M_Z^2)$. They are obtained, in a straightforward way, from the expressions given in Eqs. (B2)–(B5), putting all 1-loop terms $\tilde{\Delta}_{\alpha,lf}$, R_{lf}, $V_{\gamma Z,lf}$, $V_{Z\gamma,lf}$ equal to zero and replacing $\alpha(0)$ by $\alpha(M_Z^2)$. In the case of the leptonic cross section we used

$$\begin{aligned}\sigma_\mu^B &= \sigma_\mu^{B,\gamma\gamma} + \sigma_\mu^{B,\gamma Z} + \sigma_\mu^{B,ZZ} \\ &= \frac{4\pi\alpha^2(M_Z^2)}{3q^2} + 8\pi\alpha(M_Z^2)\frac{\Gamma_\mu}{M_Z} \\ &\quad \times \frac{q^2 - M_Z^2}{(q^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2} \frac{\tilde{v}_l^2}{1 + \tilde{v}_l^2} \\ &\quad + 12\pi \frac{\Gamma_\mu^2}{M_Z^2} \frac{q^2}{(q^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2}\end{aligned} \tag{42}$$

with $\tilde{v}_l = 1 - 4s_l^2(M_Z^2)$. For the forward-backward asymmetry we write

$$A_{FB,\mu}^B = \frac{\sigma_{FB,\mu}^B}{\sigma_\mu^B} \tag{43}$$

with

$$\begin{aligned}\sigma_{FB,\mu}^B &= \sigma_{FB,\mu}^{B,\gamma Z} + \sigma_{FB,\mu}^{B,ZZ} \\ &= 6\pi\alpha(M_Z^2)\frac{\Gamma_\mu}{M_Z} \frac{q^2 - M_Z^2}{(q^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2} \frac{1}{1 + \tilde{v}_l^2} \\ &\quad + 36\pi \frac{\Gamma_\mu^2}{M_Z^2} \frac{q^2}{(q^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2} \frac{\tilde{v}^2}{(1 + \tilde{v}^2)^2}.\end{aligned} \tag{44}$$

The choice of the constants is, to some extent, arbitrary. We have followed the rather pragmatic attitude of optimizing our approximation in the 500 GeV region (LC domain). This fixes the values [17]

$$c_\mu = -53.87, \quad c_{FB} = -20.82. \tag{45}$$

The comparison of our simple effective expressions, Eqs. (36)–(44), with TOPAZ0 are shown again in Fig. 9. As one sees, between 300 GeV and 1 TeV, the difference between the two calculations is at the permille level for both considered observables. Therefore, our approximate expressions could be safely used to perform, e.g., preliminary analyses of possible effects of competitor electroweak models.[3]

As a final comment, we would like to notice that the fact that an asymptotic formula well describes the behavior of the exact corrections in a (relatively) low energy range can be due to the absence of structure like resonances in that region (in our analysis we have ignored the small peak at the $t\bar{t}$ production threshold which is negligible in any case for the considered observables).

V. CONCLUSIONS

In this paper we have determined the energy growing electroweak logarithms of RG (UV) and Sudakov (IR) origin that appear in the asymptotic expansions of four (massless) fermion processes. We have shown that (linear and quadratic) Sudakov-type logarithms are numerically much more important than RG-driven logarithms in the TeV region. Moreover, although the overall effect on the observables is of the order of a few percent, the single contributions are of the 10% order in this region. This supports the idea that next order calculations and/or resummations are needed. On the other hand, our simple one loop expansions might reveal to be unexpectedly useful in the energy region *below* 1 TeV, since the correct energy dependence of the observables seems to be properly described.

The fact that a satisfactory description of electroweak corrections at asymptotic energies requires more complicated descriptions beyond the ''naive'' one loop approximation would not be, in our opinion, a surprising feature for a gauge theory. For QED and QCD similar problems of higher order calculations and resummation have been notoriously already preformed. Our personal impression is that a proper description of the genuine electroweak sector of the SM at the future relevant energies in the TeV region requires an analogous remarkable effort.

APPENDIX A: SUDAKOV-TYPE CONTRIBUTIONS

Final fermions $f \neq b$:

$$\begin{aligned}\tilde{\Delta}^{(S)}_{\alpha,lf}(q^2,\theta) \to & \frac{\alpha}{4\pi}[6-\delta_u-2\delta_d]\ln\frac{q^2}{M_W^2}+\frac{\alpha}{12\pi}(\delta_u+2\delta_d)\ln^2\frac{q^2}{M_W^2}+\frac{\alpha(2-v_l^2-v_f^2)}{64\pi s^2c^2}\left(3\ln\frac{q^2}{M_Z^2}-\ln^2\frac{q^2}{M_Z^2}\right)\\ & -\frac{\alpha}{2\pi}\left[\left(\ln^2\frac{q^2}{M_W^2}+2\ln\frac{q^2}{M_W^2}\ln\frac{1-\cos\theta}{2}\right)(\delta_\mu+\delta_d)+\left(\ln^2\frac{q^2}{M_W^2}+2\ln\frac{q^2}{M_W^2}\ln\frac{1+\cos\theta}{2}\right)\delta_u\right]\\ & -\frac{\alpha}{256\pi Q_f s^4c^4}\left[(1-v_l^2)(1-v_f^2)\left(\ln\frac{q^2}{M_Z^2}\ln\frac{1+\cos\theta}{1+\cos\theta}\right)\right]\end{aligned} \tag{A1}$$

$$\begin{aligned}R^{(S)}_{lf}(q^2,\theta) \to & -\frac{3\alpha}{4\pi s^2}\left[2c^2-\delta_\mu-\left(1-\frac{s^2}{3}\right)\delta_u-\left(1-\frac{2s^2}{3}\right)\delta_d\right]\ln\frac{q^2}{M_W^2}\\ & -\frac{\alpha}{4\pi s^2}\left[\delta_\mu+\left(1-\frac{s^2}{3}\right)\delta_u+\left(1-\frac{2s^2}{3}\right)\delta_d\right]\ln^2\frac{q^2}{M_W^2}-\frac{\alpha(2+3v_l^2+3v_f^2)}{64\pi s^2c^2}\left(3\ln\frac{q^2}{M_Z^2}-\ln^2\frac{q^2}{M_Z^2}\right)\\ & +\frac{\alpha c^2}{2\pi s^2}\left[\left(\ln^2\frac{q^2}{M_W^2}+2\ln\frac{q^2}{M_W^2}\ln\frac{1-\cos\theta}{2}\right)(\delta_\mu+\delta_d)+\left(\ln^2\frac{q^2}{M_W^2}+2\ln\frac{q^2}{M_W^2}\ln\frac{1+\cos\theta}{2}\right)\delta_u\right]\\ & +I_{3f}\frac{\alpha}{2\pi s^2c^2}\left(v_l v_f\ln\frac{q^2}{M_Z^2}\ln\frac{1+\cos\theta}{1+\cos\theta}\right)\end{aligned} \tag{A2}$$

[3]The simplest example would be provided by a class of models with anomalous triple gauge couplings (AGC's) [18]. Here, the effect of the models is that of introducing linear terms of the type q^2/Λ^2 [19], thus changing the energy dependence of the observables as a function of the AGC parameters. Another application could be one that requires integration over a certain fraction of the energy range involving the product of the cross section by known weight functions, like those encountered when computing QED convolution. This could be performed in a rather economical way.

$$V^{(S)}_{\gamma Z,lf}(q^2,\theta)\to\frac{\alpha}{8\pi cs}\Bigg[(3-12c^2+2c^2(\delta_u+2\delta_d))\ln\frac{q^2}{M_W^2}-\left(1+\frac{2}{3}c^2(\delta_u+2\delta_d)\right)\ln^2\frac{q^2}{M_W^2}\Bigg]$$
$$-\left(\frac{\alpha v_l(1-v_l^2)}{128\pi s^3c^3}+\frac{\alpha|Q_f|v_f}{8\pi sc}\right)\left(3\ln\frac{q^2}{M_Z^2}-\ln^2\frac{q^2}{M_Z^2}\right)+\frac{\alpha c}{2\pi s}\Bigg[\left(\ln^2\frac{q^2}{M_W^2}+2\ln\frac{q^2}{M_W^2}\ln\frac{1-\cos\theta}{2}\right)(\delta_\mu+\delta_d)$$
$$+\left(\ln^2\frac{q^2}{M_W^2}+2\ln\frac{q^2}{M_W^2}\ln\frac{1+\cos\theta}{2}\right)\delta_u\Bigg]+I_{3f}\frac{\alpha}{16\pi s^3c^3}\left(v_f(1-v_l^2)\ln\frac{q^2}{M_Z^2}\ln\frac{1+\cos\theta}{1+\cos\theta}\right) \tag{A3}$$

$$V^{(S)}_{Z\gamma,lf}(q^2,\theta)\to\frac{\alpha}{8\pi cs}\Bigg[[3-12c^2-2s^2(\delta_u+2\delta_d)]\ln\frac{q^2}{M_W^2}-\left(1-\frac{2}{3}s^2(\delta_u+2\delta_d)\right)\ln^2\frac{q^2}{M_W^2}\Bigg]$$
$$-\left(\frac{\alpha v_f(1-v_f^2)}{128\pi|Q_f|s^3c^3}+\frac{\alpha v_l}{8\pi sc}\right)\left(3\ln\frac{q^2}{M_Z^2}-\ln^2\frac{q^2}{M_Z^2}\right)+\frac{\alpha c}{2\pi s}\Bigg[\left(\ln^2\frac{q^2}{M_W^2}+2\ln\frac{q^2}{M_W^2}\ln\frac{1-\cos\theta}{2}\right)(\delta_\mu+\delta_d)$$
$$+\left(\ln^2\frac{q^2}{M_W^2}+2\ln\frac{q^2}{M_W^2}\ln\frac{1+\cos\theta}{2}\right)\delta_u\Bigg]+\frac{\alpha}{32\pi Q_f s^3c^3}\left(v_l(1-v_f^2)\ln\frac{q^2}{M_Z^2}\ln\frac{1+\cos\theta}{1+\cos\theta}\right) \tag{A4}$$

where $\delta_{\mu,u,d}=1$ for $f=\mu,u,d$ and 0 otherwise and $v_l=1-4s^2$, $v_f=1-4|Q_f|s^2$.

In each of the above equations, we have successively added the contributions coming from triangles containing one or two W, triangles containing one Z, from the WW box and finally from the ZZ box.

APPENDIX B: CONTRIBUTIONS TO THE VARIOUS OBSERVABLES

The general expression of the $l^+l^-\to f\bar{f}$ cross section can be written as

$$\frac{d\sigma_{lf}}{d\cos\theta}=\frac{4\pi}{3}\mathcal{N}_f q^2\left\{\frac{3}{8}(1+\cos^2\theta)[(1-PP')U_{11}+(P'-P)U_{21}]+\frac{3}{4}\cos\theta[(1-PP')U_{12}+(P'-P)U_{22}]\right\} \tag{B1}$$

where

$$U_{11}\frac{\alpha^2(0)Q_f^2}{q^4}[1+2\tilde{\Delta}^{(lf)}\alpha(q^2,\theta)]+2[\alpha(0)|Q_f|]\frac{q^2-M_Z^2}{q^2((q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2)}\left[\frac{3\Gamma_l}{M_Z}\right]^{1/2}\left[\frac{3\Gamma_f}{N_fM_Z}\right]^{1/2}\frac{\tilde{v}_l\tilde{v}_f}{(1+\tilde{v}_l^2)^{1/2}(1+\tilde{v}_f^2)^{1/2}}$$
$$\times\left[1+\tilde{\Delta}^{(lf)}\alpha(q^2,\theta)-R^{(lf)}(q^2,\theta)-4s_lc_l\left\{\frac{1}{\tilde{v}_l}V^{(lf)}_{\gamma Z}(q^2,\theta)+\frac{|Q_f|}{\tilde{v}_f}V^{(lf)}_{Z\gamma}(q^2,\theta)\right\}\right]+\frac{\left[\frac{3\Gamma_l}{M_Z}\right]\left[\frac{3\Gamma_f}{\mathcal{N}_fM_Z}\right]}{(q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2}$$
$$\times\left[1-2R^{(lf)}(q^2,\theta)-8s_lc_l\left\{\frac{\tilde{v}_l}{1+\tilde{v}_l^2}V^{(lf)}_{\gamma Z}(q^2,\theta)+\frac{\tilde{v}_f|Q_f|}{(1+\tilde{v}_f^2)}V^{(lf)}_{Z\gamma}(q^2,\theta)\right\}\right] \tag{B2}$$

$$U_{12}=2[\alpha(0)|Q_f|]\frac{q^2-M_Z^2}{q^2((q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2)}\left[\frac{3\Gamma_l}{M_Z}\right]^{1/2}\left[\frac{3\Gamma_f}{\mathcal{N}_fM_Z}\right]^{1/2}\frac{1}{(1+\tilde{v}_l^2)^{1/2}(1+\tilde{v}_f^2)^{1/2}}[1+\tilde{\Delta}^{(lf)}\alpha(q^2,\theta)-R^{(lf)}(q^2,\theta)]$$
$$+\frac{\left[\frac{3\Gamma_l}{M_Z}\right]\left[\frac{3\Gamma_f}{\mathcal{N}_fM_Z}\right]}{(q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2}\left[\frac{4\tilde{v}_l\tilde{v}_f}{(1+\tilde{v}_l^2)(1+\tilde{v}_f^2)}\right]\left[1-2R^{(lf)}(q^2,\theta)-4s_lc_l\left\{\frac{1}{\tilde{v}_l}V^{(lf)}_{\gamma Z}(q^2,\theta)+\frac{|Q_f|}{\tilde{v}_f}V^{(lf)}_{Z\gamma}(q^2,\theta)\right\}\right] \tag{B3}$$

$$U_{21}=2[\alpha(0)|Q_f|]\frac{q^2-M_Z^2}{q^2((q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2)}\left[\frac{3\Gamma_l}{M_Z}\right]^{1/2}\left[\frac{3\Gamma_f}{\mathcal{N}_f M_Z}\right]^{1/2}\frac{\tilde{v}_f}{(1+\tilde{v}_l^2)^{1/2}(1+\tilde{v}_f^2)^{1/2}}$$
$$\times\left[1+\tilde{\Delta}^{(lf)}\alpha(q^2,\theta)-R^{(lf)}(q^2,\theta)-\frac{4s_l c_l|Q_f|}{\tilde{v}_f}V_{Z\gamma}^{(lf)}(q^2,\theta)\right]+\frac{\left[\frac{3\Gamma_l}{M_Z}\right]\left[\frac{3\Gamma_f}{\mathcal{N}_f M_Z}\right]}{(q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2}\left[\frac{2\tilde{v}_l}{(1+\tilde{v}_l^2)}\right]$$
$$\times\left[1-2R^{(lf)}(q^2,\theta)-4s_l c_l\left\{\frac{1}{\tilde{v}_l}V_{\gamma Z}^{(lf)}(q^2,\theta)+\frac{2\tilde{v}_f|Q_f|}{(1+\tilde{v}_f^2)}V_{Z\gamma}^{(lf)}(q^2,\theta)\right\}\right] \tag{B4}$$

$$U_{22}=2[\alpha(0)|Q_f|]\frac{q^2-M_Z^2}{q^2((q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2)}\left[\frac{3\Gamma_l}{M_Z}\right]^{1/2}\left[\frac{3\Gamma_f}{\mathcal{N}_f M_Z}\right]^{1/2}\frac{\tilde{v}_l}{(1+\tilde{v}_l^2)^{1/2}(1+\tilde{v}_f^2)^{1/2}}$$
$$\times\left[1+\tilde{\Delta}^{(lf)}\alpha(q^2,\theta)-R^{(lf)}(q^2,\theta)-\frac{4s_l c_l}{\tilde{v}_l}V_{\gamma Z}^{(lf)}(q^2)\right]+\frac{\left[\frac{3\Gamma_l}{M_Z}\right]\left[\frac{3\Gamma_f}{\mathcal{N}_f M_Z}\right]}{(q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2}\left[\frac{2\tilde{v}_f}{(1+\tilde{v}_f^2)}\right]$$
$$\times\left[1-2R^{(lf)}(q^2,\theta)-4s_l c_l\left\{\frac{2\tilde{v}_l}{(1+\tilde{v}_l^2)}V_{\gamma Z}^{(lf)}(q^2,\theta)+\frac{|Q_f|}{\tilde{v}_f}V_{Z\gamma}^{(lf)}(q^2,\theta)\right\}\right]. \tag{B5}$$

Here P,P' are the *longitudinal* polarization degrees of the initial lepton and antilepton, and $\mathcal{N}_f$ is the color factor for the $f\bar{f}$ channel which includes the appropriate QCD corrections to the input.

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