

# Factorization and decay constants $f_{D_s^*}$ and $f_{D_s}$

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We calculate the decay constants of  $D_s$  and  $D_s^*$  with  $\bar{B}^0 \rightarrow D^+ \ell^- \nu$  and  $\bar{B}^0 \rightarrow D^+ D_s^{-(*)}$  decays. In our analysis we take the factorization method, considering nonfactorizable term contributions, and used two different form-factor behaviors (constant and monopole-type) for  $F_0(q^2)$ . We also consider the QCD-penguin and electroweak-penguin contributions in hadronic decays within the naive dimensional reduction renormalization scheme at next-to-leading order calculation. We estimate the decay constant of the  $D_s$  meson to be  $233 \pm 49$  MeV for a (pole/pole)-type form factor and  $255 \pm 54$  MeV for a (pole/constant)-type form factor. For the  $D_s^*$  meson, we get  $f_{D_s^*} = 346 \pm 82$  MeV, and  $f_{D_s^*}/f_{D_s} = 1.43 \pm 0.45$  for a (pole/constant)-type form factor.

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## I. INTRODUCTION

Measuring purely leptonic decays of heavy mesons provides the most clear way for the determination of weak decay constants of heavy mesons, which connect the measured quantities, such as the  $B\bar{B}$  mixing ratio, to Cabibbo-Kobayashi-Maskawa (CKM) matrix elements  $V_{cb}$ ,  $V_{ub}$ . However, currently it is not possible to determine  $f_B$ ,  $f_{B_s}$ ,  $f_{D_s}$ , and  $f_{D_s^*}$  experimentally from leptonic  $B$  and  $D_s$  decays. For instance, the decay rate for  $D_s^+$  is given by [1]

$$\Gamma(D_s^+ \rightarrow \ell^+ \nu) = \frac{G_F^2}{8\pi} f_{D_s}^2 m_\ell^2 M_{D_s} \left(1 - \frac{m_\ell^2}{M_{D_s}^2}\right)^2 |V_{cs}|^2. \quad (1)$$

Because of helicity suppression, the electron mode  $D_s^+ \rightarrow e^+ \nu$  has a very small rate. The relative widths are  $10:1:2 \times 10^{-5}$  for  $\tau^+ \nu$ ,  $\mu^+ \nu$ , and  $e^+ \nu$  final states, respectively. Unfortunately, the mode with the largest branching fraction,  $\tau^+ \nu$ , has at least two neutrinos in the final state and is difficult to detect in experiment. So theoretical calculations for decay constants have to be used. The factorization ansatz for nonleptonic decay modes provides us a good approximate method to obtain nonperturbative quantities such as form factors and decay constants which are hardly accessible in any other way [2,3].

There are many ways that the quarks produced in a nonleptonic weak decay can arrange themselves into hadrons. The final state is linked to the initial state by complicated trees of gluon and quark interactions, pair production, and loops. These make the theoretical description of nonleptonic decays difficult. However, since the products of a  $B$  meson decay are quite energetic, it is possible that the complicated QCD interactions are less important and that the two quark

pairs of the currents in the weak Hamiltonian group individually into the final-state mesons without further exchanges of gluons. The color transparency argument suggests that a quark-antiquark pair remains at a state of small size with a correspondingly small chromomagnetic moment until it is far from the other decay products.

Color transparency is the basis for the factorization hypothesis, in which amplitudes factorize into products of two current matrix elements. This ansatz is widely used in heavy quark physics, as it is almost the only way to treat hadronic decays. The factorization approximation works reasonably well for color-favored two body decays of  $B$  and  $D$  mesons.

In this paper we consider the way to determine weak decay constants  $f_{D_s}$  and  $f_{D_s^*}$  under the factorization ansatz including penguin effects. In our analysis we only consider  $\bar{B}^0 \rightarrow D^+ \ell^- \bar{\nu}$  and  $\bar{B}^0 \rightarrow D^+ D_s^{-(*)}$  for less theoretical uncertainty. We also can use  $\bar{B}^0 \rightarrow D^{+*} \ell^- \bar{\nu}$  and  $\bar{B}^0 \rightarrow D^{+*} D_s^{-(*)}$  to extract the weak decay constants, however, these processes have more theoretical ambiguity than the former one, because we need four form factors ( $A_0$ ,  $A_1$ ,  $A_2$ , and  $V$ ) for the  $B \rightarrow D^*$  transition instead of two form factors ( $F_1$  and  $F_0$ ) for the  $B \rightarrow D$  transition. We do not include in our work the analysis with  $B \rightarrow D^* \ell \nu$  and  $B \rightarrow D^* D_s^{(*)}$  in order to avoid this extra theoretical ambiguity. In Sec. II we discuss the way to extract the unknown parameter  $|V_{cb} F_1^{BD}(0)|$  from the branching ratio of the semileptonic decay  $\bar{B}^0 \rightarrow D^+ \ell^- \bar{\nu}$ . In order to check the validity of the factorization assumption, we study the nonleptonic two-body decays,  $B \rightarrow D_\rho$ ,  $D\pi$  and  $DK^{(*)}$  in Sec. III. In Sec. II we calculate  $f_{D_s}$  and  $f_{D_s^*}$  from  $\bar{B}^0 \rightarrow D^+ D_s^{-(*)}$  decay modes. In our analysis we improve the previous analysis [4] by considering the QCD-penguin and electroweak-penguin effects of about 13% for  $B \rightarrow DD_s$  and 4% for  $B \rightarrow DD_s^*$ , which are not negligible as discussed in [5]. Also we follow the gauge-independent approach to calculate the effective Wilson coefficients which was studied by perturbative QCD factorization theorem [6].

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## II. SEMILEPTONIC DECAY $\bar{B}^0 \rightarrow D^+ l^- \bar{\nu}$

From Lorentz invariance one finds the decomposition of the hadronic matrix element in terms of hadronic form factors:

$$\begin{aligned} & \langle D^+(p_D) | J_\mu | \bar{B}^0(p_B) \rangle \\ &= \left[ (p_B + p_D)_\mu - \frac{m_B^2 - m_D^2}{q^2} q_\mu \right] F_1^{BD}(q^2) \\ &+ \frac{m_B^2 - m_D^2}{q^2} q_\mu F_0^{BD}(q^2), \end{aligned} \quad (2)$$

where  $J_\mu = \bar{c} \gamma_\mu b$  and  $q_\mu = (p_B - p_D)_\mu$ . In the rest frame of the decay products,  $F_1(q^2)$  and  $F_0(q^2)$  correspond to  $1^-$  and  $0^+$  exchanges, respectively. At  $q^2=0$  we have the constraint

$$F_1^{BD}(0) = F_0^{BD}(0), \quad (3)$$

since the hadronic matrix element in Eq. (2) is nonsingular at this kinematic point.

The  $q^2$  distribution in the semileptonic decay  $\bar{B}^0 \rightarrow D^+ l^- \bar{\nu}$  is written in terms of the hadronic form factor  $F_1^{BD}(q^2)$  as

$$\frac{d\Gamma(\bar{B}^0 \rightarrow D^+ l^- \bar{\nu})}{dq^2} = \frac{G_F^2}{24\pi^3} |V_{cb}|^2 [K(q^2)]^3 |F_1^{BD}(q^2)|^2, \quad (4)$$

where the  $q^2$ -dependent momentum  $K(q^2)$  is given by

$$K(q^2) = \frac{1}{2m_B} [(m_B^2 + m_D^2 - q^2)^2 - 4m_B^2 m_D^2]^{1/2}. \quad (5)$$

In the zero lepton mass limit,  $0 \leq q^2 \leq (m_B - m_D)^2$ .

For the  $q^2$  dependence of the form factors, Wirbel *et al.* [7] assumed a simple pole formula for both  $F_1(q^2)$  and  $F_0(q^2)$  (pole/pole):

$$\begin{aligned} F_1(q^2) &= F_1(0) \left/ \left( 1 - \frac{q^2}{m_{F_1}^2} \right) \right., \\ F_0(q^2) &= F_0(0) \left/ \left( 1 - \frac{q^2}{m_{F_0}^2} \right) \right., \end{aligned} \quad (6)$$

with the pole masses

$$m_{F_1} = 6.34 \text{ GeV}, \quad m_{F_0} = 6.80 \text{ GeV}. \quad (7)$$

Korner and Schuler [8] also adopted the same  $q^2$  dependence of  $F_1(q^2)$  and  $F_0(q^2)$  given by Eqs. (6) and (7). On the other hand, the heavy quark effective theory gives in the  $m_{b,c} \rightarrow \infty$  limit the relation between  $F_1(q^2)$  and  $F_0(q^2)$  given by [9,10]

$$F_0(q^2) = \left[ 1 - \frac{q^2}{(m_B + m_D)^2} \right] F_1(q^2). \quad (8)$$

The combination of Eqs. (6) and (8) suggests that  $F_0(q^2)$  is approximately constant when we keep the simple pole dependence for  $F_1(q^2)$ . Therefore, in this paper, as well as the above (pole/pole) form factors, we will also consider the following ones (pole/const):

$$F_1(q^2) = F_1(0) \left/ \left( 1 - \frac{q^2}{m_{F_1}^2} \right) \right., \quad F_0(q^2) = F_0(0), \quad (9)$$

with

$$m_{F_1} = 6.34 \text{ GeV}. \quad (10)$$

By introducing the variable  $x \equiv q^2/m_B^2$ , which has the range of  $0 \leq x \leq (1 - m_D/m_B)^2$  in the zero lepton mass limit, Eq. (4) is written as

$$\frac{d\Gamma(\bar{B}^0 \rightarrow D^+ l^- \bar{\nu})}{dx} = \frac{G_F^2 m_B^5}{192\pi^3} |V_{cb} F_1^{BD}(0)|^2 \frac{\lambda^3[1, m_D^2/m_B^2, x]}{(1 - m_D^2/m_B^2)^2}, \quad (11)$$

$$\lambda \left[ 1, \frac{m_D^2}{m_B^2}, x \right] = \left[ \left( 1 + \frac{m_D^2}{m_B^2} - x \right)^2 - 4 \frac{m_D^2}{m_B^2} \right]^{1/2}.$$

Then the branching ratio  $\mathcal{B}(\bar{B}^0 \rightarrow D^+ l^- \bar{\nu})$  is given by

$$\begin{aligned} \mathcal{B}(\bar{B}^0 \rightarrow D^+ l^- \bar{\nu}) &= \left( \frac{G_F m_B^2}{\sqrt{2}} \right)^2 \frac{m_B}{\Gamma_B} \frac{2}{192\pi^2} |V_{cb} F_1^{BD}(0)|^2 \times I \\ &= 2.221 \times 10^2 |V_{cb} F_1^{BD}(0)|^2 \times I, \end{aligned} \quad (12)$$

where the dimensionless integral  $I$  is given by

$$I = \int_0^{(1 - m_D/m_B)^2} dx \frac{\left[ \left( 1 + \frac{m_D^2}{m_B^2} - x \right)^2 - 4 \frac{m_D^2}{m_B^2} \right]^{3/2}}{\left( 1 - \frac{m_D^2}{m_B^2} x \right)^2} = 0.121 \quad (13)$$

In obtaining the numerical values in Eqs. (12) and (13), we used the following experimental results [11]:  $m_D = m_{D^+} = 1.869 \text{ GeV}$ ,  $m_B = m_{B^0} = 5.279 \text{ GeV}$ ,  $\Gamma_B = \Gamma_{B^0} = 4.219 \times 10^{-13} \text{ GeV}$  [ $\tau_{B^0} = (1.56 \pm 0.06) \times 10^{-12} \text{ s}$ ], and  $G_F = 1.16639(2) \times 10^{-5} \text{ GeV}^{-2}$ . Since  $\mathcal{B}(\bar{B}^0 \rightarrow D^+ l^- \bar{\nu}) = (1.78 \pm 0.20 \pm 0.24) \times 10^{-2}$  was obtained experimentally, the value of  $|V_{cb} F_1^{BD}(0)|$  can be extracted from Eq. (12). Following this procedure, we obtain [12]

$$|V_{cb} F_1^{BD}(0)| = (2.57 \pm 0.14 \pm 0.17) \times 10^{-2}. \quad (14)$$

In the calculations of the next sections, we will use  $|V_{cb} F_1^{BD}(0)| = (2.57 \pm 0.22) \times 10^{-2}$  which is given by combining the statistical and systematic errors in Eq. (14).

TABLE I. The obtained values of the braching ratios with  $a_1 = 1.02$  and experimental measurements.

	$\mathcal{B}(\bar{B}^0 \rightarrow D^+ \rho^-)$ $\times 10^3$	$\mathcal{B}(\bar{B}^0 \rightarrow D^+ K^{*-})$ $\times 10^4$	$\mathcal{B}(\bar{B}^0 \rightarrow D^+ \pi^-)$ $\times 10^3$	$\mathcal{B}(\bar{B}^0 \rightarrow D^+ K^-)$ $\times 10^4$
(Pole/Pole)	$9.01 \pm 1.54$	$4.62 \pm 0.79$	$3.58 \pm 0.61$	$2.74 \pm 0.47$
(Pole/const.)	$9.01 \pm 1.54$	$4.62 \pm 0.79$	$3.57 \pm 0.61$	$2.71 \pm 0.46$
Experiments	$8.4 \pm 1.6 \pm 0.7$		$3.1 \pm 0.4 \pm 0.2$	

### III. TEST OF FACTORIZATION WITH $\bar{B}^0 \rightarrow D^+ \rho^-$ AND $\bar{B}^0 \rightarrow D^+ \pi^-$ , AND PREDICTION OF BRANCHING RATIO $\mathcal{B}(\bar{B}^0 \rightarrow D^+ K^{*(-)})$

In general, the test of factorization, independent of the numerical values of  $a_1$ ,  $a_2$ , and of the CKM parameters  $|V_{cb}|$  or  $|V_{ub}|$ , can be carried out by considering the ratios of rates for two class I or class II  $B$ -meson hadronic two-body decays. On the other hand, we can also use the relation between the semileptonic decays and the nonleptonic decays with  $a_1$  and  $a_2$  given by other sources. In our analysis we use the latter one.

Let us start by recalling the relevant effective weak Hamiltonian:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* [C_1(\mu) \mathcal{O}_1 + C_2(\mu) \mathcal{O}_2] + \text{H.c.}, \quad (15)$$

$$\mathcal{O}_1 = (\bar{d}\Gamma^\rho u)(\bar{c}\Gamma_\rho b), \quad \mathcal{O}_2 = (\bar{c}\Gamma^\rho u)(\bar{d}\Gamma_\rho b), \quad (16)$$

where  $G_F$  is the Fermi coupling constant, and  $V_{cb}$  and  $V_{ud}$  are corresponding Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and  $\Gamma_\rho = \gamma_\rho(1 - \gamma_5)$ . The Wilson coefficients  $C_1(\mu)$  and  $C_2(\mu)$  incorporate the short-distance effects arising from the renormalization of  $\mathcal{H}_{\text{eff}}$  from  $\mu = m_W$  to  $\mu = O(m_b)$ . By using the Fierz transformation under which  $V-A$  currents remain  $V-A$  currents, we get the following equivalent forms:

$$\begin{aligned} C_1 \mathcal{O}_1 + C_2 \mathcal{O}_2 &= \left( C_1 + \frac{1}{N_c} C_2 \right) \mathcal{O}_1 + C_2 (\bar{d}\Gamma^\rho T^a u)(\bar{c}\Gamma_\rho T^a b) \\ &= \left( C_2 + \frac{1}{N_c} C_1 \right) \mathcal{O}_2 + C_1 (\bar{c}\Gamma^\rho T^a u)(\bar{d}\Gamma_\rho T^a b), \end{aligned} \quad (17)$$

where  $N_c = 3$  is the number of colors and  $T^a$ 's are SU(3) color generators. The second terms in Eq. (17) involve color-octet currents. In the factorization assumption, these terms are neglected and  $\mathcal{H}_{\text{eff}}$  is rewritten in terms of ‘‘factorized hadron operators’’ [7]:

$$\begin{aligned} \mathcal{H}_{\text{eff}} &= \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* (a_1 [\bar{d}\Gamma^\rho u]_H [\bar{c}\Gamma_\rho b]_H \\ &\quad + a_2 [\bar{c}\Gamma^\rho u]_H [\bar{d}\Gamma_\rho b]_H) + \text{H.c.}, \end{aligned} \quad (18)$$

where the subscript  $H$  stands for *hadronic* implying that the Dirac bilinears inside the brackets be treated as interpolating

fields for the mesons and no further Fierz reordering need be done. The phenomenological parameters  $a_1$  and  $a_2$  are related to  $C_1$  and  $C_2$  by

$$a_1 = C_1 + \frac{1}{N_c} C_2, \quad a_2 = C_2 + \frac{1}{N_c} C_1. \quad (19)$$

From the analyses of Buras [13], the parameters  $a_1$  and  $a_2$  are determined at next-to-leading order (NLO) calculation in the naive dimensional reduction (NDR) scheme as

$$a_1 = 1.02 \pm 0.01, \quad a_2 = 0.20 \pm 0.05. \quad (20)$$

For the two-body decay, in the rest frame of initial meson the differential decay rate is given by

$$d\Gamma = \frac{1}{32\pi^2} |\mathcal{M}|^2 \frac{|\mathbf{p}_1|}{M^2} d\Omega, \quad (21)$$

$$|\mathbf{p}_1| = \frac{[(M^2 - (m_1 + m_2)^2)(M^2 - (m_1 - m_2)^2)]^{1/2}}{2M}, \quad (22)$$

where  $M$  is the mass of the initial meson, and  $m_1(m_2)$  and  $\mathbf{p}_1$  are the mass and momentum of one of the final mesons. By using Eqs. (2), (18), and  $\langle 0 | \Gamma_\mu | \rho(q, \varepsilon) \rangle = \varepsilon_\mu(q) m_\rho f_\rho$ , Eq. (21) gives the following formula for the branching ratio of  $\bar{B}^0 \rightarrow D^+ \rho^-$ :

$$\begin{aligned} \mathcal{B}(\bar{B}^0 \rightarrow D^+ \rho^-) &= \left( \frac{G_F m_B^2}{\sqrt{2}} \right)^2 |V_{ud}|^2 \frac{1}{16\pi} \frac{m_B}{\Gamma_B} a_1^2 \frac{f_\rho^2}{m_B^2} |V_{cb} F_1^{BD}(m_\rho^2)|^2 \\ &\quad \times \left\{ \left[ 1 - \left( \frac{m_D + m_\rho}{m_B} \right)^2 \right] \left[ 1 - \left( \frac{m_D - m_\rho}{m_B} \right)^2 \right] \right\}^{3/2} \\ &= 13.25 \times |V_{cb} F_1^{BD}(m_\rho^2)|^2 \times \left( \frac{a_1}{1.02} \right)^2. \end{aligned} \quad (23)$$

In obtaining the numerical values in Eq. (23), we used the experimental results given below in Eq. (13),  $m_\rho = m_{\rho^+} = 766.9$  MeV,  $f_\rho = f_{\rho^+} = 216$  MeV, and  $V_{ud} = 0.9751$  [11]. For the value of  $a_1$  we used the value given in Eq. (20). Then, by using the formula (23) with the values of  $|V_{cb} F_0^{BD}(0)|^2 [F_0^{BD}(0) = F_1^{BD}(0)]$  given in Eq. (14), we obtain the branching ratio  $\mathcal{B}(\bar{B}^0 \rightarrow D^+ \rho^-)$  presented in Table I.

For the process  $\bar{B}^0 \rightarrow D^+ K^{*-}$ , by using  $\langle 0 | \Gamma_\mu | K^*(q, \varepsilon) \rangle = \varepsilon_\mu(q) m_{K^*} f_{K^*}$ , we have

$$\begin{aligned} \mathcal{B}(\bar{B}^0 \rightarrow D^+ K^{*-}) &= \left( \frac{G_F m_B^2}{\sqrt{2}} \right)^2 |V_{us}|^2 \frac{1}{16\pi} \frac{m_B}{\Gamma_B} a_1^2 \frac{f_{K^*}^2}{m_B^2} |V_{cb} F_1^{BD}(m_{K^*}^2)|^2 \\ &\times \left\{ \left[ 1 - \left( \frac{m_D + m_{K^*}}{m_B} \right)^2 \right] \left[ 1 - \left( \frac{m_D - m_{K^*}}{m_B} \right)^2 \right] \right\}^{3/2} \end{aligned}$$

$$= 0.67 \times |V_{cb} F_1^{BD}(m_{K^*}^2)|^2 \times \left( \frac{a_1}{1.02} \right)^2, \quad (24)$$

where we used  $m_{K^*} = m_{K^{*-}} = 891.59$  MeV,  $f_{K^*} = f_{K^{*-}} = 218$  MeV, and  $V_{us} = 0.2215$  [11]. By using Eq. (24) with  $|V_{cb} F_1^{BD}(0)|^2$  in Eq. (14), we obtain the branching ratio  $\mathcal{B}(\bar{B}^0 \rightarrow D^+ K^{*-})$  presented in Table I.

By using Eqs. (2), (18), and  $\langle 0 | \Gamma_\mu | \pi(q) \rangle = i q_\mu f_\pi$ , Eq. (21) gives the following formula for the branching ratio of the process  $\bar{B}^0 \rightarrow D^+ \pi^-$ :

$$\begin{aligned} \mathcal{B}(\bar{B}^0 \rightarrow D^+ \pi^-) &= \left( \frac{G_F m_B^2}{\sqrt{2}} \right)^2 |V_{ud}|^2 \frac{1}{16\pi} \frac{m_B}{\Gamma_B} a_1^2 \frac{f_\pi^2}{m_B^2} |V_{cb} F_0^{BD}(m_\pi^2)|^2 \\ &\times \left( 1 - \frac{m_D^2}{m_B^2} \right)^2 \left\{ \left[ 1 - \left( \frac{m_D + m_\pi}{m_B} \right)^2 \right] \left[ 1 - \left( \frac{m_D - m_\pi}{m_B} \right)^2 \right] \right\}^{1/2} \\ &= 5.42 \times |V_{cb} F_0^{BD}(m_\pi^2)|^2 \times \left( \frac{a_1}{1.02} \right)^2, \end{aligned} \quad (25)$$

where we used  $m_\pi = m_{\pi^-} = 139.57$  MeV and  $f_\pi = f_{\pi^-} = 131.74$  MeV [11]. By using the formula (25) with the values of  $|V_{cb} F_0^{BD}(0)|^2 [F_0^{BD}(0) = F_1^{BD}(0)]$  in Eq. (14), we obtain the branching ratio  $\bar{B}^0 \rightarrow D^+ \pi^-$  presented in Table I.

For the process  $\bar{B}^0 \rightarrow D^+ K^-$ , by using  $\langle 0 | \Gamma_\mu | K^-(q) \rangle = i q_\mu f_{K^-}$ , we have

$$\begin{aligned} \mathcal{B}(\bar{B}^0 \rightarrow D^+ K^-) &= \left( \frac{G_F m_B^2}{\sqrt{2}} \right)^2 |V_{us}|^2 \frac{1}{16\pi} \frac{m_B}{\Gamma_B} a_1^2 \frac{f_K^2}{m_B^2} |V_{cb} F_0^{BD}(m_K^2)|^2 \\ &\times \left( 1 - \frac{m_D^2}{m_B^2} \right)^2 \left\{ \left[ 1 - \left( \frac{m_D + m_K}{m_B} \right)^2 \right] \left[ 1 - \left( \frac{m_D - m_K}{m_B} \right)^2 \right] \right\}^{1/2} \\ &= 0.41 \times |V_{cb} F_0^{BD}(m_K^2)|^2 \times \left( \frac{a_1}{1.02} \right)^2. \end{aligned} \quad (26)$$

where we used  $m_K = m_{K^-} = 493.68$  MeV,  $f_K = f_{K^+} = 160.6$  MeV [11]. By using Eq. (26) with  $|V_{cb} F_1^{BD}(0)|^2$  in Eq. (14), we obtain the branching ratio  $\mathcal{B}(\bar{B}^0 \rightarrow D^+ K^-)$  presented in Table I. It seems that the factorization method works well in  $\bar{B}^0 \rightarrow D^+ \pi^-, D^+ \rho^-$  decays. We predict branching ratios:

$$\begin{aligned} \mathcal{B}(\bar{B}^0 \rightarrow D^+ K^-) &\simeq 2.7 \times 10^{-4} \left( \frac{a_1}{1.02} \right)^2, \\ \mathcal{B}(\bar{B}^0 \rightarrow D^+ K^{*-}) &\simeq 4.6 \times 10^{-4} \left( \frac{a_1}{1.02} \right)^2, \end{aligned} \quad (27)$$

which is certainly reachable in near future.

#### IV. DETERMINATION OF $f_{D_s^*}$ AND $f_{D_s}$ FROM $\bar{B}^0 \rightarrow D^+ D_s^{*-}$ AND $\bar{B}^0 \rightarrow D^+ D_s^-$

The effective Hamiltonian for  $\Delta B = 1$  transitions is given by

$$\begin{aligned} H_{\text{eff}} &= \frac{G_F}{\sqrt{2}} \left[ V_{ub} V_{uq}^* (C_1 O_1^u + C_2 O_2^u) + V_{cb} V_{cq}^* (C_1 O_1^c + C_2 O_2^c) \right. \\ &\quad \left. - V_{tb} V_{tq}^* \sum_{i=3}^6 C_i O_i \right], \end{aligned} \quad (28)$$

where  $q = d, s$ , and  $C_i$  are the Wilson coefficients evaluated at the renormalization scale  $\mu$ , and the current-current operators  $O_1^{u,c}$  and  $O_2^{u,c}$  are

$$\begin{aligned} O_1^u &= (\bar{u}_\alpha b_\alpha)_{V-A} (\bar{q}_\beta u_\beta)_{V-A}, & O_1^c &= (\bar{c}_\alpha b_\alpha)_{V-A} (\bar{q}_\beta c_\beta)_{V-A}, \\ O_2^u &= (\bar{u}_\beta b_\alpha)_{V-A} (\bar{q}_\beta u_\beta)_{V-A}, & O_2^c &= (\bar{c}_\beta b_\alpha)_{V-A} (\bar{q}_\alpha c_\beta)_{V-A}, \end{aligned} \quad (29)$$

and the QCD penguin operators  $O_3 - O_6$  are

$$\begin{aligned}
 O_3 &= (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V-A}, \\
 O_4 &= (\bar{q}_\beta b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\alpha q'_\beta)_{V-A}, \\
 O_5 &= (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V+A}, \\
 O_6 &= (\bar{q}_\beta b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\alpha q'_\beta)_{V+A}. \quad (30)
 \end{aligned}$$

The electroweak penguin operators  $O_7-O_{10}$  are given by

$$\begin{aligned}
 O_7 &= (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} \frac{3}{2} e_{q'} (\bar{q}'_\beta q'_\beta)_{V+A}, \\
 O_8 &= (\bar{q}_\beta b_\alpha)_{V-A} \sum_{q'} \frac{3}{2} e_{q'} (\bar{q}'_\alpha q'_\beta)_{V+A}, \\
 O_9 &= (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} \frac{3}{2} e_{q'} (\bar{q}'_\beta q'_\beta)_{V-A}, \\
 O_{10} &= (\bar{q}_\beta b_\alpha)_{V-A} \sum_{q'} \frac{3}{2} e_{q'} (\bar{q}'_\alpha q'_\beta)_{V-A}. \quad (31)
 \end{aligned}$$

In Eq. (28) we consider the effects of the electroweak-penguin operators, however, we neglect the contribution of the dipole operators, since its contribution is not important in this work.

When we take  $m_t=174$  GeV,  $m_b=5.0$  GeV,  $\alpha_s(M_z)=0.118$ , and  $\alpha_{em}(M_z)=1/128$ , the numerical values of the renormalization-scheme-independent Wilson coefficients  $\bar{C}_i$  at  $\mu=m_b$  are given by [14]

$$\begin{aligned}
 \bar{C}_1 &= 1.1502, \quad \bar{C}_2 = -0.3125, \\
 \bar{C}_3 &= 0.0174, \quad \bar{C}_4 = -0.0373, \quad \bar{C}_5 = 0.0104, \\
 \bar{C}_6 &= -0.0459, \\
 \bar{C}_7 &= -1.050 \times 10^{-5}, \quad \bar{C}_8 = 3.839 \times 10^{-4}, \\
 \bar{C}_9 &= -0.0101, \quad \bar{C}_{10} = 1.959 \times 10^{-3}. \quad (32)
 \end{aligned}$$

The effective Hamiltonian in Eq. (28) for the decays  $\bar{B}^0 \rightarrow D^+ D_s^{-(*)}$  can be rewritten as

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ V_{cb} V_{cs}^* (C_1^{\text{eff}} O_1 + C_2^{\text{eff}} O_2) - V_{tb} V_{ts}^* \sum_{i=3}^{10} C_i^{\text{eff}} O_i \right], \quad (33)$$

where  $C_i^{\text{eff}}$  are given by [15]

$$C_1^{\text{eff}} = \bar{C}_1, \quad C_2^{\text{eff}} = \bar{C}_2, \quad C_3^{\text{eff}} = \bar{C}_3 - P_s/N_c,$$

$$\begin{aligned}
 C_4^{\text{eff}} &= \bar{C}_4 + P_s, \\
 C_5^{\text{eff}} &= \bar{C}_5 - P_s/N_c, \quad C_6^{\text{eff}} = \bar{C}_6 + P_s, \quad C_7^{\text{eff}} = \bar{C}_7 + P_e, \\
 C_8^{\text{eff}} &= \bar{C}_8, \\
 C_9^{\text{eff}} &= \bar{C}_9 + P_e, \quad C_{10}^{\text{eff}} = \bar{C}_{10}, \quad (34)
 \end{aligned}$$

with

$$\begin{aligned}
 P_s &= \frac{\alpha_s}{8\pi} \left[ \frac{10}{9} - G(m_q, q^2, \mu) \right] \bar{C}_2(\mu), \\
 P_e &= \frac{\alpha_{em}}{9\pi} \left[ \frac{10}{9} - G(m_q, q^2, \mu) \right] [3\bar{C}_1(\mu) + \bar{C}_2(\mu)], \quad (35) \\
 G(m_q, q^2, \mu) &= -4 \int_0^1 x(1-x) \ln \left( \frac{m_q^2 - x(1-x)q^2}{\mu^2} \right) dx, \quad (36)
 \end{aligned}$$

where  $q$  denotes the momentum of the virtual gluons appearing in the QCD timelike matrix elements, and  $N_c$  is the number of colors. Assuming  $q^2 = m_b^2/2$ , we obtain the analytic formula for  $G(m_q, q^2, \mu)$ :

$$\begin{aligned}
 &G \left( m_q, \frac{m_b^2}{2}, \mu = m_b \right) \\
 &= -\frac{2}{3} \ln \left( \frac{y}{8} \right) + \frac{10}{9} + \frac{2}{3} y \\
 &\quad + \frac{(2+y)\sqrt{1-y}}{3} \left[ \ln \left| \frac{1-\sqrt{1-y}}{1+\sqrt{1-y}} \right| + i\pi \right] \quad (37)
 \end{aligned}$$

with  $y = 8m_q^2/m_b^2$ .

By considering the nonfactorizable term contributions, the relation between the effective coefficients  $a_i^{\text{eff}}$  and the Wilson coefficients in the effective Hamiltonian are given by

$$a_{2i}^{\text{eff}} = C_{2i}^{\text{eff}} + \frac{1}{N_c^{\text{eff}}} C_{2i-1}^{\text{eff}}, \quad a_{2i-1}^{\text{eff}} = C_{2i-1}^{\text{eff}} + \frac{1}{N_c^{\text{eff}}} C_{2i}^{\text{eff}}, \quad (38)$$

where  $i=1, \dots, 5$ , and the nonfactorizable effects are absorbed into the  $N_c^{\text{eff}}$  by

$$\frac{1}{N_c^{\text{eff}}} \equiv \frac{1}{N_c} + \chi_i, \quad N_c = 3. \quad (39)$$

In order to simplify the notation, we will use the notation  $a_i^{\text{eff}}$  instead of  $a_i^{\text{eff}}$  in the equations below.

In usual factorization approach, when we consider the off-shell momentum of the external quark line, the effective Wilson coefficient has the ambiguities of the infrared cutoff and gauge dependence. As stressed by [16], the gauge and infrared dependence always appears as long as the matrix elements of operators are calculated between quark states. Recently this problem was solved by perturbative QCD

TABLE II. The values of the effective Wilson coefficient  $C_i^{\text{eff}}$  with the  $\mu = m_b(m_b) = 4.3$  GeV,  $m_c(m_b) = 0.95$  GeV in the NDR scheme at the NLO calculation.

Coefficients	Real Part	Imaginary Part
$C_1^{\text{eff}}$	1.168	0.0
$C_2^{\text{eff}}$	-0.365	0.0
$C_3^{\text{eff}}$	$2.25 \times 10^{-2}$	$4.5 \times 10^{-3}$
$C_4^{\text{eff}}$	$-4.58 \times 10^{-2}$	$-1.36 \times 10^{-2}$
$C_5^{\text{eff}}$	$1.33 \times 10^{-2}$	$4.5 \times 10^{-3}$
$C_6^{\text{eff}}$	$-4.80 \times 10^{-2}$	$-1.36 \times 10^{-2}$
$C_7^{\text{eff}}$	$2.37 \times 10^{-4}$	$-2.88 \times 10^{-4}$
$C_8^{\text{eff}}$	$4.30 \times 10^{-4}$	0.0
$C_9^{\text{eff}}$	$-1.11 \times 10^{-2}$	$-2.88 \times 10^{-4}$
$C_{10}^{\text{eff}}$	$3.75 \times 10^{-3}$	0.0

factorization theorem [6] by using the on-shell external quark. By following their approach and inserting the values for  $m_q = m_c(\mu) = 0.95$  GeV, we get the values  $C_i^{\text{eff}}$  ( $i = 1 \sim 10$ ) for  $b \rightarrow c$  given in Table II. For different combinations of  $N_c^{\text{eff}} = 2, 3$ , and 5, the values of the effective coefficients  $a_i$  ( $i = 1 \rightarrow 10$ ) are shown in Table III. Here  $(N_c)_{LL,LR} = 3$  corresponds to the naive factorization approximation without considering nonfactorizable contributions.

The decay amplitude  $\mathcal{A}(\bar{B}^0 \rightarrow D^+ D_s^-) \equiv \langle D^+ D_s^- | \mathcal{H}_{\text{eff}} | \bar{B}^0 \rangle$  is given as follows:

$$\begin{aligned}
 \mathcal{A}(\bar{B}^0 \rightarrow D^+ D_s^-) &= \frac{G_F}{\sqrt{2}} \left[ V_{cb} V_{cs}^* a_1 - V_{tb} V_{ts}^* \right. \\
 &\quad \left. \times \left( a_4 + a_{10} + 2(a_6 + a_8) \frac{m_{D_s}^2}{(m_b - m_c)(m_s + m_c)} \right) \right] \mathcal{M}_a \\
 &\simeq \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_1 R_{DD_s} \mathcal{M}_a, \quad (40)
 \end{aligned}$$

TABLE III. The values of the effective coefficients  $a_i$  with  $\mu = m_b(m_b) = 4.3$  GeV and  $m_c(m_b) = 0.95$  GeV in the NDR scheme at NLO calculations.  $a_{2i}$  and  $a_{2i-1}$  are defined by  $a_{2i-1} = C_{2i-1}^{\text{eff}} + C_{2i}^{\text{eff}}/N_c^{\text{eff}}$  and  $a_{2i} = C_{2i}^{\text{eff}} + C_{2i-1}^{\text{eff}}/N_c^{\text{eff}}$ . Here we have taken  $(N_c)_{LL}$  for the  $(V-A)(V-A)$  interaction and  $(N_c)_{LR}$  for the  $(V-A)(V+A)$  interaction.

Coeffs.	$(N_c)_{LL} = 2, (N_c)_{LR} = 2$		$(N_c)_{LL} = 2, (N_c)_{LR} = 5$		$(N_c)_{LL} = 3, (N_c)_{LR} = 3$	
	Real Part	Imag. Part	Real Part	Imag. Part	Real Part	Imag. Part
$a_1$	0.985	0.0	0.985	0.0	1.046	0.0
$a_2$	0.219	0.0	0.219	0.0	0.024	0.0
$a_3$	$-4.00 \times 10^{-4}$	$-2.30 \times 10^{-3}$	$-4.00 \times 10^{-4}$	$-2.30 \times 10^{-3}$	$7.23 \times 10^{-3}$	$-3.30 \times 10^{-5}$
$a_4$	$-3.46 \times 10^{-2}$	$-1.14 \times 10^{-2}$	$-3.46 \times 10^{-2}$	$-1.14 \times 10^{-2}$	$-3.83 \times 10^{-2}$	$-1.12 \times 10^{-2}$
$a_5$	$-1.07 \times 10^{-2}$	$2.3 \times 10^{-3}$	$3.70 \times 10^{-3}$	$1.78 \times 10^{-3}$	$-2.70 \times 10^{-3}$	$-3.33 \times 10^{-5}$
$a_6$	$-4.13 \times 10^{-2}$	$-1.14 \times 10^{-2}$	$-4.53 \times 10^{-2}$	$-1.27 \times 10^{-2}$	$-4.36 \times 10^{-2}$	$-1.21 \times 10^{-2}$
$a_7$	$-2.19 \times 10^{-5}$	$-2.88 \times 10^{-4}$	$-1.51 \times 10^{-4}$	$-2.88 \times 10^{-4}$	$-9.35 \times 10^{-5}$	$-2.88 \times 10^{-4}$
$a_8$	$3.11 \times 10^{-4}$	$-1.44 \times 10^{-4}$	$-3.82 \times 10^{-4}$	$-5.77 \times 10^{-5}$	$3.51 \times 10^{-4}$	$-9.61 \times 10^{-5}$
$a_9$	$-9.27 \times 10^{-3}$	$-2.88 \times 10^{-4}$	$-9.27 \times 10^{-3}$	$-2.88 \times 10^{-4}$	$-9.90 \times 10^{-3}$	$-2.88 \times 10^{-4}$
$a_{10}$	$-1.82 \times 10^{-4}$	$-1.44 \times 10^{-4}$	$-1.82 \times 10^{-4}$	$-1.44 \times 10^{-4}$	$3.39 \times 10^{-5}$	$-9.61 \times 10^{-5}$

where

$$R_{DD_s} = \left[ 1 + \frac{(a_4 + a_{10})}{a_1} + 2 \frac{(a_6 + a_8)}{a_1} \frac{m_{D_s}^2}{(m_b - m_c)(m_s + m_c)} \right] \quad (41)$$

and

$$\begin{aligned}
 \mathcal{M}_a &= \langle D_s^- | \bar{s} \gamma^\mu \gamma_5 c | 0 \rangle \langle D^+ | \bar{c} \gamma_\mu b | \bar{B}^0 \rangle \\
 &= -i f_{D_s} (m_B^2 - m_D^2) F_0^{BD}(m_{D_s}^2). \quad (42)
 \end{aligned}$$

On the other hand, we have

$$\begin{aligned}
 \mathcal{A}(\bar{B}^0 \rightarrow D^+ D_s^{*-}) &= \frac{G_F}{\sqrt{2}} [V_{cb} V_{cs}^* a_1 - V_{tb} V_{ts}^* (a_4 + a_{10})] \mathcal{M}_b \\
 &\simeq \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_1 R_{DD_s^*} \mathcal{M}_b, \quad (43)
 \end{aligned}$$

where

$$R_{DD_s^*} = \left( 1 + \frac{(a_4 + a_{10})}{a_1} \right) \quad (44)$$

and

$$\begin{aligned}
 \mathcal{M}_b &= \langle D_s^{*-} | \bar{s} \gamma^\mu \gamma_5 c | 0 \rangle \langle D^+ | \bar{c} \gamma_\mu b | \bar{B}^0 \rangle \\
 &= m_{D_s^*} f_{D_s^*} [\epsilon(q) \cdot (p_B + p_D)] F_1^{BD}(m_{D_s^*}^2). \quad (45)
 \end{aligned}$$

We can estimate the penguin contributions for each process, for example, in the case of  $N_{LL} = 2$  and  $N_{LR} = 5$ :

For  $\bar{B}^0 \rightarrow D^+ D_s^-$ ;

$$\left| \frac{\mathcal{A}_P}{\mathcal{A}_T} \right| = \left| \frac{(a_4 + a_{10})}{a_1} + 2 \frac{(a_6 + a_8)}{a_1} \frac{m_{D_s}^2}{(m_b - m_c)(m_c + m_s)} \right|$$

$$= 13.1\%, \quad (46)$$

$$\text{For } \bar{B}^0 \rightarrow D^+ D_s^{*-}; \quad \left| \frac{\mathcal{A}_P}{\mathcal{A}_T} \right| = \left| \frac{(a_4 + a_{10})}{a_1} \right| = 3.9\%, \quad (47)$$

where  $\mathcal{A}_T$  ( $\mathcal{A}_P$ ) stands for the amplitude of the tree diagram (penguin diagram). Here we used the values  $m_c(m_b) = 0.95$  GeV and  $m_s(m_b) = 90$  MeV. Therefore, the penguin contributions affect the extraction of the decay constants  $f_{D_s}$  and  $f_{D_s^*}$ . The penguin contributions for  $B \rightarrow DD_s$  are more than three times those for  $B \rightarrow DD_s^*$ .

From Eqs. (40) and (43) the decay constants are given by

$$f_{D_s^*} = (0.87 \times 10^{-1} \text{ GeV}) \times \frac{\sqrt{\mathcal{B}(\bar{B}^0 \rightarrow D^+ D_s^{*-})}}{|V_{cb} F_1^{BD}(m_{D_s^*}^2)|} \left( \frac{1.02}{a_1} \right) R_{DD_s^*},$$

$$f_{D_s} = (0.64 \times 10^{-1} \text{ GeV}) \frac{\sqrt{\mathcal{B}(\bar{B}^0 \rightarrow D^+ D_s^-)}}{|V_{cb} F_0^{BD}(m_{D_s}^2)|} \left( \frac{1.02}{a_1} \right) R_{DD_s}. \quad (48)$$

Browder *et al.* [17] presented the following experimental results for the branching ratios:

$$\mathcal{B}(\bar{B}^0 \rightarrow D^+ D_s^{*-}) = (1.14 \pm 0.42 \pm 0.28) \times 10^{-2}$$

$$= (1.14 \times 0.50) \times 10^{-2},$$

$$\mathcal{B}(\bar{B}^0 \rightarrow D^+ D_s^-) = (0.74 \pm 0.22 \pm 0.18) \times 10^{-2}$$

$$= (0.74 \times 0.28) \times 10^{-2}, \quad (49)$$

where we combined the statistical and systematic errors. From Eqs. (41), (44), (48), and (49), we obtain the results which are obtained by including the penguin contributions:

$$f_{D_s^*} = 346 \pm 82 \text{ MeV},$$

$$f_{D_s} = 233 \pm 49 \text{ MeV} \quad \text{for (pole/pole),}$$

$$f_{D_s^*} = 346 \pm 82 \text{ MeV},$$

$$f_{D_s} = 255 \pm 54 \text{ MeV} \quad \text{for (pole/const)}. \quad (50)$$

From Eqs. (41), (44), and (48) the ratio of the vector and pseudoscalar decay constants  $f_{D_s^*}/f_{D_s}$  is given by

$$\frac{f_{D_s^*}}{f_{D_s}} = 1.36 \frac{|V_{cb} F_0^{BD}(m_{D_s}^2)|}{|V_{cb} F_1^{BD}(m_{D_s^*}^2)|} \left[ \frac{\mathcal{B}(\bar{B}^0 \rightarrow D^+ D_s^{*-})}{\mathcal{B}(\bar{B}^0 \rightarrow D^+ D_s^-)} \right]^{1/2} \left( \frac{0.87}{0.96} \right), \quad (51)$$

which gives

TABLE IV. The obtained values of  $f_{D_s^*}$  (MeV) and  $f_{D_s}$  (MeV), and their ratio  $f_{D_s^*}/f_{D_s}$ , and the results from other theoretical calculations and existing experimental results. Here we referred the corrected  $f_{D_s}$  values for the experimental data [26–30], from Ref. [31].

	$(N_c)_{LL}$	$(N_c)_{LR}$	$f_{D_s^*}$ (MeV)	$f_{D_s}$ (MeV)	$f_{D_s^*}/f_{D_s}$
(Pole/Pole)	2	2	346 ± 82	231 ± 48	1.57 ± 0.50
	2	5	346 ± 82	233 ± 49	1.56 ± 0.49
	3	3	325 ± 77	216 ± 45	1.57 ± 0.50
(Pole/const)	2	2	346 ± 82	252 ± 53	1.44 ± 0.45
	2	5	346 ± 82	255 ± 54	1.43 ± 0.45
	3	3	325 ± 77	235 ± 50	1.44 ± 0.45
Browder <i>et al.</i> [17]			243 ± 70	277 ± 77	0.88 ± 0.35
Hwang and Kim [18]			362 ± 15	309 ± 15	1.17 ± 0.02
Cheng and Yang [19]			266 ± 62	261 ± 46	1.02 ± 0.30
Capstick and Godfrey [20]				290 ± 20	
Dominguez [21]				222 ± 48	
UKQCD [22]				212 <sup>+4+46</sup> <sub>-3-7</sub>	
BLS [23]				230 ± 7 ± 35	
MILC [24]				199 ± 8 <sup>+40+10</sup> <sub>-11-0</sub>	
Becirevic <i>et al.</i> [25]			272 ± 16 <sup>+0</sup> <sub>-20</sub>	231 ± 12 <sup>+6</sup> <sub>-0</sub>	1.18 ± 0.18
WA75 [26]				238 ± 47 ± 21 ± 48	
CLEO 1 [27]				282 ± 30 ± 43 ± 34	
CLEO 2 [28]				280 ± 19 ± 28 ± 34	
BES [29]				430 <sup>+150</sup> <sub>-130</sub> ± 40	
E653 [30]				190 ± 34 ± 20 ± 26	

$$\frac{f_{D_s^*}}{f_{D_s}} = 1.56 \pm 0.49 \quad \text{for (pole/pole),}$$

$$\frac{f_{D_s^*}}{f_{D_s}} = 1.43 \pm 0.45 \quad \text{for (pole/const).} \quad (52)$$

The decay constant is changed according to the  $q^2$  behavior of the form factor  $F_0(q^2)$ . However, the amount of change is less than 10% as shown in Eq. (50). From this we know that the decay constant is not so much dependent on the behavior of the form factor. Also, when we consider the uncertainty from nonfactorizable effects, the decay constant is changed within 10% discrepancy. In Table IV we show the results of  $f_{D_s^*}$ ,  $f_{D_s}$  and  $f_{D_s^*}/f_{D_s}$  for different nonfactorizable contributions.

As discussed in [4], when we consider the penguin contributions with nonfactorizable effects, the value of the decay constant  $f_{D_s^*}$  is increased by 8%, however, for  $f_{D_s}$  it is increased by up to 19%. So the ratio  $f_{D_s^*}/f_{D_s}$  is decreased by 9%. In Table IV we summarized the values of decay constant  $f_{D_s^*}$ ,  $f_{D_s}$ , and the ratio of  $f_{D_s^*}/f_{D_s}$  from various sources. Our result for  $f_{D_s}$  agrees well with other theoretical calculations and experimental results within errors. For the ratio  $f_{D_s^*}/f_{D_s}$ , our results have a value greater than 1, however, Browder *et al.* [17] have a value less than 1. It seems that this ratio is more likely to be greater than 1 when we consider that the decay constant of  $\rho$  mesons is 1.5 times greater than that of the  $\pi$  meson. The difference of the results by Cheng and Yang [19] comes from the different method and using different Wilson coefficients. Their values come by comparing two nonleptonic decay modes, for instance  $\mathcal{B}[B \rightarrow DD_s(D^{(*)}D_s^*)]/\mathcal{B}(B \rightarrow D\pi)$ .

## V. CONCLUSION

By including the penguin contributions and the nonfactorizable term contributions, we calculated the weak decay constants  $f_{D_s}$  and  $f_{D_s^*}$  from  $\bar{B}^0 \rightarrow D^+ \ell^- \nu$  and  $\bar{B}^0 \rightarrow D^+ \bar{D}_s^{(*)}$ . In our analysis, we consider the QCD-penguin and electroweak-penguin contributions in hadronic two-body decays within the NDR renormalization scheme at the next-to-leading order calculation. We also considered the effect of two different  $q^2$  dependences of the form factor for  $F_0^{BD}(q^2)$ . The value of  $f_{D_s}$  is changed by less than 10% for different form factors.

The penguin effects for  $B \rightarrow DD_s$  decay are quite sizable, and we obtained  $f_{D_s} = 233 \pm 49$  MeV for the monopole type of  $F_0^{BD}$ ,  $f_{D_s} = 255 \pm 54$  MeV for the constant  $F_0^{BD}$ . When we considered the nonfactorizable contributions, we obtained  $f_{D_s^*} = 346 \pm 82$  MeV for the  $D_s^*$  meson. These values will be improved vastly when the large accumulated data samples are available at CLEO II.V and III, and the Belle and BaBar experiments at the asymmetric  $B$  factory in the near future.

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