

# Pion form factor in QCD at intermediate momentum transfers

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We present a quantitative analysis of the electromagnetic pion form factor in the light-cone sum rule approach, including radiative corrections and higher-twist effects. The comparison to the existing data favors the asymptotic profile of the pion distribution amplitude and allows us to estimate the deviation:  $[\int du/u \phi_\pi(u)]/[\int du/u \phi_\pi^{\text{as}}(u)] = 1.1 \pm 0.1$  at the scale of 1 GeV. Special attention is paid to the precise definition and interplay of soft and hard contributions at intermediate momentum transfer, and to the matching of the sum rule to the perturbative QCD prediction. We observe a strong numerical cancellation between the soft (end-point) contribution and power-suppressed hard contributions of higher twist, so that the total nonperturbative correction to the usual PQCD result turns out to be of the order of 30% for  $Q^2 \sim 1 \text{ GeV}^2$ .

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## I. INTRODUCTION

There is a clear tendency for QCD-oriented experimental studies to go for more and more exclusive channels. All future plans also call for very high luminosity and would therefore be perfectly suited for the investigation of exclusive and semexclusive reactions. A problem which hinders all attempts to implement these projects is the lack of truly quantitative QCD predictions. It is widely anticipated (see e.g., [1–5]) that for experimentally accessible values of the momentum transfer, the perturbative QCD factorization for hard exclusive reactions [6] receives non-negligible corrections from the so-called soft, or end-point, contributions, which are essentially nonperturbative. One practical difficulty is that soft corrections can in many cases be mimicked (numerically) by modifying the shape of hadron distribution amplitudes. An agreement of perturbative predictions with the data cannot, therefore, be used to claim the smallness of end-point effects which have to be estimated independently using a certain nonperturbative approach. Creating a systematic framework for a study of soft end-point corrections is becoming, thus, increasingly timely.

It has been suggested [4] that the soft end-point contribution to the pion electromagnetic form factor can be estimated in a largely model-independent way within the framework of light-cone sum rules [7]. The aim of the present paper is to put this technique on a more quantitative footing. To this end we calculate the radiative correction to the light-cone sum rule, elaborate on the scale dependence, and demonstrate how the sum rule estimates of the end-point effects can naturally be combined with the next leading order (NLO) QCD perturbative calculation. In addition, we estimate the twist 6

contribution to the sum rule due to the quark condensate and find this correction to be small.

The presentation is organized as follows. In Sec. II we recall the basic ideas of the light-cone sum rule approach and derive the simplest sum rule. Section III is devoted to the calculation of the radiative correction and separation of soft and hard effects. As expected, we find that in the  $Q^2 \rightarrow \infty$  limit the form factor is dominated by the hard rescattering contribution alone, while to  $1/Q^4$  accuracy both soft and hard contributions have to be taken into account. Higher-twist corrections to the light-cone sum rule are considered in Sec. IV, while Sec. V contains the results of our numerical analysis. Matching of the sum rule with the NLO perturbative predictions is discussed in Sec. VI. Finally, in Sec. VII we summarize. The paper contains two appendixes where we collect some useful but bulky expressions and present the relevant formulas for light-cone distributions of the pion.

## II. METHOD OF LIGHT-CONE SUM RULES

The approach is based on the study of the correlation function [8]

$$T_{\mu\nu}(p, q) = i \int d^4x e^{iqx} \langle 0 | T \{ j_\mu^5(0) j_\nu^{\text{em}}(x) \} | \pi^+(p) \rangle, \quad (1)$$

where  $j_\mu^5 = \bar{d} \gamma_\mu \gamma_5 u$  and  $j_\nu^{\text{em}} = e_u \bar{u} \gamma_\nu u + e_d \bar{d} \gamma_\nu d$  is the quark electromagnetic current. With  $p^2 = m_\pi^2$  and  $Q^2 = -q^2$  fixed, the correlation function (1) depends on a single invariant variable  $s = (p - q)^2$ . The contribution of the pion intermediate state equals

$$T_{\mu\nu}(p, q) = 2if_\pi(p - q)_\mu p_\nu F_\pi(Q^2) \frac{1}{m_\pi^2 - (p - q)^2}, \quad (2)$$

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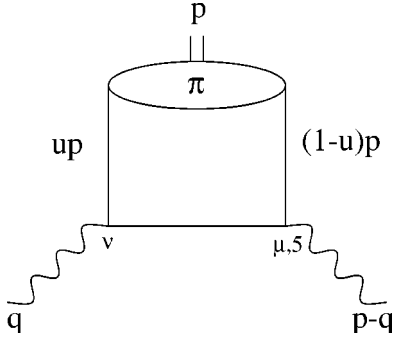


FIG. 1. The tree-level contribution to the correlation function in Eq. (1).

where  $f_\pi$  is the pion decay constant and  $F_\pi(Q^2)$  is the pion electromagnetic form factor. On the other hand, at large negative  $(p-q)^2$  and  $q^2$  the correlation function can be calculated in QCD, in full analogy with the  $\gamma^* \gamma^* \pi$  transition form factor. A common idea of all QCD sum rules is a matching between the QCD calculation at Euclidean momenta and the dispersion relation in terms of contributions of hadronic states, which allows one to estimate the hadronic quantity of interest. The specifics of the light-cone sum rules are how exactly the QCD calculation and matching are done. To illustrate this point, consider the contribution of the simplest diagram in Fig. 1:

$$T_{\mu\nu} = \frac{1}{2\pi^2} \int d^4x \frac{e^{iqx}}{x^4} \langle 0 | [e_u \bar{d}(0) \gamma_\mu \not{x} \gamma_\nu \gamma_5 u(x) - e_d \bar{d}(x) \gamma_\nu \not{x} \gamma_\mu \gamma_5 u(0)] | \pi^+(p) \rangle. \quad (3)$$

Expansion of the remaining nonlocal matrix elements around the middle point in a formal Taylor series generates the Wilson operator-product expansion in contributions of local operators of increasing dimension:

$$O_{\mu, \mu_1, \mu_2, \dots, \mu_n}^n = \bar{d}(0) \gamma_\mu \gamma_5 i \vec{D}_{\mu_1} \cdots i \vec{D}_{\mu_n} u(0), \quad (4)$$

$D_\mu = \partial_\mu - igA_\mu$  being the covariant derivative and  $\vec{D}_\mu = \vec{D}_\mu - \vec{D}_\mu$ . Restricting ourselves for the moment to operators of the lowest twist (highest Lorentz spin) we consider the relevant reduced matrix elements

$$x_{\mu_1} \cdots x_{\mu_n} \langle 0 | O_{\mu, \mu_1, \mu_2, \dots, \mu_n}^n | \pi^+(p) \rangle = if_\pi p_\mu (px)^n \langle \langle O_n \rangle \rangle + \cdots. \quad (5)$$

They are related, as first found in [6], to the moments of the pion distribution amplitude:

$$\begin{aligned} \langle 0 | \bar{d}(0) \gamma_\mu \gamma_5 u(x) | \pi^+(p) \rangle &= ip_\mu f_\pi \int_0^1 du e^{-iupx} \\ &\quad \times \varphi_\pi(u, \mu^2 \sim x^{-2}), \\ \langle \langle O_n \rangle \rangle &= \int_0^1 du (1-2u)^n \varphi_\pi(u). \end{aligned} \quad (6)$$

Substituting Eq. (5) into the expansion of Eq. (3) and integrating over  $x$ , we obtain for the contribution of Fig. 1,<sup>1</sup>

$$T_{\mu\nu} = \frac{2if_\pi p_\mu p_\nu}{Q^2 - s} \left\{ 1 + \frac{2Q^2}{Q^2 - s} \sum_{n=2,4,\dots} \langle \langle O_n \rangle \rangle \times \left( \frac{2Q^2}{Q^2 - s} - 1 \right)^{n-1} \right\} + \text{other Lorentz structures}, \quad (7)$$

where  $s = (p-q)^2$ . To construct a sum rule, we make the Borel transformation

$$\begin{aligned} \frac{1}{m_\pi^2 - s} &\rightarrow \exp[-m_\pi^2/M^2], \\ \frac{1}{(Q^2 - s)^n} &\rightarrow \frac{1}{(M^2)^{n-1} \Gamma(n)} \exp[-Q^2/M^2], \end{aligned} \quad (8)$$

introducing a new variable  $M^2$  (the Borel parameter), and equating the Borel-transformed versions of Eqs. (2) and (7). For simplicity we neglect the continuum subtraction here. Neglecting the pion mass, the result reads

$$F_\pi(Q^2) = e^{-Q^2/M^2} \left\{ 1 + \sum_{n=2,4,\dots} \langle \langle O_n \rangle \rangle \times \sum_{k=1}^n \binom{n-1}{k-1} \frac{1}{\Gamma(k+1)} \left( \frac{-2Q^2}{M^2} \right)^k \right\}. \quad (9)$$

This sum rule is, however, completely unsatisfactory.

Indeed, QCD sum rules are generally expected to hold in a certain interval of values of the Borel parameter, such that contributions of both higher resonances and higher orders of the operator product expansion (OPE) are simultaneously suppressed. It is easy to see that in the present situation these two conditions are contradictory, unless  $Q^2$  is sufficiently small. Indeed, on the one hand, one has to keep  $M^2$  small, of the order of 1–2 GeV<sup>2</sup>, to suppress the contribution of, e.g., the  $a_1$ -meson intermediate state. On the other hand, for a fixed  $M^2$  the higher-order terms on the right-hand side (RHS) of the sum rule are enhanced by factors  $(Q^2)^k$  and for  $Q^2 > M^2$  the OPE expansion breaks down.

An escape suggested in [7] is to avoid the Wilson short-distance expansion altogether and write the answer for the diagram in Fig. 1 directly in terms of the pion distribution amplitude. The expansion parameter then becomes the *twist* of the operators rather than their dimension. Using Eq. (3) and the definition of the pion distribution amplitude in Eq. (6) we obtain to leading twist accuracy, instead of Eq. (7), a compact expression

<sup>1</sup>The terms with odd  $n$  vanish because of  $G$  parity.

$$T_{\mu\nu} = 2if_{\pi}p_{\mu}p_{\nu} \int_0^1 du \frac{u\varphi_{\pi}(u)}{uQ^2 - us} + \dots, \quad (10)$$

where  $\bar{u} = 1 - u$ . Making, once again, the Borel transformation, we get the simplest *light-cone sum rule* [4]

$$F_{\pi}(Q^2) = \int_0^1 du \varphi_{\pi}(u) \exp\left(-\frac{\bar{u}Q^2}{uM^2}\right). \quad (11)$$

This sum rule is perfectly well behaved at  $Q^2 \rightarrow \infty$  and it is instructive to trace how the above-mentioned difficulties of the standard approach have been resolved. Because of the strong exponential suppression factor, the important region of integration over the momentum fraction variable  $u$  gets shifted, in the large- $Q^2$  limit, to the end-point region  $1 - u \sim M^2/Q^2$ . In this regime, the virtuality of the quark [the denominator in Eq. (10)] remains all the time of order  $M^2$ , as  $Q^2 \rightarrow \infty$ . The deficiency of the short-distance expansion is now clearly seen as originating from the wrong expansion parameter  $(Q^2 - s)/2$  [cf. Eq. (7)], corresponding, effectively, to the expansion around the symmetric point<sup>2</sup>  $u = 1/2$ .

To be somewhat more quantitative, we have to make the usual continuum subtraction. This is trivial in the case at hand, since expression (10) is easily converted to the form of a dispersion integral over  $s = (p - q)^2$ . All we have to do is to truncate this integral at a certain threshold  $s_0$ , called the interval of duality. The result [4] is that the integration over the momentum fraction is cut from below at the value

$$u_0 = Q^2/(s_0 + Q^2). \quad (12)$$

In addition, the pion distribution amplitude has to be taken at the scale corresponding to the quark virtuality:

$$\mu_u^2 = \bar{u}Q^2 + uM^2. \quad (13)$$

Implementing these small improvements, we obtain the leading-twist leading-order light-cone sum rule [4]

$$F_{\pi}(Q^2) = \int_{u_0}^1 du \varphi_{\pi}(u, \mu_u) \exp\left(-\frac{\bar{u}Q^2}{uM^2}\right). \quad (14)$$

The crucial advantage of the light-cone sum rule approach is that it allows one to incorporate the information on the end-point behavior of the pion distribution amplitude

$\varphi_{\pi}(u) \sim 1 - u$  at  $u \rightarrow 1$ . In the limit  $Q^2 \rightarrow \infty$  the integration region in Eq. (14) shrinks to a point  $u = 1$  so that one obtains

$$F_{\pi}(Q^2) \sim \frac{\varphi'_{\pi}(0, \mu^2 \sim M^2)}{Q^4} \int_0^{s_0} s ds e^{-s/M^2}, \quad (15)$$

<sup>2</sup>A similar deficiency of the short-distance expansion in the case of heavy-to-light correlation functions is demonstrated in [9].

where  $\varphi'_{\pi}(0) \equiv (d/du)\varphi_{\pi}(u)|_{u \rightarrow 0} = -\varphi'_{\pi}(1)$ . The Borel variable  $M^2$  corresponds to the (inverse) distance at which the matching is done between the parton and hadron representations.

The expressions in Eqs. (14), (15) present a typical ‘‘soft’’ or ‘‘end-point’’ contribution to the pion form factor which is sensitive to the pion wave function at a *low* normalization point and comes from large transverse distances of order  $b \sim s_0^{-1/2}$ .

To illustrate this point, write the four-dimensional integration in Eq. (1) as a product of 2 two-dimensional integrations in longitudinal and transverse (to  $p$  and  $q$ ) coordinates. Leaving the transverse integration intact, a short calculation gives for the RHS of Eq. (11)

$$\int d^2b \int_0^1 du \varphi_{\pi}(u) \frac{uM^2}{4\pi} \exp\left(-\frac{1}{4}uM^2b^2 - \frac{\bar{u}Q^2}{uM^2}\right). \quad (16)$$

The distribution of transverse distances in the diagram in Fig. 1 is, thus, Gaussian, with the average transverse size  $\langle b^2 \rangle = 4/(uM^2)$  controlled by the value of the Borel parameter. One also sees that the scale of the distribution amplitude in Eqs. (13), (14) is determined by the weighted average of the momentum transfer  $Q^2$  and the (inverse) transverse distance between the quarks, as expected on general grounds [10].

Including the continuum subtraction modifies this distribution rather significantly as the small- $b$  region is dominated by high-mass excitations and gets suppressed. After some algebra we obtain the sum rule equivalent to Eq. (14) but with an explicit separation of different transverse distances:

$$\begin{aligned} F_{\pi}(Q^2) &= \frac{1}{4\pi} \int d^2b \int_{u_0}^1 du \varphi_{\pi}(u) e^{-\bar{u}Q^2/(uM^2)} \\ &\quad \times \int_0^{us_0 - \bar{u}Q^2} dt e^{-t/(uM^2)} J_0(\sqrt{b^2 t}) \\ &\xrightarrow{Q^2 \rightarrow \infty} \frac{\varphi'_{\pi}(0)}{4\pi Q^4} \int d^2b \int_0^{s_0} ds e^{-s/M^2} \\ &\quad \times \int_0^s dt (s-t) J_0(\sqrt{b^2 t}), \end{aligned} \quad (17)$$

where  $J_0$  is the Bessel function. The resulting transverse-distance distribution (normalized to unity at  $b = 0$ ) is shown in Fig. 2. The dependence on both  $Q^2$  and the Borel parameter is actually very weak and the overall scale of transverse distances is determined almost entirely by the value of the continuum threshold. Because of this, for  $M^2 \gg s_0$  the pion distribution amplitude has to be taken at the scale  $\mu^2 \sim s_0$ , rather than at<sup>3</sup>  $\mu^2 \sim M^2$ . The width of the  $b^2$  distribution in Fig. 2 should be compared with the electromagnetic pion diameter squared:  $(2R_{\pi}^{\text{em}})^2 \sim 2 \text{ fm}^2$ .

<sup>3</sup>It can be shown that this change of scale takes into account the continuum subtraction in the running coupling; cf. [11].

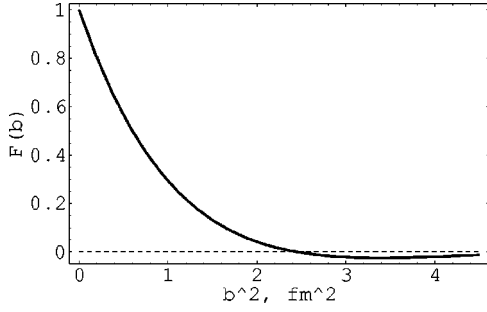


FIG. 2. The transverse-distance separation between the quark and the antiquark in the leading-order light-cone sum rule (14) in the large- $Q^2$  limit for typical values of the sum rule parameters  $s_0 = 0.7 \text{ GeV}^2$  and  $M^2 = 1.0 \text{ GeV}^2$ .

### III. RADIATIVE CORRECTIONS

#### A. General case

In order to improve the accuracy of the light-cone sum rule (14), one has to calculate the  $O(\alpha_s)$  radiative corrections to the leading-order correlation function (10). The corresponding Feynman diagrams are shown in Fig. 3. The calculation is straightforward, albeit tedious, and technically similar to the calculation of the radiative correction to the  $\gamma^* \gamma^* \pi$  transition form factor for different photon virtualities [12]. We handle ultraviolet and infrared collinear divergences by dimensional regularization in the modified minimal subtraction (MS) scheme. Because of the fact that the diagrams contain two  $\gamma_5$  matrices (one from the axial-vector vertex and one from the pion projection), there is no  $\gamma_5$  ambiguity. We have also checked that the collinear divergences are absorbed in the definition of the scale-dependent pion distribution amplitude  $\varphi_\pi$ .

Our result for the twist 2 part of the correlation function (1) to  $O(\alpha_s)$  can be represented in a form of convolution of

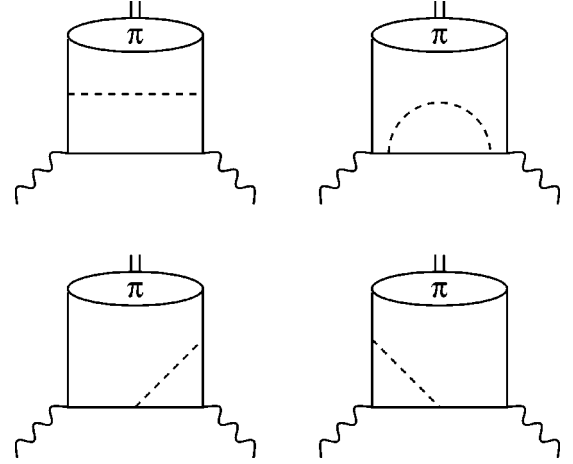


FIG. 3. The radiative corrections to the correlation function in Eq. (1). Dashed lines denote virtual gluons.

$\varphi_\pi$  with the hard scattering amplitude

$$T_{\mu\nu}^{(2)} = 2if_\pi p_\mu p_\nu \int_0^1 du \varphi_\pi(u, \mu) \left\{ H_0(Q^2, s, u) + \frac{\alpha_s C_F}{4\pi} H_1(Q^2, s, u, \mu) \right\}, \quad (18)$$

where the leading-order result was already given in Eq. (10),

$$H_0(Q^2, s, u) = \frac{u}{\bar{u}Q^2 - us}, \quad (19)$$

and the radiative correction to the hard scattering amplitude equals

$$H_1(Q^2, s, u, \mu) = \frac{Q^2}{u\bar{u}(Q^2+s)^3(\bar{u}Q^2-us)} \left[ -9u\bar{u}(\bar{u}Q^2+s)^2 - [Q^4\bar{u}(3\bar{u}-2) + Q^2s(5-6u\bar{u}) + s^2u(3u-2)] \ln\left(\frac{\bar{u}Q^2-us}{\mu^2}\right) \right. \\ \left. + [Q^4\bar{u}u + Q^2s(1+2u\bar{u}) + s^2\bar{u}u] \ln^2\left(\frac{\bar{u}Q^2-us}{\mu^2}\right) + u[-2Q^4\bar{u} + 5Q^2s + s^2(1+2\bar{u})] \ln\frac{s}{\mu^2} \right. \\ \left. - us[Q^2(1+\bar{u}) + s\bar{u}] \ln^2\frac{s}{\mu^2} + \bar{u}[Q^4(1+2u) + 5Q^2s - 2s^2u] \ln\frac{Q^2}{\mu^2} - \bar{u}Q^2[Q^2u + s(1+u)] \ln^2\frac{Q^2}{\mu^2} \right]. \quad (20)$$

To obtain the radiative correction to the light-cone sum rule for  $F_\pi$ , one has to calculate the imaginary part of  $H_1$  in the variable  $s$ . The resulting expression is presented in Appendix A. After continuum subtraction and Borel transformation, the sum rule reads

$$F_\pi^{(2)}(Q^2) = \int_0^1 du \varphi_\pi(u, \mu) [\Theta(u-u_0) \mathcal{F}_{\text{soft}}^{(2)}(u, M^2, s_0) + \Theta(u_0-u) \mathcal{F}_{\text{hard}}^{(2)}(u, M^2, s_0)], \quad (21)$$

where

$$\begin{aligned}
 \mathcal{F}_{\text{soft}}^{(2)}(u, M^2, s_0) = & \exp\left(-\frac{\bar{u}Q^2}{uM^2}\right) \left\{ 1 + \frac{\alpha_s}{4\pi} C_F \left[ -9 + \frac{\pi^2}{3} + 3 \ln \frac{Q^2}{\mu^2} + 3 \ln \frac{\bar{u}Q^2}{u\mu^2} - \ln^2 \frac{Q^2}{\mu^2} - \ln^2 \frac{\bar{u}Q^2}{u\mu^2} \right] \right\} \\
 & + \frac{\alpha_s}{4\pi} C_F \left( \int_{\bar{u}Q^2/u}^{s_0} \frac{ds Q^2 e^{-s/M^2}}{u(Q^2+s)^3} \left[ 5s + Q^2 \left( 1 + 2 \ln \frac{-\rho}{\mu^2} \right) + 2 \left( \frac{Q^2}{\bar{u}} + s \right) \ln \frac{-\rho}{s} \right. \right. \\
 & + \left. \frac{2Q^2}{u} \left( \frac{Q^2+s}{s} + \frac{2M^2+Q^2+s}{M^2} \ln \frac{-\rho}{s} \right) \ln \frac{-\rho}{\mu^2} \right] + \int_0^{\bar{u}Q^2/u} \frac{ds Q^2 e^{-s/M^2}}{u\bar{u}(Q^2+s)^3} \left\{ 2u \left( Q^2 - s + s \ln \frac{s}{\mu^2} \right) + \left[ -Q^2 + 5s \right. \right. \\
 & \left. \left. + 2(Q^2 - s) \ln \frac{s}{\mu^2} - \frac{s(Q^2+s)}{M^2} \left( -3 + 2 \ln \frac{s}{\mu^2} \right) \right] \ln \frac{\rho}{\mu^2} \right\} + 2 \frac{u_0^2}{u^2} e^{-s_0/M^2} \ln \frac{-\rho_0}{\mu^2} \ln \frac{u-u_0}{\bar{u}_0} \right) \quad (22)
 \end{aligned}$$

and

$$\begin{aligned}
 \mathcal{F}_{\text{hard}}^{(2)}(u, M^2, s_0) = & \frac{\alpha_s}{4\pi} C_F \left( \int_0^{s_0} \frac{ds Q^2 e^{-s/M^2}}{\bar{u}(Q^2+s)^3} \left\{ 2 \left( Q^2 - s + s \ln \frac{s}{\mu^2} \right) + \frac{1}{u} \left[ -Q^2 + 5s + 2(Q^2 - s) \ln \frac{s}{\mu^2} \right. \right. \right. \\
 & \left. \left. - \frac{s(Q^2+s)}{M^2} \left( -3 + 2 \ln \frac{s}{\mu^2} \right) \right] \ln \frac{\rho}{\mu^2} \right\} - \frac{u_0 \bar{u}_0}{u\bar{u}} e^{-s_0/M^2} \left( 2 \ln \frac{s_0}{\mu^2} - 3 \right) \ln \frac{\rho_0}{\mu^2} \right). \quad (23)
 \end{aligned}$$

Here  $\rho = \bar{u}Q^2 - us$  and  $\rho_0 = \bar{u}Q^2 - us_0 = (1 - u/u_0)Q^2$ . The superscript “(2)” indicates the leading twist 2 contribution. Higher-twist terms will be added in the next section. We interpret the parts of Eq. (21) with  $\mathcal{F}_{\text{hard}}^{(2)}(u, M^2, s_0)$  and  $\mathcal{F}_{\text{soft}}^{(2)}(u, M^2, s_0)$  as ‘hard’ and ‘soft’ contributions to the pion form factor, respectively, defined with the explicit cut-off in the momentum fraction  $u = u_0 \sim 1 - s_0/Q^2$ . This separation will be discussed in detail below.

### B. Study case: Asymptotic distribution amplitude

For the asymptotic shape of the pion distribution amplitude  $\varphi_\pi^{\text{as}}(u) = 6u(1-u)$  the momentum-fraction integration in the radiative correction can easily be done analytically, with the simple result

$$\begin{aligned}
 F_\pi^{\text{as}}(Q^2) = & 6 \int_0^{s_0} ds e^{-s/M^2} \frac{sQ^4}{(s+Q^2)^4} \left\{ 1 + \frac{\alpha_s C_F}{4\pi} \left[ \frac{\pi^2}{3} - 6 \right. \right. \\
 & \left. \left. - \ln^2 \frac{Q^2}{s} + \frac{s}{Q^2} + \frac{Q^2}{s} \right] \right\}. \quad (24)
 \end{aligned}$$

All the scale-dependent logarithmic terms cancel in this case, as expected. For large  $Q^2 \gg s_0$  one can expand the sum rule (24) in powers of  $1/Q^2$ :

$$\begin{aligned}
 F_\pi^{\text{as}}(Q^2) = & \frac{3\alpha_s C_F}{2\pi Q^2} \int_0^{s_0} ds e^{-s/M^2} + \frac{6}{Q^4} \int_0^{s_0} ds s e^{-s/M^2} \\
 & \times \left\{ 1 - \frac{\alpha_s C_F}{4\pi} \left[ 10 - \frac{\pi^2}{3} + \ln^2 \frac{Q^2}{s} \right] \right\} + \mathcal{O}(1/Q^6). \quad (25)
 \end{aligned}$$

To interpret the leading term, we notice that the integral  $\int_0^{s_0} ds e^{-s/M^2}$  can be related to the pion decay constant through the QCD sum rule [13]

$$\begin{aligned}
 f_\pi^2 = & \frac{1}{4\pi^2} \int_0^{s_0} ds e^{-s/M^2} \left( 1 + \frac{\alpha_s}{\pi} \right) + \frac{\langle 0 | \alpha_s / \pi G^2 | 0 \rangle}{12M^2} \\
 & + \frac{176}{81M^4} \pi \alpha_s \langle \bar{q}q \rangle^2 + \dots \quad (26)
 \end{aligned}$$

The perturbative correction and the gluon- and quark-condensate contributions involve an extra power of  $\alpha_s$  and are absent, therefore, in our approximation. Substituting  $\int_0^{s_0} ds e^{-s/M^2} \rightarrow 4\pi^2 f_\pi^2$ , we obtain

$$F_\pi^{\text{as}}(Q^2) \rightarrow \frac{8\pi\alpha_s f_\pi^2}{Q^2}, \quad (27)$$

which coincides with the classical result [6].

It is easy to see that the  $\mathcal{O}(1/Q^2)$  contribution to the form factor comes entirely from the term which we have identified as ‘hard,’ while all power-suppressed corrections involve both hard and soft contributions. In particular, we obtain, to  $\mathcal{O}(1/Q^4)$  accuracy,

$$\begin{aligned}
 F_\pi^{\text{as, hard}}(Q^2) \equiv & \int_0^{u_0} du \varphi_\pi^{\text{as}}(u) \mathcal{F}_{\text{hard}}^{(2)}(u, M^2, s_0) \\
 = & \frac{3\alpha_s C_F}{2\pi Q^2} \int_0^{s_0} ds e^{-s/M^2} \left\{ 1 - \frac{s}{Q^2} \left[ 1 + 2 \frac{s_0}{s} \right. \right. \\
 & \left. \left. + \ln \frac{s}{\mu^2} + \left( 3 - 2 \ln \frac{s}{\mu^2} \right) \ln \frac{Q^2}{s_0 - s} \right] \right\}. \quad (28)
 \end{aligned}$$



The soft part is then just what is left when this hard contribution is subtracted from the total result in Eq. (25). Notice that the separation of soft and hard contributions depends on the collinear factorization scale, even for the asymptotic distribution amplitude. We will elaborate on this dependence in what follows.

In the local duality limit  $M^2 \rightarrow \infty$  one obtains

$$F_\pi^{\text{as, hard}}(Q^2) = \frac{3\alpha_s C_F}{2\pi Q^2} s_0 \left\{ 1 - \frac{s_0}{Q^2} \left[ \frac{13}{2} - \frac{\pi^2}{6} + \ln \frac{Q^2}{s_0} \ln \frac{\mu^2}{s_0} + \ln \frac{\mu^2}{s_0} + 2 \ln \frac{Q^2}{s_0} \right] \right\} \quad (29)$$

and

$$F_\pi^{\text{as, soft}}(Q^2) = \frac{3s_0^2}{Q^4} + \frac{3\alpha_s C_F}{4\pi Q^4} s_0^2 \left\{ \frac{5}{2} + \ln^2 \frac{\mu^2}{s_0} - \ln^2 \frac{Q^2}{\mu^2} + 2 \ln \frac{\mu^2}{s_0} + 3 \ln \frac{Q^2}{s_0} \right\}, \quad (30)$$

where  $s_0 \approx 4\pi^2 f_\pi^2$ ; cf. Eq. (26). Note that the  $O(1/Q^4)$  hard contribution is large and negative, while the soft radiative correction  $O(\alpha_s/Q^4)$  is positive, unless  $Q^2 \gg s_0, \mu^2$ . This implies considerable cancellations in the sum of the soft and hard contributions so that in order to make their separation physically meaningful one must assume a low value of the factorization scale  $\mu^2 \sim s_0$ .

Finally, notice the double-logarithmic contribution  $\sim \ln^2 Q^2/s$  in Eq. (24) which is reminiscent of the Sudakov logarithms discussed in [10]. A typical size of these corrections is of the order of  $\ln^2 Q^2/s_0$  which for  $s_0 \sim 0.7 - 0.8 \text{ GeV}^2$  and  $Q^2 \sim 1 - 10 \text{ GeV}^2$  is much less than  $\ln^2 Q^2/\Lambda_{\text{QCD}}^2$  with  $\Lambda_{\text{QCD}} \sim 200 \text{ MeV}$ , as usually assumed. For this reason, exponentiation of Sudakov corrections is numerically not important in the present approach.

### C. $1/Q^2$ expansion in the general case

For a generic pion distribution amplitude the NLO light-cone sum rule (21) again simplifies considerably upon the expansion in powers of  $1/Q^2$ . We obtain, to  $O(1/Q^4)$  accuracy,

$$F_\pi(Q^2) = \frac{\alpha_s}{2\pi} C_F \int_0^{s_0} \frac{dse^{-s/M^2}}{Q^2} \int_0^1 du \frac{\varphi_\pi(u)}{\bar{u}} + \varphi'_\pi(0) \int_0^{s_0} \frac{dsse^{-s/M^2}}{Q^4} \left[ 1 + \frac{\alpha_s}{4\pi} C_F \times \left( -9 + \frac{1}{3} \pi^2 + \ln \frac{s}{\mu^2} - \ln^2 \frac{s}{Q^2} \right) \right]$$

<sup>4</sup>It is easy to see that for  $\mu^2 = Q^2$  there are double-logarithmic contributions  $\sim \ln^2(Q^2/s_0)$  to Eqs. (29) and (30) which have opposite sign and partially cancel in the sum.

$$+ \frac{\alpha_s}{4\pi} C_F \int_0^{s_0} \frac{dsse^{-s/M^2}}{Q^4} \left\{ \left( 2 \ln \frac{s}{\mu^2} - 3 \right) \times \int_0^1 du \left[ \frac{\varphi_\pi(u) - \bar{u}\varphi'(0)}{\bar{u}^2} \right] + \left( 2 \ln \frac{s}{\mu^2} - 8 \right) \int_0^1 du \frac{\varphi_\pi(u)}{\bar{u}} \right\}. \quad (31)$$

The scale dependence cancels to the required accuracy, since [14]

$$\frac{d}{d \ln \mu} \varphi'_\pi(0, \mu) = \frac{\alpha_s}{\pi} \left\{ \int_0^1 du \left[ \frac{\varphi_\pi(u) + \bar{u}\varphi'_\pi(1)}{\bar{u}^2} + \frac{\varphi_\pi(u)}{\bar{u}} \right] - \frac{1}{2} \varphi'_\pi(1) \right\}. \quad (32)$$

The contribution of hard rescattering equals

$$F_\pi^{\text{hard}}(Q^2) = \frac{\alpha_s}{2\pi} C_F \int_0^{s_0} \frac{dse^{-s/M^2}}{Q^2} \int_0^1 du \frac{\varphi_\pi(u)}{\bar{u}} + \frac{\alpha_s}{4\pi} C_F \int_0^{s_0} \frac{dsse^{-s/M^2}}{Q^4} \left\{ \left( 2 \ln \frac{s}{\mu^2} - 3 \right) \times \int_0^1 du \left[ \frac{\varphi_\pi(u) - \bar{u}\varphi'_\pi(0)}{\bar{u}^2} \right] + \left( 2 \ln \frac{s}{\mu^2} - 8 \right) \int_0^1 du \frac{\varphi_\pi(u)}{\bar{u}} \right\} + \frac{\alpha_s}{4\pi} C_F \varphi'_\pi(0) \int_0^{s_0} \frac{dse^{-s/M^2}}{Q^4} \times \left\{ s \left( 2 \ln \frac{s}{\mu^2} - 3 \right) \ln \frac{Q^2}{s_0 - s} - 2s_0 \right\}, \quad (33)$$

and the soft contribution is identified as the difference  $F_\pi^{\text{soft}}(Q^2) = F_\pi(Q^2) - F_\pi^{\text{hard}}(Q^2)$ . Note the term proportional to  $\varphi'_\pi(0) = -\varphi'_\pi(1)$  in the last line of Eq. (33) which is concentrated at the end point but enters as part of the hard contribution.

A few comments are in order concerning this expansion. First, consider the leading asymptotic  $O(1/Q^2)$  term. Substituting, as above,  $\int_0^{s_0} ds e^{-s/M^2} \rightarrow 4\pi^2 f_\pi^2$ , this contribution can be rewritten as

$$F_\pi(Q^2) = \frac{8\pi\alpha_s f_\pi^2}{9Q^2} \int_0^1 dv \frac{\varphi_\pi^{\text{as}}(v)}{\bar{v}} \int_0^1 du \frac{\varphi_\pi(u)}{\bar{u}}, \quad (34)$$

where we used that  $\int_0^1 dv \varphi_\pi^{\text{as}}(v)/\bar{v} = 3$ . This expression is similar, but does not yet coincide with the perturbative QCD result [6]

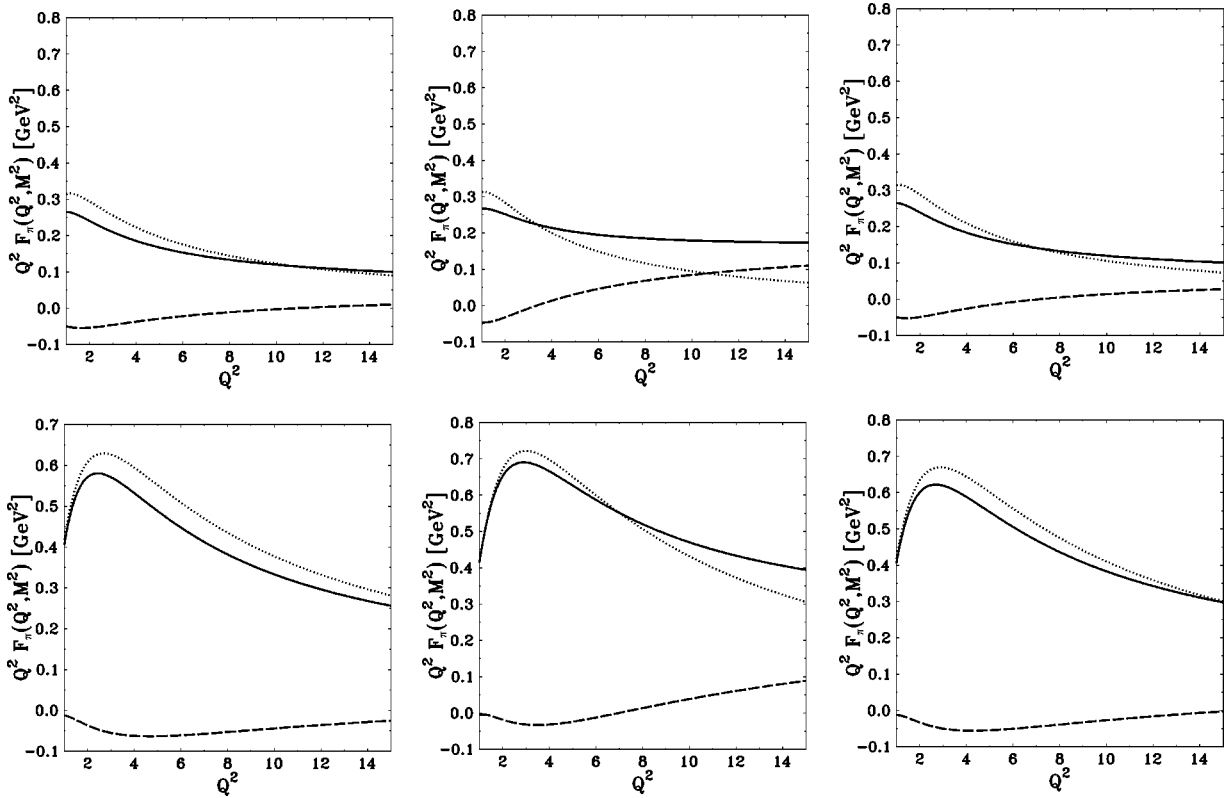


FIG. 4. The relative contributions of hard (dashed curves) and soft (dotted curves) regions in the leading-twist light-cone sum rule (21) for the pion form factor (the sum: solid lines). The upper figures correspond to asymptotic and the lower figures to the Chernyak-Zhitnitsky (CZ) pion distribution amplitude. The three figures from left to right correspond to the choice of factorization scale  $\mu^2 = Q^2$ ,  $\mu^2 = s_0$ , and  $\mu^2 = \bar{u}Q^2 + uM^2$ , respectively.

$$F_\pi(Q^2) = \frac{8\pi\alpha_s f_\pi^2}{9Q^2} \left| \int_0^1 du \frac{\varphi_\pi(u)}{\bar{u}} \right|^2. \quad (35)$$

It is easy to convince oneself that the missing corrections to the asymptotic pion distribution amplitude in the first integral in Eq. (34) as well as the missing nonperturbative corrections to  $f_\pi$  are supplied by eventual higher-order and higher-twist corrections to the sum rule. Since such corrections are difficult to evaluate directly, one may try to improve the light-cone sum rule by combining it with the known full NLO perturbative calculation. Such a possibility will be discussed in Sec. VI.

Second, the structure of the  $O(1/Q^4)$  power correction to the pion form factor is very similar to the heavy quark limit of the light-cone sum rule for  $B \rightarrow \pi e \nu$  decay considered in [14]. In particular, note the  $1/\bar{u}^2$  weight factor in the integral over the pion distribution amplitude, the structure of double logarithms, and, finally, the cancellation of the collinear factorization scale dependence by the same mechanism. In both cases, the distinction between soft and hard contributions necessitates a kind of generalized ‘‘plus-distribution’’ subtraction of divergent integrals over the pion distribution amplitude at  $u \rightarrow 1$ , as is done in the second line of Eq. (33). This feature seems to be general, whereas the distribution of finite terms  $\sim \varphi'_\pi(0)$  between the hard and the soft contributions is arbitrary. The expression for such terms in the last

line in Eq. (33) corresponds to the particular definition (21) with a rigid cutoff in momentum fraction. Although such a definition is the most intuitive one, it is not unique and, as seen from Eqs. (31), (33), introduces rather cumbersome ‘‘surface terms’’  $\sim \varphi'_\pi(0)$  which appear both in hard and soft contributions and cancel in their sum. An interesting alternative [14] which we do not pursue in detail in this work is to define the separation between hard and soft contributions order by order in the  $1/Q^2$  expansion using ‘‘plus distributions’’ to regularize the divergent momentum-fraction integrals. To the accuracy of Eq. (31), this procedure corresponds to the definition of the hard contribution as given in the first three lines in Eq. (33), omitting the ‘‘surface term.’’ The soft contribution is given then by the second line in Eq. (31).

Last but not least, having in mind that the separation of hard and soft contributions is ambiguous, one may add them together and consider their sum as a ‘total nonperturbative’ power correction to each order in the  $1/Q^2$  expansion. Inspection of Eqs. (25), (29), (30) suggests that soft and hard corrections in general have opposite signs and partially cancel in the sum. We postpone the detailed discussion of this issue to Sec. VI where we summarize our numerical results.

#### D. Numerical estimates

Results of the numerical evaluation of the sum rule (21) are shown in Fig. 4 by solid curves, for  $s_0 = 0.7 \text{ GeV}^2$  and for a typical value of the Borel parameter  $M^2 = 1.0 \text{ GeV}^2$ .

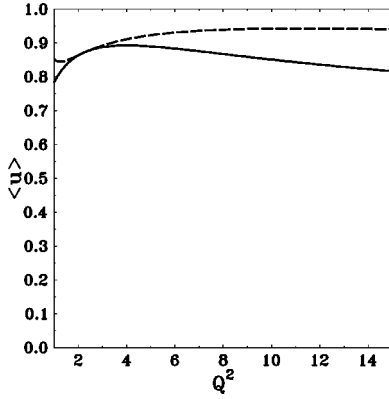


FIG. 5. Average momentum fraction  $u$  as a function of  $Q^2$  for the asymptotic (solid curve) and CZ (dashed curve) distribution amplitudes at  $M^2 = 1.0$  GeV $^2$ . The scale is  $\mu^2 = (1-u)Q^2 + uM^2$ .

(The choice of input parameters is discussed below in Sec. V.) Soft and hard contributions are shown by dotted and dashed curves, respectively. The results are plotted using the asymptotic  $\varphi_\pi^{\text{as}}(u) = 6u(1-u)$  and the CZ  $\varphi_\pi^{\text{CZ}}(u, \mu = 1 \text{ GeV}) = 30\bar{u}u(2u-1)^2$  pion distribution amplitudes, for three different choices of the factorization scale,  $\mu^2 = Q^2$ ,  $\mu^2 = s_0$  (see the discussion above), and  $\mu^2 = \mu_u^2 = \bar{u}Q^2 + uM^2$ , according to Eq. (13).<sup>5</sup> In all calculations in this paper we use the two-loop QCD running coupling with  $\Lambda_{\overline{\text{MS}}}^{(3)} = 336$  MeV corresponding to  $\alpha_s(1 \text{ GeV}) = 0.48$  and  $\alpha_s(s_0) = 0.59$ .

It is seen that hard contribution to the form factor defined with a ‘‘natural’’ momentum-fraction cutoff remains small and *negative* for the main part of the interesting region of  $Q^2$ .

Furthermore, in Fig. 5 we show the average value of the momentum fraction  $u$  in the integral in Eq. (21) calculated as a function of  $Q^2$  for the asymptotic (solid curve) and CZ (dashed curve) distribution amplitudes. This average value turns out to be very large, and, contrary to usual expectations, does not depend significantly on the shape of the pion distribution amplitude.

The negative contribution of the hard-rescattering mechanism may appear unexpected and counterintuitive. We emphasize, however, that the separation between hard and soft terms is ambiguous and depends on their definition — this is, in fact, the main lesson to be learned from our analysis. Note that the scale dependence is much more pronounced for hard and soft contributions taken separately than for their sum.

## IV. HIGHER-TWIST CORRECTIONS

### A. Twist 4 contributions

The operator-product expansion of the correlation function (1) near the light cone  $x^2 = 0$  can be continued beyond the leading twist 2 approximation (10). This procedure yields higher-twist corrections to the light-cone sum rule (21). They are suppressed by additional inverse powers of  $M^2$  and  $Q^2$ .

<sup>5</sup>If the momentum-fraction-dependent scale  $\mu_u$  is used, it is implied that  $\alpha_s(\mu_u)$  is inserted inside the  $u$  integrals.

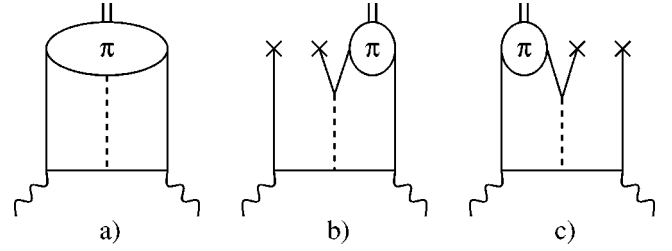


FIG. 6. The light-cone expansion of the quark propagator in the correlation function (1).

Physically, the higher-order terms of the light-cone expansion take into account both the transverse momentum of the quark-antiquark state and the contributions of higher Fock states in the pion wave function. As explained in [15], these two effects are indistinguishable due to QCD equations of motion.

Next to the leading twist 2 term, the correlation function (1) receives several twist 4 contributions.<sup>6</sup> First of all, one has to take into account the twist 4 components of the quark-antiquark matrix element  $\langle 0 | \bar{d}(0) \gamma_\mu \gamma_5 u(x) | \pi^+(p) \rangle$  in the diagram in Fig. 1. Furthermore, the gluon emission from the virtual quark should be included, yielding the diagram of Fig. 6(a) with the twist 4 quark-antiquark-gluon distribution amplitudes of the pion. To calculate this diagram, one makes use of the light-cone expansion of the quark propagator [16] given in Appendix A. The definitions of all relevant twist 4 two- and three-particle distribution amplitudes [15,17] are collected in Appendix B.

The corresponding calculation has been carried out in [4]. The twist 4 contribution to the correlation function (1) can be written in the following compact form:

$$T_{\mu\nu}^{(4)} = 2i p_\mu p_\nu f_\pi \int_0^1 du \frac{u \varphi^{(4)}(u)}{(\bar{u}Q^2 - us)^2}, \quad (36)$$

where

$$\begin{aligned} \varphi^{(4)}(u) = & -4 \left( g_1(u) - \int_0^u dv g_2(v) \right) + 2u g_2(u) \\ & + \int_0^u d\alpha_1 \int_0^{\bar{u}} d\alpha_2 \left[ \frac{\tilde{\varphi}_\parallel(\alpha_i) + 2\tilde{\varphi}_\perp(\alpha_i)}{\alpha_3} \right. \\ & + \frac{1 - 2u + \alpha_1 - \alpha_2}{\alpha_3^2} [\varphi_\parallel(\alpha_i) \\ & \left. + \varphi_\perp(\alpha_i)] \right]_{\alpha_3 = 1 - \alpha_1 - \alpha_2} \end{aligned} \quad (37)$$

is a combination of twist 4 distribution amplitudes of the pion. The explicit expression for  $\varphi^{(4)}$  is given in Appendix B.

<sup>6</sup>Twist 3 contributions to Eq. (1) are proportional to  $m_\pi^2$  and vanish in the chiral limit adopted here.



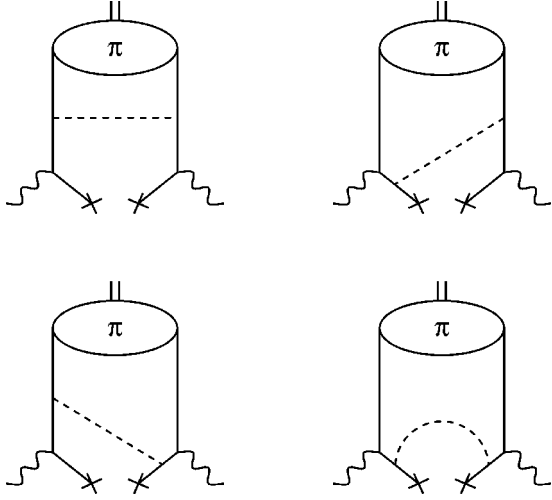


FIG. 7. Quark condensate corrections to the correlation function in Eq. (1).

The twist 4 correction to the light-cone sum rule is easily obtained by taking the imaginary part of Eq. (36) in  $s=(p-q)^2$  and subtracting the continuum above  $s_0$ . After Borel transformation one obtains

$$F_{\pi}^{(4)}(Q^2) = \int_{u_0}^1 du \frac{\varphi^{(4)}(u)}{uM^2} \exp\left(-\frac{\bar{u}Q^2}{uM^2}\right) + \frac{u_0\varphi^{(4)}(u_0)}{Q^2} e^{-s_0/M^2}. \quad (38)$$

The second term on the RHS of Eq. (38) has not been taken into account in [4]. It appears as a ‘‘surface term’’ when the correlation function with a denominator  $(q-up)^{2n} = (-\bar{u}Q^2 + us)^n$  with  $n > 1$  is converted (integrating by parts) into a canonical dispersion integral  $T_{\mu\nu}(s, Q^2) = 1/\pi \int d\tilde{s} \text{Im} T_{\mu\nu}(\tilde{s}, Q^2)/(\tilde{s}-s)$ . Adding the expression in Eq. (38) to the leading twist 2 contribution (21) one obtains the light-cone sum rule for  $F_{\pi}(Q^2)$  to twist 4 accuracy. As seen from Eq. (38), all twist 4 effects have to be identified (to our accuracy) as part of the soft contribution to the form factor. Since  $\varphi^{(4)}(u) \sim (1-u)$  at  $u \rightarrow 1$  (see Appendix B), the twist 4 corrections are of order  $1/Q^4$  in the large- $Q^2$  limit.

Assuming asymptotic expressions for the quark-antiquark-gluon distribution amplitudes [15,17,18] one obtains a compact expression

$$\varphi^{(4)}(u) = \frac{20}{3} \delta^2(\mu) u^2 \bar{u} (3u-2), \quad (39)$$

where  $\delta^2(1 \text{ GeV}) \approx 0.2 \text{ GeV}^2$  is a scale-dependent parameter determining the pion coupling to the local quark-antiquark-gluon operator (see Appendix B for the definition). The twist 4 correction to the sum rule simplifies in this case to

$$F_{\pi}^{(4)}(Q^2) = \frac{20}{3} \delta^2(\mu) \int_0^{s_0} ds e^{-s/M^2} \frac{Q^8}{(Q^2+s)^6} \times \left(1 - \frac{8s}{Q^2} + \frac{6s^2}{Q^4}\right), \quad (40)$$

revealing at  $Q^2 \rightarrow \infty$  the  $1/Q^4$  behavior. Taking in addition the local duality limit  $M^2 \rightarrow \infty$  yields an estimate

$$F_{\pi}^{(4)}(Q^2) = \frac{20\delta^2(s_0)s_0}{3Q^4} \sim \left(\frac{1.0 \text{ GeV}^2}{Q^2}\right)^2. \quad (41)$$

### B. Factorizable twist 6 contributions

An estimate of the twist 6 contribution to the light-cone sum rule presents a new result of this paper. This calculation is interesting for several reasons. As is well known [19], twist 4 operators are ‘irreducible’ in the sense that they cannot be factorized in a product of gauge-invariant operators of lower twist. This property is special and limited to twist 4. Several light-cone operators of twist 6 exist which can be factorized as a product of two gauge-invariant twist 3 operators (or, alternatively, one twist 2 and one twist 4). Sandwiched between the vacuum and one-pion state, such operators generally produce two types of contributions: Factorizable in terms of a low-twist two-particle distribution amplitude times quark (or gluon) condensate and nonfactorizable, which give rise to genuine twist 6 multiparton pion distribution amplitudes. We emphasize that factorizable contributions have to be subtracted in the construction of multiparton distribution amplitudes similarly as disconnected diagrams proportional to the quark condensate should not be taken into account in the nucleon matrix element  $\langle N|\bar{q}q|N\rangle$  corresponding to the nucleon  $\sigma$  term.

In the present context, arguments based on conformal symmetry suggest that contributions of higher Fock states are strongly suppressed at  $u \rightarrow 1$  and their contributions to the sum rule are, probably, negligible. Factorizable contributions, on the other hand, are expected to supply the missing nonperturbative corrections in the sum rule in the large- $Q^2$  limit and can be large. They are also of principal interest and indicate, as we will see, certain limitations for the light-cone sum rule approach.

Guided by the existence of large quark condensate corrections  $\sim \langle 0|\bar{q}q|0\rangle^2$  in classical QCD sum rule calculations of the pion form factor [20,21], in this paper we concentrate on factorizable contributions of twist 6 four-quark operators, e.g.,

$$\begin{aligned} & \langle 0|\bar{q}(v_1x)\gamma_{\mu}q(v_2x)\bar{q}(v_3x)\gamma_{\nu}\gamma_5q(v_4x)|\pi^+(p)\rangle \\ &= \frac{i}{12} \langle 0|\bar{q}q|0\rangle \langle 0|[\bar{q}(v_1x)\sigma_{\mu\nu}\gamma_5q(v_4x) \\ &+ \bar{q}(v_3x)\sigma_{\mu\nu}\gamma_5q(v_2x)]|\pi^+(p)\rangle - \frac{1}{12} g_{\mu\nu} \langle 0|\bar{q}q|0\rangle \end{aligned}$$

$$\times \langle 0 | [\bar{q}(v_1x)\gamma_5 q(v_4x) - \bar{q}(v_3x)\gamma_5 q(v_2x)] | \pi^+(p) \rangle, \quad (42)$$

which involve the quark condensate and the two existing two-particle pion distribution amplitudes of twist 3,  $\varphi_p$  and  $\varphi_\sigma$ ; see Appendix B. Definitions of both of them include the normalization factor

$$\mu_\pi = \frac{m_\pi^2}{m_u + m_d} = -\frac{2}{f_\pi^2} \langle \bar{q}q \rangle, \quad (43)$$

so that the corresponding contributions to the sum rule appear to be proportional to the quark condensate squared.

We start from the light-cone expansion of the quark propagator (see Appendix A) which contains contributions proportional to the covariant derivative  $D^\alpha G_{\alpha\nu}$  of the gluon field strength. They are reduced to a quark-antiquark pair due to the QCD equations of motion. One quark (antiquark) from this pair can be combined with an antiquark (quark) from the initial currents forming a quark condensate as in Figs. 6(b), 6(c).

A straightforward calculation gives

$$\begin{aligned} T_{\mu\nu}^{\text{Figs. 6(b),6(c)}} &= 2ip_\mu p_\nu \alpha_s \pi C_F \frac{\langle \bar{q}q \rangle}{N_c} f_\pi \mu_\pi \int_0^1 du \int_0^1 dv v \bar{v} \left( 2\varphi_p(u) \left\{ \frac{(1-uv)(u-2)}{[q-(1-uv)p]^6} + \frac{vu^2}{[q-uvp]^6} \right\} \right. \\ &\quad \left. + \frac{1}{3} \varphi_\sigma(u) \left\{ \frac{uv-3}{[q-(1-uv)p]^6} - \frac{uv}{[q-uvp]^6} + \frac{3q^2v(2+u)}{[q-(1-uv)p]^8} - \frac{3q^2uv}{[q-uvp]^8} \right\} \right). \end{aligned} \quad (44)$$

Another source of the factorizable twist 6 contribution is provided by the four-quark operators in the light-cone expansion of Eq. (1) with a perturbative gluon exchange between two currents (see Fig. 7). The technique of this expansion is explained in [16,22]. A lengthy but equally straightforward calculation yields

$$\begin{aligned} T_{\mu\nu}^{\text{Fig. 7}} &= 8ip_\mu p_\nu \alpha_s \pi C_F \frac{\langle \bar{q}q \rangle}{N_c} \mu_\pi f_\pi \int_0^1 du \int_0^1 dv v \left[ \varphi_p(u) \frac{\bar{u}}{[q-(1-uv)p]^6} + \frac{1}{2} \varphi_\sigma(u) \frac{1}{[q-(1-uv)p]^6} \right. \\ &\quad \left. + \varphi_\sigma(u) \frac{v(\bar{u}p - q^2)}{[q-(1-uv)p]^8} \right]. \end{aligned} \quad (45)$$

In addition, we have considered the twist 6 parts of the two- and three-particle matrix elements corresponding to the diagrams of Fig. 1 and Fig. 5(a) and have not found any factorizable contributions. The sum of Eqs. (44) and (45) represents, therefore, the complete answer for the factorizable twist 6 contributions of four-quark operators.

The corresponding correction to the light-cone sum rule can be obtained following the standard procedure, that is, taking the imaginary part in  $s=(p-q)^2$ , subtracting the continuum above  $s=s_0$  in the dispersion integral, and performing the Borel transformation. Because of large dimension of the denominators in Eqs. (44) and (45), one ends up with a rather complicated structure of surface terms at  $s=s_0$ . The final answer can be written as

$$F_\pi^{(6)}(Q^2) = \frac{\alpha_s \pi C_F}{N_c} \langle \bar{q}q \rangle \mu_\pi \int_0^\infty ds \left[ f_2(s, Q^2) \frac{d^2}{ds^2} [\Theta(s_0-s)e^{-s/M^2}] + f_3(s, Q^2) \frac{d^3}{ds^3} [\Theta(s_0-s)e^{-s/M^2}] \right], \quad (46)$$

where

$$f_{2,3}(s, Q^2) = f_{2,3}^{\text{Figs. 6(b),6(c)}}(s, Q^2) + f_{2,3}^{\text{Fig. 7}}(s, Q^2), \quad (47)$$

with

$$\begin{aligned} f_2^{\text{Figs. 6(b),6(c)}}(s, Q^2) &= \frac{s}{2Q^2(Q^2+s)} \int_{s/(Q^2+s)}^1 \frac{du}{u^3} \left( u - \frac{s}{Q^2+s} \right) \left[ 2(2-u)\varphi_p(u) + \left( 1 + \frac{2s}{3Q^2} \right) \varphi_\sigma(u) \right] \\ &\quad - \frac{1}{2(Q^2+s)} \int_{Q^2/(Q^2+s)}^1 \frac{du}{u^3} \left( u - \frac{Q^2}{Q^2+s} \right) [2u\varphi_p(u) - \frac{1}{3}\varphi_\sigma(u)], \\ f_2^{\text{Fig. 7}}(s, Q^2) &= -\frac{2s}{Q^4} \int_{s/(Q^2+s)}^1 \frac{du}{u^2} \left[ \bar{u}\varphi_p(u) + \frac{1}{2}\varphi_\sigma(u) \left( 1 - \frac{\bar{u}s}{uQ^2} \right) \right], \end{aligned} \quad (48)$$

$$\begin{aligned}
 f_3^{\text{Fig. 6(b),6(c)}}(s, Q^2) &= \frac{s^2}{6Q^4} \int_{s/(Q^2+s)}^1 \frac{du}{u^4} \left( u - \frac{s}{Q^2+s} \right) (2+u) \varphi_\sigma(u) - \frac{1}{6} \int_{Q^2/(Q^2+s)}^1 \frac{du}{u^3} \left( u - \frac{Q^2}{Q^2+s} \right) \varphi_\sigma(u), \\
 f_3^{\text{Fig. 7}}(s, Q^2) &= -\frac{2s^2}{3Q^4} \int_{s/(Q^2+s)}^1 \frac{du}{u^3} \varphi_\sigma(u) \left( 1 - \frac{\bar{u}(Q^2+s)}{2Q^2} \right). \tag{49}
 \end{aligned}$$

It is easy to see that the expressions in Eqs. (48) and (49) receive contributions from both hard  $u < u_0$  and soft  $u > u_0$  regions, which we do not write separately in this case. The hard contribution takes into account the integration region corresponding to a large momentum  $\sim Q$  flowing through the gluon line and can be thought of as part of the hard mechanism contribution to the form factor generated by product of two twist 3 distribution amplitudes, with ‘‘wrong’’ quark helicities [23,24]. In the light-cone sum rule approach one distribution amplitude is present directly, and the second one is modelled using the duality approximation, as in the leading twist.

Inserting the asymptotic expressions for the distribution amplitudes  $\varphi_p(u) = 1$  and  $\varphi_\sigma(u) = 6u(1-u)$  and integrating over  $u$  one obtains the expansions

$$\begin{aligned}
 f_2(s, Q^2) &= -\frac{1}{Q^2} + \frac{s}{Q^4} \left( 5 - 2 \ln \frac{Q^2}{s} \right) + \dots, \\
 f_3(s, Q^2) &= -\frac{s}{Q^2} + \frac{s^2}{Q^4} \left( 3 - \ln \frac{Q^2}{s} \right) + \dots, \tag{50}
 \end{aligned}$$

substitution of which in Eq. (46) yields the twist 6 correction to the light-cone sum rule to the  $O(1/Q^4)$  accuracy:

$$F_\pi^{(6)}(Q^2) = \frac{4\alpha_s \pi C_F}{N_c f_\pi^2 Q^4} \langle \bar{q}q \rangle^2 \simeq \left( \frac{0.2 \text{ GeV}^2}{Q^2} \right)^2, \tag{51}$$

much smaller than  $F_\pi^{(4)}$ .

Most importantly, the  $O(1/Q^2)$  contributions have cancelled. Inspection of Eqs. (48) and (49) reveals that all  $O(1/Q^2)$  contributions in individual diagrams originate from the ‘‘hard’’ integration region  $u < u_0$  and their cancellation involves both diagrams and both distribution functions  $\varphi_\sigma$  and  $\varphi_p$ , in agreement with [23].

The observed cancellation of  $O(1/Q^2)$  corrections is not entirely trivial. One might fear that factorization of a local quark condensate brings us back to the deficiency of the standard QCD sum rule approach discussed in Sec. II: Expansion of local operators messes up the power counting in the momentum transfer, as observed in [20,21]. In other language, factorization of the quark condensate is equivalent to using a very bad model for the distribution amplitude of the

pion created by the interpolation current in Eq. (1), corresponding to the sum of two  $\delta$  functions; see [25,26].<sup>7</sup>

## V. NUMERICAL ANALYSIS

Combining the twist 2 calculation (21) with the twist 4 corrections in Eq. (38) and twist 6 in Eq. (46), we are now in a position to evaluate the complete light-cone sum rule for the pion form factor:

$$F_\pi(Q^2) = F_\pi^{(2)}(Q^2, M^2) + F_\pi^{(4)}(Q^2, M^2) + F_\pi^{(6)}(Q^2, M^2). \tag{52}$$

To avoid misunderstanding, note that terms of higher twist are not suppressed, in general, by increasing powers of  $1/Q^2$ , but rather by increasing powers of the Borel parameter. In particular, all twists contribute to  $1/Q^4$  accuracy, with main contributions coming from the soft region, in agreement with the general wisdom (see also Sec. II) that such corrections come from large transverse distances. The (numerical) hierarchy of contributions of different twist is, therefore, a self-consistency check for the light-cone sum rule approach.

There are several input parameters which should be specified in the sum rule. First of all, the pion duality interval  $s_0 = 0.7 \text{ GeV}^2$  is determined by fitting the two-point sum rule (26) to the pion decay constant  $f_\pi = 133 \text{ MeV}$ . This sum rule is reliable for the corresponding Borel parameter  $M_{2pt}^2 = 0.7-1.0 \text{ GeV}^2$ . Having in mind that in the light-cone sum rule for the same pion channel the Borel parameter should be larger, typically of order  $M^2 \simeq M_{2pt}^2 / \langle u \rangle$ , we assume  $0.8 < M^2 < 1.5 \text{ GeV}^2$  as a fiducial interval. We have checked that changing  $s_0$  by  $\pm 0.1 \text{ GeV}^2$  does not produce a significant effect, so that we stick to the above standard value [13,20,21] in what follows.

The principal input is provided by the leading twist distribution amplitude (see Appendix B)

<sup>7</sup>A detailed comparison with the results of [23,24] goes beyond the tasks of this paper. In particular, one may ask whether light-cone QCD sum rules can be used to calculate an effective infrared cutoff in the hard scattering contribution obtained in [23,24]. To address this issue one has to construct a different sum rule, using a chiral-odd interpolating current for the pion. Interpretation of Eq. (51) in this context is difficult because of possible contamination by the  $a_1$  meson. We thank M. Beneke and G. Buchalla for the discussion which initiated our interest in this problem.

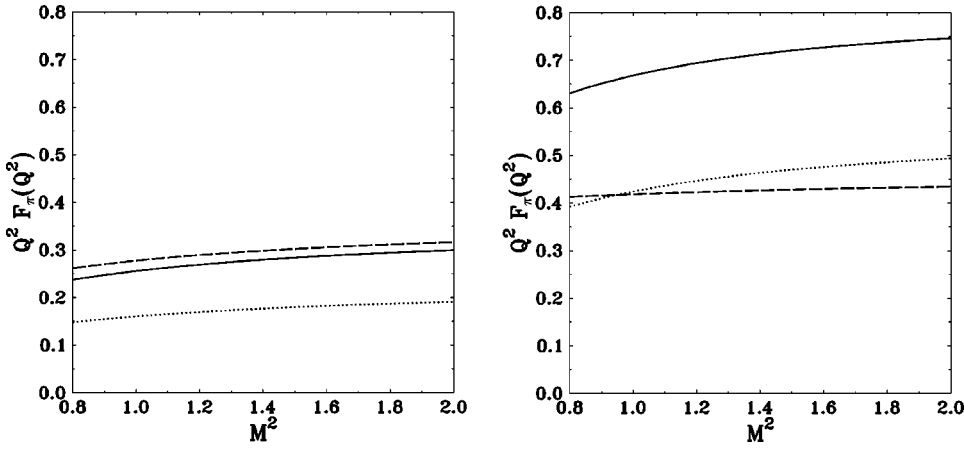


FIG. 8. The Borel parameter dependence of the light-cone sum rule for the asymptotic (left) and CZ (right) distribution amplitudes, at  $Q^2=1, 3,$  and  $10 \text{ GeV}^2$  shown by dashed, solid, and dotted curves, respectively.

$$\begin{aligned} \varphi_\pi(u, \mu) = & 6u\bar{u} [1 + a_2(\mu)C_2^{3/2}(u-\bar{u}) \\ & + a_4(\mu)C_4^{3/2}(u-\bar{u}) + \dots], \end{aligned} \quad (53)$$

where the coefficients  $a_n$  present the main nonperturbative input of interest. Taking into account the poor accuracy of the present data as well as the considerable uncertainties in the sum rules themselves, we cannot aim to distinguish between contributions of different Gegenbauer polynomials. We put, therefore, all  $a_n$ ,  $n=4,6,\dots$ , to zero and consider the values  $a_2=0$  (asymptotic distribution) and  $a_2(1 \text{ GeV})=2/3$  (CZ distribution) as the two extreme alternatives. Calculations in this section are done taking into account the anomalous dimension of  $a_2$  to one-loop accuracy; see Appendix B. Our main goal will be to determine  $a_2$  from the comparison of the sum rule results with the experimental data.

The higher-twist distribution amplitudes and relevant parameters represent another set of inputs. They are listed in Appendix B. The uncertainty in higher-twist corrections turns out to be sufficiently small and does not influence our final results.

Finally, one has to specify the renormalization and factorization scale  $\mu$ . For this numerical analysis we use the  $u$ -dependent scale  $\mu_u^2=(1-u)Q^2+uM^2$  for the leading twist 2 contributions and simply take  $\mu^2=M^2$  for the soft-

region ( $u>u_0$ ) dominated higher-twist corrections in the sum rule. The scale dependence is, in fact, rather mild.

The Borel parameter dependence of the sum rule is shown in Fig. 8 for three different values of  $Q^2$ . As can be seen from this figure, the prediction for the form factor is sufficiently stable.

The relative contributions of different twists to the sum rule are shown as a function of  $Q^2$  in Fig. 9. The twist 4 contribution does not exceed 25–30 % of the total result while the twist 6 correction is negligibly small. This hierarchy reveals a good convergence of the light-cone expansion, at least at  $Q^2>1 \text{ GeV}^2$ . For lower values of  $Q^2$  the higher-twist corrections become unstable and the approach breaks down.

Finally, in Fig. 10 we compare the light-cone sum rule calculation with the available experimental data in the interval  $1<Q^2<7 \text{ GeV}^2$  taken from [27,28]. The dashed and dotted curves correspond to the asymptotic and CZ distribution amplitudes, respectively. The solid curve presents the best fit, yielding

$$a_2^{\text{LCSR}}(\mu=1 \text{ GeV})=0.12 \pm 0.07_{-0.07}^{+0.05}, \quad (54)$$

with  $\chi^2=13.9$  for 14 degrees of freedom. The first error comes from the experimental uncertainty, whereas the second error corresponds to the variation of the Borel parameter.

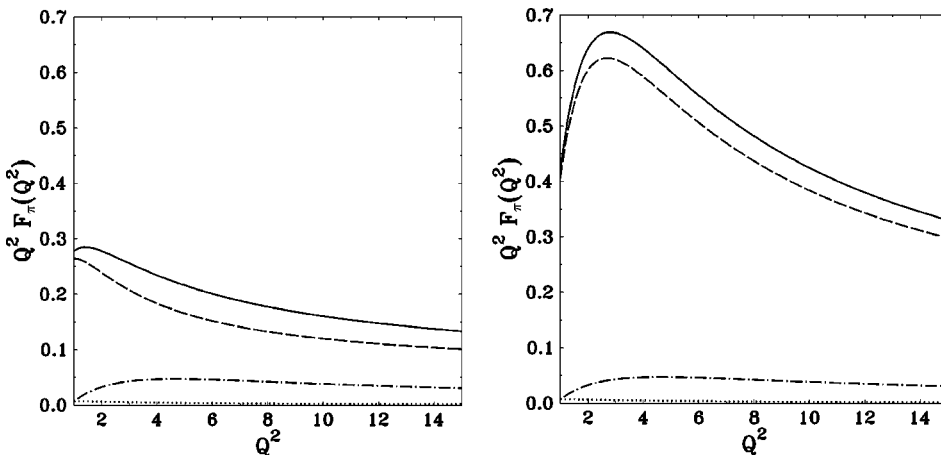


FIG. 9. The relative importance of contributions of different twist in the light-cone sum rule for the asymptotic (left) and CZ (right) twist 2 distribution amplitudes. Twist 2 (dashed curves), twist 4 (dot-dashed curves), and twist 6 (dotted curves) contributions and their sum (solid curve) are plotted at  $M^2=1 \text{ GeV}^2$ .

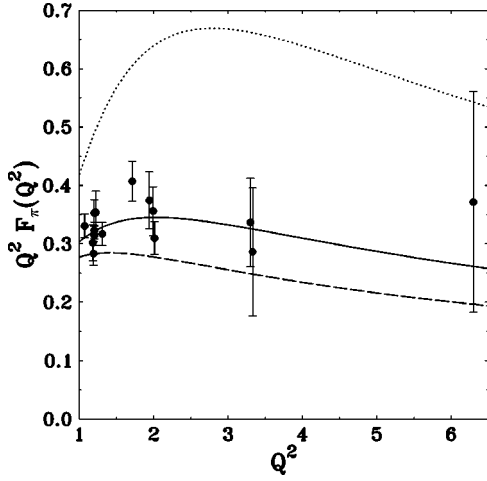


FIG. 10. The light-cone sum rule predictions for the pion electromagnetic form factor using asymptotic distribution amplitude (dashed curve), CZ distribution (dotted curve), and fit to the data (solid curve).

This value for the coefficient  $a_2$  translates to the estimate for the characteristic integral:

$$\int \frac{du}{u} \varphi_\pi(u, \mu = 1 \text{ GeV}) = 3.36 \pm 0.21_{-0.21}^{+0.15}. \quad (55)$$

## VI. MATCHING WITH THE NLO PERTURBATIVE CALCULATION

As discussed in Sec. III C the light-cone sum rule in the present form does not yet reproduce the full perturbative result in the asymptotic limit  $Q^2 \rightarrow \infty$ . The missing terms correspond to higher-order corrections in the light-cone expansion and are difficult to calculate directly. Instead, one can make a matching of the light-cone sum rule to the NLO perturbative calculation by following the standard logic of the asymptotic expansion [29]. To this end, write the twist 2 contribution to the sum rule defined in Eq. (21) as a sum of two terms, adding and subtracting the leading asymptotic expression at  $Q^2 \rightarrow \infty$  [see Eq. (31), first line]:

$$\begin{aligned} F_\pi^{(2)}(Q^2) &= F_{\text{pert}}^{(2)}(Q^2) + F_{\text{nonp}}^{(2)}(Q^2), \\ F_{\text{pert}}^{(2)}(Q^2) &= \frac{\alpha_s}{2\pi} C_F \int_0^{s_0} \frac{ds e^{-s/M^2}}{Q^2} \int_0^1 du \frac{\varphi_\pi(u)}{\bar{u}}, \\ F_{\text{nonp}}^{(2)}(Q^2) &= F_\pi^{(2)}(Q^2) - F_{\text{pert}}^{(2)}(Q^2). \end{aligned} \quad (56)$$

Following the arguments of [6] one can prove that higher-order corrections to the sum rule must assemble themselves to reproduce the factorized expression

$$F_{\text{pert}}^{(2)}(Q^2) \Rightarrow \int_0^1 dx \int_0^1 dy \varphi_\pi(x, \mu) T_H(x, y, Q^2, \mu) \varphi_\pi(y, \mu), \quad (57)$$

with

$$\begin{aligned} T_H(x, y, Q^2, \mu^2) &= \frac{2\pi C_F \alpha_s(\mu) f_\pi^2}{N_c Q^2 (1-x)(1-y)} \\ &\times \left[ 1 + \frac{\alpha_s(\mu)}{\pi} T_1(x, y, Q^2/\mu^2) + \dots \right]. \end{aligned} \quad (58)$$

The remainder  $F_{\text{nonp}}^{(2)}(Q^2)$  is suppressed by a power of  $Q^2$  and presents a true nonperturbative ‘‘higher twist’’ correction to the usual perturbative result based on collinear factorization.

Making the substitution (57) we effectively take into account all higher-order corrections to the sum rule to  $O(1/Q^2)$  accuracy and neglect such corrections for power-suppressed terms. This procedure tacitly implies that the numerical effect of the replacement (57) is more important than of uncalculated (higher-order and higher-twist) corrections to  $F_{\text{nonp}}^{(2)}(Q^2)$ . Such an assumption is natural, but in fact flawed because of potential double counting of perturbative contributions of soft regions. As one signal for this problem, one may notice that the perturbative QCD expression suffers from infrared renormalons in high orders [30], which have to be cancelled by the corresponding renormalon contributions to  $F_{\text{nonp}}^{(2)}(Q^2)$ . Using a full resummed expression for  $F_{\text{pert}}^{(2)}(Q^2)$  together with the leading-order expression for  $F_{\text{nonp}}^{(2)}(Q^2)$  destroys this intricate cancellation and is, therefore, not fully consistent theoretically. This is a usual difficulty of making a separation between ‘‘perturbative’’ and ‘‘nonperturbative’’ contributions, which has been discussed in much detail recently in context of the calculation of power corrections to deep inelastic scattering, Drell-Yan processes, event shapes in  $e^+e^-$  annihilation, and inclusive  $B$  decays [30,31].

An alternative and theoretically better defined possibility is to make a separation between soft and hard contributions to the pion form factor with an explicit cutoff, as in Sec. III, define  $F_{\text{nonp}}^{(2)}(Q^2)$  as the contribution coming from the soft region, and replace the ‘‘hard’’ contribution to the light-cone sum rule by the perturbative expression restricted to the same hard region. A difficulty in this case is that the soft-hard separation in the sum rule involves a cutoff in one momentum fraction only and becomes ambiguous when applied to the fully factorized expression (57) involving two momentum fractions.<sup>8</sup>

In the present paper we consider the first possibility because of its relative simplicity. We take into account the radiative correction to the hard-scattering kernel [32–35] and the complete NLO evolution of the pion distribution amplitude [36]; see Appendix B. To this accuracy,

$$F_{\text{pert}}(Q^2) = F_{\text{pert}}^{\text{LO}}(Q^2) + F_{\text{pert}}^{\text{NLO}}(Q^2), \quad (59)$$

<sup>8</sup>A natural solution would be to introduce a cutoff in the transverse quark-antiquark separation rather than in the momentum fraction.



where

$$F_{\text{pert}}^{\text{LO}}(Q^2) = 8\pi\alpha_s(\mu^2)\frac{f_\pi^2}{Q^2}|1+a_2^{\text{LO}}(\mu^2)+a_4^{\text{LO}}(\mu^2)|^2 \quad (60)$$

and [35]

$$\begin{aligned} F_{\text{pert}}^{\text{NLO}}(Q^2) &= \frac{16f_\pi^2}{Q^2}\alpha_s^2(\mu^2)[1+a_2^{\text{LO}}(\mu^2)+a_4^{\text{LO}}(\mu^2)] \\ &\times \left[ a_2^{\text{NLO}}(\mu^2)+a_4^{\text{NLO}}(\mu^2)+\sum_{k=3}^{\infty}a_{2k}^{\text{NLO}}(\mu^2) \right] \\ &+ 8\frac{f_\pi^2}{Q^2}\alpha_s^2(\mu^2)\left\{ \frac{2}{3}\left[ \frac{25}{6}a_2^{\text{LO}}(\mu^2)+\frac{91}{15}a_4^{\text{LO}}(\mu^2) \right] \right. \\ &\times [1+a_2^{\text{LO}}(\mu^2)+a_4^{\text{LO}}(\mu^2)]\ln\frac{\mu^2}{Q^2} \\ &+ \frac{9}{4}[1+a_2^{\text{LO}}(\mu^2)+a_4^{\text{LO}}(\mu^2)]^2\ln\frac{\mu^2}{Q^2}+6.58 \\ &+ 24.99a_2^{\text{LO}}(\mu^2)+21.43[a_2^{\text{LO}}(\mu^2)]^2 \\ &+ 32.81a_4^{\text{LO}}(\mu^2)+32.55[a_4^{\text{LO}}(\mu^2)]^2 \\ &\left. + 53.37a_2^{\text{LO}}(\mu^2)a_4^{\text{LO}}(\mu^2) \right\}. \quad (61) \end{aligned}$$

Note that we do not distinguish between the renormalization and factorization scales.

The complete expression for the form factor reads, respectively,

$$\begin{aligned} F_\pi(Q^2) &= F_{\text{pert}}(Q^2)+F_{\text{nonp}}^{(2)}(Q^2,M^2)+F_{\pi}^{(4)}(Q^2,M^2) \\ &+ F_{\pi}^{(6)}(Q^2,M^2), \quad (62) \end{aligned}$$

where we have taken into account that twist 4 and twist 6 corrections to the light-cone sum rule receive no  $1/Q^2$  contributions to our accuracy.

For the numerical analysis, we still have to specify the factorization scale. Since after the subtraction of the asymptotic  $1/Q^2$  contribution the sum rule contribution is dominated by soft contributions, we choose the fixed scale  $\mu^2 \sim M^2 = 1 \text{ GeV}^2$  for simplicity. For the perturbative contribution we use

$$\mu^2 = \kappa Q^2 + M^2, \quad M^2 = 1 \text{ GeV}^2, \quad (63)$$

with parameter  $\kappa$  in the range

$$1/4 < \kappa < 1. \quad (64)$$

Note that with small values of  $\kappa$  the scale is almost  $Q^2$  independent. Effectively, this choice amounts to doing the perturbative expansion to fixed (second) order and not at

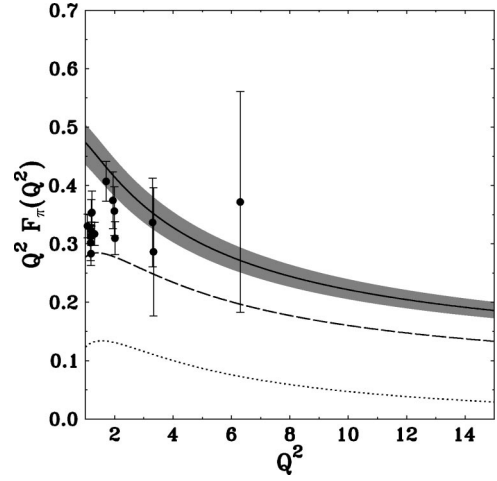


FIG. 11. The nonperturbative correction to the pion form factor (dotted curve) combined with the NLO perturbation theory (sum: solid curve), compared with the pure light-cone sum rule result (dashed curve). An asymptotic pion distribution amplitude is assumed. The gray band shows the scale dependence of the hard-scattering contribution; see text.

tempting a renormalization group resummation. This allows us to minimize the problem with the double counting of infrared regions.

The numerical results are shown in Fig. 11 assuming the asymptotic pion distribution amplitude at the scale of 1 GeV. The result of the calculation using Eq. (62) and  $\kappa=1/2$  is shown by the solid curve with the shaded band corresponding to variation of the scale parameter  $\kappa$  in the given range. The dotted curve presents the nonperturbative contribution and the dashed curve is the ‘‘pure’’ light-cone sum rule calculation with the same parameters. The difference between the solid and the dashed curves presents, therefore, the net effect of the substitution (57).

The nonperturbative (power-suppressed) contribution to the pion form factor shown by the dotted curve in Fig. 11 presents considerable interest by itself. It is, obviously, independent of whether the substitution (57) is used (cf. discussion at the end of Sec. III C), and turns out to be comfortably small. This smallness may appear to be unexpected after we have found large soft (end-point) corrections in Sec. III, and is due to a strong cancellation between the leading-order soft contribution to the sum rule [first line in Eq. (25)] and the large radiative correction [second line in Eq. (25)] corresponding to the sum of soft and hard contributions to  $1/Q^4$  accuracy. As seen from Eqs. (29), (30) the large negative hard contribution  $\sim 1/Q^4$  plays the most important role in this cancellation.

Since according to our analysis the pion distribution amplitude does not differ significantly from the asymptotic distribution, the theoretical uncertainty in the light-cone sum rule calculation of the nonperturbative correction to the pion form factor is dominated by dependence on the Borel parameter, as illustrated in Fig. 12.

With the central values of parameters, the nonperturbative correction can be parametrized in the region  $1 < Q^2$

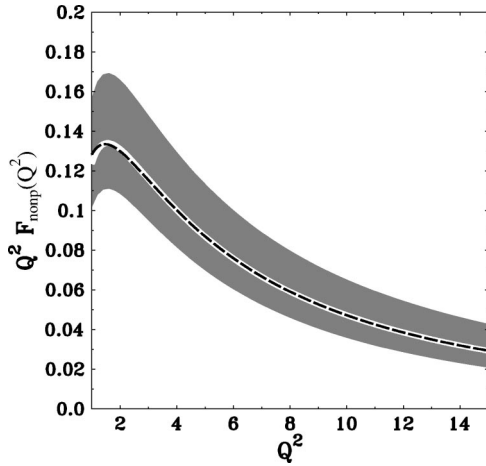


FIG. 12. The light-cone sum rule prediction for the nonperturbative correction to the pion form factor. The gray band shows the sensitivity of the result to variation of the Borel parameter within  $0.8 \text{ GeV}^2 < M^2 < 1.5 \text{ GeV}^2$ . The white line is the calculation for the standard reference value  $M^2 = 1 \text{ GeV}^2$  assumed throughout this paper and the dashed curve is the fit (65).

$< 15 \text{ GeV}^2$  as<sup>9</sup>

$$Q^2 F_{\text{nonp}}(Q^2) = Q^2 / (1.7046 + 1.0662Q^2 + 0.0219Q^4)^2 \quad (65)$$

(all numbers in GeV), and the theoretical error (the gray area in Fig. 12) roughly corresponds to the uncertainty in the overall normalization of order  $\pm 25\%$ .

Choosing, as above, a model for the distribution amplitude at the scale  $1 \text{ GeV}$  as a sum of the leading term and the second Gegenbauer polynomial, and fitting the parameter  $a_2(1 \text{ GeV})$  to the data, we find

$$a_2(\mu = 1 \text{ GeV}) = -0.06 \pm 0.24 \pm 0.03 \pm 0.03,$$

$$\int \frac{du}{u} \varphi_\pi(u, \mu = 1 \text{ GeV}) = 2.82 \pm 0.72 \pm 0.09 \pm 0.09. \quad (66)$$

The first error comes from the experimental uncertainty, the second error corresponds to uncertainty of the nonperturbative contribution (mainly dependence on the Borel parameter), and the third error is the scale dependence of the NLO perturbative result. Combining the two estimates in Eqs. (54) and (66) and adding the errors in quadrature, we obtain as our final result

$$a_2(\mu = 1 \text{ GeV}) = 0.1 \pm 0.1,$$

$$\int \frac{du}{u} \varphi_\pi(u, \mu = 1 \text{ GeV}) = 3.3 \pm 0.3. \quad (67)$$

<sup>9</sup>The given parametrization should not be used for larger values of  $Q^2$  since it has the wrong asymptotic behavior.

This determination is dominated by the ‘‘pure’’ light-cone sum rule result in which case we included the data points at lower values  $Q^2$  having higher accuracy. The situation will change when sufficiently precise data at  $Q^2 > 2-3 \text{ GeV}^2$  become available. In this region the NLO perturbative prediction complemented by the ‘‘higher-twist’’ power-suppressed correction in Eq. (65) becomes, from our point of view, a preferable description, with potential theoretical accuracy of the order of 10%. Note that the theoretical status of our result for the nonperturbative (soft + hard) correction is similar to model (or sum rule) determinations of matrix elements of higher-twist operators in deep inelastic scattering.

## VII. CONCLUSIONS

Elaborating on the earlier proposal [4] we have given in this paper a detailed quantitative analysis of the pion form factor in the region of intermediate momentum transfers in the light-cone sum rule approach and also combining this technique with a complete existing NLO perturbative calculation. Our results support the shape of the pion distribution amplitude that is close to the asymptotic expression and are inconsistent with the CZ-type distributions. Our final estimate for the parameter  $a_2$  characterizing the deviation from the asymptotic form is given in Eq. (67).<sup>10</sup>

Another important conclusion of our analysis is that the nonperturbative contribution to the pion form factor turns out to be rather moderate and does not exceed 30% in the full  $Q^2$  range; see Fig. 11 and Fig. 12. One has to have in mind, however, that separation of ‘‘perturbative’’ and ‘‘nonperturbative’’ contributions is theoretically not well defined because QCD perturbation theory is divergent [30]. A fully theoretically consistent approach necessarily has to introduce an explicit scale separation, and in particular consider soft and hard contributions to the pion form factor separately. We have presented a detailed study of the soft-hard separation implemented with a hard momentum fraction cutoff in Sec. III. One finds that soft contributions are generally very large and the smallness of the total nonperturbative correction is due to cancellations between soft and hard terms of higher twist. Thus, somewhat paradoxically, the nonperturbative effects in the pion form factor can be small and the soft contributions large, simultaneously.

To summarize, we believe that the light-cone sum rule approach presents a powerful and theoretically consistent framework to the analysis of hard exclusive reactions for intermediate momentum transfers. The main and essential assumption of the method is duality, i.e., that the pion contribution can be isolated from the correlation function by integrating the QCD spectral density in a certain energy range—the interval of duality. While the numerical accuracy of this approximation can be disputed, it satisfies all known

<sup>10</sup>The smallness of nonasymptotic contributions to  $\varphi_\pi$  is in agreement with the light-cone sum rule analysis [39] for the  $\gamma^* \gamma \pi^0$  transition form factor, compared with the CLEO data [40]. For a recent update including NLO effects see [41].

QCD constraints and provides a perfect laboratory for the study of different interaction mechanisms involving several scales. In particular, the scale dependence of the soft-hard separation studied in this work is of general validity.

### ACKNOWLEDGMENTS

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### APPENDIX A

Here we collect some useful formulas.

The imaginary part of the radiative correction  $H_1$  to the hard scattering amplitude reads

$$\begin{aligned}
\frac{1}{\pi} \operatorname{Im} H_1(Q^2, s, u, \mu) = & \left( -9 + \frac{1}{3} \pi^2 + 3 \ln[Q^2/\mu^2] - \ln[Q^2/\mu^2]^2 + 3 \ln\left[\frac{\bar{u}Q^2}{u\mu^2}\right] - \ln^2\left[\frac{\bar{u}Q^2}{u\mu^2}\right] \right) \delta(\rho) \\
& + \Theta[-\rho] \frac{Q^2\{-2Q^2 + 3\rho + 5s + 2(Q^2 + \rho + s) \ln[-\rho/\mu^2]\}}{(Q^2 + s)^3 \bar{u}u} \\
& + 2\Theta[-\rho] \frac{Q^2\{-(Q^2 + s) \ln[s/\mu^2] + u(Q^2 - s + s \ln[s/\mu^2])\}}{(Q^2 + s)^3 \bar{u}u} + 2\Theta[\rho] \frac{Q^2(Q^2 - s + s \ln[s/\mu^2])}{(Q^2 + s)^3 \bar{u}} \\
& + \frac{Q^2 s(-3 + 2 \ln[s/\mu^2])}{(Q^2 + s)^2 \bar{u}} \frac{d}{d\rho} (\ln[\rho/\mu^2] \Theta[\rho]) + 2 \frac{Q^4 \ln[s/\mu^2]}{(Q^2 + s)^2 u} \frac{d}{d\rho} (\ln[-\rho/\mu^2] \Theta[-\rho]) \\
& - 2 \frac{Q^4}{(Q^2 + s)^2 u} \frac{d}{d\rho} (\ln^2[-\rho/\mu^2] \Theta[-\rho]), \tag{A1}
\end{aligned}$$

where  $\rho = Q^2 \bar{u} - us$ .

The light-cone expansion of the quark propagator is derived in [16]:

$$\begin{aligned}
S(x, 0) = & -i \langle 0 | T \{ q(x) \bar{q}(0) \} | 0 \rangle \\
= & \frac{\Gamma(d/2) \not{x}}{2\pi^2 (-x^2)^{d/2}} + \frac{\Gamma(d/2-1)}{16\pi^2 (-x^2)^{d/2-1}} \int_0^1 du \{ \bar{u} \not{x} \sigma_{\mu\nu} G^{\mu\nu}(ux) + u \sigma_{\mu\nu} G^{\mu\nu}(ux) \not{x} \\
& + 2i \bar{u} u \not{x} x_\rho D_\lambda G^{\rho\lambda}(ux) \} - \frac{\Gamma(d/2-2)}{16\pi^2 (-x^2)^{d/2-2}} \int_0^1 du \left\{ i \left( \bar{u} u - \frac{1}{2} \right) D_\mu G^{\mu\nu}(ux) \gamma_\nu + \frac{i}{2} \bar{u} u (1-2u) x_\mu \not{D} D_\nu G^{\mu\nu}(ux) \right. \\
& \left. + \frac{1}{2} \bar{u} u \epsilon_{\mu\nu\alpha\beta} x_\mu D^\alpha D_\lambda G^{\lambda\beta} \gamma^\nu \gamma_5 \right\} + \dots, \tag{A2}
\end{aligned}$$

where  $G^{\mu\nu} = g_s G^{\mu\nu a} (\lambda^a/2)$ ,  $\operatorname{Tr}(\lambda^a \lambda^b) = 2\delta^{ab}$ , and  $d$  is the space-time dimension. Only the terms proportional to the one-gluon-field strength and its first covariant derivative are shown for brevity.

### APPENDIX B

Here we define the light-cone distribution amplitudes of the pion and specify their parameters. The leading twist 2 amplitude  $\varphi_\pi(u)$  and the twist 4 amplitudes  $g_1(u)$  and  $g_2(u)$  enter the light-cone expansion of the matrix element

$$\langle 0 | \bar{d}(0) \gamma_\mu \gamma_5 u(x) | \pi^+(p) \rangle = i p_\mu f_\pi \int_0^1 du e^{-iupx} [\varphi_\pi(u) + x^2 g_1(u)] + f_\pi \left( x_\mu - \frac{x^2 p_\mu}{px} \right) \int_0^1 du e^{-iupx} g_2(u). \tag{B1}$$

The QCD equations of motions relate  $g_1$  and  $g_2$  to the quark-antiquark-gluon twist 4 distributions  $\varphi_\parallel$ ,  $\varphi_\perp$ ,  $\tilde{\varphi}_\parallel$ , and  $\tilde{\varphi}_\perp$ . The latter are defined by the matrix elements [15]

$$\begin{aligned} \langle 0 | \bar{d}(-x) \gamma_\mu \gamma_5 G_{\alpha\beta}(vx) u(x) | \pi^+(p) \rangle &= p_\mu \frac{P_{\alpha X \beta} - P_{\beta X \alpha}}{px} f_\pi \int \mathcal{D}\alpha_i \varphi_{\parallel}(\alpha_i) e^{-ipx\tau(\alpha_i)} \\ &+ (g_{\mu\alpha}^\perp p_\beta - g_{\mu\beta}^\perp p_\alpha) f_\pi \int \mathcal{D}\alpha_i \varphi_{\perp}(\alpha_i) e^{-ipx\tau(\alpha_i)}, \end{aligned} \quad (\text{B2})$$

$$\begin{aligned} \langle 0 | \bar{d}(-x) \gamma_\mu i \tilde{G}_{\alpha\beta}(vx) u(x) | \pi^+(p) \rangle &= p_\mu \frac{P_{\alpha X \beta} - P_{\beta X \alpha}}{px} f_\pi \int \mathcal{D}\alpha_i \tilde{\varphi}_{\parallel}(\alpha_i) e^{-ipx\tau(\alpha_i)} \\ &+ (g_{\mu\alpha}^\perp p_\beta - g_{\mu\beta}^\perp p_\alpha) f_\pi \int \mathcal{D}\alpha_i \tilde{\varphi}_{\perp}(\alpha_i) e^{-ipx\tau(\alpha_i)}, \end{aligned} \quad (\text{B3})$$

where  $\tilde{G}_{\alpha\beta} = \frac{1}{2} \epsilon_{\alpha\beta\rho\lambda} G^{\rho\lambda}$  and the following abbreviations are used:

$$\tau(\alpha_i) = \alpha_1 - \alpha_2 + v\alpha_3,$$

$$\mathcal{D}\alpha_i = d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3),$$

and

$$g_{\alpha\beta}^\perp = g_{\alpha\beta} - \frac{x_\alpha p_\beta + x_\beta p_\alpha}{px}.$$

The distribution amplitudes are usually constructed using the formalism of the conformal expansion [15]. To achieve a reasonable accuracy one tries to retain a few first terms of this expansion in addition to the leading asymptotic term. The most familiar example is the twist 2 pion distribution [6]

$$\begin{aligned} \varphi_\pi(u, \mu) &= 6u\bar{u} [1 + a_2(\mu) C_2^{3/2}(u - \bar{u}) + a_4(\mu) C_4^{3/2}(u - \bar{u}) \\ &+ \dots], \end{aligned} \quad (\text{B4})$$

where two orders of the conformal expansion in Gegenbauer polynomials  $C_n^{3/2}$  are explicitly shown, with

$$\begin{aligned} C_2^{3/2}(x) &= \frac{3}{2} (5x^2 - 1), \\ C_4^{3/2}(x) &= \frac{15}{8} (21x^4 - 14x^2 + 1). \end{aligned} \quad (\text{B5})$$

The coefficients  $a_n$  determine the nonasymptotic part of  $\varphi_\pi$ . Their scale dependence is given in the leading order by

$$a_n^{\text{LO}}(\mu_2) = \left( \frac{\alpha_s(\mu_2)}{\alpha_s(\mu_1)} \right)^{-\gamma_n^{(0)}/\beta_0} a_n^{\text{LO}}(\mu_1), \quad (\text{B6})$$

where  $\beta_0 = 11 - \frac{2}{3} N_F$ , and the anomalous dimensions are

$$\gamma_n^{(0)} = C_F \left[ 3 + \frac{2}{(n+1)(n+2)} - 4 \left( \sum_{k=1}^{n+1} \frac{1}{k} \right) \right]. \quad (\text{B7})$$

In the numerical analysis in this paper we use, in particular, the asymptotic distribution (all  $a_n = 0$ ) and the CZ distribu-

tion [ $a_2(1.0 \text{ GeV}) = 2/3$ ,  $a_{n>2} = 0$ ]. In next-to-leading order [36] the evolution requires an infinite sum of coefficients:

$$\begin{aligned} a_2^{\text{NLO}}(\mu^2) &= a_2^{\text{LO}}(\mu^2) P_2(\mu^2) + Q_{20}(\mu^2), \\ a_4^{\text{NLO}}(\mu^2) &= a_4^{\text{LO}}(\mu^2) P_4(\mu^2) + Q_{40}(\mu^2) \\ &+ a_2^{\text{LO}}(\mu^2) Q_{42}(\mu^2), \\ a_{2k}^{\text{NLO}}(\mu^2) &= Q_{2k0}(\mu^2) + a_2^{\text{LO}}(\mu^2) Q_{2k2}(\mu^2) \\ &+ a_4^{\text{LO}}(\mu^2) Q_{2k4}(\mu^2), \quad k \geq 3, \end{aligned} \quad (\text{B8})$$

with the following notation:

$$\begin{aligned} P_k(\mu^2) &= \frac{1}{4} \left( \frac{\gamma_k^{(1)}}{2\beta_0} + \frac{\beta_1}{\beta_0^2} \gamma_k^{(0)} \right) \left( 1 - \frac{\alpha_s(\mu_0^2)}{\alpha_s(\mu^2)} \right), \\ Q_{kn}(\mu^2) &= \frac{(2k+3)}{(k+1)(k+2)} \\ &\times \frac{(n+2)(n+1)}{2(2n+3)} C_{kn}^{(1)} S_{kn}(\mu^2), \\ S_{kn}(\mu^2) &= \frac{\gamma_k^{(0)} - \gamma_n^{(0)}}{\gamma_k^{(0)} - \gamma_n^{(0)} + \beta_0} \\ &\times \left[ 1 - \left( \frac{\alpha_s(\mu_0^2)}{\alpha_s(\mu^2)} \right)^{1 + (\gamma_k^{(0)} - \gamma_n^{(0)})/\beta_0} \right], \end{aligned}$$

$$\begin{aligned} C_{kn}^{(1)} &= (2n+3) \left[ \frac{\gamma_n^{(0)} - \beta_0 + 4C_F A_{kn}}{(k-n)(k+n+3)} \right. \\ &+ \left. \frac{2C_F [A_{kn} - \psi(k+2) + \psi(1)]}{(n+1)(n+2)} \right], \end{aligned}$$

$$\begin{aligned} A_{kn} &= \psi \left( \frac{k+n+4}{2} \right) - \psi \left( \frac{k-n}{2} \right) + 2\psi(k-n) \\ &- \psi(k+2) - \psi(1), \end{aligned} \quad (\text{B9})$$

where

$$\psi(z) = \frac{d}{dz} [\ln \Gamma(z)], \quad \beta_1 = 102 - \frac{38}{3} N_F,$$

$$\gamma_0^{(1)} = 0, \quad \gamma_2^{(1)} = 111.03, \quad \gamma_4^{(1)} = 150.28. \quad (\text{B10})$$

Expressions for the twist 4 distributions including the next-to-leading corrections in conformal spin have been derived in [15,17]:

$$g_1(u) = \frac{5}{2} \delta^2 \bar{u}^2 u + \frac{1}{2} \delta^2 \epsilon \left[ \bar{u} u (2 + 13 \bar{u} u) + 10 u^3 \left( 2 - 3u + \frac{6}{5} u^2 \right) \ln u + 10 \bar{u}^3 \times \left( 2 - 3\bar{u} + \frac{6}{5} \bar{u}^2 \right) \ln \bar{u} \right],$$

$$g_2(u) = \frac{10}{3} \delta^2 \bar{u} u (u - \bar{u}), \quad (\text{B11})$$

$$\varphi_{\parallel}(\alpha_i) = 120 \delta^2 \epsilon (\alpha_1 - \alpha_2) \alpha_1 \alpha_2 \alpha_3,$$

$$\varphi_{\perp}(\alpha_i) = 30 \delta^2 (\alpha_1 - \alpha_2) \alpha_3^2 \left[ \frac{1}{3} + 2 \epsilon (1 - 2 \alpha_3) \right],$$

$$\tilde{\varphi}_{\parallel}(\alpha_i) = -120 \delta^2 \alpha_1 \alpha_2 \alpha_3 \left[ \frac{1}{3} + \epsilon (1 - 3 \alpha_3) \right],$$

$$\tilde{\varphi}_{\perp}(\alpha_i) = 30 \delta^2 \alpha_3^2 (1 - \alpha_3) \left[ \frac{1}{3} + 2 \epsilon (1 - 2 \alpha_3) \right]. \quad (\text{B12})$$

To this accuracy, the specific combination (37) of twist 4 distribution amplitudes reads

$$\varphi^{(4)}(u) = \frac{20}{3} \delta^2 u^2 \bar{u} (3u - 2) - 4 \delta^2 \epsilon u (2 + 11u - 26u^2 + 13u^3) - 8 \delta^2 \epsilon [u^3 (10 - 15u + 6u^2) \ln(u) + \bar{u}^3 (1 + 3u + 6u^2) \ln(1 - u)]. \quad (\text{B13})$$

The normalizations of all these distributions are determined by a single nonperturbative parameter  $\delta^2$  defined as

$$\langle \pi | g_s \bar{d} \tilde{G}_{\alpha\mu} \gamma^{\alpha} u | 0 \rangle = i \delta^2 f_{\pi} q_{\mu}. \quad (\text{B14})$$

The second parameter  $\epsilon$  in Eqs. (B11) and (B12) is responsible for the first nonasymptotic corrections. QCD sum rule estimates yield  $\delta^2 \approx 0.2 \text{ GeV}^2$  [37,38] and  $\epsilon \approx 0.5$  [15]. The scale dependence of these parameters is given by [15]

$$\delta^2(\mu_2^2) = \left( \frac{\alpha_s(\mu_2^2)}{\alpha_s(\mu_1^2)} \right)^{32/(9\beta_0)} \delta^2(\mu_1^2),$$

$$(\delta^2 \epsilon)(\mu_2^2) = \left( \frac{\alpha_s(\mu_2^2)}{\alpha_s(\mu_1^2)} \right)^{10/\beta_0} (\delta^2 \epsilon)(\mu_1^2). \quad (\text{B15})$$

Finally, we should include in our list the twist 3 distribution amplitudes  $\varphi_p$  and  $\varphi_{\sigma}$  used in the calculation of the twist 6 corrections. These distributions parametrize the following matrix elements:

$$\langle 0 | \bar{d}(0) i \gamma_5 u(x) | \pi^+(p) \rangle = f_{\pi} \mu_{\pi} \int_0^1 du e^{-iupx} \varphi_p(u),$$

$$\langle 0 | \bar{d}(0) \sigma_{\alpha\beta} \gamma_5 u(x) | \pi^+(p) \rangle = \frac{i}{6} (p_{\alpha} x_{\beta} - p_{\beta} x_{\alpha}) f_{\pi} \mu_{\pi} \times \int_0^1 du e^{-iupx} \varphi_{\sigma}(u), \quad (\text{B16})$$

where  $\mu_{\pi} = m_{\pi}^2 / (m_u + m_d)$ . The well-known asymptotic form of these distributions

$$\varphi_p(u) = 1, \quad \varphi_{\sigma}(u) = 6u\bar{u}, \quad (\text{B17})$$

is sufficient for the approximation adopted in this paper. The relation (43), together with the standard value of the quark condensate  $\langle \bar{q}q \rangle(1 \text{ GeV}) = (-240 \text{ MeV})^3$ , yields  $\mu_{\pi}(\mu = 1 \text{ GeV}) \approx 1.56 \text{ GeV}$ . Note that the normalization of the twist 6 correction is effectively determined by the product  $\alpha_s(\mu) \langle \bar{q}q \rangle^2(\mu)$  having in total a negligible anomalous dimension.

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- [1] N. Isgur and C. H. Llewellyn Smith, Phys. Lett. B **217**, 535 (1989).  
[2] A. V. Radyushkin, Nucl. Phys. A**527**, 153C (1991); A**532**, 141 (1991).  
[3] R. Jakob and P. Kroll, Phys. Lett. B **315**, 463 (1993); J. Bolz *et al.*, Z. Phys. C **66**, 267 (1995).  
[4] V. M. Braun and I. Halperin, Phys. Lett. B **328**, 457 (1994).  
[5] B. Chibisov and A. R. Zhitnitsky, Phys. Rev. D **52**, 5273 (1995).  
[6] V. L. Chernyak and A. R. Zhitnitsky, JETP Lett. **25**, 510

- (1977); Yad. Fiz. **31**, 1053 (1980); [Sov. J. Nucl. Phys. **31**, 544 (1980)]; A. V. Efremov and A. V. Radyushkin, Phys. Lett. **94B**, 245 (1980); Theor. Math. Phys. **42**, 97 (1980); G. P. Lepage and S. J. Brodsky, Phys. Lett. **87B**, 359 (1979); Phys. Rev. D **22**, 2157 (1980); V. L. Chernyak, V. G. Serbo, and A. R. Zhitnitsky, JETP Lett. **26**, 594 (1977); Sov. J. Nucl. Phys. **31**, 552 (1980).  
[7] I. I. Balitsky, V. M. Braun, and A. V. Kolesnichenko, Nucl. Phys. B**312**, 509 (1989); V. M. Braun and I. E. Filyanov, Z. Phys. C **44**, 157 (1989); V. L. Chernyak and I. R. Zhitnitskii,



- Nucl. Phys. **B345**, 137 (1990).
- [8] N. S. Craigie and J. Stern, Nucl. Phys. **B216**, 209 (1983).
- [9] V. M. Belyaev, V. M. Braun, A. Khodjamirian, and R. Rückl, Phys. Rev. D **51**, 6177 (1995).
- [10] H. Li and G. Stermann, Nucl. Phys. **B381**, 129 (1992).
- [11] I. I. Balitsky, A. V. Kolesnichenko, and A. V. Yung, Yad. Fiz. **41**, 282 (1985) [Sov. J. Nucl. Phys. **41**, 178 (1985)].
- [12] E. Braaten, Phys. Rev. D **28**, 524 (1983); E. P. Kadantseva, S. V. Mikhailov, and A. V. Radyushkin, Yad. Fiz. **44**, 507 (1986) [Sov. J. Nucl. Phys. **44**, 326 (1986)].
- [13] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979).
- [14] E. Bagan, P. Ball, and V. M. Braun, Phys. Lett. B **417**, 154 (1998).
- [15] V. M. Braun and I. E. Filyanov, Z. Phys. C **48**, 239 (1990).
- [16] I. I. Balitsky and V. M. Braun, Nucl. Phys. **B311**, 541 (1989).
- [17] P. Ball, J. High Energy Phys. **01**, 010 (1999).
- [18] V. L. Chernyak and A. R. Zhitnitsky, Phys. Rep. **112**, 173 (1984); A. R. Zhitnitsky, I. R. Zhitnitsky, and V. L. Chernyak, Sov. J. Nucl. Phys. **41**, 284 (1985).
- [19] E. V. Shuryak and A. I. Vainshtein, Nucl. Phys. **B199**, 451 (1982).
- [20] B. L. Ioffe and A. V. Smilga, Phys. Lett. **114B**, 353 (1982); Nucl. Phys. **B216**, 373 (1983).
- [21] V. A. Nesterenko and A. V. Radyushkin, Phys. Lett. **115B**, 410 (1982).
- [22] I. I. Balitsky, Phys. Lett. **124B**, 230 (1983).
- [23] B. V. Geshkenbein and M. V. Terentev, Phys. Lett. **117B**, 243 (1982); Yad. Fiz. **39**, 873 (1984) [Sov. J. Nucl. Phys. **39**, 554 (1984)]; **40**, 758 (1984) [**40**, 487 (1984)].
- [24] G. A. Miller and J. Pasupathy, Z. Phys. A **348**, 123 (1994).
- [25] A. V. Radyushkin and R. T. Ruskov, Nucl. Phys. **B481**, 625 (1996).
- [26] P. Ball and V. M. Braun, Phys. Rev. D **55**, 5561 (1997).
- [27] C. J. Bebek *et al.*, Phys. Rev. D **17**, 1693 (1978).
- [28] NA7 Collaboration, S. R. Amendolia *et al.*, Nucl. Phys. **B277**, 168 (1986).
- [29] J. C. Collins, D. E. Soper, and G. Sterman, in *Perturbative QCD*, edited by A. H. Mueller (World Scientific, Singapore, 1989).
- [30] M. Beneke, Phys. Rep. **317**, 1 (1999).
- [31] B. R. Webber, in “Zuoz 1994, Proceedings, Hadronic aspects of collider physics,” pp. 49–77, hep-ph/9411384; Nucl. Phys. B (Proc. Suppl.) **71**, 66 (1999); V. M. Braun, in “Moriond 1995: Hadronic,” pp. 0271–278, hep-ph/9505317; in “Balholm 1997: Beyond the standard models,” pp. 341–346, hep-ph/9708386; R. Akhoury and V. I. Zakharov, Nucl. Phys. B (Proc. Suppl.) **54A**, 217 (1997); V. I. Zakharov, Prog. Theor. Phys. Suppl. **131**, 107 (1998); G. Sterman, hep-ph/9806533; hep-ph/9905548.
- [32] R. D. Field, R. Gupta, S. Otto, and L. Chang, Nucl. Phys. **B186**, 429 (1981).
- [33] F. M. Dittes and A. V. Radyushkin, Yad. Fiz. **34**, 529 (1981) [Sov. J. Nucl. Phys. **34**, 293 (1981)]; A. V. Radyushkin and R. S. Khalmuradov, *ibid.* **42**, 458 (1985).
- [34] E. Braaten and S. Tse, Phys. Rev. D **35**, 2255 (1987).
- [35] B. Melic, B. Nizic, and K. Passek, Phys. Rev. D **60**, 074004 (1999).
- [36] D. Muller, Phys. Rev. D **49**, 2525 (1994); **51**, 3855 (1995).
- [37] V. A. Novikov, M. A. Shifman, A. I. Vainshtein, M. B. Voloshin, and V. I. Zakharov, Nucl. Phys. **B237**, 525 (1984).
- [38] V. L. Chernyak, A. R. Zhitnitsky, and I. R. Zhitnitsky, Sov. J. Nucl. Phys. **38**, 645 (1983).
- [39] A. Khodjamirian, Eur. Phys. J. C **6**, 477 (1999).
- [40] CLEO Collaboration, J. Gronberg *et al.*, Phys. Rev. D **57**, 33 (1998).
- [41] A. Schmedding and O. Yakovlev, hep-ph/9905392.